

**Figure 4.** (a) Temperature dependence of the electron spin polarization at  $\nu = 2/3$  and  $V_{SB} = 0$  for different magnetic field values in the vicinity of the spin transition  $B_{2/3}^C = 2.1T$ . (b) Arrhenius plots  $\ln(\gamma_e)$  vs.  $1/T$  (for  $B < B_{2/3}^C$ , squares) and  $\ln(1 - \gamma_e)$  vs.  $1/T$  (for  $B > B_{2/3}^C$ , circles) at several magnetic fields. (c) Magnetic field dependence of the spin-flip activation energy for  $\nu = 2/3$  around  $B_{2/3}^C$ . In the insets the CF spin-split Landau level diagrams are presented.

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## Magnetocapacitance studies of two-dimensional electron systems with long-range potential fluctuations

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**Abstract.** We report on magnetocapacitance study of the quantum Hall effect (QHE) states. Capacitance minima width was found to be independent of magnetic field and to be the same for even, odd and fractional QHE states when measured as a function of the average electron density. This result indicates that the width of capacitance minima in the samples investigated are governed by long-range carrier density fluctuations. At low temperatures, the amplitudes of the minima decrease linearly with the temperature increase. All our experimental results for the integer QHE states are quantitatively explained by introducing unbroadened magnetic levels and dispersion of the electron density along the sample. The energy gaps at even filling factors obtained from fitting the experimental data are found to be close to the known cyclotron gaps. At odd fillings  $\nu = 1, 3$ , and 5, the energy gaps appear to be enhanced in comparison with the Zeeman splitting, with the enhancement decreasing with filling factor.

The capacitance minima are argued to originate from the motion of incompressible regions along a sample caused by the gate voltage variation. We derive the condition for the appearance and motion of such regions for the case of gated samples with long-range fluctuations of density of charged donors.

The appearance of narrow magnetocapacitance peaks when a dc current is passed through the sample is reported. We hypothesize that these peaks are due to the current percolation along incompressible regions.

## 1. Introduction

The method of capacitance spectroscopy, which implies precise measurement of electric capacitance  $C$  of a parallel-plate capacitor formed from a two-dimensional electron system (2DES) and a parallel metal film (FET gate), is one of a few experimental methods for detecting thermodynamic characteristics of 2DES. It can also be used to investigate distribution of current under the conditions corresponding to the quantum Hall effect.

In this paper we will consider details of the application of the method for studies of 2DES with long-range potential fluctuations in the quantum Hall regime. An example of electron systems of this type is the most perfect semiconducting heterostructures with selective doping. We will show that

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the shape and the temperature dependence of minima on the capacitance curve, caused by jumps of the chemical potential at integer QHE states, are quantitatively described in terms of the electron density dispersion related to the fluctuating potential. Our data demonstrate that Landau levels broadening due to short-range potential fluctuations is insignificant in the studied samples. The jumps of the chemical potential observed in the experiments are consistent with the spin gaps enhanced by exchange interaction and with the cyclotron energy. We will show that the microscopic mechanism responsible for capacitance minima is the motion of strips of the incompressible electron phase, corresponding to the regions with an integer filling factor of Landau levels. We will also investigate peaks on the capacitance curve, caused by the current flowing through the sample under the QHE conditions and will show that these peaks indicate pinching of the current in narrow strips, which move over the sample as the average electron density changes. The latter circumstance proves the current to flow in the bulk of the sample.

The method of capacitance spectroscopy is based on the existence of the contact potential difference between a 2DES and a gate, which is equal to the difference in their work functions. For this reason the difference in the corresponding electric potentials is not equal to the applied voltage  $V_g$ . As a result, the experimentally measured capacitance  $C \equiv dQ/dV_g$  of such a structure appears to depend on the density of electron states at the Fermi level [1] (here  $Q$  is the charge of the 2DES). When the 2DES is not homogeneous the capacitance depends on the derivative  $d\mu_s/dn_s$  of the average chemical potential  $\mu_s$  with respect to the average electron density  $n_s$ , the potential being reckoned from the bottom of the lowest subband of the size quantization. This dependence can be written as [1–3]:

$$\frac{1}{C} = \frac{1}{C_g} + \frac{1}{C_z} + \frac{1}{Se^2} \frac{d\mu_s}{dn_s}, \quad (1)$$

where  $S$  is the area of the 2DES under the gate,  $C_g = \kappa S/4\pi d$  is the geometrical capacitance of the sample, determined by the thickness  $d$  of the dielectric layer between the 2DES and the gate, and  $\kappa$  is the dielectric constant of the layer. The capacitance  $C_z$  depends on the electron density in the 2DES and is of the order of  $\kappa_s S/4\pi z_0$ , where  $z_0$  is the 2DES thickness specified by the size of the electron wave function in the direction perpendicular to the 2DES, while  $\kappa_s$  is the dielectric constant in the vicinity of the 2DES. The last two terms in Eqn (1) are usually small corrections to the first one. Expression (1) is valid when a change in the gate voltage affects only the charge in the 2DES and the gate, leaving intact the charge of impurities in the heterostructure. In this case the second term in Eqn (1) does not depend on magnetic field. This condition is normally fulfilled at low temperatures in FET based on GaAs/AlGaAs heterostructures. In particular, it is perfectly fulfilled in the samples studied in the work. According to Eqn (1), the chemical potential jumps between the Landau levels in the QHE states result in minima on the capacitance curve, which can serve as a measure of the jumps.

## 2. Samples and the experimental technique

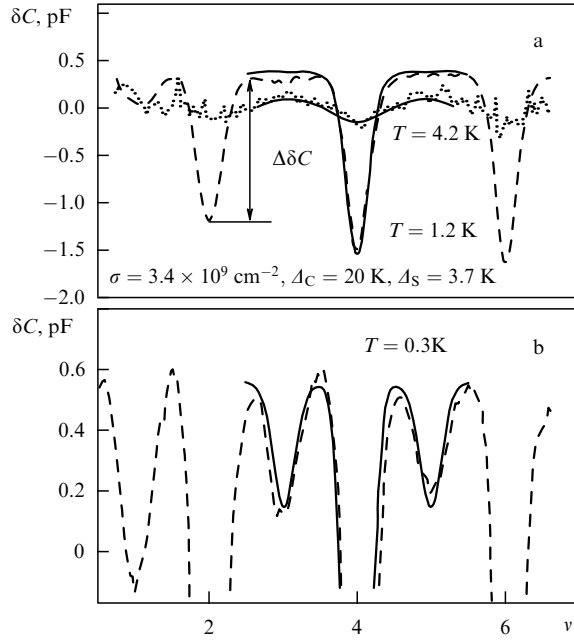
In this paper we present experimental data measured in FET samples, which, at zero gate voltage, had  $n_s = 1.4 \times 10^{11} \text{ cm}^{-2}$  and the electron mobility equal to  $1.2 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ . The

samples were prepared from GaAs/AlGaAs heterostructures grown by the molecular-beam epitaxy. They had the following sequence of layers in the order of growth: GaAs – AlGaAs (70 nm),  $\delta$ -layer of Si – AlGaAs (500 nm), and  $\delta$ -layer of Si – GaAs (10 nm). The 2DES was formed at the lowest heterojunction in the GaAs layer due to ionization of silicon impurity centers. The thickness of the first undoped AlGaAs layer (spacer) was about 70 nm so that the typical scale of potential fluctuations in the 2DES, caused by fluctuations of concentration of charged donors outside the spacer, exceeded greatly the magnetic length and the average distance between electrons. A metal film of size  $0.4 \times 2.3 \text{ mm}^2$  was deposited on the top of the heterostructure, forming the Schottky barrier. The electron density  $n_s$  in the 2DES determined from the period of the Shubnikov–de Haas oscillations was proportional to the voltage between the 2DES and the gate. The corresponding coefficient of proportionality coincided with the value extracted from the sample capacitance within 1% accuracy, indicating that impurity centers were not recharged and Eqn (1) can be used to evaluate the experimental data. To measure the capacitance  $C$ , we modulated the dc voltage  $V_g$ , which controlled the concentration  $n_s$ , by an ac voltage at frequency 9.2 Hz. We recorded two components of the current flowing through the sample: the component shifted by  $90^\circ$  relative to the modulation voltage, and the in-phase component. The former is a measure of the sample capacitance, while the latter characterizes conductivity along the 2DES. At low conductivity the capacitor can hardly be charged and precise measurement of the capacitance becomes impossible. Since the diagonal components of magnetoconductivity tensor decrease exponentially with lowering temperature, the capacitance spectroscopy method described above can be used only in a limited temperature range depending on the energy gap of the relevant QHE state. To treat experimental data quantitatively, we used only the results, which did not reveal resistive effects, that was verified by the absence of the in-phase component of the current.

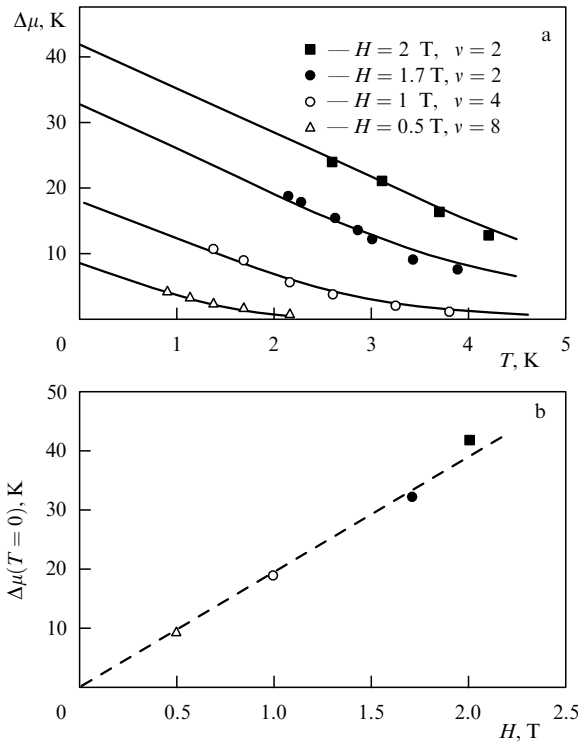
## 3. Temperature dependence of capacitance

We have investigated dependencies  $C(n_s, T)$  in magnetic fields perpendicular to the 2DES, which lead to the formation of the QHE states in the 2DES at integer filling factors  $\nu = n_s/N_0$  for the Zeeman sublevels of the Landau levels ( $N_0 = eH/hc$  is the degeneracy of the sublevel). Figure 1 shows an example of the dependencies measured at  $H = 1 \text{ T}$  and various temperatures. The minima on the capacitance curve arising at integer  $\nu$  correspond to jumps of the chemical potential in the QHE state. The deep minima at even filling factors at  $T = 0.3 \text{ K}$ , however, are due to the resistive effects. The width of the minima of the first type appears to be independent of the filling factor and magnetic field within 10% of the experimental accuracy (for even filling factors this effect was found in the fields varying by a factor of 6) and does not depend on the type (integer or fractional) of the QHE state (see also [4]). This fact suggests that at low temperatures the broadening of the magnetocapacitance minima in our samples is determined by the inhomogeneous electron concentration rather than by broadening of energy levels due to short-range potential. In this paper we focus on the temperature dependence of the amplitude of the minima, which is found to be nearly linear as shown in Fig. 2.

We were able to describe the shape of the minima and their temperature dependence, on the basis of a phenomenological



**Figure 1.** Experimental (dotted and dashed lines) and calculated (solid lines) dependencies of the difference  $\delta C(n_s) = C(H = 1 \text{ T}, n_s) - C(H = 0, n_s)$  vs. the filling factor  $\nu$  at various temperatures indicated at the curves. The model curves are calculated with the following fitting parameters:  $\Delta_s = 3.7 \text{ K}$ ,  $\Delta_c = 20 \text{ K}$  and  $\sigma = 3.4 \times 10^9 \text{ cm}^{-2}$ . Also shown is the method for determination of the amplitude of the minimum  $\Delta\delta C = \delta C(\nu = p + 1/2) - \delta C(\nu = p)$  ( $p$  is an integer number).



**Figure 2.** (a) Experimental (symbols) and calculated (solid lines) temperature dependencies of  $\Delta\mu(T) = [\Delta\delta C(T)]e^2 S(2\pi)^{1/2} \sigma / C^2$  at various magnetic fields. At  $T = 0 \text{ K}$  the calculated value of  $\Delta\mu(T)$  coincides with the gap in the 2DES spectrum. (b)  $\Delta\mu(H)|_{T=0}$  extracted from the data plotted in Fig. 2a. The dashed line corresponds to the cyclotron splitting  $\hbar\omega_c = \hbar eH/m^*c$  at  $m^* = 0.067m_0$ .

approach [2, 5]. Within this approach the electron density distribution in the sample is taken to be Gaussian, and the chemical potential jump  $\Delta\mu$  is to be averaged over this distribution. As a result [2], the expression for the minimum at the integer filling factor  $i$  takes the form

$$\Delta C = -\frac{C_g^2}{S} \frac{\Delta\mu}{e^2} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(iN_0 - n_s)^2}{2\sigma^2}\right]. \quad (2)$$

Here  $\sigma$  is the dispersion of the Gaussian distribution. Below we will demonstrate that this formula coincides with the results [4] obtained by studying the motion of strips of the incompressible electron phase. In contrast to [2, 5] we have also taken into account the Fermi distribution of electrons over energy levels. In such a model, there are three parameters: the dispersion  $\sigma$  of the density distribution, the cyclotron ( $\Delta_c$ ) and the spin ( $\Delta_s$ ) gaps. In GaAs/AlGaAs samples the latter is much less than the former. These parameters can easily be extracted from the experimental data. The dispersion is determined from the width of minima, after that the amplitude of the minimum depends only on the gap where the Fermi level lies. The results of the fitting are shown in Figs. 1 and 2 by solid lines. The values of parameters  $\Delta_c$  and  $\Delta_s$  are found to differ slightly for various minima. This effect takes place mainly at odd  $\nu$ , when the relevant spin gap decreases substantially with the number of filled Landau levels. Figure 1 plots the curve calculated for  $\nu = 3$  and  $\nu = 5$  with the same value of the spin gap; one can see that the experimentally measured amplitude at  $\nu = 3$  is higher while that at  $\nu = 5$  is smaller than the calculated ones. In order to fit the data we used the difference in capacitances at zero and quantizing magnetic fields,

$$\delta C(n_s) = C(H, n_s) - C(H = 0, n_s).$$

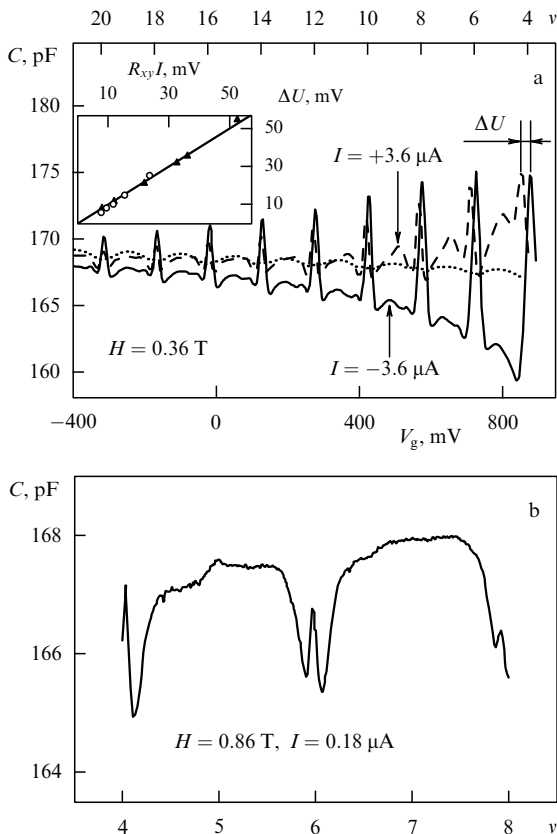
Calculating this difference we replaced the zero-field derivative  $d\mu_s/dn_s$  by its value for noninteracting electrons,  $\pi\hbar^2/m^*$ , where  $m^* = 0.067m_e$  is the effective electron mass in GaAs. The corresponding correction is 0.65 pF in our case. Using this fitting procedure, we found not only the amplitudes of the minima but also the whole dependence  $\delta C(n_s)$  plotted in Fig. 1. The chemical potential jumps obtained by the fitting procedure at even filling factors are shown in Fig. 2b. The jumps are found to be close to the values for noninteracting electrons with  $m^* = 0.067m_e$ . Note that the spin gaps at the filling factors 3 and 5, and magnetic field  $H = 1 \text{ T}$  (Fig. 1) exceed by an order of magnitude the Zeeman splitting equal to 0.3 K for the g-factor ( $-0.44$ ) of electrons in GaAs. This fact reflects the known effect of enhancement of spin gaps due to exchange interaction. It is worth mentioning that our model predicts the linear temperature dependence of the amplitude of the minimum at low temperatures (such a dependence can also be found analytically [6]).

We emphasize that our data demonstrate the absence of significant broadening of the Landau levels by the short-range potential. The quality of the fitting procedure was found to be strongly reduced by introducing a finite width  $\Gamma > 0.1\hbar\omega_c$  of the levels. Probably, Landau levels were locally narrow, but long-range fluctuations were also dominant in the experiments [1, 7–10], whose data were used to draw conclusions about high density of states between the Landau levels, contradicting the theoretical predictions.

#### 4. Influence of current pinching on FET capacitance

The effects considered in this section lead to a rather important conclusion about the distribution of currents over the sample in the QHE regime, shedding light on this still unsolved problem (see, for example, [11]). There are two approaches to solve this problem. They are based on consideration of currents carried by various electron states lying either exactly at or below the Fermi level. The most widely discussed example of the first type of states are the edge states [12] arising in the regions with non-integer local filling factors of Landau levels when the potential changes smoothly near the sample edge. The second type of states occurs in the strips [13, 14] of incompressible electron phase with integer filling factors.

A moderate dc current  $I$  flowing through a sample of the Hall bar geometry results in the appearance of the peaks near the minima on the capacitance curve (Fig. 3b). As the current increases, these peaks get larger, broadened and shifted (Fig. 3a). Changing polarity of the current leads to the change in the direction of the shift, with the difference in the gate voltage between the centers of the peaks observed at opposite currents being equal to  $R_{xy}I$  (where  $R_{xy}$  is the Hall resistance at the corresponding integer filling factor) (see inset



**Figure 3.** Dependence of capacitance on the gate voltage with dc current  $I$  flowing through the sample. (a)  $I = \pm 3.6 \mu\text{A}$ ,  $H = 0.36 \text{ T}$  (solid and dashed lines); dotted line shows to the capacitance at zero current. (b)  $I = 0.18 \mu\text{A}$ ,  $H = 0.86 \text{ T}$ . The inset shows the dependence of the difference  $\Delta U$  in the positions of peaks at opposite currents  $I$  on the Hall voltage  $IR_{xy}$  measured by changing the filling factor (circles) or the magnitude of the current (triangles). The straight line corresponds to the  $IR_{xy} = \Delta U$ .

in Fig. 3a). We think that these peaks on the capacitance curve can be explained only by current pinching inside narrow regions whose positions depend on the average electron density. A similar effect at high currents was carefully considered in [15, 16], the analytical solution to the nonlinear problem being obtained in [15] under the assumption that the dissipative conductivity depends on the electron density as

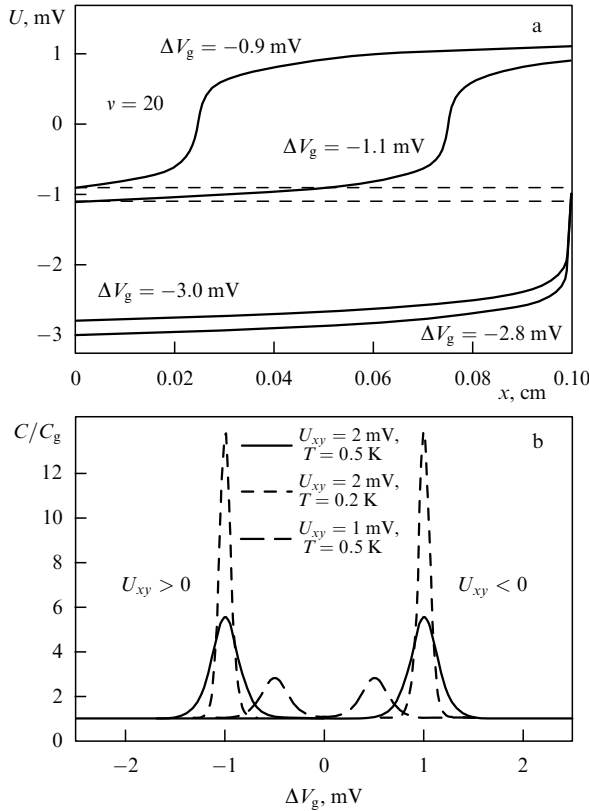
$$\begin{aligned}\sigma_{xx} &= \sigma_0 \exp\left(-\frac{\Delta\mu}{2T}\right) \cosh \frac{\epsilon_F}{T} \\ &= \sigma_0 \exp\left(-\frac{\Delta\mu}{2T}\right) \cosh \frac{\Delta n}{DT}.\end{aligned}\quad (3)$$

Here  $\epsilon_F$  is the Fermi energy reckoned from the center of the gap  $\Delta\mu$  between the Landau levels,  $\sigma_0$  is the conductivity, usually of the order of  $e^2/h$ ,  $\Delta n = n - iN_0$ , and  $D$  is the density of states in the center of the gap. Such a dependence arises in the model considering narrow strips of delocalized states near the centers of Landau levels. This dependence is confirmed by experimental data (see, for example [17] and references therein) and numerical calculations for the 2DES with long-range disorder [18], provided that the electron density  $n$  is replaced by its average value  $n_s$ , and the density of states  $D$  is replaced by  $dn_s/d\mu_s$ . Due to the inhomogeneity of the electron density, the value of  $dn_s/d\mu_s$  at integer filling factors is finite and weakly depends on  $n_s$ . Since the current flowing through the 2DES leads to the spatial redistribution of electrons required to produce the Hall voltage, the problem of current distribution is nonlinear. In the gated samples with the density dependence of the conductivity following Eqn (3), the problem can be solved analytically within the capacitor approximation. The latter relates the potential difference  $U$  between the gate and the 2DES with the electron concentration,  $n = C_0 U/e$ , where  $C_0 = C_g/S$  is the capacitance per unit area. The corresponding solution for a long rectangular sample is given by [15]

$$U(x) = \gamma T \operatorname{arsinh} \left[ \frac{x}{w} \sinh(z+s) + \left(1 - \frac{x}{w}\right) \sinh(z-s) \right] + V_g^0.\quad (4)$$

Here  $s = IR_{xy}/2\gamma T$ ,  $z = (V_g - V_g^0 + IR_{xy}/2)/\gamma T$ ,  $w$  is the width of the conducting channel of the 2DES,  $V_g$  is the potential difference between the gate and the point  $x = 0$  of the layer,  $V_g^0$  is the gate voltage corresponding to the QHE state, and  $\gamma = e/(C_0 d\mu_s/dn_s)$ . The distribution of the potential calculated by this formula is shown in Fig. 4a. The current pinch corresponds to sharp changes in  $U$ , it is located in the vicinity of the point where the QHE state ( $n = iN_0$ ) is formed and the dissipative conductivity has the minimum. A small change in the gate voltage displaces the pinch. The area between two curves corresponding to different gate voltages is proportional to the change in the charge of the sample. When the current pinch does not move (bottom pair of solid curves) this change is seen to be much less and does not differ significantly from that at zero current (the area between the horizontal dashed lines). Using Eqn (4), one can easily calculate the dependence of the capacitance on the gate voltage:

$$C = C_g \left( \frac{s \coth s}{\cosh^2 z} + \tanh^2 z \right).\quad (5)$$

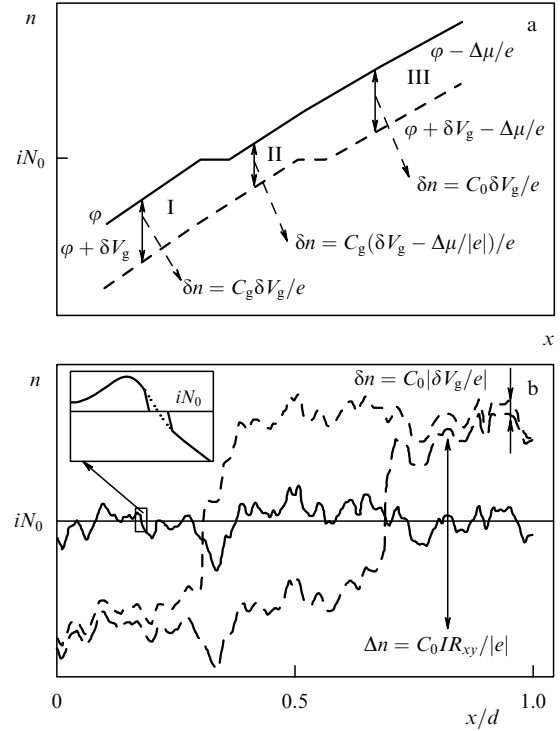


**Figure 4.** (a) Dependence of the voltage  $U$  between the gate and the 2DES on the  $x$  coordinate perpendicular to the current, calculated from Eqn (4) for two close values of  $V_g$ . The voltage is reckoned with respect to  $V_g^0$ . The value of  $\Delta V_g = V_g - V_g^0$  is indicated at the curves. The calculations were performed for  $\gamma = 0.36 \text{ mV K}^{-1}$ ,  $w = 1 \text{ mm}$ ,  $T = 0.5 \text{ K}$ ,  $IR_{xy} = U_{xy} = 2 \text{ mV}$ . (b)  $C(V_g)$  obtained from Eqn (5) at various values of the Hall voltage  $U_{xy}$  and temperature indicated in the figure. All the other parameters are the same as in Fig. 4a.

This relation describes the peak on the capacitance curve (Fig. 4b) at  $V_g = V_g^0 - IR_{xy}/2$ , when the pinch passes through the center of the sample. This explains the shift in the peak position by  $IR_{xy}$  with respect to the gate voltage on changing the polarity of the current, that was observed experimentally (Fig. 3a). The amplitude of the peak is inversely proportional to temperature and increases with the current  $I$ . At rather high current and low density of states  $D$  the capacitance can exceed by several times the geometrical capacitance  $C_g$  of the sample.

Equation (4) had been derived in [15] under assumption that the current density is proportional to the electric field, i.e., it neglects the difference in the chemical potentials across the pinch, equal to the energy gap  $\Delta\mu$  between the neighboring Landau levels. This approximation is valid when the potential difference across the pinch greatly exceeds  $\Delta\mu$ . The opposite situation will be qualitatively discussed below. Recently [4] we have demonstrated that, in the absence of dc current, the motion of incompressible strips [13, 14], caused by a change of the average ( $n_s$ ) electron density in FET with long-range fluctuations of  $n$ , leads to the minima on the capacitance curve, described by Eqn (2).

First we consider the case when the electron density changes only in the direction perpendicular to the long channel (coordinate  $x$  in Fig. 5a). A strip of incompressible phase arises in the region where the integer number  $i$  of Landau levels is filled ( $n = iN_0$ ). We assume here that the



**Figure 5.** A schematic dependence of the electron density on the coordinate in the presence of long-range disorder. (a) Variation of  $n(x)$  and the electric potential ( $\phi$ , etc.) in the vicinity of an incompressible strip due to variation of the gate voltage by  $\delta V_g$  at  $I = 0$ . (b) Variation of the electron density over the 2DES in the presence of a moderate current  $I$  caused by the variation of the gate voltage  $\delta V_g$ .

width of the strip is much less than the distance  $d$  between the 2DES and the gate. Since  $\mu + e\phi = \text{const}$ , the electric potential ( $\phi$ ) jump across the strip equal to  $\Delta\mu/e$  [13]. When the distance between neighboring strips greatly exceeds  $d$ , the capacitor approximation is valid far away from the strips. As the gate voltage changes by  $\delta V_g$ , the strip is displaced. The change in the carrier density can easily be calculated in the case of large  $\delta V_g$ , when the displacement of the strip is much greater than  $d$ . In the regions I and III (see Fig. 5a) this change is given by  $C_0\delta V_g/e$ , while in the region II through which the strip moves it is smaller:  $C_0(\delta V_g - \Delta\mu/|e|)/e$ . In the case of arbitrary number of strips the change in the charge of the capacitor is  $\delta Q \approx C_0[(S - \delta S)\delta V_g + \delta S(\delta V_g - \Delta\mu/|e|)]$ . Here  $\delta S$  is the change in the area of regions with  $n < iN_0$  caused by the change in the gate voltage. If the electron density distribution in the sample is described by a function  $f(n, n_s)$ , then  $\delta S \approx Sf(iN_0, n_s)C_0\delta V_g/|e|$ , and for the Gaussian distribution we obtain the expression for the capacitance which coincides with Eqn (2).

Now let us turn to the case of finite currents. A current  $I$  flowing in the strip should alter the electrochemical potential difference across the strip by  $IR_{xy}$ . This will increase or decrease the electrical potential jump as dictated by the sign of the current. The motion of strips with increasing (decreasing) potential difference should enhance (diminish) the capacitance minimum. The Hall voltage induced by the current results in that the average electron densities at the opposite edges in the sample get shifted in the opposite directions with respect to  $iN_0$ . In this case the number of strips with potential difference less than  $\Delta\mu$  exceeds the number of the strips of another type by unity. Hence the

current flowing through the sample should decrease the minimum on the capacitance curve. The number of strips decreases with the current, and, ultimately, at very high currents, only one strip (cf. Fig. 5b) will survive in the bulk of the sample, which is equivalent to the pinch discussed above. This is a qualitative description. The quantitative consideration should be based on a self-consistent calculation of the distribution of potentials and electron density around the incompressible current-carrying strip.

We would also like to discuss the case of low currents when the peak is narrow and arises near the center of the minimum (Fig. 3b). The width of the minimum is determined by the dispersion of the electron density in the sample [4] and, hence, by the range of the parameters at which strips of incompressible electron phase exist in a sample, whereas the width of the peak is determined by another range of the parameters at which current flows only through the incompressible strips, i.e., at which percolation over the regions with incompressible phase takes place.

We believe that the peaks observed on the capacitance curve suggest that the current flows through the regions in the bulk of the sample, whose positions change as the gate voltage varies. The peaks can hardly be explained in terms of edge currents.

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## The problem of Coulomb interactions in the theory of the quantum Hall effect

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**Abstract.** We summarize the main ingredients of a unifying theory for abelian quantum Hall states. This theory combines the Finkel'stein approach to localization and interaction effects with the topological concept of an instanton vacuum as well as Chern–Simons gauge theory. We elaborate on the meaning of a new symmetry ( $\mathcal{F}$  invariance) for systems with an infinitely ranged interaction potential. We address the renormalization of the theory and present the main results in terms of a scaling diagram of the conductances.

### 1. Introduction

In this contribution we discuss some of the recent advancements in the theory of the quantum Hall effect [1–3]. In particular, we address some of the main steps in the development of a theory [4] that combines the *instanton vacuum* approach to spin polarized, free electrons [5] with the *Finkel'stein treatment* of the Coulomb interactions [6] in the disordered systems.

The electron gas with an infinite-range interaction potential is, in many ways, very different from what we know about the theory of free electrons. This class of problems belongs to a different universality class of quantum transport phenomena and it is characterized by a typical interaction symmetry which we term  $\mathcal{F}$  invariance [1].  $\mathcal{F}$  invariance is intimately related to the electrodynamic  $U(1)$  gauge invariance and it has major consequences for the renormalization of the theory [2].

The main physical objective of our theory is to unify the different aspects of (abelian) quantum Hall states originated from different sources have been studied over the years independently. They include the quantum critical behavior of the quantum Hall plateau transitions [7], composite fermion theory or the Chern–Simons mapping between integral and fractional quantum Hall states [8], the Luttinger liquid theory of quantum Hall edge excitations [9], as well as the stability or robustness of the quantization phenomenon due to the disorder [10]. For a detailed exposure we refer the reader to the literature and here, we only present a brief introduction to the subject.

### 2. Matrices in frequency and replica space

Diffusive modes are encoded in the unitary matrix field variables  $Q_{nm}^{\beta\gamma}$  [6]. Here, the superscripts represent the replica indices ( $\beta, \gamma = 1, 2, \dots, N_r$  where  $N_r \rightarrow 0$  at the end of all calculations) and the subscripts denote the Matsubara frequency indices ( $n, m = 0, \pm 1, \pm 2, \dots, \pm N'_{\max}$  where the cutoff  $N'_{\max}$  is sent to infinity in the end). The matrices  $Q$  generally obey the constraints

$$Q^\dagger = Q, \quad Q^2 = 1, \quad \text{tr } Q = 0$$

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