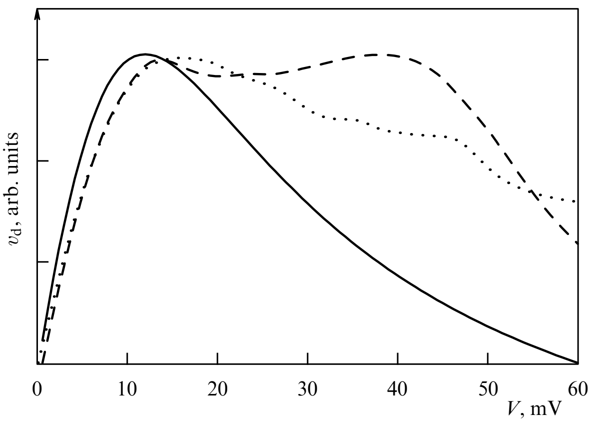

Figure 5. $v_d(V)$ curves calculated for $B = 5$ T, $\theta = 0°$ (solid curve), $B = 1.5$ T, $\theta = 45°$(dashed curve) and $B = 5$T, $\theta = 45°$ (dotted curve).

$B = 1.5$ T (5 T). For $V \lesssim V_B$, the shape of both of these curves is similar to that for $\theta = 0°$. Moreover, in this low voltage regime, tilting the magnetic field has little effect on the magnitude of v_d. This is because when V is small, the electrons travel such a short distance before scattering that it is hard to distinguish between the regular ($\theta = 0°$) and chaotic ($\theta \neq 0°$) trajectories, and so both types of orbit have similar drift velocities. In a tilted magnetic field, v_d is slightly lower because the field component parallel to the potential barriers deflects the electron trajectories, thus reducing the average velocity along the SL axis [26]. At high voltages, by contrast, the mean free path of the electrons is long enough for the differences between regular and chaotic orbits to influence the transport properties of the SL. Electrons in spatially extended chaotic trajectories (Fig. 2c) travel further along the SL before scattering, and therefore have higher drift velocities. This should raise the electrical conductivity measured in electron transport experiments.

In a tilted magnetic field, the $v_d(V)$ curves contain weak oscillatory structure which can be seen most easily in the dotted (5 T, 45°) trace in Fig. 5. The origin of these oscillations seems to involve resonances between the classical cyclotron and Bloch frequencies when $\theta = 0°$, and will be analyzed in detail elsewhere [27].

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Quasiclassical memory effects: anomalous transport properties of two-dimensional electrons and composite fermions subject to a long-range disorder

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Abstract. We have studied the ac response and magnetoresistance of a two-dimensional electron gas in high-mobility samples in the presence of smooth disorder, with emphasis on the composite-fermion description of a half-filled Landau level. We have found that the low-ω behavior of the ac conductivity $\sigma(\omega)$ is governed by memory effects associated with return processes that are neglected in Boltzmann transport theory: the return-induced correction to $\mathrm{Re}\,\sigma$ exhibits a kink $\propto |\omega|$. It is shown that the non-Markovian quasiclassical kinetics leads to a strong magnetoresistance $\Delta\rho_{xx}$. We argue that the quasiclassical memory effects account for the positive $\Delta\rho_{xx}$ observed at small deviations from half-filling. At a larger deviation, the positive magnetoresistance is followed by a sharp falloff of ρ_{xx}.

Recently, there has been a revival of interest in *quasiclassical* transport properties of a two-dimensional electron gas (2DEG). This is motivated by the realization that the classical dynamics in a disordered system constitutes in fact far more than the idealized Drude picture and, to describe the transport properties of the system, one has sometimes to completely abandon theories based on the Boltzmann equation. In Boltzmann transport theory, formulated in terms of a collision integral, quasiclassics leads to the Drude results: analytical behavior of the ac conductivity $\sigma(\omega)$ at $\omega \to 0$, zero magnetoresistance (MR), etc. It has been demonstrated, however, that quasiclassical *memory effects*, neglected in the conventional Boltzmann approach, yield a

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wealth of anomalous transport properties of a 2DEG subject to a *long-range* disorder. In particular, the non-Markovian kinetics gives rise to a quasiclassical zero-frequency anomaly [1] in the ac response of a disordered 2DEG, associated with return processes in the presence of smooth inhomogeneities. Specifically, the return-induced correction to $\mathrm{Re}\,\sigma(\omega)$ exhibits a kink $\propto|\omega|$. Another manifestation of the non-Markovian kinetics is a strong positive MR [2] in low magnetic fields, which explains [3] the otherwise puzzling positive MR observed near half-filling of the lowest Landau level in the fractional quantum Hall regime. The strength of the above anomalies depends on a ratio d/l, where d is the correlation radius of disorder, l is the mean free path, and grows with increasing d as a power of this parameter. Since quantum corrections are governed by another small parameter $1/k_{\mathrm{F}}l \ll 1$, where k_{F} is the Fermi wavevector, it is the long-range correlations of disorder with $k_{\mathrm{F}}d \gg 1$ that reveal the quasiclassical anomalies. The condition $k_{\mathrm{F}}d \gg 1$ is typically very well satisfied in high-mobility semiconductor heterostructures, where charged impurities, located in a layer separated by a large spacer from the 2DEG plane, create a smooth disorder in the electron system.

Here, we summarize recent work on (i) quasiclassical zero-frequency anomaly and (ii) quasiclassical magnetoresistance. We focus on the case of smooth (allowing for a quasiclassical treatment) Gaussian (in the sense of statistics of fluctuations) disorder. As we will show, the amplitude of the anomalies is sensitive to time-reversal symmetry of disorder. Specifically, in the limit of weak disorder, the anomalies appear to be strongly enhanced for a random magnetic field (RMF) as compared to a random scalar potential. We will consider both types of disorder. The case of a smoothly varying RMF is of major interest particularly because of the relevance of the problem to the composite-fermion (CF) description of the transport properties of a half-filled Landau level [4]. Also, a long-range RMF has been realized in semiconductor heterostructures by attaching superconducting or ferromagnetic overlayers or by prepatterning the sample (randomly curving the 2DEG layer).

We begin by considering the zero-frequency anomaly. Our starting point is the disorder-averaged quasiclassical expression for the conductivity in terms of the exact Liouville operator L:

$$\sigma(\omega) = e^2 \nu v_F^2 \int \frac{\mathrm{d}\phi}{2\pi} \left\langle \cos\phi \frac{1}{L} \cos\phi \right\rangle , \qquad (1)$$

where ν is the density of states at the Fermi energy ϵ_{F}, v_{F} is the Fermi velocity, ϕ is the velocity angle on the Fermi surface. The Liouville operator is represented as $L = L_0 + \delta L$, where $L_0 = -\mathrm{i}\omega + v_{\mathrm{F}}\mathbf{n}\nabla$, $\mathbf{n} = (\cos\phi, \sin\phi)$, and δL is a disorder-induced correction. In the case of a RMF $B(\mathbf{r})$, the fluctuating correction is $\delta L_B = (e/mc)B(\mathbf{r})\partial_\phi$. For a scalar random potential $V(\mathbf{r})$, it is given by

$$\delta L_V = \delta v(\mathbf{r})\mathbf{n}\nabla + [\nabla \delta v(\mathbf{r})](\hat{\mathbf{z}} \times \mathbf{n})\partial_\phi ,$$

where $\delta v(\mathbf{r}) = v(\mathbf{r}) - v_{\mathrm{F}}$ is the fluctuation of the local velocity $v(\mathbf{r}) = \{(2/m)[\epsilon_{\mathrm{F}} - V(\mathbf{r})]\}^{1/2}$ and $\hat{\mathbf{z}}$ is a unit vector in z direction. Expanding Eqn (1) in δL and resumming the series, we obtain the ac conductivity $\sigma(\omega) = \sigma_0/\tau(-\mathrm{i}\omega + M)$ in terms of a self-energy $M(\omega)$ ('memory function'). Here σ_0 is the dc Drude conductivity and τ is the momentum relaxation time. In the Boltzmann (collision integral) approximation $M_0 = \tau^{-1}$. Introducing the self-energy allows us to construct a *classical* diagrammatic technique [1] for the return-induced correction ΔM in terms of classical diffusion propagators (see also [5]), close in essence to the quantum diagrammatic technique in the weak-localization theory.

The leading contribution to the nonanalytic in ω correction ΔM comes from return processes involving one diffusion propagator. In the simplest case of a weak RMF, it is given by

$$\Delta M = 2\left(\frac{e}{mc}\right)^2 \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \frac{\mathrm{d}\phi}{2\pi} \frac{\mathrm{d}\phi'}{2\pi} \times \sin\phi \; W_B(q) P_D(\mathbf{q}, \phi, \phi') \sin\phi' , \qquad (2)$$

where $P_D(\mathbf{q}, \phi, \phi') = \gamma(\mathbf{q}, \phi)(-\mathrm{i}\omega + Dq^2)^{-1}\gamma(\mathbf{q}, \phi')$ is the diffusion propagator with the vertex corrections $\gamma(\mathbf{q}, \phi)$ and $W_B(q)$ is the Fourier transform of the correlator of RMF. At $ql \ll 1$, $\gamma(\mathbf{q}, \phi) \simeq 1 - \mathrm{i}ql\cos(\phi - \phi_q)$. For CFs in a half-filled Landau level $W_B(q) = (2hc/e)^2 n_i \exp(-2qd)$, where n_i is the concentration of charged impurities. In Eqn (2) the characteristic (from the infrared side) $q \sim (|\omega|/D)^{1/2}$, so that in doing the integral one can take the correlator $W_B(q)$ at $q = 0$. The result is

$$\frac{\Delta \mathrm{Re}\,\sigma(\omega)}{\sigma_0} = -\frac{\pi}{2}\frac{d}{l}|\omega|\tau , \quad |\omega|\tau \ll 1 . \qquad (3)$$

The nonanalytic frequency dependence of $\sigma(\omega)$ reflects long-time tails in the velocity–velocity correlator $\langle \mathbf{v}(t)\mathbf{v}(0)\rangle \sim (d/l)v_{\mathrm{F}}^2(\tau/t)^2$ (familiar from the Lorentz gas model [6]), which should be contrasted with the exponential decay in Boltzmann theory. The long-memory effect is related to self-intersections of a diffusion trajectory: traveling through the same local configuration of disorder twice introduces correlations of scattering acts, totally absent in the Boltzmann description. The larger the parameter d/l, the stronger the anomaly. Note that for CFs at half-filling $d/l \sim 1$ [3, 4].

Calculating ΔM for scalar disorder is somewhat trickier. The fluctuating term δL_V contains spatial derivatives, which yields the first-order contribution to ΔM:

$$\Delta M_1 = \frac{2}{p_{\mathrm{F}}^2} \int \frac{\mathrm{d}\phi}{2\pi} \frac{\mathrm{d}\phi'}{2\pi} \frac{\mathrm{d}^2 q}{(2\pi)^2} \sin\phi \sin(\phi - \phi_q) q^2 W_V(q) \times P_D(\mathbf{q}, \phi, \phi') \sin\phi' \sin(\phi' - \phi_q) , \qquad (4)$$

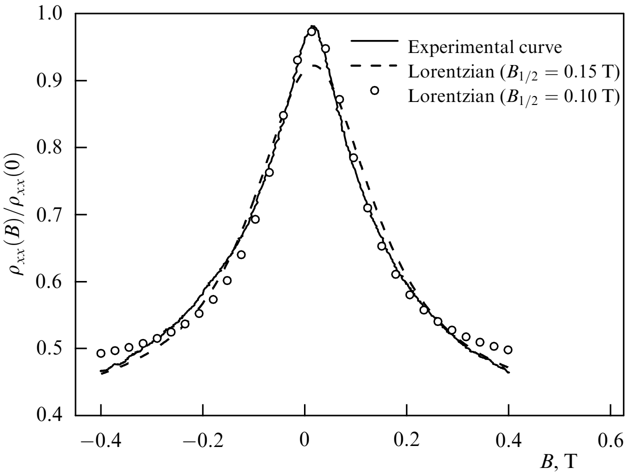
where $W_V(q)$ is the correlator of a random potential,

$$W_V(q) = (\pi\hbar^2/m)^2 n_i \exp(-2qd)$$

for charged impurities. Note that, unlike the case of RMF, this integral does not require taking the vertex corrections $\gamma(\mathbf{q}, \phi)$ into account. Equation (4) gives

$$\frac{\Delta \mathrm{Re}\,\sigma(\omega)}{\sigma_0} = \pi\left(\frac{d}{l}\right)^3 |\omega|\tau .$$

In fact, however, the leading contribution to ΔM in the case of scalar disorder comes from second-order processes. Specifically, the second-order term $\Delta M_2/M_0 \propto (d/l)^2$ scales with a smaller power of d/l. This, at first glance, counterintuitive feature is related to the anomalous smallness of ΔM_1 in the otherwise regular expansion in powers of d/l (third- and higher-order terms in ΔM are small compared to ΔM_2). The



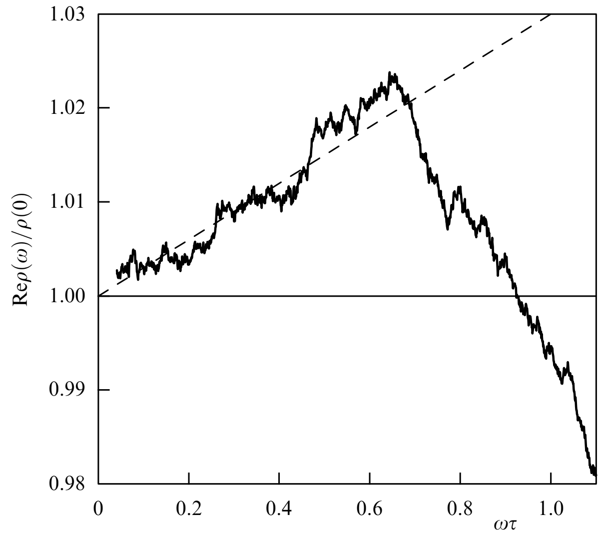


Figure 1. Real part of the ac resistivity in a random magnetic field with $d/l = 0.5$ normalized to its $\omega = 0$ value. The dashed line is a guide for the eye, $\Delta\mathrm{Re}\,\rho(\omega)/\rho(0) \propto |\omega|$.

point is that the $|\omega|$ anomaly is related to the integration over small q of the form

$$\int \frac{\mathrm{d}^2 q q^2}{-\mathrm{i}\omega + Dq^2} ,$$

where the numerator of the integrand tends to zero as q^2 at $q \to 0$. To second order, the large momenta associated with the spatial derivatives in L_V are 'disentangled' from the small momentum carried by the diffuson, so that the leading q^2 term comes from the vertex corrections $\gamma(\mathbf{q}, \phi)$. Let us now count powers of l: two factors $\gamma(\mathbf{q}, \phi)$ yield $q^2 l^2$, whereas one loses only l^{-1} when going to second order, which explains the total gain of one power of l/d as compared to Eqn (3). Evaluation of the second-order diagrams [1] gives

$$\frac{\Delta\mathrm{Re}\,\sigma(\omega)}{\sigma_0} = \frac{3\pi}{8}\left(\frac{d}{l}\right)^2 |\omega|\tau , \quad |\omega|\tau \ll 1 . \qquad (5)$$

The anomaly for scalar disorder is seen to be much weaker (at $d/l \ll 1$) compared to the case of RMF and has opposite sign.

Equations (3), (5) describe the limit of weak disorder at zero external magnetic field $\overline{B}$. We examined also the percolation-type problem that arises [3, 7] in a weakly disordered system with smooth disorder at strong $\overline{B}$. The dc conductivity σ_{xx} then falls off exponentially with growing $\overline{B}$ because of the increasing adiabaticity of the electron motion and related quasiclassical localization. It is worth stressing that the exponential suppression of the chaotic dynamics for CFs starts in fact at a small deviation from half-filling, namely at $|\nu_e - 1/2| \sim 1/k_F d \ll 1$, where ν_e is the electron filling factor. In the limit where transport is governed by the quasiclassical localization we found [3] that

$$\Delta\mathrm{Re}\,\sigma(\omega)/\sigma(0) \sim |\omega|\tau_s ,$$

where τ_s is a characteristic time of traversal of a link of the percolation network. For CFs τ_s scales as $\sigma_{xx}^{-7/3}$. Note also that the ac conductivity behaves similarly in a system with strong RMF ($d/l \gg 1$) at zero $\overline{B}$: dc transport in this system is due to percolating 'snake states' [3] and $\tau_s \sim (d/v_F)(d/l)^{7/9}$.

In addition to the analytical methods we have employed computer simulations of the classical motion of a particle in RMF. The results obtained for the memory function at

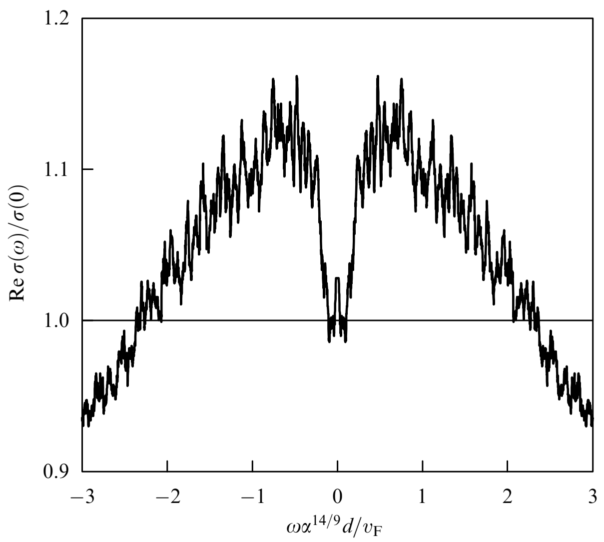


Figure 2. Real part of the ac conductivity in a random magnetic field at $d/l \simeq 30$. A nonanalytic dip around $\omega = 0$ is clearly seen.

$d/l = 0.5$ are shown in Fig. 1. The magnitude of the $|\omega|$ correction is noticeably smaller than Eqn (3) would predict. We attribute this discrepancy to the fact that Eqn (3) was derived for $d/l \ll 1$. A smaller value of the coefficient at $d/l = 0.5$ is consistent with the fact that at $d/l \sim 1$ the coefficient changes sign and the correction to the conductivity becomes positive. In Figure 2 we show the numerical data for $d/l \simeq 30$. A pronounced dip in the ac conductivity around $\omega = 0$ in the expected range $|\omega| \sim 1/\tau_s$ confirms the analytical results.

Let us consider now the effect of electron–electron interaction on the zero-frequency anomaly, which is twofold. First, it leads to inelastic scattering at finite temperature and one might wonder if the inelasticity cuts off the singular behavior of $\sigma(\omega)$ at $\omega \to 0$. The answer is no [1], in contrast to the quantum singular corrections, since although the inelastic scattering does make the Liouville operator massive it does not destroy the pole in the classical diffusion propagator. Second, the interaction yields dynamical screening, which requires a delicate treatment in the present problem. It turns out that the screening has a profound effect on the zero-frequency anomaly since the Coulomb interaction destroys the diffusion pole in the return-induced correction ΔM. In terms of the standard diagrammatic technique, this means that although the conductivity is a response to the screened (total) electric field and, therefore, is given by unscreened (irreducible with respect to the Coulomb interaction) density–density correlator, one should use screened diffusion propagators in all internal blocks. Specifically, instead of the bare diffuson $\Gamma(\omega, q) = (2\pi\nu\tau_0^2)^{-1}(-\mathrm{i}\omega + Dq^2)^{-1}$ one should use the screened (RPA) propagator

$$\Gamma_{\mathrm{scr}}(\omega, q) = \Gamma(\omega, q) \times \left[1 + \Gamma(\omega, q)(2\pi\nu\tau_0)^2\left(-\frac{\mathrm{i}\omega}{2\pi}\right)V_{\mathrm{scr}}(\omega, q)\right]. \quad (6)$$

The dynamically screened Coulomb interaction is given by

$$V_{\mathrm{scr}} = \frac{2\pi e^2}{q}\,\frac{-\mathrm{i}\omega + Dq^2}{-\mathrm{i}\omega + Dq^2 + v_{\mathrm{M}} q},$$

where $v_{\mathrm{M}} = 2\pi\sigma_0$ is the Maxwell velocity of the charge spreading in two dimensions. The screening 'kills' the diffusion pole responsible for the $|\omega|$ anomaly,

$$\Gamma_{\mathrm{scr}} \to \frac{1}{2\pi\nu\tau_0^2}\,\frac{v_{\mathrm{M}}}{Dq}\,\frac{1}{-\mathrm{i}\omega + v_{\mathrm{M}} q}.$$

It follows, in particular, that the interaction makes the zero-frequency anomaly sensitive to the screening by external gates, since a weak short-range interaction does not affect the memory effects. Hence, we predict the quasiclassical anomaly in gated structures. As for the half-filling problem, CFs interact with each other not only by the Coulomb interaction but also through the transverse gauge field fluctuations [4]. One can show, however, that by symmetry the latter do not have any effect on the $|\omega|$ anomaly, whereas the Coulomb interaction leads to results similar to a Fermi liquid at zero magnetic field.

We turn now to another phenomenon associated with the quasiclassical memory effects, the quasiclassical MR. To begin with, we recall that MR is zero in Boltzmann theory only in the case of a white-noise disorder, when $\langle \mathbf{v}(t)\mathbf{v}(0)\rangle = v_{\mathrm{F}}^2 \exp[-S(t)]$ with $S(t) = t/\tau$ at all t down to $t = 0$ and τ does not depend on $\overline{B}$. In fact, there exists a finite MR even within the collision-integral approximation if disorder is correlated on a finite spatial scale d. The source of MR is a cyclotron bending of trajectories within this correlation radius, which gives [8, 9] a small negative MR $\Delta\rho_{xx}/\rho_0$ of the order of $(d/R_{\mathrm{c}})^2$, where R_{c} is the cyclotron radius. A remarkable feature, which we address here, is that the non-Markovian kinetics leads to a much stronger positive MR [2], which may even be much larger than unity. We believe that it is this positive MR, counted from the resistance at half-filling, that has been observed in the fractional quantum Hall regime. More specifically, since the memory effects depend on time-reversal symmetry of disorder, in the case of a weak RMF the return-induced MR dominates (compared to the negative MR above) at all $\overline{B}$, thus explaining the positive MR of CFs, whereas in the case of a weak scalar disorder this only occurs at large enough $\overline{B}$. The point, however, is that the non-Markovian kinetics gives a leading contribution to MR at fields that are still much weaker than the field at which the quasiclassical localization [3, 7] shows up and ρ_{xx} starts to fall off with increasing $\overline{B}$. The strong positive MR is related to correlations of scattering acts at the points where quasiclassical trajectories self-intersect and may be considered as a precursor of the adiabatic localization.

Let us outline the derivation of MR. The starting point is the expression for the 2×2 conductivity matrix in terms of the exact Liouville operator, similar to Eqn (1). We then proceed along the same lines as in the derivation of the zero-frequency anomaly to introduce the memory-function matrix and the return-induced correction $\Delta M_{xx}\tau = \Delta\rho_{xx}/\rho_0$. An essential difference is that the characteristic q in the integral determining ΔM_{xx} [cf. Eqn (2)] now does not tend to zero at $\omega \to 0$ and grows with increasing $\overline{B}$. Let us focus on the limit $\omega_{\mathrm{c}}\tau \gg 1$, where ω_c is the cyclotron frequency, in which case the characteristic $q \gg l^{-1}$. Clearly, at such large q we can no longer expand the average Liouville propagator in terms of diffusion modes and should treat the dynamics on the ballistic scale more carefully. Since we deal with a long-range disorder, it is appropriate to approximate the stochastic motion of particles by a Fokker–Planck equation corresponding to the diffusion in *momentum* space. For the case of RMF we get

$$\Delta M_{xx} = 2\left(\frac{e}{mc}\right)^2 \int \frac{\mathrm{d}^2 q}{(2\pi)^2}\,\frac{\mathrm{d}\phi}{2\pi}\,\sin\phi\, W_B(q)\, g_D(\omega, \mathbf{q}, \phi), \quad (7)$$

where g_D is the solution of

$$(-\mathrm{i}\omega + \mathrm{i}\mathbf{v}\mathbf{q} + \omega_{\mathrm{c}}\partial_\phi - \tau^{-1}\partial_\phi^2) g_D(\omega, \mathbf{q}, \phi) = \sin\phi. \quad (8)$$

The result is

$$\frac{\Delta\rho_{xx}}{\rho_0} = \left(\frac{\overline{B}}{B_0}\right)^2 = 2\,\frac{d}{l}\,(\omega_{\mathrm{c}}\tau)^2, \quad (9)$$

where we used the correlator of RMF [given above Eqn (3)] describing disorder in the CF system, B_0 is the rms amplitude of the fluctuations. Equation (9) is valid for $\overline{B} \ll B_0$, whereas the adiabatic regime begins at $\overline{B} \sim B_0(l/d)^{1/6}$ [3]. In the intermediate range, the positive MR gets large, $\Delta\rho_{xx}/\rho_0 \gg 1$. In this region, the effective scattering rate is renormalized $\tau^{-1} \to \tau^{-1} + \Delta M_{xx}$, which yields

$$\frac{\rho_{xx}}{\rho_0} = \frac{1}{2} + \left[\frac{1}{4} + \left(\frac{\overline{B}}{B_0}\right)^2\right]^{1/2}. \quad (10)$$

This result describes MR up to $\overline{B}/B_0 \sim (l/d)^{1/6}$, where the resistivity reaches its maximum; in still higher fields ρ_{xx} drops rapidly due to the adiabatic character of motion.

Following the same route for the case of a random scalar potential, we obtain

$$\frac{\Delta\rho_{xx}}{\rho_0} = \frac{2\zeta(3/2)}{\pi}\left(\frac{d}{l}\right)^3 (\omega_{\mathrm{c}}\tau)^{9/2}. \quad (11)$$

In contrast to RMF, the adiabatic falloff starts already when this expression becomes of order unity.

We have performed numerical simulations of MR for both types of disorder. In Figure 3 the results for RMF are shown, for three different strength of disorder. At $d \ll l$, the

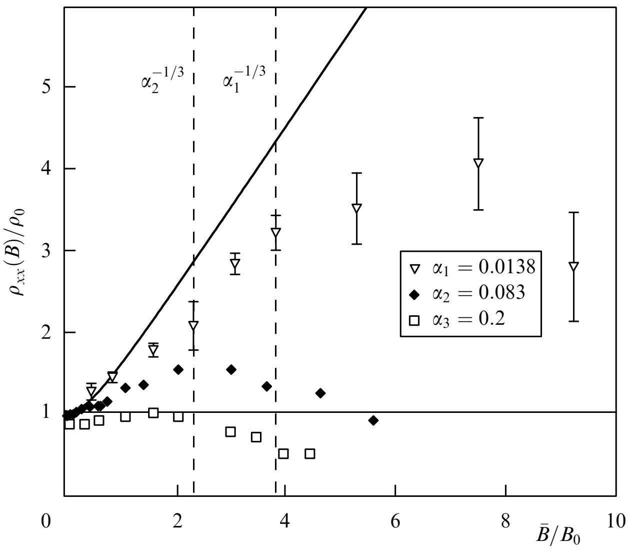


Figure 3. Magnetoresistivity (normalized by the Drude value) in a random magnetic field from numerical simulations for three different strengths of disorder; the full line corresponds to Eqn (10). Parameter $\alpha = (d/2l)^{1/2}$.

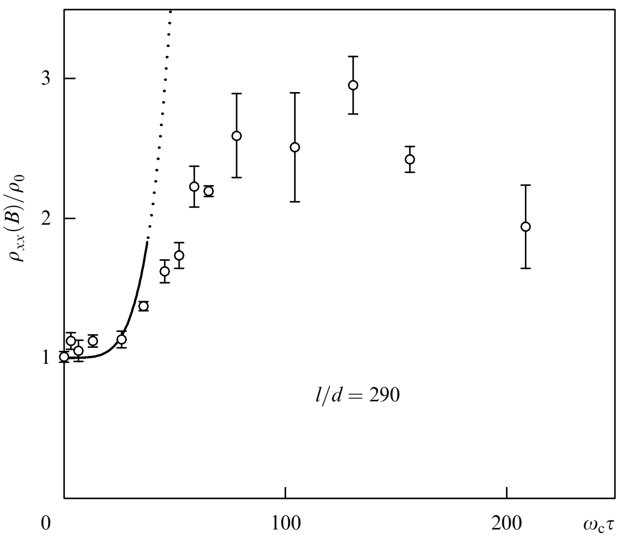


Figure 4. Magnetoresistivity in a random potential from computer simulations in comparison with Eqn (11).

theoretical prediction of a strong positive MR (10) crossing over to a negative one at $\overline{B} \sim B_0(l/d)^{1/6}$ is confirmed by the data. At moderately small d/l the positive MR still exists, but becomes weak; this is the region of d/l relevant to the CF system. The numerically calculated MR for $d/l \sim 0.1-0.2$ agrees well [3] with the experimental data [10] around $\nu_e = 1/2$. At sufficiently large d/l the region of positive MR disappears, and ρ_{xx} drops monotonously with $\overline{B}$ [3]. The numerically found MR for the random potential case (Fig. 4) shows good agreement with the theoretical result (11) up to $\Delta\rho_{xx}/\rho_0 \sim 1$. At larger $\overline{B}$, ρ_{xx} deviates from (11) and starts to decrease, as expected.

In conclusion, we have analyzed non-Markovian effects in quasiclassical transport of a two-dimensional electron gas subject to a long-range Gaussian disorder. Particular attention has been paid to the composite-fermion description of transport in a half-filled Landau level. We have shown that the ac conductivity has a nonanalytic correction $\propto |\omega|$. We have also calculated the quasiclassical magnetoresistance: the memory effects lead to a strong positive magnetoresistance, which we argue to have been observed in the composite-fermion system near half-filling.

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Density of states near the Anderson transition in the $(4-\epsilon)$-dimensional space

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Abstract. The calculation of the density of states for the Schrödinger equation with a Gaussian random potential is equivalent to the problem of a second-order transition with a 'wrong' sign of the coefficient of the quartic term in the Ginzburg – Landau Hamiltonian. The special role of the dimension $d = 4$ for such Hamiltonian can be seen from different viewpoints but fundamentally is determined by the renormalizability of the theory. Construction of ϵ-expansion in direct analogy with the phase transitions theory gives rise to a problem of a 'spurious' pole. To solve this problem, the proper treatment of the factorial divergency of the perturbation series is necessary. In $(4-\epsilon)$-dimensional theory, the terms of the leading order in $1/\epsilon$ should be retained for $N \sim 1$ (N is an order of the perturbation theory) while all degrees of $1/\epsilon$ are essential for large N in view of the fast growth of their coefficients. The latter are calculated in the leading order in N from the Callan – Symanzik equation with results of Lipatov method using as boundary conditions. The qualitative effect consists in shifting of the phase transition point to the complex plane. This results in elimination of the 'spurious' pole and in regularity of the density of states for all energies.

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