relationship between the maximal and minimal distances between the EFs and the QPC. The simplest case, which is quite realistic, is when these distances are of the same order of magnitude. When $\Gamma_T^{-1} \ll |\tau| \ll \Gamma_{\min}^{-1}$ the noise intensity can be expressed as (cf. Ref. [11])

$$S(\tau) \approx \left(\frac{2e^2 V}{h}\right)^2 \left(\frac{4\pi P_0 kTA_0}{3}\right)^2 \times \left[\frac{\ln(1/\Gamma_{\min}|\tau|)}{\ln(\Gamma_T/\Gamma_{\min})}\right]^2 .$$

By obtaining estimates for $\Gamma_{T/\min}$ from noise spectra in the normal state one can, in principle, estimate the coupling parameter A_0. A key point is to make measurements of both the MAQI interference pattern and the normal-state noise spectra in a rather large frequency range. This combination does not look too simple.

To conclude, we have presented a method for investigating the influence of noise in bias and gate voltage of a SQPC on coherent Andreev states. This is done by estimating the effect of the fluctuations on the so-called microwave-activated quantum interferometer [3]. Finally, we note that this paper together with work in Ref. [12] presents a framework which can be used to investigate the coupling of a SQPC to its electromagnetic environment.

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Spin-entangled electrons in mesoscopic systems

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Abstract. Entanglement acts as a fundamental resource for many applications in quantum communication. We propose and theoretically analyze methods for preparing and detecting entanglement between the spins of electrons in a mesoscopic environment. The entanglement production mechanism which we present is based on two quantum dots coupled to a superconductor from which paired electrons are injected via Andreev tunneling. The spin-correlated electrons can then hop from the quantum dots into normal leads. For detection we propose to measure the shot noise which is produced by the entangled electrons after they have passed a beam splitter. The enhancement of the noise by a factor of two turns out to be a unique signature for the spin singlet, a maximally entangled state. In a different setting, the entangled *ground state* in two tunnel-coupled quantum dots is detected via the Aharonov–Bohm oscillations in the co-tunneling current.

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1. Introduction

The recently demonstrated injection of spin-polarized electrons into semiconductor material [1, 2] is an important progress towards replacing the spatial (charge) degrees of freedom of the electron by its spin as the carrier of information in electronics [3]. Moreover, Kikkawa et al. [4] have found very long quantum coherence times for the electron spins in GaAs, which makes them candidates for carriers of quantum information (qubits) [5]. The long-term goal of implementing quantum information into physical systems is building a quantum computer, a device that could efficiently solve some problems for which there is no efficient classical algorithm (for a recent review, see [6]). However, there are also other ideas, e.g. in quantum communication, which seem to be more feasible with the presently available technology. One of the fundamental resource for many applications in quantum communication are pairs of entangled particles [7]. Two qubits (spins) are called entangled if their state cannot be expressed as a tensor product of states of the two qubits (spins). Well-known examples of maximally entangled states of two qubits are the spin singlet and triplet (with $m_z = 0$) of two spin-1/2 particles. In quantum optics, violations of Bell inequalities and quantum teleportation with photons have been investigated [8, 9], while so far, no corresponding experiments for electrons in a solid state environment are reported. This reflects the fact that it is very hard to produce and to measure entanglement of electrons in solid state.

One possibility for producing entangled states from product states is using the quantum gates which are the building blocks of quantum computers [5, 10]. In this paper, we present and theoretically analyze another proposal for the production of spin entangled electron pairs in mesoscopic systems, which uses the properties of the superconducting condensate and the simultaneous tunneling of a Cooper pair into a pair of quantum dots [11]. After this process, the entangled pair of electrons can hop from the dots into normal Fermi leads. We then discuss the persistence of this entanglement during electron transport in the Fermi leads where a large number of other electrons are present and interact with the entangled electrons. Furthermore, we propose an interference experiment, in which the EPR pairs produced in this way can be unambiguously tested for entanglement [12]. Here, the indicator for entanglement is the shot noise at the outgoing arm of a beam splitter into which the electrons to be tested are injected. Finally, it is known that the two-electron ground state of a pair of quantum dots coupled by a tunneling barrier is a spin singlet at zero magnetic field, which can cross over into a spin triplet at finite magnetic fields [10]. We discuss a recently proposed detection scheme [13] for these entangled ground states, which involves the Aharonov–Bohm phase in the co-tunneling current in the Coulomb blockade regime.

2. Andreev entangler

In this section, we consider an s-wave superconductor which acts as a natural source of spin-entangled electrons, since the electrons form Cooper pairs with singlet spin-wavefunctions [14]. We assume that the superconductor is held at the chemical potential μ_S, and is weakly coupled by tunnel barriers to two separate quantum dots D_1 and D_2 which are themselves weakly coupled to Fermi liquid leads L_1 and L_2, respectively, both held at the same chemical potential $\mu_1 = \mu_2$ (see Fig. 1). The tunneling amplitudes between superconductor and dots, and dots and leads, are denoted by T_{SD} and T_{DL}, respectively (for simplicity we assume them to be equal for both dots and leads).

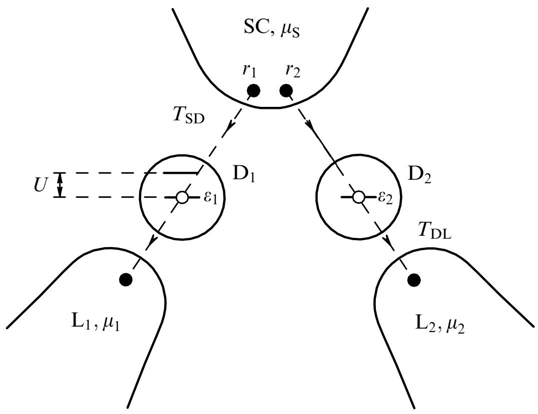

Figure 1. Entangler setup: Two entangled electrons initially forming a Cooper pair can tunnel with amplitude T_{SD} from two points $\mathbf{r}_1$, $\mathbf{r}_2$ of the superconductor, SC, (distance $\delta r = |\mathbf{r}_1 - \mathbf{r}_2|$) to two dots, D_1 and D_2 by means of Andreev tunnelling. The dots are coupled to normal leads L_1, L_2 with tunnelling amplitude T_{DL}. In order to maximize the efficiency of the entangler, we require asymmetric barriers with $|T_{SD}|/|T_{DL}| \ll 1$. The superconductor and leads are kept at chemical potentials μ_l and μ_S, respectively.

In general, the tunnel-coupling of a superconductor to a normal region allows for coherent transport of two electrons of opposite spin due to Andreev tunneling, while single-electron tunneling is suppressed [15]. In the present setup, we envision a situation where the two electrons are forced to tunnel coherently into *separate* leads rather than both into the same lead, which can be enforced by two intermediate quantum dots in the Coulomb blockade regime [16] so that the state with double occupation of one quantum dot is strongly suppressed, and thus tunneling into separate dots and subsequently separate leads is preferred.

A bias voltage $\Delta\mu = \mu_S - \mu_l > 0$ is applied in order to obtain transport of entangled electrons from the superconductor via the dots to the leads. The chemical potentials ϵ_1 and ϵ_2 of the two quantum dots can be tuned by external gate voltages [16] such that the coherent tunneling of two electrons into different leads is at resonance, described by a product of two Breit–Wigner resonances peaked at $\epsilon_1 = \mu_S$ and $\epsilon_2 = \mu_S$. In contrast, we will see that the resonance for the coherent tunneling of two electrons into the same lead is suppressed by the on-site Coulomb repulsion U of a quantum dot.

Before presenting our result, we introduce the relevant parameters describing the proposed device and specify their regime of interest. We work at the resonances $\epsilon_1 = \epsilon_2 = \mu_S$ since then the total current and the desired suppression for tunneling into the same lead is maximized. Also, the desired injection of the two electrons into separate leads at the *same orbital energy* is then achieved; this turns out to be crucial for the detection of entanglement which we propose in Section 3.2. It is most convenient to work in the regime where the dot levels ϵ_n have vanishing occupation probability. For this purpose we require that the quantum dot-lead coupling is much stronger than the superconductor-quantum dot coupling, i.e. $|T_{SD}| \ll |T_{DL}|$, so that electrons which enter the quantum dots from the superconductor will leave the quantum dots to the leads much faster than new electrons can be provided from the superconductor. The stationary occupation due to the coupling to the leads is indeed exponentially small if $\Delta\mu > k_B T$, T being the temperature and k_B the Boltzmann constant. In this asymmetric barrier case, the resonant dot levels ϵ_n can be occupied only during a virtual process.

Next, the quantum dots are allowed to contain an arbitrary but even number of electrons, N_D = even, with total spin zero in the ground state (i.e. antiferromagnetic filling of the dots). An odd number N_D must be excluded since a simple spin flip on the quantum dot would be possible in the transport process and as a result the desired entanglement would be lost. Moreover, we have to make sure that also spin-flip processes of the following kind are excluded. Consider an electron that tunnels from the superconductor into a given dot. In principle, it is possible (e.g. in a sequential tunneling process [16]) that another electron with the opposite spin leaves the dot and tunnels into the lead, and, again, the desired entanglement would be lost. However, such spin-flip processes will be excluded if the energy level spacings of the quantum dots, $\delta\epsilon$, (assumed to be similar for both dots) exceeds both, temperature $k_B T$ and bias voltage $\Delta\mu$. A serious mechanism for the loss of entanglement is given by electron hole-pair excitations out of the Fermi sea of the leads during the resonant tunneling events. However, one can show that such many-particle contributions are suppressed if the resonance width $\gamma_l = 2\pi\nu_l |T_{DL}|^2$ is smaller than $\Delta\mu$ (for $\epsilon_n \simeq \mu_S$), where ν_l is the density of states (DOS) per spin of the leads at the chemical potential μ_l.

Finally, an additional energy scale that enters the consideration is the superconducting energy gap Δ, which is the minimal energy it costs to break up a Cooper pair into two quasi-particles. This gap energy also characterizes the time delay between the subsequent coherent Andreev tunneling events of the two electrons of a Cooper pair. In order to exclude single-electron tunneling where the creation of a quasi-particle in the superconductor is a final excited state we require that $\Delta \gg \Delta\mu, k_B T$. Summarizing all above inequalities, we can specify the following regime of interest for entanglement production [11]

$$\Delta,\ U,\ \delta\epsilon > \Delta\mu > \gamma_l,\ k_B T, \quad \text{and} \quad \gamma_l > \gamma_S\,. \tag{1}$$

In this regime, we have calculated and compared the stationary charge current of two entangled electrons for two competing transport channels, first for the desired transport of the two entangled electrons each into *different* leads (current I_1) and second for the unwanted transport of both electrons into the *same* lead (current I_2). We have calculated the currents I_1, I_2 by making use of a T-matrix approach which is well-adopted for describing Breit–Wigner resonances. The final result for the ratio of the two currents is [11]

$$\frac{I_1}{I_2} = \frac{2\mathcal{E}^2}{\gamma^2}\left[\frac{\sin(k_\mathrm{F}\delta r)}{k_\mathrm{F}\delta r}\right]^2 \exp\left(-\frac{2\delta r}{\pi\xi}\right), \tag{2}$$

where

$$\frac{1}{\mathcal{E}} = \frac{1}{\pi\Delta} + \frac{1}{U}, \qquad \gamma = \gamma_1 + \gamma_2 \,. \tag{3}$$

The current I_1 becomes exponentially suppressed with increasing distance $\delta r = |\mathbf{r}_1 - \mathbf{r}_2|$ between the tunneling points on the superconductor, the scale given by the superconducting coherence length ξ. This does not pose severe restrictions for a conventional s-wave material with ξ typically being on the order of μm. In the important case $0 \le \delta r \sim \xi$ the suppression is only polynomial $\propto 1/(k_\mathrm{F}\delta r)^2$, with k_F being the Fermi wavevector in the superconductor. On the other hand, we see that the effect of the quantum dots consists in the suppression factor $(\gamma/\mathcal{E})^2$ for tunneling into the same lead. Thus in addition to Eqn (1) we have to impose the condition $k_\mathrm{F}\delta r < \mathcal{E}/\gamma$. We would like to stress that the suppression (rather than only the absolute current) is maximized by working around the resonance $\epsilon_\mathrm{n} \simeq \mu_\mathrm{S} = 0$. We remark that incoherent transport (sequential tunneling) is negligible as long as the scattering rate Γ_φ is much smaller than γ_1 since $(I_\mathrm{seq}/I_\mathrm{coh}) \simeq (\Gamma_\varphi/\gamma_1)$ [17].

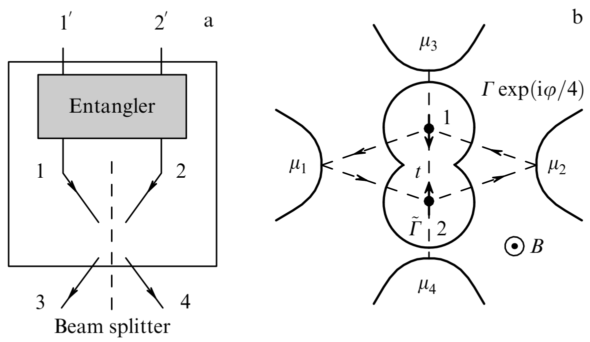

Figure 2. (a) Setup for measuring the noise of entangled states. Uncorrelated electrons are fed from leads $1', 2'$ into the entangler, which produces entangled electron pairs, injecting them separately into leads $1, 2$. The entanglement is detected in the noise in lead 3 or 4 after the beamsplitter. (b) Double-dot (DD) containing two electrons, weakly coupled to leads $1, \ldots, 4$, at chemical potentials $\mu_1, \ldots, \mu_4$. The tunnelling amplitudes from dots to leads are Γ, $\tilde\Gamma$. Tunnelling (t) between the dots results in a singlet (triplet) ground state. The closed tunnelling path between dots and leads $1, 2$ encloses the area A.

3. Transport and detection of entangled electrons

We first consider the situation shown in Fig. 2a. The entangler is assumed to be a device which can generate entangled electrons, one possible implementation being the one discussed in the previous Section. The beam splitter makes sure that the electrons leaving the entangler are interchanged with some finite amplitude. We will study the current–current correlations (noise) $\langle\delta I_\alpha \delta I_\beta\rangle$ measured in the outgoing leads $\alpha, \beta = 3, 4$ of the beam splitter.

Particles with symmetric wave functions show bunching behavior [18, 19] in their noise correlations, whereas for particles with antisymmetric wave functions one observes antibunching. Such antibunching effects for electrons in the normal state were studied theoretically [20, 21] and experimentally [22]. The noise is sensitive to the symmetry of only the *orbital part* of the wave function in the absence of spin scattering processes [23]. According to the Pauli principle, however, the antisymmetric spin wave function of the spin singlet requires a symmetric orbital wave function, therefore leading to particle bunching and thus an enhancement of the noise. Accordingly, we expect antibunching for spin triplet states. Therefore, we can distinguish spin singlets from triplets by measuring the correlations of the outgoing current of the beam splitter. We first study the transport of entangled electrons in metallic leads and then extend the standard scattering matrix approach [20, 21] to entangled states.

3.1 Transport

When electrons are injected from the entangler into the leads $1, 2$ with the filled Fermi sea ψ_0, we obtain the state

$$|\psi^{t/s}_{\mathbf{n}\mathbf{n}'}\rangle = \frac{1}{\sqrt{2}}\left(a^\dagger_{\mathbf{n}\uparrow} a^\dagger_{\mathbf{n}'\downarrow} \pm a^\dagger_{\mathbf{n}\downarrow} a^\dagger_{\mathbf{n}'\uparrow}\right)|\psi_0\rangle\,, \tag{4}$$

with $\mathbf{n} = (\mathbf{q}, l)$, $\mathbf{q}$ the momentum of an electron, and l the lead number. The operators $a^\dagger_{\mathbf{n}\sigma}$ and $a_{\mathbf{n}\sigma}$ create or annihilate an electron in state $\mathbf{n}$ with spin σ. The propagation of the triplet or singlet, interacting with all other electrons in the Fermi sea, can be described by the Green's function

$$G^{t/s}(\mathbf{12}, \mathbf{34}; t) = \langle\psi^{t/s}_{\mathbf{12}}, t|\psi^{t/s}_{\mathbf{34}}\rangle\,. \tag{5}$$

If we prepare a singlet (triplet), $|G^{t/s}(\mathbf{12}, \mathbf{12}; t)|$ tells us how large the amplitude is to find a singlet (triplet) after time t. In order to evaluate this quantity, a perturbative calculation was performed, invoking the time- and spin-independent Hamiltonian, $H = H_0 + \sum_{i<j} V_{ij}$, where H_0 describes the free motion of the electrons, and V_{ij} is the bare Coulomb interaction between electrons i and j. The problem of evaluating the two-particle Green's function defined above can be reduced to evaluating the single-particle Green's function $G_{1,2}$ if the Coulomb interaction between lead 1 and 2 is neglected and only interactions within the leads are taken into account. This is a reasonable assumption if the leads 1 and 2 are sufficiently separated. The (time-ordered) single-particle Green's function close to the Fermi surface is given by [24]

$$G_{1,2}(\mathbf{q}, t) \approx -\mathrm{i} z_\mathbf{q} \Theta(\epsilon_\mathbf{q} - \epsilon_\mathrm{F}) \exp(-\mathrm{i}\epsilon_\mathbf{q} t - \Gamma_\mathbf{q} t)\,,$$

for times in the range smaller than the quasi-particle lifetime $1/\Gamma_\mathbf{q}$. Here, $\epsilon_\mathbf{q} = q^2/2m$ denotes the energy of the quasi-particle, and ϵ_F is the Fermi energy. In two dimensions, $\Gamma_\mathbf{q} \propto (\epsilon_\mathbf{q} - \epsilon_\mathrm{F})^2 \log(\epsilon_\mathbf{q} - \epsilon_\mathrm{F})$ [25] within the random phase approximation (RPA). We find $|G^{t/s}(\mathbf{12}, \mathbf{12}; t)| = z_\mathrm{F}^2$, and have thus further evaluated the quasi-particle weight at the Fermi surface, defined by

$$z_\mathrm{F} = \left[1 - \frac{\partial\,\mathrm{Re}\Sigma(k_\mathrm{F}, \omega = 0)}{\partial\omega}\right]^{-1},$$

where $\Sigma(q, \omega)$ is the irreducible self-energy. For a two-dimensional electron system (2DES), we obtain within RPA

$$z_\mathrm{F} = 1 - r_\mathrm{s}\left(\frac{1}{2} + \frac{1}{\pi}\right), \tag{6}$$

in leading order of the interaction parameter $r_s = 1/k_F a_B$, where $a_B = \epsilon_0 \hbar^2/me^2$ is the Bohr radius. In a GaAs 2DES we have $a_B = 10.3$ nm, and $r_s = 0.614$, and thus we obtain $z_F = 0.665$ [26, 27]. The amplitude of recovering a singlet or triplet state after injecting it into an interacting Fermi sea is reduced by a factor of $z_F^{-2} \approx 2$. Except for this renormalization, the entanglement of the singlet or triplet state is not affected by the interacting electrons in the filled Fermi sea. Therefore we can study the noise of entangled electrons using the standard scattering theory for quasi-particles in a Fermi liquid.

3.2 Detection

When calculating the noise correlations for scattering with the entangled incident state $|\pm\rangle \equiv |\psi_{\mathbf{12}}^{t/s}\rangle$, we set $\mathbf{n} = (\varepsilon_n, n)$, now using the electron energies ε_n instead of the momentum as the orbital quantum number in Eqn (4). Correspondingly, the operator $a_{\alpha\sigma}^\dagger(\varepsilon)$ creates an incoming electron in lead α with spin σ and energy ε. The theory for the current correlations in a multiterminal conductor as given in Ref. [20] is valid for uncorrelated Fermi leads. It has to be slightly generalized in order to be applicable for the case of correlated (entangled) incoming particles. The current operator for lead α of a multiterminal conductor is

$$I_\alpha(t) = \frac{e}{h\nu} \sum_{\varepsilon\varepsilon'\sigma} \left[a_{\alpha\sigma}^\dagger(\varepsilon) a_{\alpha\sigma}(\varepsilon') - b_{\alpha\sigma}^\dagger(\varepsilon) b_{\alpha\sigma}(\varepsilon') \right] \times \exp \frac{\mathrm{i}(\varepsilon - \varepsilon')t}{\hbar}, \qquad (7)$$

where the operators $b_{\alpha\sigma}(\varepsilon)$ for the outgoing carriers are given by $b_{\alpha\sigma}(\varepsilon) = \sum_\beta s_{\alpha\beta} a_{\beta\sigma}(\varepsilon)$, where $s_{\alpha\beta}$ denotes the (spin- and energy-independent) scattering matrix, and ν is the density of states in the leads. The scattering matrix allows us to write Eqn (7) as

$$I_\alpha(t) = \frac{e}{h\nu} \sum_{\varepsilon\varepsilon'\sigma} \sum_{\beta\gamma} a_{\beta\sigma}^\dagger(\varepsilon) A_{\beta\gamma}^\alpha a_{\gamma\sigma}(\varepsilon') \exp \frac{\mathrm{i}(\varepsilon - \varepsilon')t}{\hbar}, \qquad (8)$$

$$A_{\beta\gamma}^\alpha = \delta_{\alpha\beta}\delta_{\alpha\gamma} - s_{\alpha\beta}^* s_{\alpha\gamma}. \qquad (9)$$

The spectral density of the current fluctuations (noise) $\delta I_\alpha = I_\alpha - \langle I_\alpha \rangle$ between the leads α and β,

$$S_{\alpha\beta}(\omega) = \lim_{T\to\infty} \frac{h\nu}{T} \int_0^T \mathrm{d}t \exp(\mathrm{i}\omega t) \langle \pm | \delta I_\alpha(t) \delta I_\beta(0) | \pm \rangle, \quad (10)$$

can now be evaluated, using the scattering matrix for the beam splitter (Fig. 2a) $s_{31} = s_{42} = r$, and $s_{41} = s_{32} = t$, where r and t denote the reflection and transmission amplitudes. In the absence of backscattering $s_{12} = s_{34} = s_{\alpha\alpha} = 0$. For the noise correlations, we obtain

$$S_{33} = S_{44} = -S_{34} = 2\frac{e^2}{h\nu} T(1 - T)(1 \mp \delta_{\varepsilon_1\varepsilon_2}), \qquad (11)$$

at zero frequency and

$$S_{\alpha\alpha}(\omega) = S_{\alpha\alpha}^{\mathrm{FS}}(\omega) + \frac{e^2}{h\nu} \left[(1 - \delta_{\omega,0}) + T(1 - T)(2\delta_{\omega,0} \mp \delta_{\omega,\epsilon_1-\epsilon_2} \mp \delta_{\omega,\epsilon_2-\epsilon_1}) \right] \qquad (12)$$

at finite frequencies, where $S_{\alpha\alpha}^{\mathrm{FS}}(\omega)$ denotes the noise contribution due to the Fermi sea. Here, the upper (lower) sign refers to the spin triplet (singlet), $T = |t|^2$ is the transmittivity of the beam splitter, and ν the density of states in the leads. The average current in lead α, $|\langle I_\alpha \rangle| = e/h\nu$, is not sensitive to the orbital symmetry of the wave function. Our result (11) implies that if two electrons with the same energies, $\varepsilon_1 = \varepsilon_2$, in the singlet state are injected into the leads 1 and 2, then the zero frequency noise is enhanced by a factor of two, compared to the shot noise of uncorrelated particles [20, 28, 29], $2e^2 T(1 - T)/h\nu$. We emphasize that the entangling mechanism presented in the previous section produces pairs of entangled electrons with *equal energy*, and thus satisfies the requirement $\varepsilon_1 = \varepsilon_2$. The predicted enhancement of noise arises because the electrons arrive in the outgoing leads in 'bunches' (preferably zero or two electrons) due to their symmetric orbital wave function. On the other hand, the triplet states exhibit antibunching, leading to a complete suppression of the noise, $S(\omega = 0) = 0$. Since the noise enhancement for the singlet is a *unique* signature for entanglement (there exists no unentangled state with the same symmetry), the entanglement can be observed by measuring the noise power in the outgoing arms of the beam splitter.

4. Probing entanglement in a double dot

The double-dot (DD) system (see Fig. 2b) contains four metallic leads which are in equilibrium with associated reservoirs kept at the chemical potentials μ_i, $i = 1, \ldots, 4$. The leads are weakly coupled to the dots with tunneling amplitudes Γ and $\tilde{\Gamma}$, and the leads $1, 2$ are coupled to *both* dots and play the role of probes where the currents I_i are measured. The leads 3 and 4 are feeding electrodes to manipulate the electron filling in the dots. The quantum dots contain one (excess) electron each, and are coupled to each other by the tunneling amplitude t, which leads to a level splitting [5, 10] $J = E_t - E_s \sim 4t^2/U$ in the DD, with U being the single-dot Coulomb repulsion energy, and $E_{s/t}$ are the singlet/triplet energies. We recall that for two electrons in the DD (and for weak magnetic fields) the ground state is given by a spin singlet. For convenience we count the chemical potentials μ_i from E_s. The coupling $\tilde{\Gamma}$ to the feeding leads can be switched off while probing the DD with a current. Here we assume that $\tilde{\Gamma} = 0$.

Using a standard tunneling Hamiltonian approach [24], we write $H = H_0 + V$, where the first term in $H_0 = H_D + H_1 + H_2$ describes the DD and $H_{1,2}$ the leads (assumed to be Fermi liquids). The tunneling between leads and dots is described by the perturbation $V = V_1 + V_2$, where

$$V_n = \Gamma \sum_s \left[D_{n,s}^\dagger c_{n,s} + c_{n,s}^\dagger D_{n,s} \right],$$

$$D_{n,s} = \exp\left(\pm \frac{\mathrm{i}\varphi}{4}\right) d_{1,s} + \exp\left(\mp \frac{\mathrm{i}\varphi}{4}\right) d_{2,s}, \qquad (13)$$

and where the operators $c_{n,s}$ and $d_{n,s}$, $n = 1, 2$, annihilate electrons with spin s in the nth lead and in the nth dot, respectively. The Peierls phase φ in the hopping amplitude accounts for an AB or Berry phase (see below) in the presence of a magnetic field. The upper sign belongs to lead 1 and the lower to lead 2. Finally, we assume that spin is conserved in the tunneling process. For the outgoing currents we have

$$I_n = \mathrm{i}e\Gamma \sum_s \left[D_{n,s}^\dagger c_{n,s} - c_{n,s}^\dagger D_{n,s} \right].$$

The observable of interest is the average current through the DD system, $I = \langle I_2 \rangle$.

From now on we concentrate on the CB regime where we can neglect double (or higher) occupancy in each dot for all transitions including virtual ones, i.e. we require $\mu_{1,2} < U$. Further we assume that $\mu_{1,2} > J, k_B T$ to avoid resonances which might change the DD state. The lead-dot coupling Γ is assumed to be weak so that the state of the DD is not perturbed; this will allow us to retain only the first non-vanishing contribution in Γ to I. Formally, we require $J > 2\pi\nu_t \Gamma^2$, where ν_t is the tunneling density of states of the leads. In analogy to the single-dot case [30], we refer to above CB regime as co-tunneling regime.

Continuing with our derivation of I, we note that the average $\langle \ldots \rangle \equiv \mathrm{Tr}\rho\{\ldots\}$ is taken with respect to the equilibrium state of the *entire* system set up in the distant past before V is switched on [24]. Then, in the interaction picture, the current is given by

$$I = \langle U^\dagger I_2(t) U \rangle, \quad U = T \exp\left[-\mathrm{i} \int_{-\infty}^{t} \mathrm{d}t' V(t') \right]. \tag{14}$$

The leading contribution in Γ to the co-tunneling current involves the tunneling of one electron from the DD to, say, lead 2 and of a second electron from lead 1 to the DD (see Fig. 2b). This contribution is of order $V_2 V_1^2$, and thus $I \propto \Gamma^4$, as is typical for co-tunneling [30]. Taking the trace over Fermi leads, we arrive then at the following compact expression for the co-tunneling current

$$I = \frac{1}{2} e\pi\nu_t^2 \Gamma^4 \sum_{i,f,s,s'} \rho_i \, |\langle i | D_{2,s'}^\dagger D_{1,s} | f \rangle|^2 \, \frac{\Delta_{i,f}\theta(\Delta_{i,f})}{\mu_1 \mu_2}\,,$$

$$\Delta_{i,f} = \mu_1 - \mu_2 + E_i - E_f\,. \tag{15}$$

This equation shows that in the co-tunneling regime the initial state $|i\rangle$ (with weight ρ_i) of the DD is changed into a final state $|f\rangle$ by the traversing electron. However, due to the weak coupling Γ, the DD will have returned to its equilibrium state before the next electron passes through it.

For small bias, $|\mu_1 - \mu_2| < J$, only elastic co-tunneling is allowed, i.e. $E_i = E_f$. However, this regime is not of interest here since singlet and triplet contributions turn out to be identical and thus indistinguishable. We thus focus on the opposite regime, $|\mu_1 - \mu_2| > J$, where inelastic co-tunneling† occurs with singlet and triplet contributions being different. In this regime we can neglect the dynamics generated by J compared to the one generated by the bias ('slow spins'), and drop the energies E_i and E_f in Eqn (15). Finally, using $1 = \sum_f |f\rangle\langle f|$ we obtain

$$I = e\pi\nu_t^2 \Gamma^4 C(\varphi) \frac{\mu_1 - \mu_2}{\mu_1 \mu_2}\,, \tag{16}$$

$$C(\varphi) = \sum_{s,s'} \left[\langle d_{1s'}^\dagger d_{1s} d_{1s}^\dagger d_{1s'} \rangle + \cos\varphi \langle d_{1s'}^\dagger d_{1s} d_{2s}^\dagger d_{2s'} \rangle \right]. \tag{17}$$

For the purpose of our analysis we assume that the DD is in its ground state. Equation (16) shows that the co-tunneling current depends on the properties of the ground state of the DD through the coherence factor $C(\varphi)$ given in (17). The first term in C is the contribution from the topologically trivial tunneling path which runs from lead 1 through, say, dot 1 to lead 2 and back. The second term (phase-coherent part) in C is the ground state amplitude of the exchange of electron 1 with electron 2 via the leads 1 and 2 such that a closed loop is formed enclosing an area A (see Fig. 2b). Thus, in the presence of a magnetic field B, an AB phase factor $\varphi = ABe/h$ is acquired.

Next, we evaluate $C(\varphi)$ explicitly in the singlet-triplet basis. Note that only the singlet $|S\rangle$ and the triplet $|T_0\rangle$ are entangled EPR pairs while the remaining triplets $|T_+\rangle = |\uparrow\uparrow\rangle$, and $|T_-\rangle = |\downarrow\downarrow\rangle$ are not (they factorize). Assuming that the DD is in one of these states we obtain the important result:

$$C(\varphi) = \begin{cases} 2 - \cos\varphi\,, & \text{for singlet,} \\ 2 + \cos\varphi\,, & \text{for all triplets.} \end{cases} \tag{18}$$

Thus, we see that the singlet and the triplets contribute with *opposite sign to the phase-coherent part of the current*. One has to distinguish, however, carefully the entangled from the non-entangled states. The phase-coherent part of the entangled states is a genuine *two-particle* effect, while the one of the product states cannot be distinguished from a phase-coherent *single-particle* effect. Indeed, this follows from the observation that the phase-coherent part in C factorizes for the product states $T_\pm$ while it does not so for S, T_0. Also, for states such as $|\uparrow\downarrow\rangle$ the coherent part of C vanishes, showing that two different (and fixed) spin states cannot lead to a phase-coherent contribution since we *know* which electron goes which part of the loop. Finally we note that due to the AB phase the role of the singlet and triplets can be interchanged which is to say that we can continually transmutate the statistics of the entangled pairs S, T_0 from fermionic to bosonic (like in anyons): the symmetric orbital wave function of the singlet S goes into an antisymmetric one at half a flux quantum, and vice versa for the triplet T_0.

We would like to stress that the amplitude of the AB oscillations is a direct measure of the phase coherence of the entanglement, while the period via the enclosed area $A = h/eB_0$ gives a direct measure of the nonlocality of the EPR pairs, with B_0 being the field at which $\varphi = 1$. The triplets themselves can be further distinguished by applying a directionally inhomogeneous magnetic field (around the loop) producing a Berry phase Φ^{B} [31], which is positive (negative) for the triplet $m = 1(-1)$, while it vanishes for the EPR pairs S, T_0. Thus, we will eventually see beating in the AB oscillations due to the positive (negative) shift of the AB phase Φ by the Berry phase, $\varphi = \Phi \pm \Phi^{\mathrm{B}}$.

5. Conclusions

In summary, we have presented methods for producing and detecting spin-entangled electrons (EPR pairs) in mesoscopic structures such as wires and dots which could be used as a resource for quantum communication. For entanglement production (Section 2), we have proposed to use the Andreev tunneling process from an s-wave supercondutor into two quantum dots which are coupled to normal Fermi leads, and we have specified in which regime, Eqn (1), this process is possible. In this regime, entangled electrons with equal energies are produced. Moreover, we have calculated the ratio between the current produced by electron pairs going into different leads (useful EPR pairs) and the electron pairs going into the same lead (useless as EPR pairs). The result is

† Note that the AB effect is not suppressed by this inelastic co-tunnelling, since the *entire* co-tunnelling process involving also the leads is elastic: the initial and final states of the *entire* system have the same energy.

given in Eqn (2) and allows to optimize this process of creating entangled electrons.

In Section 3.1 we have discussed the propagation of entangled electrons in Fermi leads, i.e. in the presence of many other (identical) electrons interacting with the electrons belonging to the entangled pair. We find that the entanglement becomes reduced by a factor z_F^2 due to the transport through such an environment, where z_F denotes the quasiparticle weight factor of the host material. For a two-dimensional electron gas, we explicitly calculate z_F, see Eqn (6). Then, in Section 3.2, we discuss a method for detecting entangled electrons which were produced, e.g., using the method from Section 2. We consider a scattering setup with a beam splitter, where electrons to be tested are injected in the two ingoing arms, and the current noise is measured in one of the outgoing arms. For the maximally entangled singlet and triplet states of electrons with equal energies (such as those produced by the method presented in Section 2) we find the resultant Eqn (11), predicting an enhancement by a factor of two of noise for the singlet, and a complete reduction for the three triplets. We conclude that the enhancement of noise unambiguously indicates an entangled state (the spin singlet).

Finally, in Section 4 we analyze a different situation, in which the entanglement of the *ground state* of a double dot is probed. This is done by measuring the Aharonov–Bohm oscillations in the co-tunneling current which are predicted in Eqns (16) and (17). It is found that the phase-coherent part (17) which distinguishes spin singlets from triplet factorizes in the expression (16) for the co-tunneling current.

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Unpaired Majorana fermions in quantum wires

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Abstract. Certain one-dimensional Fermi systems have an energy gap in the bulk spectrum while boundary states are described by one Majorana operator per boundary point. A finite system of length L possesses two ground states with an energy difference proportional to $\exp(-L/l_0)$ and different fermionic parities. Such systems can be used as qubits since they are intrinsically immune to decoherence. The property of a system to have boundary Majorana fermions is expressed as a condition on the bulk electron spectrum. The condition is satisfied in the presence of an arbitrary small energy gap induced by proximity of a three-dimensional p-wave superconductor, provided that the normal spectrum has an odd number of Fermi points in each half of the Brillouin zone (each spin component counts separately).

1. Introduction

Implementing a full-scale quantum computer is a major challenge to modern physics and engineering. Theoretically, this goal should be achievable due to the possibility of fault-tolerant quantum computation [1]. Unlimited quantum computation is possible if errors in the implementation of each gate are below a certain threshold [2–5]. Unfortunately, for conventional fault-tolerance schemes the threshold appears to be about 10^{-4}, which is beyond the reach of current technologies. It has been also suggested that fault-tolerance can be achieved at the physical level (instead of using quantum error-correcting codes). The first proposal of these kind [6] was based on non-Abelian anyons in two-dimensional systems. A mathematical result concerning universal quantum computation with certain type of anyons has been recently obtained [7], but, generally, this approach is

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