This demonstrates the potential of these loops for further work on macroscopic quantum coherence and solid-state quantum computing. This requires quantum state control with pulsed microwaves and development of measurement schemes that are less invasive. Multiple qubit circuits with controlled coupling are within reach using present-day technology.

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Quantum Andreev interferometer in an environment

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Abstract. The influence of a noisy environment on coherent transport in Andreev states through a point contact between two superconductors is considered. The amount of dephasing is estimated for a microwave-activated quantum interferometer. Possibilities of experimentally investigating the coupling between a superconducting quantum point contact and its electromagnetic environment are discussed.

1. Introduction

The assumption of coherent transport in Andreev states in a superconducting quantum point contact (SQPC) is widely used in theoretical work, see, e.g., the items of Ref. [1]. However, in realistic systems, interactions with a dynamical environment will always introduce some amount of dephasing, see the items of Ref. [2] for a review.

The so-called microwave-activated quantum interferometer (MAQI) [3] is a device proposed as a tool to study the dynamics of Andreev levels (ALs), present in a superconducting point contact. It is based on a short, single-mode, weakly biased SQPC which is subject to microwave irradiation. Confined to the contact area there are current-carrying Andreev states. The corresponding energy levels — Andreev levels — are found in pairs within the superconductor energy gap Δ, one below and one above the Fermi level. If an SQPC is short ($L \ll \xi_0$ where L is the length of the junction while ξ_0 is the superconductor coherence length), there is only one pair of Andreev levels and their positions depend on the order parameter phase difference, ϕ, across the contact as

$$E_{\pm} = \pm E(\phi) = \pm \Delta \sqrt{1 - D \sin^2\left(\frac{\phi}{2}\right)}\,. \qquad (1)$$

The two states carry current in opposite directions and in equilibrium at low temperature only the lower state is populated. The applied bias, V, through the Josephson relation $\dot{\phi} = 2eV/\hbar$, forces the Andreev levels to move adiabatically within the energy gap with a period of $T_{\mathrm{p}} = \hbar\pi/eV$, see Fig. 1.

The microwave field induces Landau – Zener (LZ) transitions between the Andreev levels (indicated by wavy lines in Fig. 1). If the upper level is populated after the second transition, a delocalized quasi-particle excitation will be created when this Andreev level merges with the continuum. The result will be a dc contribution to the current. Further, this current exhibits an interference pattern since there are two paths with different phase gains available to the upper

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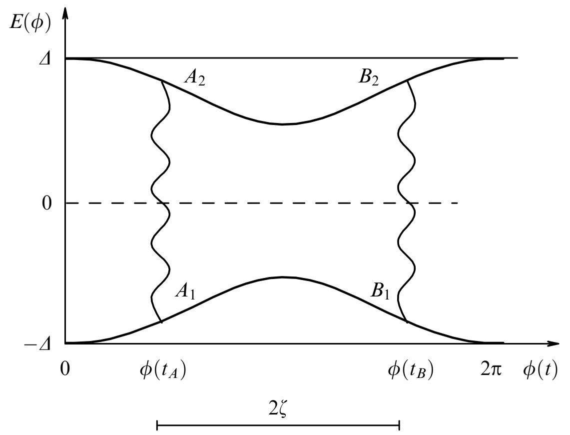

Figure 1. Time evolution of the Andreev levels within the energy gap of a single-mode SQPC. The wavy lines connecting points A_1, A_2 and B_1, B_2 symbolize resonant transitions between the levels induced by applied microwave field. The symbol ζ defines the position of the resonances.

level. It is this 'interference effect' which is utilized in the MAQI for Andreev-level spectroscopy. This is an example of a *coherent* quantum state created by transitions separated in time.

In the previous work [3], full coherency of Andreev states is assumed and a method for Andreev-level spectroscopy is presented. The spectroscopy is based on the interference pattern in the dc current through an SQPC induced by coherent microwave field. If dephasing is present, this interference pattern will deteriorate. This connection between dephasing and current makes MAQI a suitable system to study the effect of low-frequency electrical noise (the so-called *flicker noise*) in the junction transparency on transport through Andreev states. It also provides an excellent opportunity to probe the coupling between an SQPC and its electromagnetic environment.

The Andreev level positions in an SQPC depend on the order parameter phase difference ϕ across the contact, as well as on its transparency D. Consequently, there are two sources of dephasing — fluctuations in the bias voltage and in the transmission coefficient of SQPC. A fluctuation in the bias voltage V changes the ramping phase velocity $\dot{\phi} = 2eV/\hbar$, which in turn influences the phase accumulated between the subsequent Landau–Zener transitions. The second source of decoherence changes the positions of the Andreev levels through variations in the transparency D, and in this way the accumulated phase is changed again. The variation of the transparency D can be caused by the presence of an impurity atom close to the junction which has two states of almost equal energy to choose from. When the atom tunnels between its two states, the junction transparency will fluctuate. Another source of D fluctuations is the tunneling of an electron between impurity atoms in a doped region. If there are two such neighboring defects with available states, a hybrid two-level state is formed and the electron can hop between the two. This hopping will then add a fluctuation to the junction transparency. The amplitude of these fluctuations depend on the distance between the defects and the junction. From now on we will refer to these dynamic defects as two-level elementary fluctuators (EFs). It is known that EFs are responsible for the flicker noise.

In the following we briefly discuss the role of bias voltage fluctuations and then concentrate on the dephasing induced by flicker noise in the normal state junction transparency D of an SQPC in the transport through Andreev states.

2. Theory

Consider a short single-mode SQPC which is subject to a high frequency microwave field ($\hbar\omega = 2E(\phi) \leqslant 2\Delta$). Let the contact, placed at $x = 0$, be characterized by an energy-independent transparency D. A weak bias, $eV \ll \Delta$, is applied across the junction. We choose to describe the quasi-particles in the contact region with the following wave function:

$$\Psi(x, t) = u_+(x, t)\exp(\mathrm{i}k_\mathrm{F}x) + u_-(x, t)\exp(-\mathrm{i}k_\mathrm{F}x)\,,$$

where the envelope functions $u_\pm(x, t)$, left and right movers, are two-component vectors in electron-hole space. To simplify notation we introduce the four-component vector $\mathbf{u} = [u_+, u_-]$. This vector satisfies the time-dependent Bogoliubov–de Gennes equation $\mathrm{i}\hbar\partial\mathbf{u}/\partial t = [\mathcal{H}_0 + \sigma_z V_g(t)]\mathbf{u}$ where

$$\mathcal{H}_0 = -\mathrm{i}\hbar v_\mathrm{F}\sigma_z\tau_z\,\partial/\partial x + \Delta\left\{\sigma_x\cos\left[\frac{\phi(t)}{2}\right] + \mathrm{sgn}(x)\sigma_y\sin\left[\frac{\phi(t)}{2}\right]\right\},$$

σ_i and τ_i denote Pauli matrices in electron-hole space and in $\pm$ space, respectively, while $V_\mathrm{g}(t) = V_\omega\cos(\omega t)$ is the time-dependent gate potential. We assume $eV_\omega \ll \Delta$. The boundary condition at $x = 0$ is

$$\mathbf{u}(+0) = D^{-1/2}\left[1 - \tau_y\,(1 - D)^{1/2}\right]\mathbf{u}(-0)\,.$$

As a result of the applied high frequency field, there will be resonant transitions between the Andreev levels. These transitions introduce a mixed state which can be described within the resonance approximation as

$$\mathbf{u}(x, t) = \sum_\pm b^\pm(t)\,\mathbf{u}^\pm(x)\exp\left(\mp\frac{\omega t}{2}\right),$$

where $\mathbf{u}^+(x)$ and $\mathbf{u}^-(x)$ are the envelope functions of the upper and lower Andreev states, while b^+ and b^- are the corresponding probability amplitudes. The final result is a dc current through the SQPC [3],

$$I_\mathrm{dc} = 2I_0\sin^2(\Theta + \Phi)\,,$$
$$I_0 \equiv \frac{2e}{\hbar\pi}\,r^2(1 - r^2)(2\Delta - \hbar\omega)\,, \qquad (2)$$

where r is the LZ transition amplitude, which depends on the bias voltage and the amplitude of the perturbation (Θ is the phase of the LZ transition, which can be considered constant). The phase Φ which inhibits the interference is calculated through

$$\Phi = \frac{1}{2eV}\int_{\phi_\mathrm{A}}^{\phi_\mathrm{B}}\mathrm{d}\phi\left[E(\phi) - \frac{\hbar\omega}{2}\right].$$

3. Noise in the bias voltage

One of the basic assumptions for the interferometer is a stable bias. Let us now assume that $V(t) = V + V_\mathrm{f}(t)$ where $V_\mathrm{f}(t) \ll V$ is the fluctuation. The corresponding contribution to the accumulated phase is

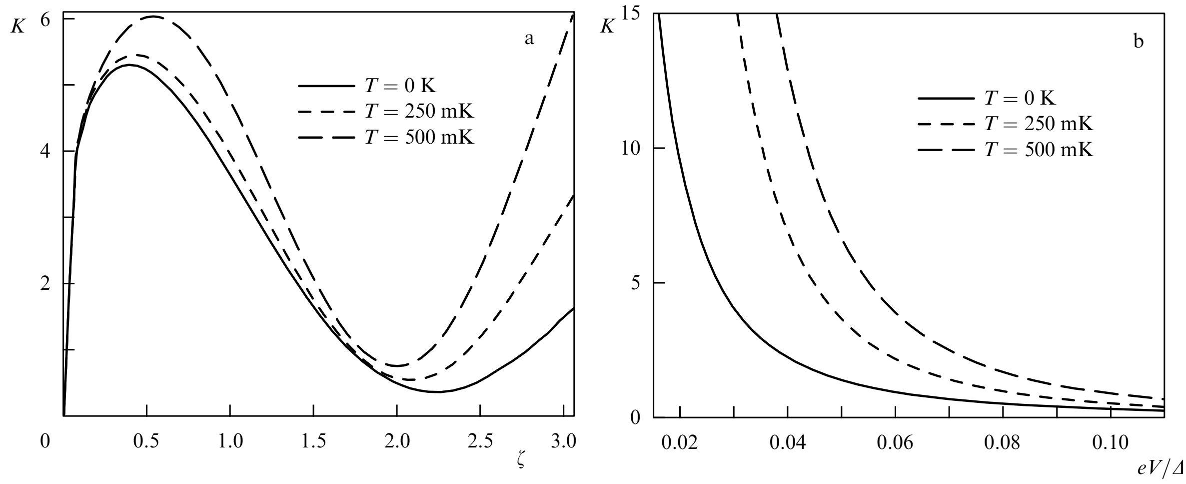

Figure 2. Dephasing as a result of voltage fluctuations appears with a factor of $W = \exp(-K)$. Here K is calculated as a function of the resonance position ζ (a) when $eV/\Delta = 0.1$ and as a function of the bias voltage (b) for $\zeta \approx 2.2$ $(\hbar\omega/2\Delta \approx 0.95)$. $Z_0 = 200\ \Omega$ and $C = 0.5$ fF.

$$\Phi_{\rm f} = -\frac{1}{2eV}\int_{\phi_{\rm A}}^{\phi_{\rm B}} {\rm d}\phi\, g(\phi)\, V_{\rm f}\left(\frac{\hbar\phi}{2eV}\right),$$

$$g(\phi) = \frac{E(\phi) - \hbar\omega/2}{eV}.$$

Inserting the fluctuating part of the phase into the expression (2) for the current and averaging over the fluctuations of bias voltage we find

$$I_{\rm dc} = I_0[1 - W\cos(2\Phi + 2\Theta)],\quad W = {\rm Re}\langle \exp 2{\rm i}\Phi_{\rm f}\rangle. \quad (3)$$

By introducing the quantity

$$\Psi(\phi) = -\frac{1}{V}\int_{-\infty}^{\phi} {\rm d}\phi' g(\phi') V_{\rm f}\left(\frac{\hbar\phi'}{2eV}\right)$$

and assuming the distribution of $V_{\rm f}$ to be Gaussian, we can write W as $W = \exp(-K)$, with

$$K(\phi_{\rm A}, \phi_{\rm BH}) = -\langle[\Psi(\phi_{\rm BH}) - \Psi(\phi_{\rm A})]\Psi(\phi_{\rm A})\rangle.$$

This correlation function depends on the environment through the averages, $\langle V_{\rm f}(t)V_{\rm f}(t')\rangle$, and we calculate K by expanding the periodic function $g(\phi)$ in a Fourier series, g_k and g_l, and further applying the fluctuation-dissipation theorem. The result is $K = \sum_{kl} g_k g_l \mathcal{K}_{kl}$ with

$$\mathcal{K}_{kl} = \frac{8}{R_{\rm q}}\int \frac{\nu\,{\rm d}\nu\ {\rm Re}\,[Z_t(\nu\omega_{\rm J})]\ \sin^2[(k + l + 2\nu)\zeta/2]}{(\nu + k)(\nu + l)\ \tanh(\hbar\omega_{\rm J}\nu/2T)}. \quad (4)$$

Here $\zeta = (\phi_{\rm BH} - \phi_{\rm A})/2$, $R_{\rm q} = h/e^2$ and $\nu = \omega/\omega_{\rm J}$, $\omega_{\rm J} = 2eV/\hbar$ is the Josephson frequency, while $Z_{\rm t}$ is the effective impedance of the circuit.

Concrete results were obtained for the so-called infinite transmission line model of the environment which is rather close to a realistic situation. The impedance in this case is expressed as $Z(\omega) = [(R_0 + {\rm i}\omega L_0)/{\rm i}\omega C_0]^{1/2}$ where L_0 is the inductance and C_0 is the capacitance, both per unit length. To calculate the impedance seen by the junction we need to include the capacitance C of the junction itself in parallel with $Z(\omega)$,

$$\mathrm{Re}\,[Z_{\rm t}(\omega)] = \frac{Z_0}{1 + \gamma\nu^2},$$

$$Z_0 = \left(\frac{L_0}{C_0}\right)^{1/2},\quad \gamma = \frac{C^2 L_0}{C_0\omega_{\rm J}^2}.$$

Here we have neglected R_0 which is usually small. An example of results is shown in Fig. 2.

The nonmonotonic behavior of K when traced as a function of ζ follows from the Fourier series expansion of $g(\zeta)$. These terms reflect the dependence of the Andreev spectra on ϕ. Around this point it can be interesting to vary the parameters of the environment and the bias. The dependence on the bias voltage at the above mentioned point, $\zeta \approx 2.2$, is shown in Fig. 2b. An increased bias should give a weaker dephasing, since a higher bias gives, through the Josephson relation, a shorter time between the resonances.

4. Flicker noise in the junction transparency

The main topic of this work is to study the effect of fluctuations in the junction transparency on the MAQI. For simplicity we choose to model the sources of these fluctuations, the EFs, with the so-called random telegraph process. This process is characterized by a random quantity $\xi(t)$ which has the value $+1$ or -1 depending on whether the upper or lower EF state is occupied. We assume that the probability of each state is the same, namely 1/2. This is acceptable since EFs with interlevel distances, $E_i \ll T$, will be 'frozen' — they behave as static impurities which do not affect the dynamic fluctuations of D. In this model the EF switches between its two states randomly in time. Physically, switching is a result of interactions between the EF and phonons or electrons in the contact area.

In the presence of EFs the junction transparency will be modulated. In other words, $D \to D + D_{\rm f}(t)$, where $D_{\rm f}(t)$ is assumed to be small. Generally, $D_{\rm f}(t) = \sum_i A_i\xi_i(t)$, with A_i being the coupling strength of the ith EF. We assume that the random processes in different EFs are not correlated. Consequently, after a change in variables from t to ϕ we can specify the random telegraph processes $\xi_i(t)$ through the correlation function,

$$\langle \xi_i(\phi_1)\xi_j(\phi_2)\rangle = \delta_{ij}\exp(-2\gamma_i|\phi_2 - \phi_1|),$$

where γ_i is the switching rate of i-th EF in units of the Josephson frequency. It is related to the dimensional switching rate Γ as $\gamma \equiv \hbar\Gamma/2eV$.

Depending on the construction of the SQPC there can be any number of EFs which are 'in range' to influence the transparency. In junctions which are very small it is probable that only one single EF will be in the vicinity of the contact. In this case, the coupling constant A and the switching rate Γ can be directly evaluated from the measured telegraph noise intensity in the normal state, $S(\tau) \equiv \langle I(t+\tau)I(t)\rangle_t - \langle I(t)\rangle_t^2$. Indeed, the current through a single mode QPC at low temperatures can be expressed, according to the Landauer formula, as $I(t) = 2e^2VD(t)/h$. Consequently, the random telegraph noise intensity is equal to

$$S(\tau) = \left(\frac{2e^2VA}{h}\right)^2 \exp(-2\Gamma\tau)\,,$$

and both A and Γ can be extracted from measured $S(\tau)$. A possible approach for extracting these model parameters from noise measurements in the case of many fluctuators in the QPC area will be discussed later.

5. Small contact — single EF

In the case of a very small contact it is possible to consider only one EF and put $i = 1$. We start by decomposing Eqn (1) as

$$E(\phi) = E_+(\phi) + E_-(\phi)\xi(\phi)\,,$$
$$E_\pm \equiv \frac{1}{2}\left[E(\phi|D_1) \pm E(\phi|D_{-1})\right],$$

where $D_{\pm 1}$ are the two different values the transparency fluctuates between. Further, we assume that both γ and $A \equiv \bar{E}_-/eV$ are much smaller than the reduced interlevel distance

$$E_+ \approx \frac{\Delta}{eV}\,, \quad \bar{E}_- = \frac{1}{2\pi}\int_0^{2\pi} E_-(\phi)\,\mathrm{d}\phi\,.$$

This means that all deviations in time are much longer than the Andreev level formation time, which is of the order $\hbar/2\Delta$.

Fortunately, the expression above is linear in ξ and we can write the effect of the EF as an additive contribution to the accumulated phase without making any approximations. Namely,

$$\Phi_{\mathrm{f}} = \frac{1}{2}\int_{\phi_{\mathrm{A}}}^{\phi_{\mathrm{BH}}} g_{\mathrm{fs}}(\phi)\,\xi(\phi)\,\mathrm{d}\phi\,.$$

After averaging over the realizations of the random process $\xi(t)$, the expression (2) for the MAQI current is replaced by Eqn (3). To facilitate the calculation of the dephasing term, W, we define the auxiliary function,

$$\Psi(\phi) = \left\langle \exp\left[\mathrm{i}A\int_{\phi_{\mathrm{A}}}^{\phi} \mathrm{d}\phi'\,g_{\mathrm{fs}}(\phi')\,\xi(\phi')\right]\right\rangle, \qquad (5)$$
$$g_{\mathrm{fs}}(\phi) = \frac{E_-(\phi)}{\bar{E}_-}\,.$$

The quantity of interest, W, is related to $\Psi(\phi)$ as $W \equiv \Psi(\phi_{\mathrm{BH}})$. The function Ψ satisfies the differential equation, cf. with Ref. [4],

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}\phi^2} + \left(2\gamma - \frac{\mathrm{d}\ln g_{\mathrm{fs}}}{\mathrm{d}\phi}\right)\frac{\mathrm{d}\Psi}{\mathrm{d}\phi} + A^2 g_{\mathrm{fs}}^2(\phi)\Psi = 0\,, \qquad (6)$$

with the initial conditions $\Psi(\phi_{\mathrm{A}}) = 1$, $\mathrm{d}\Psi/\mathrm{d}\phi|_{\phi=\phi_{\mathrm{A}}} = 0$.

If the transparency D is not too close to 1, then the function $g_{\mathrm{fs}}(\phi)$ is rather smooth and the qualitative results can be obtained assuming $g_{\mathrm{fs}}(\phi) = 1$. Then

$$W = \exp(-2\gamma\zeta) \times \left[\cosh\left(2\zeta\sqrt{\gamma^2 - A^2}\right) + \frac{\gamma}{\sqrt{\gamma^2 - A^2}}\sinh\left(2\zeta\sqrt{\gamma^2 - A^2}\right)\right]. \qquad (7)$$

We observe that the result depends on the dimensionless parameter γ/A. It is practical to consider the following two limiting cases: (i) the 'slow EF', $\gamma \ll A$, which corresponds to low temperatures, and (ii) the 'fast EF', $\gamma \gg A$, which corresponds to relatively high temperatures.

In the low-temperature limit the EF will slowly switch between its two states, and $\gamma \to 0$. Then

$$W\cos(\Phi) = \frac{\cos(\Phi + \Phi_\zeta) + \cos(\Phi - \Phi_\zeta)}{2}\,.$$

For a constant $g_{\mathrm{fs}} = 1$, $\Phi_\zeta = 2\zeta$, where $\zeta = (\phi_{\mathrm{BH}} - \phi_{\mathrm{A}})/2$ is equal to half the distance between the resonance positions, see Fig. 1. In the general case these quantities are increasing functions of ζ. Thus, at $\gamma = 0$, the current is split into two interference patterns of equal magnitude shifted by the phase Φ_ζ and there is no dephasing. This splitting into two patterns of equal magnitude follows from the assumption that the occupation probability is the same for the two EF states. The general case of arbitrary probabilities for the EF states can be solved numerically. At finite γ, dephasing takes place and the amplitude of the interference oscillations decreases by $\exp(-2\gamma\zeta)$. The physical reason for dephasing is the finite lifetime of an EF in a given state. In the case of *fast* switching, $W = \exp(-K)$, $K = A^2\zeta/\gamma$. Here we also find an exponential decay of the interference term, however, the decay rate is $\propto \gamma^{-1}$. This effect is similar to the well known *motional narrowing* of spectral lines [5]. When the EF fluctuates rapidly enough compared to the 'energy resolution' $E_-/\hbar$, influence from the difference between two EF states is smeared and dephasing will be of a diffusive character, with an effective, time-dependent, diffusion constant $A^2/2\gamma$.

The two above limiting cases match at $\gamma \approx A$. In general, when g_{fs} is a pronounced function of ϕ, one has to solve Eqn (6) numerically. Our analysis shows that the above qualitative conclusions remain valid if the SQPC transparency, D, is not very close to 1.

6. Large contact — many EFs

Let us consider a large number of EFs with varying switching rates distributed in the contact area. For simplicity, we shall assume that only the fluctuators with interlevel spacings $U_i \lesssim T$ are important, and that their distribution is uniform, $\mathcal{P}_U(U) = P_0\mathcal{V}$. Here $\mathcal{V}$ is the sample volume. Further, we assume that the switching rates γ_i are the same for both transition directions (up and down) between the EF's levels. This assumption is natural because the ratio between the corresponding transition rates is $\exp(-U_i/T)$. Within the assumptions discussed above, the final results are substantially simplified while preserving the essential dependence on temperature and the resonance position. These approximations agree with a general theory developed in Ref. [6] for the case of dephasing by two-level systems (TLS) in glasses.

The first step now is to linearize the SQPC's transparency with respect to ξ_i as $D \to D + D_f(t)$. This allows us to once again find an additive contribution to the accumulated phase, which in this case will be

$$\Phi_f \approx \sum_i A_i \int_{\phi_A}^{\phi_{BH}} \xi_i(\phi)\, g_{fm}(\phi)\, d\phi .$$

Here we have defined

$$A_i = \frac{1}{2eV}\delta D_i \frac{d\overline{E(\phi)}}{dD}, \quad g_{fm}(\phi) = \left[\frac{d\overline{E(\phi)}}{dD}\right]^{-1} \frac{dE(\phi)}{dD} .$$

In the same manner as above, we can express the modified MAQI current through expression (3) with $W \to \mathcal{W}$ given by the expression

$$\mathcal{W} = \left\langle \exp\left[i \sum_i A_i \int_{\phi_A}^{\phi_{BH}} g_{fm}(\phi)\, \xi_i(\phi)\, d\phi \right]\right\rangle_{A,\gamma,\xi} .$$

To approximate this average we use the Holtsmark method [7] which is valid in the limit of many fluctuators, $N = P_0 \mathcal{V} kT \gg 1$. This allows us to rewrite W as the average over the contributions (7) from single EFs as $\mathcal{W} = \exp(-\mathcal{K})$ with

$$\mathcal{K} \approx P_0 \mathcal{V} kT \langle 1 - W(A,\gamma)\rangle_{A,\gamma} . \tag{8}$$

Since the number of EFs is assumed to be large, to keep dephasing at a reasonable level it is important to keep $\langle 1 - W\rangle$ small.

With known solutions for W found above the average $\langle 1 - W\rangle$ remains to be calculated. To calculate this average one has to specify the distributions of the parameters A and γ. The simplest and most natural assumption is that these two quantities are not correlated. Consequently, the distribution $\mathcal{P}(A,\gamma)$ can be decoupled as $\mathcal{P}_A(A)\,\mathcal{P}_\gamma(\gamma)$. To specify the distribution $\mathcal{P}_A$ let us assume that the EFs are uniformly distributed in space. An EF behaves like a dipole, either electric or elastic, this allows us to specify its interaction strength as $A(r) = A_0/r^3$, where r is the distance between the contact and a given EF [8], while A_0 is a coupling constant dependent on a specific interaction mechanism. Note that the quantity A_0 has dimension of volume. Within this model we arrive at the normalised distribution function $\mathcal{P}_A(A) = 4\pi A_0/3\mathcal{V}A^2$. The distribution $\mathcal{P}_\gamma(\gamma)$ is specified in a manner which is commonly used in glasses. Namely, the *logarithm* of γ is assumed to be uniformly distributed. Hence, $\mathcal{P}_\gamma(\gamma) \propto \gamma^{-1}$. To normalise it let us take into account that for a given energy spacing U there is a *maximal* switching rate. Since we are interested in the fluctuators with $U_i \lesssim T$, we can specify the maximal switching rate as γ_T, which is a function of temperature. The actual temperature dependence is determined by the specific interaction mechanism between the EF and its environment. If the transitions between the EF states are caused by interaction with phonons, then $\gamma_T \propto T^3$ [9], while if the transitions are caused by the electron excitations, then $\gamma_T \propto T$ [10]. Therefore, the normalised distribution can be specified as $\mathcal{P}_\gamma(\gamma) = (\mathcal{L}\gamma)^{-1}$, where $\mathcal{L} = \ln(\gamma_T/\gamma_{min}) \gg 1$. Here we introduced the minimal switching rate, γ_{min}. To express the decay in a more clear form let us introduce the dimensionless frequency ν_d corresponding to the interaction strength for an EF separated from the contact by an average distance to the active fluctuators, $\bar{r} \equiv (4\pi P_0 kT/3)^{-1/3}$, divided by the Josephson energy $2eV$. We can specify ν_d as

$$\nu_d = \frac{4\pi P_0 kT A_0}{3} = \frac{A_0}{\bar{r}^3} . \tag{9}$$

The decay rate $\mathcal{K} = -\ln \mathcal{W}$ is then given by the expression

$$\mathcal{K} = \frac{\nu_d}{\mathcal{L}} \int_0^\infty \frac{dA}{A^2} \int_{\gamma_{min}}^{\gamma_T} \frac{d\gamma}{\gamma} [1 - W(A,\gamma)] . \tag{10}$$

To estimate the amount of dephasing let us use a simplified expression (7) for W obtained for smooth $g_{fs}(\phi)$. One can see that the most important are EFs with $A \approx \gamma$, or located at $r_\gamma \approx (A_0/\gamma)^{1/3}$ from the contact. As a result, the interference pattern decays exponentially with $\mathcal{K} \approx 3\nu_d\zeta$. In general case, this expression is modified by a factor of the order 1 which is a smooth function of ζ.

7. Nonoptimal EFs

In the previous consideration it was assumed that the system size is much larger than r_γ. A consequence of this assumption is that, independent of temperature, the EFs which have the strongest effect on the junction transparency will always be included in the estimates. A further point is that the rate γ is confined to the interval between γ_{min} and γ_T. Thus we have actually assumed that the size of the region where EFs reside is larger than $r_{max} \approx (A_0/\gamma_{min})^{1/3}$, and that there is no 'excluded region' without EFs near the contact with the size less that $r_{min} \approx (A_0/\gamma_T)^{1/3}$. Both r_{min} and r_{max} decrease with increasing temperature.

What happens if this 'optimal' EF is out of the range? This can occur if the system is limited in size, or if there is a specifically pure region around the contact. If $R \lesssim r_{max}$ no 'optimal' EFs are present in the contact area, and one has to look for the most efficient, however nonoptimal ones. Concrete results depend upon the relationship between the system size R and r_{min}, they will be published and discussed in detail elsewhere. In any case, the decoherence is slower than that for $R > r_{max}$.

8. Discussion and conclusions

To estimate the dephasing rate one needs information on EF properties and distribution. For a small SQPC, when a single EF is important, the necessary parameters A and Γ can be determined from measured random telegraph noise in the normal state, as discussed earlier. The case of many EFs requires much more information, and at present time we can discuss only qualitative predictions. Generally, dephasing will increase with temperature, as well as with the interval ζ between sequential resonances. However, at large enough temperatures, when γ_{min} appears large enough, the temperature dependence of dephasing will slow down. The free parameter of the theory, A_0, can be estimated only roughly through comparison with the noise measurements in the normal state. To map the parameter A_0 to the noise, one can apply the theory of flicker noise in a QPC [11] to the case of a single mode contact. According to that theory, results for the noise intensity $S(\tau)$ are substantially dependent on the relationship between the maximal and minimal distances between the EFs and the QPC. The simplest case, which is quite realistic, is when these distances are of the same order of magnitude. When $\Gamma_T^{-1} \ll |\tau| \ll \Gamma_{\min}^{-1}$ the noise intensity can be expressed as (cf. Ref. [11])

$$S(\tau) \approx \left(\frac{2e^2 V}{h}\right)^2 \left(\frac{4\pi P_0 kTA_0}{3}\right)^2 \times \left[\frac{\ln(1/\Gamma_{\min}|\tau|)}{\ln(\Gamma_T/\Gamma_{\min})}\right]^2 .$$

By obtaining estimates for $\Gamma_{T/\min}$ from noise spectra in the normal state one can, in principle, estimate the coupling parameter A_0. A key point is to make measurements of both the MAQI interference pattern and the normal-state noise spectra in a rather large frequency range. This combination does not look too simple.

To conclude, we have presented a method for investigating the influence of noise in bias and gate voltage of a SQPC on coherent Andreev states. This is done by estimating the effect of the fluctuations on the so-called microwave-activated quantum interferometer [3]. Finally, we note that this paper together with work in Ref. [12] presents a framework which can be used to investigate the coupling of a SQPC to its electromagnetic environment.

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Spin-entangled electrons in mesoscopic systems

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Abstract. Entanglement acts as a fundamental resource for many applications in quantum communication. We propose and theoretically analyze methods for preparing and detecting entanglement between the spins of electrons in a mesoscopic environment. The entanglement production mechanism which we present is based on two quantum dots coupled to a superconductor from which paired electrons are injected via Andreev tunneling. The spin-correlated electrons can then hop from the quantum dots into normal leads. For detection we propose to measure the shot noise which is produced by the entangled electrons after they have passed a beam splitter. The enhancement of the noise by a factor of two turns out to be a unique signature for the spin singlet, a maximally entangled state. In a different setting, the entangled *ground state* in two tunnel-coupled quantum dots is detected via the Aharonov–Bohm oscillations in the co-tunneling current.

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1. Introduction

The recently demonstrated injection of spin-polarized electrons into semiconductor material [1, 2] is an important progress towards replacing the spatial (charge) degrees of freedom of the electron by its spin as the carrier of information in electronics [3]. Moreover, Kikkawa et al. [4] have found very long quantum coherence times for the electron spins in GaAs, which makes them candidates for carriers of quantum information (qubits) [5]. The long-term goal of implementing quantum information into physical systems is building a quantum computer, a device that could efficiently solve some problems for which there is no efficient classical algorithm (for a recent review, see [6]). However, there are also other ideas, e.g. in quantum communication, which seem to be more feasible with the presently available technology. One of the fundamental resource for many applications in quantum communication are pairs of entangled particles [7]. Two qubits (spins) are called entangled if their state cannot be expressed as a tensor product of states of the two qubits (spins). Well-known examples of maximally entangled states of two qubits are the spin singlet and triplet (with $m_z = 0$) of two spin-1/2 particles. In quantum optics, violations of Bell inequalities and quantum teleportation with photons have been investigated [8, 9], while so far, no corresponding experiments for electrons in a solid state environment are reported. This reflects the fact that it is very hard to produce and to measure entanglement of electrons in solid state.

One possibility for producing entangled states from product states is using the quantum gates which are the building blocks of quantum computers [5, 10]. In this paper, we present and theoretically analyze another proposal for the production of spin entangled electron pairs in mesoscopic systems, which uses the properties of the superconducting condensate and the simultaneous tunneling of a Cooper pair into a pair of quantum dots [11]. After this process, the entangled pair of electrons can hop from the dots into normal Fermi leads. We then discuss the persistence of this entanglement during electron transport in the Fermi leads where a large number of other electrons are present and interact with the entangled electrons. Furthermore, we propose an interference experiment, in which the EPR pairs produced in this way can be unambiguously tested for entanglement [12]. Here, the indicator for entanglement is the shot noise at the outgoing arm of a beam splitter into which the electrons to be tested are injected. Finally, it is known that the two-electron ground state of a pair of quantum dots coupled by a tunneling barrier is a spin singlet at zero magnetic field, which can cross over into a spin triplet at finite magnetic fields [10]. We discuss a recently proposed detection scheme [13] for these entangled ground states, which involves the Aharonov–Bohm phase in the co-tunneling current in the Coulomb blockade regime.