Physics – Uspekhi **44** (12) 1283 – 1298 (2001)

FROM THE HISTORY OF PHYSICS

PACS numbers: **01.65.+g**, **04.20.–q**

On the discovery of the gravitational field equations by Einstein and Hilbert: new materials

V P Vizgin

DOI: 10.1070/PU2001v044n12ABEH001014

Contents

1. **Introduction** **1283**
2. **Genesis of the tensor geometrical concept of gravitation** **1284**
3. **Rejection of the general covariant to equations of gravitational field, and the search for a noncovariant solution to the problem of field equations** **1286**
4. **November 1915: Einstein's and Hilbert's roads to the equations of gravitation** **1288**
5. **Hilbert's "Foundations of physics (first note)": the background** **1289**
6. **"Foundations of physics (first note)": Hilbert's way of bringing together gravitation, electromagnetism and matter** **1291**
7. **Hilbert's "Foundations of physics (first note)": differences between the proof and the published version** **1293**
8. **Conclusions** **1294**

References **1297**

Abstract. This article describes the history of discovery of the equations of gravitational field by Albert Einstein and David Hilbert in November 1915. The proof sheet of Hilbert's lecture report, made on 20 November 1915 and published in March 1916, rediscovered in 1997 in the archive of the university of Göttingen, throws new light on the history of this discovery. We also discuss the early history of the general theory of relativity that led to the expression of the general covariant equations of gravitational field.

1. Introduction

From the time of Newton and classical mechanics, forever linked with his name, it has been justly held that the basis, the core, the heart of the fundamental physical theories are the differential equations of motion of the appropriate physical systems. As Heinrich Hertz put it, "Maxwell's theory is the Maxwell equations" [1, p. 23]. Leonid Mandelstam in his *Lectures on Optics, Relativity and Quantum Mechanics* [2] in 1939 in connection with the "structure of any physical theory" said that a theory consists of "two complementary parts": the physical interpretation of the theory, and the main equations — as a rule, differential (the equations of Maxwell, Newton, Schrödinger, etc.). "Without the first part, — said Mandelstam, — the theory is illusory, empty. Without the second part, there is no theory at all" [2, pp. 326 – 327].

V P Vizgin S I Vavilov Institute of Natural Sciences and Technology
Staropanskiĭ per. 1/5, 103012 Moscow, Russian Federation
Tel. (7-095) 928 19 69
Fax (7-095) 925 99 11
E-mail: vizgin@ihst.ru

Received 4 April 2001
Uspekhi Fizicheskikh Nauk **171** (12) 1347 – 1363 (2001)
Translated by A S Dobroslavskiĭ; edited by M S Aksent'eva

Later Richard Feynman said, referring to the discovery by Paul Dirac of the main laws of relativistic quantum mechanics, that "guessing equations seems to be a very good way to discover new laws" [3, p. 57]. As a rule, equations of motion or field equations in fundamental theories derive from the variational principles (namely, the Hamilton principle), and hence the theoreticians tend to assume that "the theory is all set when the Lagrangian is written" [4, p. 38].

Thus, the issue of the discovery of the main equations of a physical theory is the central point in the development of such a theory. Such equations in the general theory of relativity (GTR), or the contemporary relativistic theory of gravitation, are the famous Einstein (or Einstein – Hilbert)[1] equations of gravitational field. The theory itself is unanimously credited to Einstein, who steadily worked towards his target from 1907, completing the theory in November 1915. At the final stage, however, when the correct general covariant equations of gravitation were expressed, Einstein was joined by the leader of mathematical Göttingen, David Hilbert, who found these equations almost simultaneously with Einstein[2]. Hilbert's contribution was always noted in the early classical relativistic literature. The most exact and detailed evaluation was given by Wolfgang Pauli in his famous encyclopedic article, "Simultaneously with Einstein and independently of

[1] In most textbooks and monographs on general relativity, including the classics (H Weyl, A Eddington, V A Fock, L D Landau and E M Lifshitz, etc.), these equations bear the name of Einstein. However, there are some exceptions (see, for example, the recent book by A A Logunov [5], where they are called 'the Hilbert – Einstein equations').

[2] It is interesting that this was the second direct and efficient invasion of mathematicians into the development of the relativistic theory of gravitation. The first occurred three years earlier, when M Grossmann, a mathematician from Zurich and a university friend of Einstein's, helped him to master Riemann's geometry and became his co-author in the first publication on the tensor-geometrical theory of gravitation, which still lacked the correct general covariant equations of gravitation (the Einstein – Grossmann theory).

him, the general covariant field equations were established by Hilbert... Hilbert's presentation, however, was not quite comfortable for the physicists, because in the first place he axiomatically defined the variational principle, and, which is more important, his equations were expressed not for an arbitrary material system, but were based on Mie's theory of matter'' [6, p. 211] [3].

Subsequently, this accomplishment of the Göttingen scholar was not often remembered, and Einstein himself hardly ever mentioned it. A new wave of discussions of Hilbert's role in the development of GTR arose in the 1970s [8 – 11] [4]. It was commonly held that the published version of Hilbert's report entitled ''Foundations of physics'' is very close to his presentation made on 20 November 1915, the only difference consisting in the reference to the latest paper of Einstein dated November 25th and containing Einstein's first version of the correct general covariant equations of gravitation. The authors of these works emphasized the dissimilarity if not the polarity of the approaches of Einstein and Hilbert, and eventually agreed with Pauli that the two scientists expressed the correct equations of gravitational field almost simultaneously and independently from each other.

Such was the situation until 1978, when the correspondence exchanged between Einstein and Hilbert in November 1915 was unearthed [12]. It turned out that even though the discovery was almost simultaneous, it definitely was not independent. By contrast, our heroes actively discussed the problems of gravitation and unified field theory, and exchanged proof sheets of their November reports. This sensational finding was publicized in the early 1980s (see, for example, Refs [7, 13]). From the correspondence it followed that before his last November report Einstein had received the text of Hilbert's communication and hence should have seen the correct equations of gravitation, but nevertheless failed to give credit to Hilbert. The balance shifted to Hilbert's advantage, although certain alibis could be found for Einstein. So the general verdict remained the same [5], notwithstanding the linkage discovered between the paths of our heroes, and the resulting passions.

In 1997, working on a project concerned with the history of GTR, sponsored by the Max Planck Institute of the History of Science, Leo Corry from the Cohn Institute of History and Philosophy of Science of the Tel Aviv university found in the Göttingen archive the proof sheet of Hilbert's report delivered on 20 November 1915 [14]. A detailed comparison of this draft dated 6 December 1915 with the published version of 31 March 1916 brought Corry and his co-authors, noted GTR historians Jurgen Renn and John Stachel, to the conclusion that, firstly, the initial version of Hilbert's theory that supposedly contained the correct form of the gravitation equations actually failed to achieve general covariance, and secondly, that the correct general covariant equations of gravitational field were not written out explicitly by Hilbert, and so Einstein could not have seen them before his last report in November [15]. The balance swung back to Einstein's advantage. A dozen important works appeared in the wake of this dramatic discovery, concerned with Hilbert's general contribution to physical science, and in particular with the analysis of Hilbert's unified field theory as the first step in the realization of the program of the field unification of physics [16 – 24] [6].

Some time ago V L Ginzburg drew my attention to the article by John Stachel in the *Journal of Astrophysics and Astronomy* [24], and knowing my interest in the history of GTR suggested writing an article for *Uspekhi Fizicheskikh Nauk*. I decided to build on my commentary to ''Foundations of physics'', published in the second volume of Hilbert's *Selected Works* [18], paying due attention to the papers of Corry, Renn, Sauer, Stachel of 1998 – 1999 [19 – 24]. I shall introduce the reader to the events preceding the November finale. Then I give a brief account of the November events, and, following the authors mentioned above, mark the changes that the Göttingen discovery has made to the history of GTR, and in particular to the issue of priority in the discovery of the equations of gravitation.

At the end of this introduction I would like to express my gratitude to V L Ginzburg who stimulated the writing of this article, and to A N Parshin, the editor of Hilbert's *Selected Works* in Russian, who introduced me to the first publication by Corry, Renn and Stachel [15] and obtained for me a photocopy of Hilbert's proof sheets from Göttingen [14]. I thank Jurgen Renn, Leo Corry and their co-authors for sending me the reprints of their papers, and Mr Rolfing, the curator of manuscripts at the Göttingen library, who made the proof sheets available for investigation, even though they have not yet been published. I also thank K A Tomilin for his assistance in the preparation of this article.

2. Genesis of the tensor geometrical concept of gravitation

The first few years after the creation of the special theory of relativity (STR) may be called the period of 'total relativization'. The collective effort led by Albert Einstein, Max Planck, Hermann Minkowski, Arnold Sommerfeld, Max Laue, Max Born and others, resulted in the creation of

[3] A similar view was held by other authorities closely linked with the Göttingen community and concerned with the problems of general relativity, such as Felix Klein and Hermann Weyl. Klein wrote in 1920, ''There is no priority issue at all, since the two authors (Einstein and Hilbert — **V V**) followed entirely different trains of thought (and in such a way that their results did not seem compatible at first). Einstein works by induction and deals with arbitrary material systems. Hilbert works by deduction, introducing the above-mentioned (related to Mie's theory — **V V**) electrodynamic constraint, from the higher variational principle'' [7, p. 107]. Weyl — almost 30 years after the events — recalled, ''In his investigations on general relativity Hilbert combined Einstein's theory of gravitation with G Mie's program of pure field physics. For the development of the theory of general relativity at that stage, Einstein's more sober procedure, which did not couple the theory with Mie's highly speculative program, proved the more fertile. Hilbert's endeavors must be looked upon as a forerunner of a unified field theory of gravitation and electromagnetism...'' [45, p. 283].

[4] Anticipating the events, we should note that E Guth even then (in 1969 – 1970) believed that the documents would be found to confirm the mutual linkage between the November developments by Einstein and Hilbert. He also interpreted this linkage, so to say, to Einstein's advantage [8, p. 205; 7, p. 311].

[5] This is, for example, how A Pais viewed the situation: ''... It is true that Hilbert's paper contained the trace term which Einstein had yet to introduce (until November 25 — **V V**)''. But, considering the entire path of Einstein towards GTR from 1907, and his first three papers of November 1915, one may state that ''Einstein was the sole creator of the physical theory of general relativity and that both he and Hilbert should be credited for the discovery of fundamental equation... (of the gravitational field — **V V**)'' [13, p. 260].

[6] In this connection I should mention my earlier book on the history of unified field theories [25] and its revised translation into English [26]. Incidentally, Corry had published a comprehensive work on Hilbert's physical studies before his momentous archival discovery [27].

relativistic mechanics of discrete systems, relativistic hydrodynamics and theory of elasticity, phenomenological electrodynamics, etc. In fact, STR turned out to be a strong and universal research program that advanced rapidly. Naturally, the need was felt for the relativistic theory of gravitation. Henri Poincaré was the first to extend the law of gravity to four dimensions (1905 – 1906); later more detailed and clear-cut variants of such an extension were discussed by Minkowski, Sommerfeld, Hendrik Lorentz, Willem de Sitter (1907 – 1911).

Einstein himself encountered this problem in 1907, writing a large review on STR commissioned by Johannes Stark for his *Jahrbuch für Radioaktivität und Elektronik*. From the outset Einstein was not satisfied with the 'quasi-long-range approach' of Poincaré and Minkowski. He believed that the relativistic generalization must be applied not to the elementary force law, but rather to the differential equation of the gravitation field — namely, the Poisson equation. He paid attention to the remarkable equality of inertial and gravitational masses — the equality that the classical theory treated as something empirical and fortuitous. At first Einstein tried to construct a relativistic (four-dimensional) extension of the Poisson equation that would include this relation and would, incidentally, explain the anomalous precession of the perihelion of Mercury. However, he was not able to do this — moreover, he concluded that the Lorentz-covariant approach is incompatible with the equality of masses. Combining this fact with the relativistic program, Einstein formulated the equivalence principle, which allowed a purely kinematic treatment of homogeneous gravitational fields. In fact, this also implied the extension of the initial relativistic program.

From his equivalence principle Einstein predicted two new optico-gravitational effects that initially seemed beyond experimental verification: the gravitational bending of light, and the dependence of the clock rate on the gravitation potential ('the red shift'). The equivalence principle and the implicated effects indicated that the relativistic analysis of gravitational fields (even the simplest homogeneous fields) calls for a radical extension of STR and the relativistic program: firstly, the noninertial (accelerated) reference frames had to be included into consideration, and secondly, the speed of light depended on the gravitational potential.

This led to fundamental difficulties associated with the extension of the equivalence principle to nonhomogeneous fields. Obviously, the relativistic program had to be extended, and the Lorentz group ought to be replaced by a broader group of transformations (most likely, nonlinear), but the character of this extension — given the arbitrary fields — remained unclear. In addition, the transition to accelerated frames deprived the coordinates of their straightforward metric meaning, which subverted the operational-measurement basis of the relativistic program. There was a somewhat hazy vision of the extended relativistic program, but it was not clear how to proceed from the simplest case of homogeneous fields to arbitrary gravitational fields.

Meeting with these difficulties, Einstein resorted to the quantum theory for some time (1908 – 1910). He kept looking for such a generalization of Maxwell's theory as would describe both the corpuscular and quantum aspects of radiation. With luck, the generalized equations might admit a group of transformations broader that the Lorentz group, and this could prompt the required extension of the relativistic program and help with resolving the difficulties in the way of applying the equivalence principle to the nonhomogeneous fields. In 1911, not having succeeded in his efforts, Einstein returned to gravitation. He reconsidered the equivalence principle in a more clear-cut and consistent manner, and derived once again the two effects mentioned above, pointing out their possible astronomical observation. He also emphasized the essentially local nature of the equivalence principle.

The idea that the speed of light depends on the gravitation potential was used by Max Abraham, who tried to combine it with the four-dimensional extension of the Poisson equation (1912). Einstein, however, saw the inconsistency of such a combination, and decided to extend the equivalence principle only to static nonhomogeneous fields. He identified the scalar gravitational potential first with the velocity of light, and then with the square root of the latter ($\Delta c = \varkappa c \rho$ and $\Delta\sqrt{c} = (\varkappa/2)\sqrt{c}\,\rho$, where ρ is the mass density, and $\varkappa$ is the gravitation constant).

Abraham then attempted to improve his theory, using Einstein's identification of the potential with $\sqrt{c}$. However, the rejection of Lorentz covariance in the scalar approach prevented the natural extension of the equivalence principle to nonhomogeneous fields, and did not provide for the extension of the relativistic program. Gunnar Nordström suggested returning to Lorentz covariance, keeping the scalar potential and postulating the equality of inertial and gravitational masses, although not in the form of the equivalence principle (1912). Einstein, however, who had already tried out this approach in 1907 – 1908 and who was faithful to his equivalence principle, would not accept the retreat to the Lorentz-covariant version of the theory. Later, when Einstein was already developing his tensor-geometrical concept of gravitation, Nordström and Gustav Mie continued working on the scalar Lorentz-covariant theories of gravitation (1913 – 1914).

At the same time, the practice of construction of scalar theories and their discussion (especially the polemics with Abraham) not only helped to recognize and understand the pitfalls on the way towards extension of the equivalence principle to nonhomogeneous fields, but also paved the way to the construction of the tensor-geometrical theory:

1. The futility of vector and scalar approaches, which pointed to the tensor character of the potential.

2. The local validity of STR and the related infinitesimal concept of space – time geometry.

3. The associated idea of using non-Euclidean geometry in accelerated frames, and hence in the presence of gravitation.

4. The extension of the class of admissible frames (suggested by the equivalence principle) in the case of arbitrary gravitation fields (in accordance with Mach's criticism of space – time absolutes) led to a radical extension of relativity associated with arbitrary continuous transformations of coordinates.

5. The idea of nonlinearity of field equations.

Combination of items 2, 3, 4 implied that some kind of metric must replace the coordinates which lose their metric property. The metric typical of STR

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$$

in the simplest cases had to be replaced by a metric with a variable speed of light

$$\mathrm{d}s^2 = c^2(x, y, z)\,\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2\,.$$

Given that this implied a transition from the 'planar' four-dimensional space – time ('Minkowski's world') to a curved space – time, it was natural to go over to Riemann geometry with the metric

$$ds^2 = g_{ik}\, dx_i\, dx_k .$$

This immediately opened the way for a natural extension of the law of motion in STR

$$\delta \int ds = 0$$

when gravitation was included: one simply had to go over from 'flat' to Riemann's metrics. The metric tensor g_{ik} then had a dual interpretation: as the main space – time characteristic, and as the gravitational potential. This constituted the essence of the tensor-geometrical concept of gravitation. Simultaneously realized was the idea of a radical extension of relativity: general relativity was interpreted as a general covariance characteristic of the arbitrarily curved Riemann geometry.

In this connection Wolfgang Pauli said, "This fusion of two previously quite disconnected subjects — metric and gravitation — must be considered as the most beautiful achievement of the general theory of relativity" [6b, p. 215; 6c, p. 148]. The scheme on the page 1287 illustrates the way from the equivalence principle to the tensor-geometrical concept of gravitation. The boxes in the middle correspond to the chain: early Lorentz-covariant attempts to solve the problem of gravitation → equivalence principle → difficulties in the way of its extension to nonhomogeneous fields → scalar theories and associated discussions → prerequisites of the tensor-geometrical concept of gravitation → the Einstein – Grossmann theory that implements this concept, and rejection of the general-covariant equations of gravitational field → search for the non-general-covariant equations of gravitational field → establishment of complete general covariance and discovery of correct general-covariant equations of gravitational field. Boxes on the left and on the right describe the factors that influenced this process. Especially emphasized are the ideas of Ernst Mach, which were important for Einstein at all stages of formulation of GTR [32]. The factors of utmost importance were also the relativistic program, associated with STR supplemented by the equivalence principle and thus extended by Einstein, who required covariance in a sense broader than Lorentzian; the experimental aspects of the theory and the system of 'first principles', or the methodological principles of physics (principles of symmetry, conservation, causality, correspondence, observability, etc.).

The three upper central boxes represent the final stage of formation of GTR, the most dramatic in our story: rejection of the general covariance of equations of gravitation and the search for a non-general-covariant solution, and the return to the general covariance of field equations and discovery of correct general-covariant equations of gravitation in tight competition with David Hilbert in November 1915.

3. Rejection of the general covariant to equations of gravitational field, and the search for a noncovariant solution to the problem of field equations

The tensor-geometrical concept of gravitation formed the core of the relativistic theory of gravitation. One just needed to find the correct equations of the gravitational field. Theoretical invariance considerations combined with the correspondence principle led to equations of the form

$$R_{ik} = -\varkappa T_{ik} ,$$

where R_{ik} is the Ricci tensor, $\varkappa$ is the constant of gravitation, and T_{ik} is the tensor of energy-momentum of matter. As follows from our scheme, these equations are very close to the correct field equations shown in the top box; they only differ by the 'one-half' term $(1/2)g_{ik}R$, or $(1/2)g_{ik}T$. As a matter of fact, the 'short' field equations were already in Einstein's 'Zurich notebook' on pages relating to 1912 [33, 34]. In 1912 and 1913, however, Einstein (apparently together with Grossmann) decided that these general-covariant field equations in the limit of weak static fields do not reduce to the Poisson equation (that is, R_{ik} does not reduce to $\Delta\varphi$), and thus do not comply with the correspondence principle.

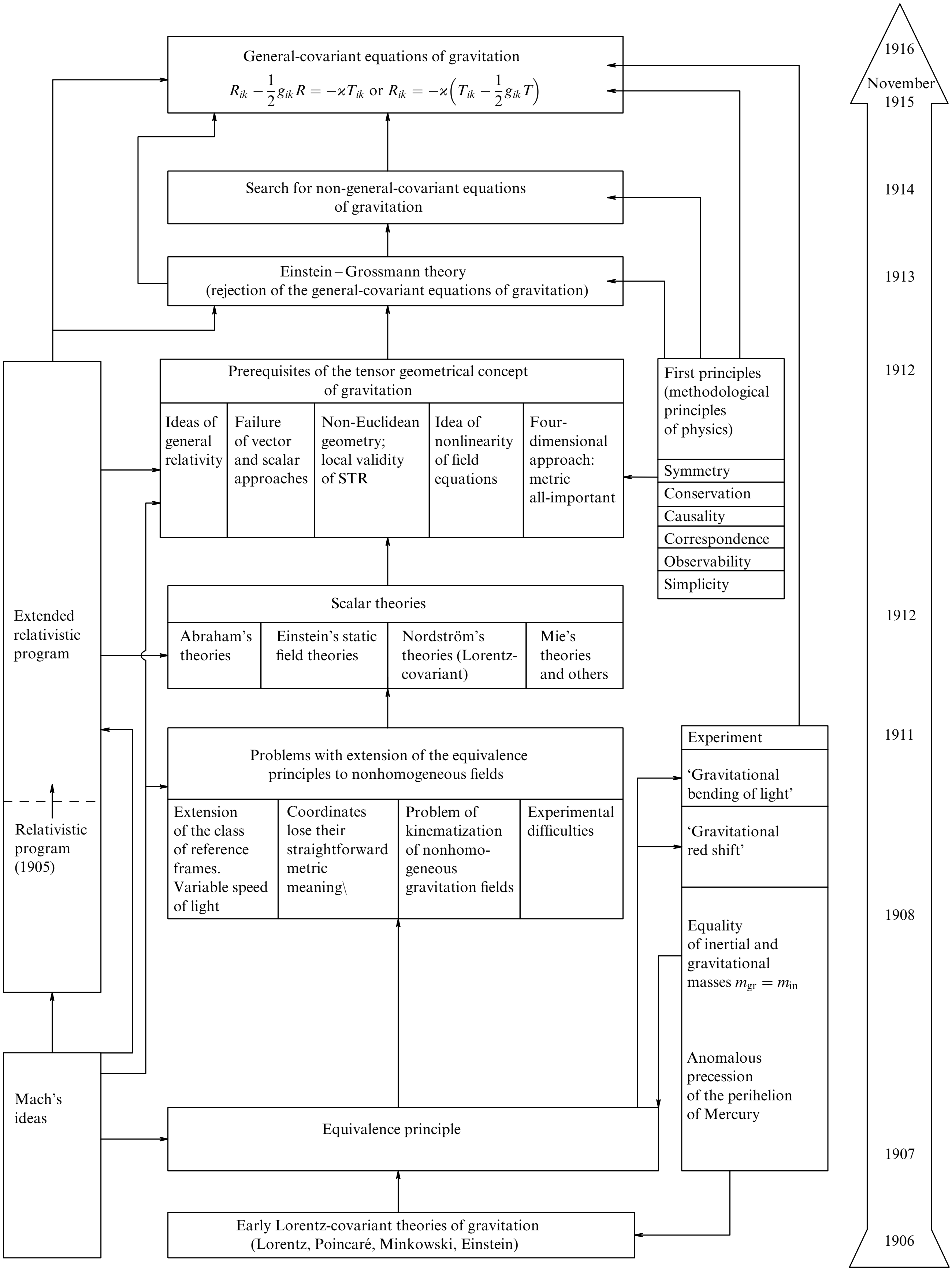
In addition, Einstein soon found additional arguments against the general-covariant field equations. One of them was related to the violation of the causality principle (a kind of 'Gedankenexperiment' known as 'the hole argument'). Another argument concerned the law of conservation of energy-momentum, which, according to Einstein, ought to be formulated in the divergent form that only took place in case of limited (to wit, linear) covariance. Einstein embarked on the non-covariant course, looking for the appropriate equations of gravitation. As a matter of fact, the dual covariance of the theory made it logically inconsistent, especially since the linear covariance of the gravitation equations suggested a violation of the initial principles of the theory: equivalence and general relativity.

In the beginning of 1914 Einstein (in a joint work with Adrian Fokker) employed the general-covariant approach for deriving the field equations in Nordström's theory, and for some time returned to the idea of "deriving the Einstein – Grossmann gravitation equations irrespective of the physical assumptions" [7, p. 250] — that is, based on the Ricci tensor. This statement is supplied with a note indicating that by that time Einstein had found a way to reconcile the general-covariant equations with the correspondence principle.

Towards the end of 1914 Einstein was looking for constraints on the class of admissible reference frames associated with the variational formulation of the field equations. He believed that the appropriate constraint (a suitable coordinate restriction 'the condition of adaptation') considerably extend the linear covariance; in particular, the class of admissible frames includes rotating frames of reference. Later this approach turned out to be a dead end. The selection of a Lagrangian quadratic in the first derivatives of g_{ik} was rather arbitrary, and the suitable transformations reduced to linear transformations [7].

Till July, or maybe even till mid-October, Einstein apparently kept to the dual covariance of the theory: in the general-covariant conception of the theory the gravitation equations only admitted limited, 'suitable' covariance.

[7] J Norton rightly noted that if it were not for Einstein's prejudice against the general covariance of equations, he could have taken the scalar curvature for the Lagrangian, and then used the known technique of elimination of the term with total divergence to express the Lagrangian of the desired quadratic form. In such a case the conditions of adaptation (suitability) agree with the general covariance, and in fact reduce to the contracted Bianchi identities [30, 33].



Genesis of the general theory of relativity

At the same time, from the remarks made by Einstein in his papers and letters of 1914 – 1915, one may conclude that two out of three arguments against the general covariance of the field equations did not seem convincing to him any longer. We have already mentioned the argument related to the Newtonian approximation. The argument based on the law of conservation of energy-momentum, judging by the note on the description of the 'hale argument' in Section 2 of the joint paper by Einstein and Grossmann published in the late 1914 (see Ref. [7, p. 284]), did not seem too important any longer, because it was not necessary to ascribe the tensor properties to the energy-momentum of gravitational field.

4. November 1915: Einstein's and Hilbert's roads to the equations of gravitation

In this section we give the chronology of the eventful November of 1915 (see below), including some prehistory and some posthistory related to December of 1915. Of course, we shall supply brief comments on this chronicle. Our last section will be devoted to the proof sheets of Hilbert's report mentioned in the introduction. Before moving on to our chronology, we would like to note that we have only made partial use of the new material relating to the previously unpublished texts and letters of Einstein, located in several recently issued volumes of his *Selected Works* [35 – 38].

Chronicle of events:
General covariant equations of gravitation

June 1913. Einstein and Grossmann "Project..." ("Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation"). Tensor-geometrical conception of gravitation. Rejection of the general-covariant equations of gravitation of the type $R_{ik} = -\varkappa T_{ik}$.

22 October 1913. Mie's letter to Hilbert (one of the first indications of Hilbert's interest in Mie's theory of matter).

19 November 1914. Einstein's "Formal foundations of the general theory of relativity" ("Die formale Grundlage der allgemeinen Relativitätstheorie") (suitable 'angepaßte' covariance of the gravitation equations).

Summer semester of 1915. Hilbert's "Lectures on the structure of matter" (without any reference to Mie's theory of matter).

29 June – 7 July 1915. Einstein's six two-hour lectures on the theory of relativity and gravitation at the Göttingen mathematical society.

7 July 1915. Einstein's letter to H Zangger: "Now everything has become clear with the theory of gravitation..."

15 July 1915. Einstein's letter to A Sommerfeld: "Hilbert has completely won me over. He is an outstanding person!".

17 July 1915. Hilbert's letter to K Schwarzschild: Einstein's lectures at Göttingen were a remarkable event.

August 1915. Einstein's letter to W de Haas: "To my great satisfaction, I was able to convince Hilbert and Klein...".

12 October 1915. Einstein's letter to H A Lorentz: "In my paper ["Formal foundations..."] I irresponsibly made the assumption of the linear covariance of the gravitational Lagrangian..." — a first indication of Einstein's return to the general covariance of field equations.

4 November 1915. Einstein's "To the general theory of relativity" (published on 11 November 1915). Return to the general covariance of field equations (restricted by the condition of unimodularity $\sqrt{-g} = \text{inv.}$): $R_{ik} = -\varkappa T_{ik}$ (where R_{ik} is 'one-half of Ricci's tensor' G_{ik}).

7 November 1915. Einstein sends Hilbert the proof sheet of his report of 4 November 1915 with a letter "...I am sending to you the galley-proof of a paper of mine in which I have modified my gravitation equations because about 4 weeks ago I understood the illusions of my former argumentation. Sommerfeld wrote to me that you also found a hair in my soup, which has made it completely unacceptable for you".

8 – 9 November 1915. Hilbert's letter to Einstein (not found).

11 November 1915. Einstein's "To the general theory of relativity" "Addendum" (published on 18 November 1915). General-covariant field equations: $G_{ik} = -\varkappa T_{ik}$ with the condition $T = 0$. Hypothesis of the electromagnetic nature of matter and fundamental role of gravitation in the structure of matter.

12 November 1915. Einstein's letter to Hilbert: "If my current modification is justified, then gravitation must play a fundamental role in the structure of matter".

13 November 1915. Hilbert's letter to Einstein, in which he briefly describes the essentials of his unified field theory and invites him to his presentation at Göttingen, which was first scheduled to take place on 16 November 1915, but then postponed till the 20th. Among other things, Hilbert wrote: "... I consider my theory to be mathematically perfect, although my mathematical constructions do not seem quite transparent, and, strictly speaking, do not comply with the axiomatic method... The equations of electrodynamics based on one common theorem are a mathematical implication of the gravitation equations. In this way, gravitation and electromagnetism are no longer entirely different species. The concept of energy... forms the basis for the subsequent construction... The four missing 'space – time equations' follow from this concept on the basis of a very simple axiom...".

15 November 1915. Einstein's letter to Hilbert, in which he apologizes for not being able to hear his report at Göttingen on account of feeling tired and not well. In particular, he writes: "Your study interests me very much, the more so since I was close to insanity trying to bridge the gap between gravitation and electromagnetism... What you have written to me instills great hopes... To gratify my impatience, please send me if possible the printer's proof of your paper".

18 November 1915. Einstein's "Explanation of motion of the perihelion of Mercury in the general theory of relativity (published 25 November 1915). Equation $G_{ik} = 0$ or $R_{ik} = 0$ (under the condition $\sqrt{-g} = 1$) is used for calculating the gravitational bending of light and explaining the anomalous precession of the perihelion of Mercury.

18 November 1915. Einstein's letter to Hilbert, which confirms the receipt of proof sheets of Hilbert's report postponed till the 20th of November. Einstein, in particular, writes, "The set of field equations proposed by you, as far as I can judge, coincides with the set that I found in the past weeks and reported at the Academy... The difficulty was not in finding the general-covariant equations for $g^{\mu\nu}$; this could be easily done with Riemann's tensor. It was very difficult to understand that these equations are a simple and natural extension of Newton's law... I was only able to do that in the past weeks... Today I submitted to the Academy my paper in which, without any additional assumptions, I use the general theory of relativity for calculating the precession of the perihelion of Mercury discovered by Leverrier".

19 November 1915. Hilbert's letter to Einstein, in which he writes: "...My heartiest congratulations on solving the problem of Mercury's motion. If I could count as fast as you, the electron ought to capitulate in front of my equations, and the hydrogen atom would apologize for failing to radiate...''.

20 November 1915. Hilbert's ''Foundations of physics''. Unified theory of gravitational and electromagnetic fields. Field equations are expressed from the general-covariant variational principle: $\delta \int (K+L)\sqrt{g}\, d\omega = 0$, where K is the scalar curvature, and L is the Lagrangian of Mie's nonlinear electrodynamics. Equations of electrodynamics based on the analogue of Nöther's second theorem (Hilbert's 'Theorem 1') may be regarded as an implication of the equations of gravitation. General covariance of the theory is achieved. Equations of gravitation in the form $K_{\mu\nu} - (1/2) g_{\mu\nu} K = -\varkappa T_{\mu\nu}$, where $K_{\mu\nu}$ is Ricci's tensor, and $T_{\mu\nu}$ is the energy-momentum tensor of Mie's theory.

20 November 1915. (Version of events revised after the discovery of proof sheets of Hilbert's report.) The proof sheet is dated 6 December 1915 and is considerably at variance with the published text (the text was published on 31 March 1916). The general covariance of the theory is not achieved. Axiom III, not present in the publication, delimits the class of admissible reference frames on the basis of conservation of energy-momentum, and reconciles the field equations with the causality principle. The general-covariant variational principle with scalar curvature is used. However, the equations of gravitation are written in the 'undecoded' form

$$\left[\sqrt{g}\,K\right]_{\mu\nu} + \frac{\partial \sqrt{g}\,L}{\partial g^{\mu\nu}} = 0\,,$$

where $[\ ]_{\mu\nu}$ is the variational derivative.

25 November 1915. Einstein's ''Equations of gravitational field'' (published 2 December 1915). ''...One can do without the assumption of the tensor of the energy of matter, if it is introduced into the field equations in a somewhat different way... Then the equations for vacuum, used for explaining the motion of the perihelion of Mercury, remain the same''.

The general-covariant equations of gravitation have the form

$$G_{im} = -\varkappa\left(T_{im} - \frac{1}{2}\,g_{im} T\right).$$

''...Our added term (that is, $(1/2) g_{im} T$ — **V V**) results in the tensors of energy of gravitational field and matter entering equation (9) (that is, the conservation law of energy-momentum — **V V**) in the same way...''.

26 November 1915. Einstein's letter to H Zangger, in which he writes of Hilbert's desire to 'nostrify' GTR: ''...A theory of unrivaled beauty. However, only one colleague really understands it, and he is cleverly trying to 'nostrify' it (as M Abraham put it)''.

28 November 1915. Einstein's letter to Sommerfeld, which gives the reasons for the return to general covariance and informs of the expression of the correct equations of gravitation: ''...During the past month I lived through one of the most exciting and stressful periods in my life, but at the same time the most fruitful''. The letter quotes the general-covariant field equations with the 'one-half' term.

6 December 1915. This is the date on the proof sheet of Hilbert's report, delivered on 20 November 1915. The text in the proof is considerably different from the published version of the report (31 March 1916), which until now has been assumed to be the same as reported on November 20, 1915.

9 December 1915. Einstein's letter to Sommerfeld contains one of the first express references to Hilbert's theory: ''As far as I know Hilbert's theory, it employs an approach to the electrodynamic phenomena that is closely related to Mie's theory. Such a specialized approach cannot be justified from the standpoint of the GTR''.

20 December 1915. Einstein's letter to Hilbert (conciliatory!): ''We had a certain misunderstanding (eine gewisse Verstimmung), the cause of which I do not want to analyze. I fought with my vexation, and with complete success. I think of you again with peaceful friendliness (ungetrübter Freundlichkeit), and ask you to think of me in the same way. Indeed, it is a shame when two real guys (zwei wirkliche Kerle), who somehow escaped from this miserable world, are not pleasant to each other''.

This chronicle of events related to the final stage of construction of GTR — namely, with the expression of the correct general-covariant equations of gravitation — is based on numerous publications of the last two decades [7, 12, 13, 15–38]. Quotations from papers and letters are taken mainly from the Princeton edition [35–37] or from Einstein's *Collected Works* in Russian [39]. Some letters of Einstein and Hilbert are quoted from Refs [12, 13, 21, 22]. We also used the publications of Einstein's correspondence with Sommerfeld and Besso [40, 41]. Excerpts from the correspondence between Einstein and Hilbert are given mainly in my translation into Russian. Hilbert's ''Foundations of physics (first note)'' were published in Russian three times [42–44], the last edition came out in 1998 (my comments on this edition, referring to the discovery of the proof sheet of Hilbert's report, can be found in Ref. [18]). The event of 20 November 1915, Hilbert's ''Foundations of physics'' is entered into this chronicle twice: how it was viewed before the discovery of the proof sheet, and how it looks now, after the discovery. The reader who is familiar with this problem from the literature [7, 12, 13, 25, 26, 29, 30, 33], or at least from the well-known book of A Pais [13], will appreciate the importance of this archival find. Now we shall turn to a more detailed comparison of the proof sheet and the published version of Hilbert's report, largely relying on the published works [15–24].

5. Hilbert's ''Foundations of physics (first note)'': the background

We have already twice (in the introduction and in the chronicle) followed the works [15, 19–24] to describe the main results of the comparison of the proof sheet of Hilbert's report, which is supposed to reflect closely the text pronounced by Hilbert on 20 November 1915, and the version that appeared in the *Göttingen Nachrichten* in March 1916 (but also dated 20 November 1915). Naturally, we emphasized the differences concerned with the general covariance and the equations of gravitational field. One might surmise, however, that Hilbert did not view these differences as important. He certainly believed that all the essential aspects of his unified theory remained the same. This is probably why he kept the date of 20 November, and did not mention that the original text of the report had been updated.

The success with the axiomatic construction of Euclidean geometry, achieved by Hilbert in 1899 (the year of publication of his *Foundations of Geometry*) had made him an ardent advocate of the axiomatic method not only in mathematics, but in science in general. ''Hilbert is the champion of axiomatics, — wrote H Weyl. — The axiomatic attitude seemed to him one of universal significance, not only for mathematics, but for all sciences. His investigations in the field of physics are conceived in the axiomatic spirit. In his lectures he liked to illustrate the method by examples taken from biology, economics, and so on'' [45, p. 274].

Hilbert saw that theoretical physics in the works of Maxwell, Boltzmann, Hertz, Lorentz, Planck and other scientists achieves such level of mathematical perfection that it is quite ripe for axiomatic representation. Among the 23 most important mathematical problems selected by the Göttingen scientist in his famous lecture entitled ''Mathematical problems'' and delivered on 8 August 1900 at a meeting of the 2nd International mathematical congress in Paris, number 6 was the problem of ''Mathematical representation of axioms of physics'' [46b, pp. 415 – 416]. Hilbert's words, especially in the light of the subsequent development of relativistic theories, sounded quite prophetic about those mathematical structures that would come forward in the solution of this problem. ''In order to construct physical systems on the model of the axioms of geometry, one must first try to use a small number of axioms for covering a class of physical phenomena as broad as possible, and then add new axioms to include the more specialized theories, and then the classification principle may possibly arise that would be capable of using the comprehensive theory of infinite groups of Lie transformations'' [46b, p. 416].

It is one thing, however, to axiomatize one physical theory — for example, mechanics or thermodynamics — but axiomatizing all of physics is a totally different business. This would involve not only the axiomatic construction itself, but also the development of some kind of unified physical theory. This could be done on the model of the electromagnetic field theory, which unified the theories of light, electricity and magnetism on the basis of Maxwell equations, and laid the foundation for the electromagnetic field program, or the electromagnetic view of the world. Hilbert saw the real possibility for the construction of such theory in 1913, when Mie's electromagnetic theory of matter (nonlinear electrodynamics) and Einstein – Grossmann's tensor-geometrical theory of gravitation appeared almost simultaneously. Both were field theories, and were based on the theory of relativity. This paved the way for the realization of the 'field-theoretical ideal of unity of physics', and at the same time the axiomatic construction of physics[8].

Without going into the details of Mie's theory, we shall describe it in the same way as it was perceived by Hilbert, noting that his understanding was based on the reformulation of the theory by Max Born [47]. The most important thing was that the theory reduced to finding the Lorentz-covariant electromagnetic-field Lagrangian, which would be a generalization of the Maxwell expression that 'gives the electron' — that is, allows the calculation of the mass of the electron and the expression of its law of motion. In accordance with terminology used by Mie, Hilbert referred to this Lagrangian as the 'world function' and considered it in the form

$$L = \alpha Q + f(q)\,,$$

or even in a more specialized form also found in Mie's paper:

$$L = \alpha Q + \beta q^3\,,$$

where

$$Q = \sum_{k,l,m,n} M_{mn} M_{lk} g^{mk} g^{nl}$$

is the general-covariant generalization of Maxwell's Lagrangian; $M_{ks} = q_{sk} - q_{ks}$ is the tensor of the electromagnetic field; $q_{ks} = \partial q_k / \partial w_s$, q_k is the electromagnetic potential; g^{mk} is the metric tensor (or the gravitational potential); $q = \sum_{k,l} q_k q_l g^{kl}$ is a scalar quadratic with respect to potentials; and α and β are constants.

It was the nonlinear added terms $f(q)$ or βq^3, exhibiting an explicit dependence on the electromagnetic potentials, that gave the possibility of reducing the elementary charged particles (and hence the matter as such) to the electromagnetic field. Hilbert even hoped that, with the appropriate selection of the nonlinear term, the generalized electrodynamics would also describe the quantum features in the behavior of particles and radiation[9]. Substantial criticism of Mie's project was given a few years later, primarily in the famous monographs of Weyl [49] and Pauli [6][10].

The tensor-geometrical theory of Einstein and Grossmann introduced the idea of general covariance and the tensor potentials of the gravitation field, and the concept of space – time as the four-dimensional Riemann manifold. However, there were no equations of gravitational field that would be adequate to this approach; moreover, it seemed to Einstein that there were fundamental arguments against the general covariance of the equations of gravitational field, related to the principle of causality and the conservation of energy-momentum. Therefore, in 1913 – 1914 (and in the first half of 1915), the equations for the potentials g_{ik} and their first and second derivatives, which possessed a limited covariance, were used for the field equations. Later it was found, however, that this covariance was restricted to linear transformations, and therefore in principle could not include the transitions between accelerated frames, and therefore poorly agreed (or did not agree at all) with the equivalence principle.

[8] Hilbert's earlier attempts at axiomatization of physics on the basis of mechanics and the special theory of relativity were fragmentary. They were not based on the 'field-theoretical ideal of unity of physics', and did not touch upon the theory of gravitation at all. See Refs [16, 20].

[9] Heuristically this could stem from the fact that Planck's constant h and the constant e^2/c, which both have the dimension of action, are close in the order of magnitude. In 1909 Einstein wrote that ''from this relation it follows that the modification of the theory that gives the elementary quantum e will also include the quantum structure of radiation'' [48, p. 178].

[10] The main drawback of Mie's conception, which was seen by Mie himself and was emphasized especially clearly by Pauli, was its gauge (or gradient) noninvariance. For both the Lagrangian of the theory and the field equations contained the absolute values of the potentials. Therefore, if φ were a solution of the basic equations, the potential $(\varphi + \mathrm{const})$ could not be a solution. This could be interpreted as the impossibility of a material particle being able ''to exist in a constant external potential field'' [6b, p. 278; 6c, p. 102]. At the same time Pauli highly appreciated Mie's conception and believed that, notwithstanding these shortcomings, one ''cannot reject Mie's electrodynamics'' (ibid.). Mie himself quite prophetically noted (especially in the light of the Aaronov – Bohm experiment) that ''the most immediate problem to which we are directed by the theory is the investigation of whether it is possible to find, in very strong electric or magnetic fields, or even in regions in which the field strengths are zero but have very large values of potentials, derivations from Maxwell's laws that hold in the ideal vacuum'' [26, p. 35].

Such was the situation with the theory of gravitation when Hilbert, who was already enthralled by physics and apparently had some ideas towards the realization of the 'field-theoretical ideal of unity of physics', invited Einstein in the summer of 1915 to Göttingen so as to get a firsthand knowledge of the relativistic tensor-geometrical theory of gravitational field.

In the period from 28 June to 5 July Einstein gave six lectures. There are notes of some of these lectures taken by an unknown listener. The 11-page document is entitled "Einstein. 28/6 – 5/7, Göttingen". It was found in Hilbert's archive in Göttingen by L Corry; in 1996 it was published in the 6th volume of the Princeton edition of Einstein's Collected Papers [37, pp. 587 – 590]. Judging by these notes, Einstein was speaking of the equivalence principle and the tensor-geometrical concept of gravitation. Unfortunately, the notes do not cover any material related to the field equations. Several of Einstein's and one of Hilbert's letters of July and August of 1915 are known, in which they enthusiastically (if not excitedly) spoke of the Göttingen meeting and of each other (see chronicle).

On the way to the momentous and dramatic events of November 1915 they both visited the island of Rügen in the Baltic Sea. It is possible, although not too likely, that they visited the resort at the same time (see, for example, Ref. [21, p. 540]. Most probably, however, Einstein was there four or six weeks before Hilbert. There are grounds to believe that Einstein started the fateful November drive in the early days of October, and Hilbert in mid-October (see chronicle and Ref. [21, pp. 540 – 541]).

The chronicle gives a clear idea of the first November results of Einstein, which he reported to the Prussian academy in Berlin on 4, 11 and 18 of November, and of the correspondence between Einstein and Hilbert. In fact, Einstein returns to the idea of the general-covariant equations of gravitational field based on the Ricci tensor, and seeks the equations in the form

$$G_{ik} = -\varkappa T_{ik}\,.$$

Obviously, by this time he understood that the arguments against the general-covariant field equations, related to the violation of the causality principle and the conservation of energy-momentum, actually miss the target. Now he was willing to achieve at all costs the general covariance of the entire theory, including the field equations. He was even prepared (in the paper dated 11 November) to assume the electromagnetic nature of matter (setting the spur of tensor $T_{\mu\nu}$ equal to zero: $T = 0$). A convincing proof of validity of the general-covariant approach to the field equations was the result of a particular case (for empty space, $T_{\mu\nu} = 0$)

$$G_{\mu\nu} = 0\,,$$

which lead to the correct calculation of the anomalous precession of the perihelion of Mercury reported on 18 November.

6. "Foundations of physics (first note)": Hilbert's way of bringing together gravitation, electromagnetism and matter

At this very time (on the 20th of November) Hilbert made a report at Göttingen ambitiously titled "Foundations of physics". Exactly one week before in his letter to Einstein of 13 November he very briefly describes the essence of his synthesis, referring to 'one general theorem' (see chronicle). It seems that he saw this as his main achievement, together with the fact that this theorem (and the entire construction) was based on two axioms. This part of his work is present both in the proof sheet and in the published version of the report.

In the spirit of Minkowski he refers to the space-time coordinates of the most general form w_s ($s = 1, 2, 3, 4$) as the 'world parameters', and postulates that any physical event is determined by 14 variables: 10 gravitation potentials $g_{\mu\nu}$ and 4 electrodynamic potentials q_s, where μ, $\nu = 1, 2, 3, 4$ and $s = 1, 2, 3, 4$, and by two axioms. The first axiom that Hilbert calls 'Mie's axiom of the world function' is nothing other than the Hamilton principle for the electromagnetic-gravitation system in question:

$$\delta \int H\sqrt{g}\,\mathrm{d}\omega = 0\,.$$

Here $g = |g_{\mu\nu}|$, $\mathrm{d}\omega = \mathrm{d}w_1\,\mathrm{d}w_2\,\mathrm{d}w_3\,\mathrm{d}w_4$, and the Lagrangian H is a function of the potentials $g_{\mu\nu}$ and q_s, of the first and second derivatives of $g_{\mu\nu}$ with respect to w_s, and of the first derivatives of q_s with respect to w_s.

The second axiom is the 'axiom of general invariance', which states that the world function H must be 'invariant with respect to any transformation of the world parameters w_s' [44, p. 368; 42a, p. 396]. This axiom ought to be credited to Einstein, who came to the idea of general covariance towards the end of 1912, and jointly with Grossmann laid the foundation of the tensor-geometrical concept of gravitation. Hilbert's interpretation of this idea is very similar to Einstein's. Hilbert remarks: "Axiom II is the simplest mathematical expression of the fact that the linkage between the potentials itself is completely independent of the way used to establish correspondence between world points and world coordinates" [44, p. 368; 42a, p. 396]. A similar formulation of Einstein's of early 1916 runs as follows: "Since all our physical experimental findings can eventually be reduced to such coincidences (that is, the 'space-time coincidence of two pointlike events' — **V V**), there is no reason to give an *a priori* advantage to some frame over the other frames — that is, we come to the requirement of general covariance" [50, p. 460]. At the same time, before 1916 Einstein had never considered the variational proof of gravitation equation with the general-covariant Lagrangian, and this was the reason why Hilbert did not associate this axiom with the name of Einstein (see Hilbert's Note 4 [44, p. 368; 42a, p. 396]).

Then follows 'Theorem I', which is a particular case of Emmie Nöther's second theorem of the invariant variation problems (Nöther's 2nd theorem) proved some two and a half years later [51]: "If expression J is invariant with respect to arbitrary transformations of the four world parameters and contains n variables and their derivatives, and if from the condition

$$\delta \int J\sqrt{g}\,\mathrm{d}\omega = 0$$

one constructs n Lagrangian variational equations in these n variables, then in this invariant system of n differential equations four equations always follow from the remaining $n - 4$ in the sense that four mutually independent linear combinations of these n differential equations and their total derivatives are satisfied identically" [44, p. 368; 42a, p. 397].

For comparison, let us give the formulation of Nöther's 2nd theorem: "If integral J is invariant with respect to group $G_{\infty\rho}$ (that is, a continuous group that depends on ρ arbitrary functions — **V V**), which contains derivatives up to the σth order, then there exist ρ identity relations between the Lagrangian expressions and their derivatives up to the σth order" [51a, p. 239; 51b, p. 613]. 'Lagrangian expressions' mean the left-hand sides of the Lagrange – Euler equations. In the special case of Hilbert's 'Theorem I', these are the 10 equations of the gravitational field:

$$\frac{\partial\sqrt{g}H}{\partial g^{\mu\nu}} - \sum \frac{\partial}{\partial w_k}\,\frac{\partial\sqrt{g}H}{\partial g_k^{\mu\nu}} + \sum \frac{\partial^2}{\partial w_k \partial w_l}\,\frac{\partial\sqrt{g}H}{\partial g_{kl}^{\mu\nu}} = 0\,,$$

that derive from the variation of the 10 gravitational potentials $g_{\mu\nu}$, and the four equations of thermodynamics:

$$\frac{\partial\sqrt{g}H}{\partial q_h} - \sum \frac{\partial}{\partial w_k}\,\frac{\partial\sqrt{g}H}{\partial q_{hk}} = 0\,,$$

that derive from the variation of the 4 electromagnetic potentials q_s.

Denoting the former by $[\sqrt{g}H]_{\mu\nu} = 0$, and the latter by $[\sqrt{g}H]_h = 0$, on the strength of 'Theorem I' Hilbert concludes that the latter four equations $[\sqrt{g}H]_h = 0$, which are the extended Maxwell equations, are the implication of the former ten equations $[\sqrt{g}H]_{\mu\nu} = 0$, which comprise the equation of gravitation.

This is the essence of Hilbert's unified field theory. It is in this sense that 'the electrodynamic phenomena are a consequence of gravitation' [44, p. 369; 42a, p. 397]. This is why he calls his 'Theorem I' the 'leitmotiv' of the entire construction. In this part the proof sheet and the published report coincide. The difference is small: just two or three places in which the published text emphasizes Einstein's priority in certain points, or somewhat downplays the novelty and importance of Hilbert's contribution. Very informative in this regard are additions and corrections made in Hilbert's own hand on the first two pages of printer's proof[11].

Then the published version of the report carries a rather extensive section full of mathematics, which is devoted to two theorems (Theorems II and III), which, as it becomes clear only at the end of the section, are only necessary for the correct introduction of the concept of energy and energy conservation. The logic of Hilbert's presentation is not at once obvious. The main equations of the theory are not yet written out, and the form of the world function is not yet determined, while the author engages in very formal and sophisticated mathematics in order to construct a rather obscure expression for the energy of the 'energy vector' e^l, which strangely depends 'linearly on the arbitrary vector p^s'.

In this part the published report and the proof sheet are very different. This somewhat mysterious circumstance is clarified, however, when we look at the proof sheet. We shall discuss this shortly, but now let us turn to the last part of the report, which moved from printer's proof to the published report almost without change. This 'almost', however, is very important, and concerns the equations of gravitational field.

Here at last Hilbert specifies the form of the world function of his theory (in fact, this is another axiom):

$$H = K + L\,,$$

where $K = \sum_{\mu,\nu} g^{\mu\nu} K_{\mu\nu}$ is the 'invariant of Riemann's tensor (curvature of the four-dimensional manifold)', and L is the electromagnetic part of the world function that remains rather uncertain (the only assumption is that it is a function of q_s, q_{sk}, $g^{\mu\nu}$).

From considerations of general covariance it appears that the derivatives of the potentials only enter L in the form of the 'skew-symmetrical invariant tensor — that is, the so-called electromagnetic six-vector':

$$M_{ks} = q_{sk} - q_{ks}\,.$$

Incidentally, "this result, which only now defines the nature of Maxwell's equations, – emphasizes Hilbert, — is obtained here as the result of general covariance — that is, on the basis of Axiom II" [44, p. 374; 42a, p. 403]. Another important conclusion made by the author in this part is the proof that the tensor of energy-momentum of matter (in this case, in the framework of Mie's generalized theory) is obtained through 'differentiation of invariant L with respect to gravitation potentials' (or, as Pauli put it later, 'by variation of the G-field in the integral of action' [6b, p. 231; 6c, p. 158]). The fact that upon transition to the limit $g_{\mu\nu} = 0$ ($\mu \neq \nu$) and $g_{\mu\mu} = 1$ the expression $(\partial\sqrt{g}L)/(\partial g^{\mu\nu})$ coincides with the tensor of energy-momentum of Mie's Lorentz-covariant theory, as noted by Hilbert, "was the first indication of the close relationship between Einstein's general theory of relativity and Mie's electrodynamics, and gave me the proof of validity of the theory under development" [44, p. 375; 42a, p. 404].

As a matter of fact, however, this result has a universal value in the general theory of relativity, and is not associated with any specific form of the Lagrangian, and, in particular, with Mie's theory.

Next in both versions of Hilbert's report goes the substantiation of the conclusion that, in accordance with 'Theorem I', the equations of electrodynamics follow from the equations of gravitation. Namely, Hilbert obtains "four mutually independent linear combinations... of the main electrodynamic equations and their first derivatives":

$$\sum_m \left(M_{m\nu}[\sqrt{g}L]_m + q_\nu \frac{\partial}{\partial w_m}[\sqrt{g}L]_m \right) = 0 \quad (\nu = 1, 2, 3, 4)\,.$$

The proof of these identities is followed by Hilbert's conclusion: "This is the exact mathematical expression of the above statement to the effect that electrodynamics is a consequence of gravitation" [44, p. 377; 42a, p. 406]. If the potentials explicitly enter the Lagrangian and the equations of electrodynamics — that is, when the theory is not gauge-invariant (as in the general case of Mie's theory), then, as demonstrated by L Renn and J Stachel, the mere fact that the field equations hold on the initial hypersurface does not warrant the general validity of the equations [22, p. 40]. When the theory is gauge-invariant and we are dealing with

[11] Hilbert apparently learned about Einstein's grievance (the chronicle see Einstein's letter to Zangger of 26 November 1915 and the subsequent conciliatory letter from Einstein to Hilbert in December), and wanted to emphasize his contribution to the development of the unified theory. For example, speaking of the gravitational potentials $g_{\mu\nu}$ he adds 'first introduced by Einstein'. Similar small corrections in Hilbert's hand in two other places were omitted in the published version.

the conventional Maxwell equations, then the latter do follow from the above relation [12].

Thus, in the case of the non-gauge-invariant Mie's theory with a Lagrangian of the type

$$L = \alpha Q + \phi(q)$$

one must generally use not the generalized Maxwell equations

$$[\sqrt{g}L]_m = 0\,,$$

but rather the equations

$$\sum_m \left(M_{mv}[\sqrt{g}L]_m + q_v \frac{\partial}{\partial w_m}[\sqrt{g}L]_m \right) = 0\,.$$

It is this method of reducing the equations of electrodynamics to equations of gravitation on the strength of 'Theorem I' (that is, a special case of Nöther's second theorem proved later) that Hilbert regarded as his main achievement. Also important was the fact that the entire construction relied on two simple axioms regarding the common variational structure of the field equations, and their universal general covariance. The entire scheme of construction and the results described above proceeded almost entirely from the proof sheet into the version published in March 1916. This is apparently the reason why Hilbert kept the date of his report (20 November 1915), without caring to mark the changes made after 6 December 1915.

7. Hilbert's "Foundations of physics (first note)": differences between the proof and the published version

Going to analyze these differences, we must emphasize that they concern that very part which relates to the general-

[12] In this connection it would be worthwhile to quote a passage from the three-volume treatise on gravitation by C Misner, K Thorne, and J Wheeler [52, § 20.6]. The authors point to the fact that, from today's standpoint, the implication of Nöther's second theorem, or the contracted Bianchi identities, is the vanishing of the covariant derivative of the tensor of energy-momentum of matter T_{ik}, interpreted as the conservation of energy-momentum of matter (in the presence of gravitation):

$$\nabla T = 0\,, \quad \text{or} \quad T^{ik}_{;k} = 0\,.$$

Substituting here the expression for T_{ik} of the electromagnetic field

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\alpha} g_{\alpha\beta} F^{\nu\beta} - \frac{1}{4} g^{\mu\nu} F_{\sigma\tau} F^{\sigma\tau} \right)\,, \qquad (*)$$

we get the four equalities

$$F^{\mu}_{\beta} F^{\beta\nu}_{;\nu} = 0\,. \qquad (**)$$

In a sufficiently general situation, when the determinant composed of the components F^{μ}_{β} and equal to $-(\bar{E},\bar{B})^2$ is nonzero, hence follow the relations

$$F^{\beta\nu}_{;\nu} = 0\,, \qquad (***)$$

which are the Maxwell equations. The authors of the cited treatise gives the following comments on this: " At all normal points (of space-time — **V V**) the solution of the four linear equations (20.38) [that is our equation $(**)$ — **V V**) with their nonvanishing determinant gives identically zero for the four unknowns (20.39) (that is, $F^{\mu\nu}_{;\nu}$ — **V V**): that is to say, Maxwell "equations of motions"

$$F^{\beta\nu}_{;\nu} = 0\,,$$

are fulfilled and must be fulfilled as a straight consequence of Einstein's field equations (20.32)(that is, equations $R_{\mu\nu} - 1/2 g_{\mu\nu} R = -\varkappa T_{ik}$ — **V V**) — plus expression 20.32 [that is, our expression $(*)$ — **V V**] for the stress-energy tensor. Special cases admit counterexamples (see exercise 20.8), but in the generic case one need not invoke Maxwell's equations of motion; one can deduce them from Einstein's field equation." [52a p. 473; 52b, p. 111].

covariant equations of the gravitational field. Observe that in the published version of his report Hilbert selects for the gravitational part of the Lagrangian the scalar curvature

$$K = \sum_{\mu,\nu} g^{\mu\nu} K_{\mu\nu}\,,$$

where $K_{\mu\nu}$ is the Ricci tensor, and varies the gravitational potential first to obtain the equations of gravitational field in the form

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial \sqrt{g}L}{\partial g^{\mu\nu}} = 0\,.$$

Then he goes on to say: "The first term on the left-hand side is transformed in the following way:

$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}\left(K_{\mu\nu} - \frac{1}{2} K g_{\mu\nu} \right)\,,$$

as is easily found from the fact that $K_{\mu\nu}$, apart from $g_{\mu\nu}$, is the only tensor of the second rank, and K is the only invariant that can be expressed exclusively from the first and second derivatives $g^{\mu\nu}_k$, $g^{\mu\nu}_{kl}$.

The differential equations obtained in this way seem to me to accord with the grandiose general theory of relativity proposed by Einstein in his recent works (at this point Hilbert makes reference to all November papers of Einstein, including the last one received on 25 November and published on 2 December 1915 — **V V**)" [44, p. 375; 42a, p. 404].

At the corresponding place of the proof sheet we only find the equations of gravitation in the form with the variation derivative

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial \sqrt{g}L}{\partial g^{\mu\nu}} = 0\,,$$

while the equations of gravitation with the variational derivative $[\sqrt{g}K]_{\mu\nu}$ converted into the famous form of $(K_{\mu\nu} - 1/2 g_{\mu\nu} K)$ are not found in the proof.

In addition, in the proof we find another clue that is not present in the published version but has left some trace there. In the final version of Hilbert's report the condition of general covariance (Axiom II) is unconditional; it is not restricted in any way also with respect to the equations of gravitational field. The form of the general-covariant equations of gravitation with the 'one-half' term is final, and is not subject to any specialization.

At the same time, in the proof, after the general conclusion based on Hilbert's first theorem to the effect that "electrodynamic phenomena are a consequence of gravitation" we see a very important passage. It states that the general-covariant field equations actually lead to a fundamental ambiguity.

Hilbert argues that this ambiguity can be removed if the general covariance is restricted by a certain physical condition, the most natural choice being the conservation of energy-momentum. Let us quote this passage in full: "As follows from our mathematical theorem (that is, Hilbert's Theorem I — **V V**), the above-formulated Axioms I and II for 14 potentials can be used for finding only ten essentially independent equations; on the other hand, keeping the condition of general invariance, it is not at all possible to obtain more than ten essentially independent equations for the 14 potentials $g^{\mu\nu}$, q_s. Therefore, if we wish to preserve the deterministic character of the main equations of physics in accordance with Cauchy's theory of differential equations, we must supplement equations (4) and (5) (that is, the main field equations — **V V**) by four additional noninvariant equations. In order to find these equations I first of all give some definition of the concept of energy'' [14, p. 3–4][13].

Then goes a rather long passage devoted to formulation of the concept of energy, and the corresponding conservation law. Referring the reader for the details to papers of J Renn and J Stachel, as well as T Sauer [21, 22], here we shall just cite the final result, formulated by Hilbert in his Axiom III ('axiom of space and time'):

''Axiom III (axiom of space and time). *Space-time coordinates are such preferential* (*besondere*) *world parameters for which the energy law* (15) *holds*'' [14, p. 7]. Equation (15) is the divergence relation

$$\sum_l \frac{\partial e_s^l}{\partial w_l} = 0\,,$$

where

$$e_s^l = \sum \left(\frac{\partial \sqrt{g} H}{\partial g_k^{\mu\nu}}\, g_s^{\mu\nu} + \frac{\sqrt{g} H}{\partial g_{kl}^{\mu\nu}}\, g_{sl}^{\mu\nu} + g_s^{\mu\nu} \frac{\partial}{\partial w_l} \frac{\partial \sqrt{g} H}{\partial g_{kl}^{\mu\nu}} \right).$$

Then Hilbert concludes that the divergence relation of Axiom III is satisfied when and only when a certain expression e_s goes to zero. This gives rise to the desired noncovariant conditions

$$e_s = \frac{\mathrm{d}^{(g)} \sqrt{g} H}{\mathrm{d} w_s} = 0\,;$$

here the total derivative with respect to world parameters is implied, and the superscript (g) indicates that only the gravitational potentials and their derivatives are taken into account, whereas the electromagnetic potentials are disregarded.

In this way – Hilbert deduces — ''...these four differential equations (16) supplement the gravitation equations (4) to make up the set of 14 equations for the 14 potentials $g^{\mu\nu}$, q_s — that is, the set of the *main equations of physics*. Since the number of equations equals the sought number potentials, all physical phenomena also satisfy the causality principle (das Kausalitätsprinzip); this demonstrates the close relationship between the energy law and the principle of causality, which accommodate each other'' [14, p. 7].

As a result we come to a somewhat paradoxical situation that resembles the dual covariance of the Einstein–Grossmann theory. The entire construction is based on the general covariance, which is then restricted in order to reconcile the theory with the causality principle. However, by contrast with Einstein's attempts (in part jointly with Grossmann) at constructing the non-general-covariant field equations, Hilbert destroys the general covariance of the field equations by supplementing them with four noncovariant equations associated with the energy conservation law. It is interesting that while Einstein in November 1915 steadily moves towards complete general covariance of the theory, Hilbert, on the contrary, starts with general covariance as the foundation of his unified theory and the basis for constructing the field equations, but later essentially destroys the covariance by supplementing the main field equations with noncovariant conditions.

In the published version of the report all arguments related to the restriction of general covariance because of the causality principle are absent, and Axiom III is absent as well. It would be natural to surmise that having read Einstein's last paper of November, which proclaimed the complete general covariance of the theory, Hilbert saw the fallacy of his noncovariant restrictions and deleted the corresponding passages from the published version of his report. It is also quite possible that Einstein's equations of gravitation with the 'one-half' term $1/2 g_{ik} T$, which appeared in Einstein's last paper of November, prompted to Hilbert the equivalent form of these equations with a similar 'one-half' term $1/2 g_{\mu\nu} R$, which could be derived in different ways — in particular, by procedure of varying of action using scalar curvature for the Lagrangian. However, this proof is not found in the published version — instead, we see there certain not-too-correct invariance theorizations (see previous page).

Thus, Hilbert removed from his report all material related to the restrictions of general covariance, the principle of causality, and Axiom III. However, a somewhat modified part dealing with the formulation of the concept of energy remained in the final version. Without the proof sheet it was not at all clear what relation to Hilbert's axiomatic project of unified theory this rather lengthy part that takes up almost one half of the entire paper bears. The proof sheet dispels the mystery: this is the trace of his search for a reasonable noncovariant restriction, which was necessary for reconciling the theory with the causality principle.

Observe that later, in the second part of his ''Foundations of physics'' — or, more precisely, in his paper entitled ''Foundations of physics (Second note)'' — Hilbert gave a comprehensive treatment of the problem of causality in the general-covariant theory, and completely recovered from his delusion of November 1915 [53a, p. 57–63; 53b, p. 382–386][14].

8. Conclusions

In this way, the comparison of the first proof sheet of Hilbert's report with its published version allows us to draw the following conclusions:

[13] Let us quote the final words of this very important argument in the German original: ''...So ist, wofern wir der Cauchyschen Theorie der Differentialgleichungen entsprechend den Grundgleichungen der Physik den Charakter der Bestimmtheit bewahren wollen, die Forderung von vier weiteren zu (4) und (5) hinzutretenden nicht invarianten Gleichungen unerlässlich. Um zo diesen Gleichungen zu gelangen, stelle ich zunächst eine Definition des Energiebegriffes auf''.

[14] Wolfgang Pauli in his encyclopedic article clearly explained Hilbert's position: ''...The general solution of field equations must contain four arbitrary functions. Hence the ten field equations... in ten unknowns g_{ik} must contain four identities. In general, in the relativistic theory (that is, in the general-covariant theory — **V V**) for m unknowns there must be only $(m-4)$ independent equations. The contradiction with the causality principle is only apparent, because all possible solutions of field equations differ from one another just formally, remaining entirely equivalent. The considerations presented here were formulated by Hilbert'' [6b, p. 233; 6c, p. 160]. For some reason Pauli makes reference to the first note of the 'Foundations of physics', although Hilbert gave his explanation of the causality problem in the general-covariant theories only in the second note. It is interesting that Pauli's criticism in Einstein's address [''...For some time Einstein held a wrong belief that this ambiguity of solutions leads to the conclusion that the equations of gravitation cannot exhibit general covariance (see Berl. Ber., 1914)'' [6b, p. 233; 6c, p. 160] can be wholly addressed to Hilbert in November 1915.

1. Both versions of the report present the basic concept of axiomatic construction of a unified theory of gravitational and electromagnetic fields and matter, based on the general-covariant principle of action with the Lagrangian constructed as the sum of scalar curvature and the Lagrangian of Mie's electromagnetic theory of matter. The unification of fields is based on the theorem formulated by Hilbert, which is a special case of Nöther's 2nd theorem proved at a later time. The equations of electrodynamics, or rather a certain generalization of them, are interpreted as a consequence of the equations of gravitation. The tensor of energy-momentum of matter (electromagnetic field) is found as the result of variation of the 'material' Lagrangian with respect to the gravitational potentials. As a matter of fact, all this construction and all these results that Hilbert considered pivotal for his theory went on from the proof to the published version of the report. This is probably why Hilbert kept the original date on his report.

2. At the same time, all material related to the equations of gravitational field and their general covariance is treated in the final version in an entirely different way than in the original proof sheet. First of all, the original version of the report (20 November 1915), which must be close to the text in the proof dated 6 December 1915, states that the general-covariant equations of gravitation, which ought to derive from the variational principle with scalar curvature for the Lagrangian, are incorrect, since they do not agree with the causality principle (this is similar to Einstein's argument against the general covariance of field equations prior to November 1915). The equations of gravitation in the conventional form (with Ricci tensor) are not written out at all, but only occur in the general form of the Lagrange – Euler equations that follow from the variational principle. Almost half of the proof sheet is taken up by the construction of the 'energy vector', and the corresponding law of conservation of energy-momentum. This is explained by the fact that it is the energy-momentum conservation, formalized in four non-general-covariant equations, that restricts the general covariance of field equations in such a way that the causality principle is respected. Thus, Hilbert's theory, in which the main equations are the equations of gravitation (while the equations of electromagnetic field are their implications), conceived as a general-covariant theory, becomes noncovariant. The overall general covariance is either sacrificed or not achieved.

As far as the correct gravitation equations (the general-covariant form with the 'one-half' term) are concerned, Hilbert does not write them out explicitly, probably assuming that these field equations will have to be modified anyway so as to accommodate the noncovariant conditions related to the energy-momentum conservation law.

3. This naturally calls for certain amendments to the issue of priority in the discovery of the correct general-covariant equations of gravitation. The latter were not present in Hilbert's original report, and only appeared in the version published on 31 March 1916, almost four months after the publication of Einstein's fourth paper of November, received on 25 November 1915 and issued on December the 2nd. Accordingly, there are grounds after all for returning to the original appellation of these equations, associated with Einstein's name alone.

4. In preparing the final version of his report, Hilbert had certainly read Einstein's last paper of November, and was probably aware of his grudge and his suspicions that Hilbert wanted to 'nostrify' the general theory of relativity. In the wake of Einstein, Hilbert understood the fallacy of arguments against general-covariant equations of gravitation related to the apparent violation of causality, and accepted them as the correct equations of gravitation. He considerably modified the part of the report which dealt with the energy-momentum conservation. This law was no longer regarded as a non-covariant restriction of the general covariance of the field equations. Having lost its fundamental value, the chapter on the energy-momentum conservation took up almost one half of the final volume of the report, which is explained by its importance in the original version of the report.

Willing to make amends, Hilbert tried to emphasize the priority of Einstein's achievements. Einstein could have learned of Hilbert's attempts, which may explain his conciliatory letter to Hilbert in December.

As a result, in the published version of Hilbert's report the general covariance was restored, and the general-covariant equations of gravitational field, similar to Einstein's, were explicitly written out. However, the variational proof of the equations is not given, and the correct form of the equations (with the 'one-half' term) is explained (not quite rigorously!) by the uniqueness of the Ricci tensor and scalar curvature as the general-covariant quantities that only depend on $g^{\mu\nu}$ and its first and second derivatives. The very pithy variational proof of these equations was given by Hilbert in a subsequent edition of his "Foundations of physics" in 1924 [54] (see also Ref. [22]). Incidentally, the overall general covariance was very important for Hilbert's theory itself, because the idea of reducing electrodynamics to gravitation relies essentially on the general covariance of the action of the theory in accordance with Hilbert's Theorem I (a special case of Nöther's 2nd theorem.

5. Now if Hilbert's project did not result in the creation of a nontrivial unified physical theory (based on the 'field-theoretical ideal of unity of physics', and the priority in the discovery of the correct general-covariant equations of gravitation ought to be credited to Einstein, then how do we appreciate Hilbert's contribution (namely, his first note in the "Foundations of physics") to theoretical physics, and to the general theory of relativity in particular? This contribution may be split into two parts. The first part is heuristic; it is the impact of Hilbert's approach on the future development of theoretical physics. In this work Hilbert was able to demonstrate the importance of the axiomatic method, based on such mathematical theories (structures) as geometry and the theory of continuous groups, as well as the calculus of variations. In a certain sense, Hilbert's theory was the starting point in the chain of unified field theories based on the general theory of relativity. It certainly influenced Hilbert's student H Weyl, who constructed the first model theory of this kind in 1918.

The second part is what was assimilated by GTR and thus became the 'golden treasure' of theoretical physics. This is something that has been tested by time and can be associated with the name of Hilbert. First of all it is the derivation of the gravitation equations from the general-covariant variational principle with scalar curvature for the Lagrangian. It would be natural to give Hilbert's name to the corresponding variational principle and Lagrangian ('Hilbert's Lagrangian', 'Hilbert's variational principle'). It is true, of course, that the 'material' part of the Lagrangian used by Hilbert was specialized and was associated with Mie's theory. Incidentally, Hilbert was also the first to demonstrate that the tensor of energy-momentum of 'matter' is found as the derivative of the 'material' Lagrangian with respect to the gravitation potentials (or, in the formulation of Pauli, "the tensor of energy-momentum of matter is obtained by variation of the G-field in the integral of action" [6b, p. 231; 6c, p. 158]). This result could be named "Hilbert's tensor of energy-momentum" (or "Hilbert's presentation of the tensor of energy-momentum of matter").

Finally, Hilbert was the first to formulate a nontrivial special case of Nöther's 2nd theorem, related to the group of general covariance, and to use it for getting the relations of identity that he interpreted as the equations of motion of matter (in the case of Hilbert's theory it was a certain extension of Maxwell's equations). Hilbert also stimulated Nöther's interest in the invariant variational problems, and she formulated a generalization of Hilbert's Theorem I and proved it.

On top of that, Hilbert's Theorem III, as was pointed out earlier (see Ref. [55], and also Refs [21, 22]), leads straight to the Bianchi identities that are so important for the general theory of relativity. Hilbert seems to have been the first to derive them from the variational principle [55].

6. The issue of who influenced whom and how is now settled in favor of Einstein. Beyond doubt, in mastering the tensor-geometrical theory of gravitation Hilbert relied on the works of Einstein and Grossmann of 1913, and on Einstein's fundamental "Formal foundations of the general theory of relativity" of 1914. Einstein's lectures at Göttingen in late June and early July 1915 were certainly very important for Hilbert, who liked to learn physical theories from direct conversations with physicists. Quite plausible, however, would be the assumption that Hilbert rejected Einstein's approach to the expression of noncovariant equations, which was associated with the rejection (in 1913–1914) of the general-covariant approach, Ricci tensor, etc. Judging by the original version of his report, Hilbert preferred to get the most out of his two axioms, including the axiom of general covariance. For example, in the original version of his report Hilbert first tries to express the general-covariant field equations, and the theoretical invariance considerations almost unambiguously point to the scalar curvature as the gravitation Lagrangian. And even though Hilbert agreed with Einstein's idea of 1913–1914 that the general-covariant field equations do not agree with the principle of causality, he believed that the restriction of the general covariance of field equations was the next step. Accordingly, he introduces the corresponding axiom (Axiom III) in the middle of his paper, when the sketch of the general-covariant theory has already been made (including the correct gravitational Lagrangian, and hence, in principle, the corresponding equations). As a matter of fact, this approach fits in well with Hilbert's application of the axiomatic method to physics: "...try to use a small number of axioms for covering the class of physical phenomena as broadly as possible, and then add new axioms to include the more specialized theories..." [46b, p. 416]. So Einstein's lectures at Göttingen and his discussions with Hilbert could well have been mutually stimulating. And at about the same time (in October 1915) they started a new spurt to the correct equations of gravitation (Einstein) and the axiomatically established unified theory of gravitation and electromagnetic field (Hilbert).

In early November 1915, however, Einstein was the first to submit and publish three seminal papers on the general theory of relativity, in which he returns to the firm ground of general covariance. He mails the printer's proof sheets of his publications to Hilbert, and starts exchanging letters. In these works he is very close to achieving general covariance and expressing the correct general-covariant field equations. In this respect, Einstein's influence on Hilbert ought to have been very strong[15].

Einstein had received the original version of Hilbert's report (printer's proof) one week before he submitted his last November paper. This paper, however, bears no trace of any influence from Hilbert and his theory whatsoever. Einstein finally achieves the overall general covariance of the gravitation equations without resorting to his former $T = 0$ hypothesis of the structure of matter. On the contrary, Hilbert starts with general covariance, and then restricts it from those considerations that Einstein had used in 1913–1914 but eventually rejected altogether.

In preparing the final version of his report, Hilbert certainly relied on Einstein's last paper of November, and referred to it explicitly. He dropped the idea of restricting the general covariance of the field equations to accommodate the causality principle and the conservation of energy-momentum, and expressly wrote out the correct general-covariant equations of gravitation with the 'one-half' term. Incidentally, preparing his "Foundations of physics (second note)" [53] for publication in 1916, Hilbert actually drifted away from his unified theory and concerned himself with the 'general issues of both logical and physical nature' of the general theory of relativity itself. In particular, he gives a detailed analysis of the problem of causality, considers certain problems of measurements in the general theory of relativity, and then goes on to the integration of Einstein's equations of gravitation, referring to the works of Einstein and Schwarzschild of 1916. Hilbert's analysis of causality in the general theory of relativity was highly appreciated by Pauli and other relativists. On Hilbert's detour to the studies in the general theory of relativity and on his contribution to this theory please see Refs [18, 22].

On the last page of "Foundations of physics (second note)" we see an 'anticipatory reference' to V Frederiks, who was preparing a publication on Schwarzschild's problem [53, p. 53]. This is the Russian physicist Vsevolod Konstantinovich Frederiks, who was interned in Germany during WWI, and who was hired by Hilbert as his assistant (which was not at all easy to do). In 1918 Frederiks returned to Russia. In 1921, in one of the first issues of *Uspekhi Fizicheskikh Nauk* he published one of the first accounts of the general theory of relativity in the Russian language, more according to Hilbert than to Einstein [56] (see also Ref. [18].

[15] The situation may be more complicated, because we know that Hilbert responded to Einstein's letter of 7 November and the proof of his first November paper with a letter (presumably on the 7th or 8th of November) that has not yet been found. This letter could contain some ideas concerning the reduction of electromagnetism (and the structure of matter) to gravitation. This could then stimulate Einstein (or pluck a unison string) to build on the idea of the electromagnetic character of matter ($T = 0$) and on the role of gravitation in the structure of matter, which leads to $(T + t) \neq 0$, where t is the spur of the similar tensor of energy-momentum of gravitational field. It is less likely that this idea in Einstein's letter to Hilbert on 12 November could have had an impact on Hilbert, since on 13 November Hilbert in his letter to Einstein gives a rather detailed account of his theory. By the time Einstein had received the proof of Hilbert's report, which was sent to him of his request, he (Einstein) had already departed from his $T = 0$ hypothesis, as indicated by footnote 4 in Einstein's paper on the perihelion of Mercury, dated 18 November.

This, however, is another story, the story of reception (recognition) of the theory of relativity in Russia and in the USSR.

References

1. Hertz H *Untersuchungen über die Ausbreitung der elektrischen Kraft* (Leipzig: J.A. Barth, 1892)
2. Mandelstam L I "Lektsii po osnovam kvantovoĭ mekhaniki (teoriya kosvennykh izmereniĭ)" (Lectures on foundations of quantum mechanics (theory of indirect measurements)) (1939), in Mandelstam L I *Lektsii po Optike, Teorii Otnositel'nosti i Kvantovoĭ Mekhanike* (Lectures on Optics, Relativity and Quantum Mechanics) (Moscow: Nauka, 1972) pp. 325 – 388
3. Feynman R *The Character of Physical Law* (Cambridge: M.I.T. Press, 1965) [Translated into Russian *Kharakter Fizicheskikh Zakonov* (Moscow: Mir, 1968) p. 57]
4. Kobzarev I Yu, Manin Yu I *Elementary Particles: Mathematics, Physics and Philosophy* (Dordrecht: Kluwer Acad. Publ., 1989); *Elementarnye Chastitsy. Dialogi Fizika i Matematika* (Moscow: Fazis, 1997) p. 38
5. Logunov A A *Teoriya Gravitatsionnogo Polya* (Theory of Gravitational Field) (Moscow: Nauka, 2000)
6. (a) Pauli W *Relativitätstheorie* (Encyklopädie der Mathematischen Wissenschaften, Bd. 19, Ed. A Sommerfeld) (Leipzig: Teubner, 1921) pp. 539 – 775; (b) [Translated into Russian *Teoriya Otnositel'nosti* (Moscow – Leningrad: GITTL, 1947)]; (c) [Translated into English *Theory of Relativity* (London: Pergamon, 1958)]
7. Vizgin V P *Relyativistskaya Teoriya Tyagoteniya* (*Istoki i Formirovanie, 1900 – 1915*) (Relativistic Theory of Gravitation (Origins and Development, 1900 – 1915) (Moscow: Nauka, 1981)
8. Guth E "Contribution to the history of Einstein's geometry as a branch of physics", in *Relativity: Proc. Relativity Conf., Midwest, Cincinnati, 1969* (Eds M Carmeli, S I Fickler, L Witten) (New York: Plenum Press, 1970) p. 161
9. Vizgin V P *Razvitie Vzaimosvyazi Printsipov Invariantnosti s Zakonami Sokhraneniya v Klassicheskoĭ Fizike* (Development of Interconnection Between the Principles of Invariance and the Laws of Conservation in Classical Physics) (Moscow: Nauka, 1972)
10. Pyenson L R "The Goettingen reception of Einstein's general theory of relativity", Ph.D. Dissertation (Baltimore, Maryland: The Johns Hopkins University, 1974)
11. Mehra J *Einstein, Hilbert, and the Theory of Gravitation. Historical Origins of General Relativity* (Dordrecht: D. Reidel Publ. Co., 1974)
12. Earman J, Glymour C "Einstein and Hilbert: Two months in the history of general relativity" *Arch. Hist. Exact Sci.* **19** 291 (1978)
13. (a) Pais A "*Subtle is the Lord...*". *The Science and the Life of Albert Einstein* (Oxford: Oxford Univ. Press, 1982); (b) [Translated into Russian *Nauchnaya Deyatel'nost' i Zhizn' Al'berta Eĭnshteĭna* (Moscow: Nauka, 1989)]
14. Hilbert D "Die Grundlagen der Physik" (Erste Mitteilung; Erste Korrektur. 6 Dez. 1915, Göttingen), Staats-und Universitätsbibliothek Goettingen (Handschriftenabteilung), Cod. Ms. D. Hilbert 634
15. Corry L, Renn J, Stachel J "Belated decision in Hilbert – Einstein priority dispute" *Science* **278** 1270 (1997)
16. Corry L "David Hilbert and the axiomatization of physics (1894 – 1905)" *Arch. Hist. Exact Sci.* **51** 83 (1997)
17. Corry L "Hilbert on kinetic theory and radiation theory (1912 – 1914)" *Math. Intell.* **20** (3) 52 (1998)
18. Vizgin V P Kommentarii i primechaniya k "Osnovaniyam fiziki" (Pervoe i vtoroe soobshcheniya) (Comments and notes to "Foundations of physics" (First and second notes)), in Hilbert D *Izbrannye Trudy* (Selected Works) (Ed. A N Parshin) (Moscow: Faktorial, 1998) p. 563
19. Corry L "From Mie's electromagnetic theory of matter to Hilbert's unified foundations of physics" *Stud. Hist. Philos. Mod. Phys. B* **30** (2) 159 (1999)
20. Corry L "David Hilbert between mechanical and electromagnetic reductionism (1910 – 1915)" *Arch. Hist. Exact Sci.* **53** 489 (1999)
21. Sauer T "The relativity of discovery: Hilbert's first note on the foundations of physics" *Arch. Hist. Exact Sci.* **53** 529 (1999)
22. Renn J, Stachel J "Hilbert's foundation of physics: From a theory of everything to a constituent of general relativity", Preprint Max-Planck-Inst. für Wissenschaftsgeschichte № 118 (Berlin: Max-Planck-Inst. für Wissenschaftsgeschichte, 1999) 113 p.
23. Renn J, Sauer T "Heuristics and mathematical representation in Einstein's search for a gravitational equation", in *The Expanding Worlds of General Relativity* (Einstein Studies, Vol. 7, Eds H Goenner, J Renn, J Ritter, T Sauer) (Boston: Birkhäuser, 1999) p. 87 – 125
24. Stachel J "New light on the Einstein – Hilbert priority question" *J. Astrophys. Astron.* **20** 91 (1999)
25. Vizgin V P *Edinye Teorii Polya v Pervoĭ Treti XX Veka* (Unified Field Theories in the First Third of the 20th Century) (Moscow: Nauka, 1985)
26. Vizgin V P *Unified Field Theories in the First Third of the 20th Century* (Basel: Birkhäuser, 1994) XIX, 335 p.
27. Corry L "Hilbert and Physics (1900 – 1915)", Preprint Max-Planck-Inst. für Wissenschaftsgeschichte № 43 (Berlin: Max-Planck-Inst. für Wissenschaftsgeschichte, 1996)
28. Norton J "What was Einstein principle of equivalence?", in *Einstein and the History of General Relativity: The Proc. of the 1986 Osgood Hill Conf., North Andover, Mass., May 1986* (Einstein Studies, Vol. 1, Eds D Howard, J Stachel) (Boston: Birkhäuser, 1989) p. 5
29. Stachel J "Einstein's search for general covariance, 1912 – 1915", in *Einstein and the History of General Relativity: The Proc. of the 1986 Osgood Hill Conf., North Andover, Mass., May 1986* (Einstein Studies, Vol. 1, Eds D Howard, J Stachel) (Boston: Birkhäuser, 1989) p. 63
30. Norton J "How Einstein Found His Field Equations, 1912 – 1915", in *Einstein and the History of General Relativity: The Proc. of the 1986 Osgood Hill Conf., North Andover, Mass., May 1986* (Einstein Studies, Vol. 1, Eds D Howard, J Stachel) (Boston: Birkhäuser, 1989) p. 101
31. Vizgin V P "Genezis tenzorno-geometricheskoĭ kontseptsii gravitatsii (1912 – 1913)" (Genesis of tensor-geometrical concept of gravitation), in *Istoriko-Matematicheskie Issledovaniya* (Studies in History of Mathematics) Vol. XXVI (Moscow: Nauka, 1982) p. 235
32. Vizgin V P "Rol' ideĭ E Makha v genezise obshcheĭ teorii otnositel'nosti (Role of Mach's ideas in the genesis of general relativity), in *Eĭnshteĭnovskiĭ Sbornik 1986 – 1990* (Moscow: Nauka, 1990) p. 49
33. Norton J D "Bor'ba Eĭnshteĭna za obshchuyu kovariantnost" (Einstein's struggle with general covariance), in *Eĭnshteĭnovskiĭ Sbornik 1982 – 1983* (Moscow: Nauka, 1986) p. 57; *Rivista Storia Sci.* **2** 191 (1985)
34. Renn J, Sauer T "Einsteins Züricher Notizbuch. Die Entdeckung der Feldgleichungen der Gravitation im Jahre 1912", Preprint Max-Planck-Inst. für Wissenschaftsgeschichte № 28 (Berlin: Max-Planck-Inst. für Wissenschaftsgeschichte, 1995)
35. Einstein A *The Collected Papers of Albert Einstein* Vol. 4 *The Swiss Years: Writings, 1912 – 1914* (Eds M J Klein et al.) (Princeton, N.J.: Princeton Univ. Press, 1995)
36. Einstein A *The Collected Papers of Albert Einstein* Vol. 5 *The Swiss Years: Correspondence, 1902 – 1914* (Eds M J Klein et al.) (Princeton, N.J.: Princeton Univ. Press, 1993)
37. Einstein A *The Collected Papers of Albert Einstein* Vol. 6. *The Berlin Years: Writings, 1914 – 1917* (Eds A J Kox et al.) (Princeton, N.J.: Princeton Univ. Press, 1996)
38. Einstein A *The Collected Papers of Albert Einstein* Vol. 8 *The Berlin Years: Correspondence, 1914 – 1918* (Eds R Schulmann et al.) (Princeton, N.J.: Princeton Univ. Press, 1998)
39. Einstein A *Sobranie Nauchnykh Trudov* (Collected Papers) Vol. 1 (Eds I E Tamm et al.) (Moscow: Nauka, 1965)
40. Einstein A *Briefwechsel. 60 Briefe aus dem golden Zeitalter der modernen Physik von Albert Einstein und Arnold Sommerfeld* (Hrsg. A Hermann) (Basel: Stuttgart Schwabe, 1968) [Translated into Russian "Iz perepiski Zommerfel'da s Eĭnshteĭnom" (From correspondence between Sommerfeld and Einstein), in Sommerfeld A *Puti Poznaniya v Fizike* (Moscow: Nauka, 1973) p. 191]
41. Einstein A, Besso M *Correspondence 1903 – 1955* (Ed. P Speziali) (Paris: Hermann, 1972) [Translated into Russian "Perepiska A Eĭnshteĭna s M Besso, 1903 – 1955" (Correspondence between A Einstein and M Besso, 1903 – 1955), in *Eĭnshteĭnovskiĭ Sbornik, 1974* (Moscow: Nauka, 1976) p. 5]

42. (a) Hilbert D "Die Grundlagen der Physik (Erste Mitteilung)" *Göttingen Nachr. Math.-Phys. Klasse* 395 (1915); (b) [First Translated into Russian "Osnovaniya fiziki (Pervoe soobshchenie)" (Foundations of physics (First note)) (Transl. by D V Zharkov), in *Variatsionnye Printsipy Mekhaniki* (Variational Principles of Mechanics) (Ed. L S Polak) (Moscow: GIFML, 1959) p. 589 – 598]
43. Hilbert D "Osnovaniya fiziki (Pervoe soobshchenie)" (Foundations of physics (First note)), in *A Eĭnshteĭn i Teoriya Gravitatsii* (A Einstein and Theory of Gravitation) (Moscow: Mir, 1979) p. 133
44. Hilbert D "Osnovaniya fiziki (Pervoe soobshchenie)" (Foundations of physics (First note)) (Transl. by D V Zharkov), in *Izbrannye Trudy* (Selected Works) Vol. 2 (Moscow: Faktorial, 1998) p. 367
45. Weyl H "David Hilbert and his mathematical works", in Ried C *Hilbert* (Berlin: Springer-Verlag, 1970) p. 245; *Bull. Am. Math. Soc.* **50** 612 (1944) [Translated into Russian "David Hilbert i ego matematicheskie trudy", in *Izbrannye Trudy* (Selected Works) Vol. 2 (Appendix) (Moscow: Faktorial, 1998) p. 480]
46. (a) Hilbert D "Mathematische Probleme" *Göttingen Nachr. Math.-Phys. Klasse* 253 – 297 (1900); (b) [Translated into Russian "Matematicheskie problemy", in *Izbrannye Trudy* (Selected Works) Vol. 2 (Appendix) (Moscow: Faktorial, 1998) p. 401]; (c) [Translated into English "Mathematical Problems" *Bull. Am. Math. Soc.* **8** 437 – 479 (1901/1902)]
47. Born M "Der Impuls-Energie-Satz in der Elektrodynamik von Gustav Mie" *Göttingen Nachr. Math.-Phys. Klasse* 23 (1914)
48. (a) Einstein A "Zum gegenwärtigen Stand des Strahlungsproblems" *Phys. Z.* **10** 185 (1909); (b) [Translated into Russian "K sovremennomu sostoyaniyu problemy izlucheniya" (On contemporary status of the problem of radiation), in *Sobranie Nauchnykh Trudov* (Collected Papers) Vol. 3 (Moscow: Nauka, 1966) p. 164]
49. (a) Weyl H *Raum-Zeit-Materie* 5th, revised ed. (Berlin: J. Springer, 1923); (b) [Translated into Russian *Prostranstvo, Vremya, Materiya* (Transl. from German by V P Vizgin) (Moscow: Yanus, 1996)]; (c) [Translated into English *Space – Time – Matter* (London: Methuen & Co., 1922)]
50. (a) Einstein A "Die Grundlage der allgemeinen Relativitätstheorie" *Ann. Phys.* (Leipzig) **49** 769 (1916); (b) [Translated into Russian "Osnovy obshcheĭ teorii otnositel'nosti" (Foundations of general relativity), in *Sobranie Nauchnykh Trudov* (Collected Papers) Vol. 1 (Moscow: Nauka, 1966) p. 452]
51. (a) Nöther E "Invariante Variationsprobleme" *Göttingen Nachr. Math.-Phys. Klasse* 235 (1918); (b) [Translated into Russian "Invariantnye Variatsionnye Zadachi" (Invariant Variation Problem), in *Variatsionnye Printsipy Mekhaniki* (Variation Principles of Mechanics) (Ed. L S Polak) (Moscow: GIFML, 1959) p. 611]
52. (a) Misner C W, Thorne K S, Wheeler J A *Gravitation* (San Francisco: W. H. Freeman, 1973); (b) [Translated into Russian Vol. 2 *Gravitatsiya* (Ed. V B Braginskiĭ) (Moscow: Mir, 1977)]
53. (a) Hilbert D "Die Grundlagen der Physik (Zweite Mitteilung)" *Göttingen Nachr. Math.-Phys. Klasse* 53 (1917); (b) [Translated into Russian "Osnovaniya fiziki (Vtoroe soobshchenie)" (Foundations of physics (Second note)) (Transl. by D V Zharkov), in *Izbrannye Trudy* (Selected Works) Vol. 2 (Moscow: Faktorial, 1998) p. 379]
54. Hilbert D "Die Grundlagen der Physik" *Math. Ann.* **92** 1 (1924)
55. Logunov A A "Predislovie nauchnogo redaktora perevoda" (Foreword by the science editor of the translation), in Pais A *Nauchnaya Deyatel'nost' i Zhizn' Al'berta Eĭnshteĭna* (Moscow: Nauka, 1989) p. 5
56. Frederiks V K "Obshchiĭ printsip otnositel'nosti Eĭnshteĭna" (Einstein's general principle of relativity) *Usp. Fiz. Nauk* **2** (2) 162 (1921); Reprinted: *Usp. Fiz. Nauk* **169** 1339 (1999) [*Phys. Usp.* **42** 1235 (1999)]