

Stationary radiation of objects with scattering media

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Abstract. The radiation observed inside or outside a stationary radiator with a scattering medium is a sum of components, each being determined by, first, the primary radiation from some part of the radiator and, second, the probability of this radiation reaching the region where it is observed. In this review, general and rather simple relations between these components are discussed. These relations, unlike the components themselves, are independent of the specific optical characteristics of the object as well as of its geometry, inhomogeneity, etc. In deriving the relations, the situations in which geometrical optics is either applicable or inapplicable to radiation in a scattering medium are considered. For the case where geometrical optics does apply, stationary relations are derived from the probabilistic stationarity condition for radiation passing through the medium, i.e., from the fact that all radiation emitted in a stationary regime disappears with probability unity. Equilibrium relations are derived from the stationary relations in the particular case of a thermal radiator in an isothermal cavity. To derive the stationary relations in the geometrical optics approximation, we obtain general solutions of the linear equation of transfer using the Green function

approach. If geometrical optics cannot be applied to a scattering and radiating medium, only relations for the components of outgoing thermal radiation are obtained, and the generalized Kirchhoff law, obtained by Levin and Rytov using statistical radio-physics methods, is employed. In this case, stationary relations are also derived from a probabilistic stationarity condition; the equilibrium relations follow from the stationary ones as well as from the equilibrium condition for radiation in the isothermal cavity. The quantities involved in all the relations obtained are a subject of experimental and computational spectroscopic studies. Examples of current and potential applications are given. The relations have been successfully used in diverse spectroscopic experiments — in studies of the effects of macroscopic particles on the emission line profiles in dusty plasmas and in temperature measurements in strongly scattering solid porous materials.

1. Introduction

The present paper analyzes stationary radiation from objects for which scattering by inhomogeneities in the medium is important. Each scattering event may change the direction of radiation. Radiation therefore travels through the scattering medium in a zigzag manner. This substantially complicates the passage of radiation from the point of origin to the point of observation, and for this reason various regions in the radiator or outside it may contribute to the observed radiation. This is illustrated in Fig. 1, which presents a schematic of an experimental setup for the observation of radiation from a scattering medium. Let the surface S enclose the region containing the scattering medium, and let Sp be a

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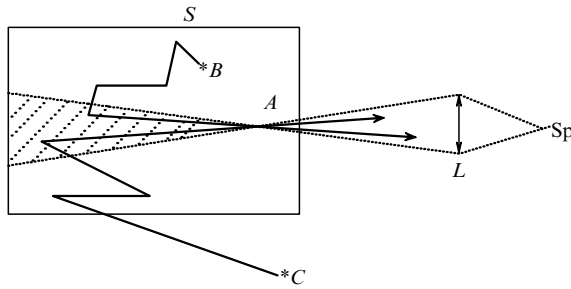


Figure 1. Schematic diagram showing how radiation from a scattering medium arrives at the detecting apparatus: *S*, surface enclosing the scattering radiator; *L*, focusing optical equipment; *Sp*, entrance to the detecting device; *A*, focusing region; *B*, *C*, radiation emergence points. The device's field of view is dashed.

spectral or optical device focused on the point of observation *A*. Radiation arrives at the device from the field of view determined by the focusing optical system *L*. In the absence of scattering, radiation can reach the device only in the case of having been emitted in the field of view in the direction toward the device, within the entrance aperture. In the presence of scattering, radiation can find its way to the device after the last scattering event in the field of view, independent of whether this radiation first appeared inside (point *B*) or outside (point *C*) the scattering medium.

In practice, it is often necessary to know how much the individual components originating in various parts of the object contribute to the total radiation observed. Such problems arise, for example, in experimental spectral diagnostics, which determines, based on the observed radiation, the primary radiation from the individual components or regions of the medium under study. It is this primary radiation that gives the direct and most complete information on the object under investigation.

In some cases, contributions from individual primary radiators can be determined from specially designed experiments in which some of the primary radiators are eliminated while the others remain intact. Normally, however, the separation of the observed radiation into components is based on solving linear equations that describe the propagation of radiation in the scattering medium. In the approximation of geometrical optics, these are the radiation-transfer equations; in the general case, the Maxwell equations.

Linear equations are adequate to describe radiation if both the emission mechanism and the parameters of the radiation–material interaction are specified and do not depend on the radiation considered. In this case, in spite of the general complexity of the propagation pattern, the radiation that appears at some point arrives at an arbitrary point of observation independently of whether or not other parts of the object radiate. This implies that, for any arbitrarily chosen frequency, the observed radiation is the sum (or integral) of the components corresponding to the individual primary radiators.

The total radiation fluxes and their components can be obtained by solving the radiation propagation equations, provided that numerous optical and geometrical characteristics of the radiator are known. Clearly, the results should vary widely, depending on the radiator conditions. Over many years, stationary solutions to the radiation-transfer equations for various specific cases have been continuously published.

The present review examines a different approach to radiation from objects with scattering media. In recent years, general and fairly simple relations between radiation components originating from different primary radiators have been established. These relations do not depend on the specific optical (absorption and scattering) characteristics, nor on the shape of the radiator, inhomogeneities in the medium, etc. All these features influence the magnitude of the components but not the general relations between them. It is this fact that makes these relations especially useful in performing radiation experiments and calculations for a wide variety of physical conditions. In particular, the general relations help in separating the observed radiation into components due to different primary sources.

General relations of this kind cannot be obtained simply by analyzing or solving the equations describing the process under study. Additionally fairly general conditions not contained in these equations are needed. Thus, one time-honored approach to deriving general relations between various physical quantities is to consider these quantities under thermodynamic-equilibrium conditions. This approach led, among other things, to the classical Kirchhoff–Clausius law, a relation between the emissivity and absorptivity of a body, which is quite general and independent of the specific radiator characteristics determining either of the quantities separately.

There are two different methods by which the radiation component relations to be studied here have been derived in the previous work. The earliest method also employed thermodynamic equilibrium conditions (see Section 3.1). The system considered was an isothermal radiator with arbitrary optical and geometrical characteristics, surrounded by an opaque wall at the same temperature. The primary sources were assumed to be thermal, i.e., the primary radiation was determined by the temperature of the source. The equilibrium relations, i.e., those derived from the thermodynamic-equilibrium conditions, involve only the relative magnitudes of all radiation components. Since these relative magnitudes do not depend on the temperature directly, the relations can also be applied (within certain restrictions) to non-isothermal objects, whose resulting radiation is not equilibrated. In this case, the thermal radiator should be surrounded by a radiating surface.

The second method for obtaining the relations, which came later, is not commonly accepted and is based on the probabilistic formulation of the stationarity condition for radiation. To clarify the essence of the method, we note the following two points. First, the propagation of radiation in a medium is a statistical process, and each radiation component is determined by the intensity of the primary radiation and by the probability that it will reach the point of observation. Second, in the present context, the stationarity of radiation is of course taken into account by the fact that the time derivative in the radiation equations is zero. However, the stationarity condition also puts restrictions on the probability of radiation passage through the medium. Note that, in a stationary radiator, the appearance and disappearance of radiation are normally simultaneous processes. Radiation results from the energy of matter being converted into that of radiation or, alternatively, is due to the illumination of the radiator from the outside. The disappearance of radiation involves the reverse processes: the energy of radiation is either absorbed by the material or escapes to the radiator's exterior. Note that radiation may

preferentially appear in some particular parts of the radiator and disappear in others. The equilibrium radiator is a special case of the stationary one, the appearance of radiation in it being fully compensated by its disappearance at every point. At the same time, in any stationary radiator the radiation emitted at some rate completely disappears at the same rate. In other words, the disappearance of the emerging radiation is a certain event.

According to the probability theory, the probability of a certain event is unity. This general statement can be applied, in particular, to the disappearance of radiation in a stationary regime. The fact that the probability of disappearance of radiation produced under stationary conditions equals unity provides a general condition necessary to obtain the required relations. The probability of disappearance is determined by the probabilities that the radiation will arrive at one part of the radiator or another and, subsequently, will either be absorbed or will pass out. Consequently, the probabilities for the passage of radiation should also satisfy this condition.

The probability condition for the unavoidable disappearance of radiation can be modified to include the relative radiation components using the relationship between these components and the radiation-passage probabilities. The stationary relations include the sum of all the relative components as well as the probability of radiation disappearing in the non-radiating parts of the object. From the stationary relations for the special case of a thermal radiator surrounded by an opaque radiating surface, one can also obtain the equilibrium relations that were earlier obtained from the thermodynamic-equilibrium conditions. In this context, we describe here the method based on the probability condition for the disappearance of radiation as the basic technique for obtaining the relations of interest.

The relations between the radiation components will be derived here both for scattering media where the approximation of geometrical optics is valid and for those where it fails. The relations will be obtained both for the general case and for special cases of practical significance. While the former will be written in an integral form, an algebraic form will be employed for the latter. The relations contain only quantities measured in spectral and optical experiments, such as the radiation intensities, optical densities, temperature, or the relative populations of the radiator's energy levels.

Sections 2–4 deal with media to which the geometrical optics approximation can be applied. The propagation of radiation is studied based on the equations of transfer describing the intensity of radiation. An equation for the response (Green) function is constructed in a fairly general form, and the general solutions necessary to obtain the relations of interest are written using this function (Section 2.3). This makes it possible to represent an individual radiation component as a product of the primary radiation intensity with the probability of the primary radiation reaching the point of observation. The solutions obtained are valid for arbitrary radiators, in particular, for thermal ones, with the primary radiation given by the classical Kirchhoff–Clausius law. Both elastic (frequency-conserving) and inelastic (frequency-nonconserving) scattering are considered. Multi-phase media (e.g., dusty gases) are allowed. In Section 3, stationary relations between the intensity components are derived. In Section 3.2, radiation disappearance probabilities are discussed in detail, equations for them are formulated, and a condition on the total probability of

disappearance is obtained. In Section 3.3, the equations for radiation disappearance probabilities are compared with those for the radiation intensity. It is shown that introducing intensity components into the probabilistic disappearance condition leads to the required relations, provided the reversibility conditions are valid for the medium and reciprocity relations are obeyed in every scattering event. A general form of the stationary relation is derived in Section 3.3, and special cases are treated in Section 3.4.

Section 4 considers specific examples of how the relations obtained in the paper can be applied to dusty plasmas. In Section 4.2, the application of equilibrium relations is illustrated. The application of stationary relations to an actual experimental study of the effect of dust particles on the profiles of dusty-gas emission lines is described in Section 4.3. Next, the possibility of experimentally determining the relative populations of atomic levels in a dusty plasma with the use of a stationary relation is discussed (Section 4.4).

Section 5 discusses a more complex case, where geometrical optics is not valid inside the scattering medium. It is applicable only outside the medium, in the region where the radiation from a medium is usually observed experimentally, for example, where the lens L and the spectrograph Sp are located (see Fig. 1). For this case, only thermal radiation is considered, whose description relies on the generalized Kirchhoff law derived by Levin and Rytov using statistical radiophysics methods, i.e., from Maxwell's equations. The generalized Kirchhoff law immediately leads to general expressions for the radiation energy in the region of observation in terms of primary sources. The contribution from each radiating part of the object to the radiation energy at a given frequency is determined by the primary thermal radiation from this part and by the probability that it reaches the point of observation. Based on the generalized Kirchhoff law, equilibrium (Section 5.3) and stationary (Section 5.4) relations between the components of the energy and radiation flux in the region of observation are obtained. Only elastic scattering in a single-phase medium is considered. Section 5.5 describes the application of the equilibrium and stationary relations to the measurement of the temperature of a strongly scattering solid porous material inside which the geometrical optics approximation is not applicable.

2. Radiation intensity expressed in terms of primary sources

Linear radiation-transfer theory for scattering media where geometrical optics is valid is based on the integro-differential equation of transfer, which expresses the balance of radiative energy (or photons) in a physically infinitesimal region. The foundations of radiation-transfer theory were laid in the middle of the 20th century, mainly by astrophysicists [1–3]. The theory was then developed at an explosive rate by numerous workers from such diverse fields as astrophysics [4–6], heat exchange [7–9], and plasma [10] and neutron [11–13] physics. In recent years, numerical solutions to the equation transfer have attracted much attention. New methods and calculations for various specific conditions are constantly being reported; see, for example, Refs [14–23]. There is an extensive literature that justifies the phenomenological radiation-transfer theory and establishes its limits. We only mention here one review article [24] and three monographs [10, 25, 26].

2.1 Characteristics of the radiation–matter interaction

Before writing out the transfer equations, it is necessary to discuss those characteristics of the interaction between radiation and matter that are used in the theory of transfer. This interaction is basically as follows. First, the energy of the radiation flux is absorbed by the medium, i.e., is converted into the energy of matter. The radiation energy may turn into the energy of the chaotic motion of particles or into the energy of the bound states in the material (which can be called true absorption). Second, scattering by inhomogeneities of various kinds causes the radiation to escape from the ray. The theory of radiation transfer assumes individual scattering events to be independent of each other.

Scattering may be either elastic, without a change in the frequency, or inelastic, with such a change. For example, scattering by water droplets or dust particles in the Earth's atmosphere is normally elastic. In contrast, the photoexcitation of an atom with a subsequent emission within the same broadened spectral line is among the numerous examples of inelastic scattering.

Let us introduce the local characteristics of the above types of radiation–matter interactions. Suppose an emitting, absorbing, and scattering medium is within a volume v enclosed by the surface S (Fig. 2). The local characteristics are allowed to depend on the point within the volume (determined by the radius vector \mathbf{r}), as well as on the radiation direction (the unit vector \mathbf{u}) and frequency ν . A dependence on \mathbf{r} refers to inhomogeneous media, whereas a dependence on \mathbf{u} characterizes anisotropic media.

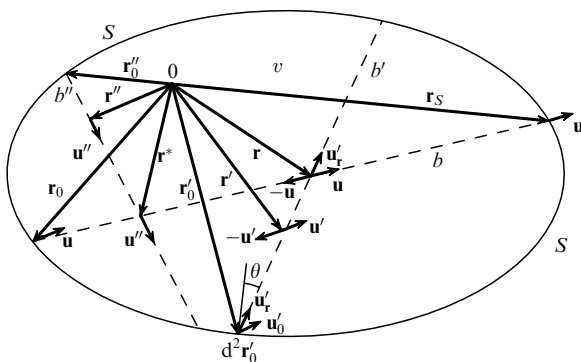


Figure 2. Schematic diagram of the passage of radiation in a scattering medium: b, b', b'' , light rays; $\mathbf{r}, \mathbf{r}^*, \mathbf{r}', \mathbf{r}''$, radius vectors of points within the volume v ; $\mathbf{r}_0, \mathbf{r}'_0, \mathbf{r}''_0, \mathbf{r}_S$, radius vectors of points on the surface S ; \mathbf{u} , unit vectors; θ , angle of deviation from the normal to the surface element $d^2r'_0$.

Let $k(\mathbf{r}, \mathbf{u}, \nu)$ be the absorption coefficient; then the product $k(\mathbf{r}, \mathbf{u}, \nu) d\mathbf{r}$ equals the probability that radiation of frequency ν traveling in the direction of \mathbf{u} will be absorbed over the length element $d\mathbf{r}$ in the vicinity of \mathbf{r} . Now let $\alpha(\mathbf{r}, \mathbf{u}, \nu)$ be the extinction coefficient describing attenuation of radiation due to both absorption and scattering. The product $\alpha(\mathbf{r}, \mathbf{u}, \nu) d\mathbf{r}$ is then the probability that radiation of frequency ν traveling in the \mathbf{u} direction will interact with the medium over the length element $d\mathbf{r}$ in the vicinity of \mathbf{r} .

The optical distance t between two arbitrary points \mathbf{r}_1 and \mathbf{r}_2 in an absorbing and scattering medium can be expressed in terms of the extinction coefficient as

$$t(\mathbf{r}_1 \rightarrow \mathbf{r}_2) = \left| \int_{\mathbf{r}_1}^{\mathbf{r}_2} \alpha(\mathbf{r}', \mathbf{u}', \nu) d\mathbf{r}' \right|. \quad (2.1)$$

Here, the integration runs along the ray passing through the points \mathbf{r}_1 and \mathbf{r}_2 . Recall that the quantity $\exp[-t(\mathbf{r}_1 \rightarrow \mathbf{r}_2)]$ is the probability of radiation passing unabsorbed and unscattered from \mathbf{r}_1 to \mathbf{r}_2 .

Let $A(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu \rightarrow \nu') d\mathbf{u}' d\nu'$ be the probability that radiation of frequency ν that travels in the \mathbf{u} direction and undergoes a single event of interaction with the medium in the vicinity of the point \mathbf{r} will acquire a direction in the interval $[\mathbf{u}', \mathbf{u}' + d\mathbf{u}']$ and a frequency in the interval $[\nu', \nu' + d\nu']$. Here $d\mathbf{u}'$ is a solid-angle element. The product $\alpha(\mathbf{r}, \mathbf{u}, \nu) A(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu \rightarrow \nu') d\mathbf{r} d\mathbf{u}' d\nu'$ is the probability that radiation of frequency ν that travels along \mathbf{u} and interacts with the medium over the length $d\mathbf{r}$ will acquire a direction in the interval $[\mathbf{u}', \mathbf{u}' + d\mathbf{u}']$ and a frequency in the interval $[\nu', \nu' + d\nu']$.

The probability of scattering with arbitrary final directions and frequencies (survival probability λ) can be obtained from A by integrating over all angles and all finite frequencies. Let $\Delta\nu$ be the range of possible frequency changes due to scattering; then

$$\lambda(\mathbf{r}, \mathbf{u}, \nu) = \int_{4\pi} d\mathbf{u}' \int_{\Delta\nu} d\nu' A(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu \rightarrow \nu'). \quad (2.2)$$

The absorption and extinction coefficients are related to the survival probability in the following manner:

$$k(\mathbf{r}, \mathbf{u}, \nu) = \alpha(\mathbf{r}, \mathbf{u}, \nu) [1 - \lambda(\mathbf{r}, \mathbf{u}, \nu)]. \quad (2.3)$$

If the scattering is elastic, we have

$$A(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu \rightarrow \nu') = A(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu) \delta(\nu - \nu'), \quad (2.4)$$

where

$$A(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu) = \frac{\lambda(\mathbf{r}, \mathbf{u}, \nu) \chi(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu)}{4\pi}, \quad (2.5)$$

$\delta(\nu - \nu')$ is the Dirac function, and $\chi(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu)$ is the scattering indicatrix with the normalization

$$\int_{4\pi} \frac{\chi(\mathbf{r}, \mathbf{u} \rightarrow \mathbf{u}', \nu)}{4\pi} d\mathbf{u}' = 1. \quad (2.6)$$

If there is no scattering, then $A = \lambda = 0$, $\alpha = k$. If there is no absorption, $k = 0$, $\lambda = 1$. In many cases, A and χ depend only on the difference $|\mathbf{u} - \mathbf{u}'|$, i.e., on the absolute magnitude of the scattering angle rather than on each of the directions \mathbf{u} and \mathbf{u}' separately. If the indicatrix is independent of the scattering angle, the scattering is said to be isotropic. We have $\chi = 1$ in this case.

Scattering in various kinds of media is often due to small macroscopic particles. They form patterns of dustiness, clouds, fogs, etc. In recent years, there has been substantial interest in phenomena occurring in so-called dusty plasmas [27–34].

In all such cases, treating radiation transfer requires a knowledge of the absorption or elastic scattering of radiation by individual particles. Theoretically, this is gained by solving the Maxwell equations for electromagnetic waves traveling in a region containing a single particle. The first studies along these lines date back to the early 20th century [35]. Since then, calculations of absorption and scattering by various particles have been done repeatedly. A wealth of material on this subject is presented in a number of monographs (see

Refs [36–41]). A brief summary of the results is contained in Ref. [42]. In recent years, computer codes have actively been developed and employed to calculate the optical characteristics of small particles under various conditions (see, e.g., Ref. [43]).

There is an important point to note about the treatment of radiation in the theory of transfer, namely, the fact that the propagation of radiation between two scattering events is considered to be independent of the scattering events themselves [26]. The propagation of radiation in between is determined by the refractive index of the medium n , which can vary only smoothly. As to the scattering, it is caused by sharp inhomogeneities, which are in fact considered to be pointlike. The dielectric constant of the medium through which radiation travels between the scattering events may differ considerably from that characterizing the vicinity of a sharp inhomogeneity. This permits the use of the ray concept to describe the radiation propagation. The behavior of rays depends on the refractive index n . If n does not vary in the medium, the rays of light are straight. If the refractive index varies (depends on \mathbf{r}), they are bent. If the medium is anisotropic along the path between scattering events (n depends on \mathbf{u}), the ray-refraction index is employed to describe the behavior of rays [10]. This quantity should be used when treating radiation in, for example, solid anisotropic materials or in plasmas in sufficiently strong magnetic fields. In the following, for the sake of simplicity, we will assume that the refraction index n is uniform and independent of the location (\mathbf{r}) and direction (\mathbf{u}).

The coefficients introduced above obey the reciprocity relations that link the forward-scattering and backscattering characteristics. These scattering events are shown schematically in Fig. 3. Let a direction- and frequency-changing scattering event occur in the path l . The scattering process follows the scheme

$$(\mathbf{u}, \nu) \rightarrow (\mathbf{u}', \nu').$$

We will consider this to be a forward-scattering event. Then the reverse to it is the scattering event that occurs in the path 2 according to the scheme

$$(-\mathbf{u}', \nu') \rightarrow (-\mathbf{u}, \nu).$$

The initial and final directions in such forward- and backscattering events are opposite, and the frequency changes are also opposite.

Usually, the reciprocity relations and the radiation–matter interaction characteristics themselves are obtained simultaneously from the equations describing the scattering process. Thus, in the case of elastic scattering by small macroscopic particles, the reciprocity relations can be

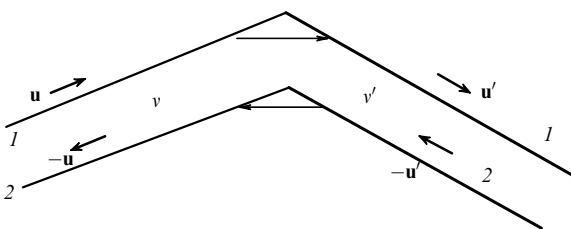


Figure 3. Forward (1) and back (2) single scattering events. The light lines refer to frequency ν ; the heavy lines, to ν' .

obtained solving the Maxwell equations [41]. The reciprocity relations for scattering by microparticles (atoms, molecules) in an isotropic medium result from solving the Schrödinger equation [44].

In this paper, we will employ the reciprocity relations obtained from the detailed-balancing principle [45]. For the elastic scattering at an arbitrary frequency ν , we have

$$\alpha(\mathbf{u})\lambda(\mathbf{u})\chi(\mathbf{u} \rightarrow \mathbf{u}') = \alpha(-\mathbf{u}')\lambda(-\mathbf{u}')\chi(-\mathbf{u}' \rightarrow -\mathbf{u}). \quad (2.7)$$

Suppose that the scattering is inelastic but the frequency changes it causes are small enough for the following approximate equalities to be true:

$$\frac{I^0(T, \nu)}{h\nu} \cong \frac{I^0(T, \nu')}{h\nu'},$$

$$n^2(\nu) \cong n^2(\nu').$$

Here,

$$I^0(T, \nu) = \left[\exp\left(\frac{h\nu}{kT}\right) - 1 \right]^{-1} h\nu \frac{2\nu^2}{c^2} \quad (2.8)$$

is the Planck function; h and k are the Planck constant and the Boltzmann constant, respectively; c is the speed of light; and T is the temperature.

In accord with Ref. [45], we have

$$\alpha(\mathbf{u}, \nu) A(\mathbf{u} \rightarrow \mathbf{u}', \nu \rightarrow \nu') \cong \alpha(-\mathbf{u}', \nu') A(-\mathbf{u}' \rightarrow -\mathbf{u}, \nu' \rightarrow \nu) \quad (2.9)$$

in this case.

Equations (2.7) and (2.9) imply that the probabilities of single forward- and backscattering events are equal if the scattering is elastic or if the inelasticity is such that the frequency changes are sufficiently small.

If the medium is a mixture of different components, each k th component can be characterized by the corresponding extinction coefficients α_k and absorption coefficients k_k , as well as by the scattering probabilities A_k or λ_k , $\chi_k/(4\pi)$. The general characteristics of the mixture are then given by the following expressions:

$$\alpha = \sum_k \alpha_k, \quad k = \sum_k k_k, \quad A = \sum_k \frac{\alpha_k A_k}{\alpha}, \quad (2.10)$$

$$\lambda = \sum_k \frac{\alpha_k \lambda_k}{\alpha}, \quad \chi = \sum_k \frac{\lambda_k \alpha_k \chi_k}{\alpha \lambda}.$$

The reciprocity relations (2.7) and (2.9) hold both for the general characteristics of the mixture and for those of each k th component.

To conclude this section, we recall that, for the geometrical-optics approximation to be applicable, rather stringent conditions should be met. It is worthwhile to indicate the conditions necessary for the applicability of the theory of transfer and hence of the characteristics introduced above. First, as usual, the radiation fluxes should not vary considerably over a distance comparable to the wavelength l . This requirement is met if $kl \ll 1$ and $\alpha l \ll 1$. Second, the scatterers should be separated widely in the sense that the distances between them should greatly exceed both the wavelength and the distance from a scatterer to the wave zone $2\pi d^2/l$, d being the size of the scatterer [26].

The following two examples will demonstrate how the above conditions put a limit on the concentration of particles in the medium.

First, consider the radiation at the center of the resonant sodium atomic line $l = 589 \text{ nm} \cong 6 \times 10^{-5} \text{ cm}$ in a gaseous medium. This line has been widely used for years in the diagnostics of the combustion products for various kinds of fuel. Let N_{Na} be the concentration of sodium atoms and let the radiation absorption cross section be σ_{Na} . Then the coefficient of absorption by sodium atoms can be written in the form $k_{\text{Na}} = N_{\text{Na}} \sigma_{\text{Na}}$. At the center of the 589 nm line, under the atmospheric pressure, $\sigma_{\text{Na}} = 2 \times 10^{-12} \text{ cm}^2$ [46]. From the first condition of the applicability of geometrical optics, we obtain the following restriction on the concentration of sodium atoms:

$$k_{\text{Na}} l = N_{\text{Na}} (2 \times 10^{-12}) (6 \times 10^{-5}) \ll 1,$$

or

$$N_{\text{Na}} \ll 10^{16} \text{ cm}^{-3}.$$

Second, consider the scattering of radiation at a wavelength of $l = 0.5 \mu\text{m}$ by particles of size $d = 5 \mu\text{m}$. The distance between particles can be estimated as $l_{\text{p-p}} \sim N_{\text{p}}^{-1/3}$, where N_{p} is the particle concentration. The inequality $2\pi d^2/l \ll l_{\text{p-p}}$ leads to the condition $N_{\text{p}} \ll 10^5 \text{ cm}^{-3}$.

Both of these inequalities considerably reduce the applicability of geometrical optics to practical calculations.

2.2 The radiation-transfer equation

As radiation is treated in the framework of linear transfer theory, two characteristics of radiation are used. The first one, intensity, is the flux of radiant energy per unit frequency passing through the medium per unit time, per unit solid angle, and per unit area perpendicular to the flux $[I(\mathbf{r}, \mathbf{u}, \nu)]$. The second is the radiation flux emitted by the medium per unit time, per unit solid angle per unit frequency, and per unit volume $[j(\mathbf{r}, \mathbf{u}, \nu)]$. The emission of radiation by the medium occurs due to two factors — the primary emission and the scattering of the radiation that comes from all directions.

If the refractive index n determining the propagation of radiation between scattering events is uniform, the stationary radiation-transfer equation can be written in the form [10]

$$\frac{\partial}{\partial b} [I(\mathbf{r}, \mathbf{u}, \nu)] = j(\mathbf{r}, \mathbf{u}, \nu) - \alpha(\mathbf{r}, \mathbf{u}, \nu) I(\mathbf{r}, \mathbf{u}, \nu). \quad (2.11)$$

This is the energy-balance equation for the radiation flux in a ray tube. The differentiation is carried out along the ray b passing through the point \mathbf{r} in the \mathbf{u} direction (see Fig. 2).

We now determine the boundary conditions for the problem. We assume that the intensity I_S of the radiation coming into the radiator from the enclosing surface S is specified. The radiation that comes from the volume v onto the surface S does not affect the magnitude of I_S . In other words, the radiation from the volume v completely disappears as it arrives at the surface S . For this boundary condition to be satisfied, the surface S should be constructed in an appropriate way. This can usually be done without difficulties, and we will take advantage of this below. Then, from the transfer equation (2.11), the following expression for the radiation intensity in terms of the flux j (see Fig. 2) can be

derived:

$$I(\mathbf{r}, \mathbf{u}, \nu) = \int_{\mathbf{r}_0}^{\mathbf{r}} j(\mathbf{r}^*, \mathbf{u}, \nu) \exp[-t(\mathbf{r}^* \rightarrow \mathbf{r}, \nu)] d\mathbf{r}^* + I_S(\mathbf{r}_0, \mathbf{u}, \nu) \exp[-t(\mathbf{r}_0 \rightarrow \mathbf{r}, \nu)]. \quad (2.12)$$

The first term is the contribution to $I(\mathbf{r}, \mathbf{u}, \nu)$ due to the radiation the medium emits in the neighborhood of the ray b . It is along this ray that the integration is performed. The second term is the contribution from the radiation that comes from the surface directly, without interacting with the medium. Here $I_S(\mathbf{r}_0, \mathbf{u}, \nu)$ is the intensity at the point \mathbf{r}_0 in the direction \mathbf{u} of the ray b .

In a similar way, the intensity at the same point but in a different direction \mathbf{u}' (along the ray b') and at a different frequency ν' can be written as

$$I(\mathbf{r}, \mathbf{u}', \nu') = \int_{\mathbf{r}'_0}^{\mathbf{r}} j(\mathbf{r}', \mathbf{u}', \nu') \exp[-t(\mathbf{r}' \rightarrow \mathbf{r}, \nu')] d\mathbf{r}' + I_S(\mathbf{r}'_0, \mathbf{u}', \nu') \exp[-t(\mathbf{r}'_0 \rightarrow \mathbf{r}, \nu')]. \quad (2.13)$$

Let p_ν be the specific flux of the intrinsic primary radiation per unit volume of the medium. Adding the flux due to the scattering of omnidirectional radiation to p_ν makes it possible to express the radiation flux j in terms of the intensity I :

$$j(\mathbf{r}, \mathbf{u}, \nu) = p_\nu(\mathbf{r}, \mathbf{u}, \nu) + \int_{4\pi} d\mathbf{u}' \int_{\Delta\nu} d\nu' I(\mathbf{r}, \mathbf{u}', \nu') \alpha(\mathbf{r}, \mathbf{u}', \nu') A(\mathbf{r}, \mathbf{u}' \rightarrow \mathbf{u}, \nu' \rightarrow \nu). \quad (2.14)$$

Here, the integration in the second term on the right extends over all possible frequencies ν' and over all angles, $d\mathbf{u}'$ being a solid-angle element. The equation for j can be obtained by substituting Eqn (2.13) into Eqn (2.14) and written in the following general form:

$$j(\mathbf{r}, \mathbf{u}, \nu) = Vj(\mathbf{r}', \mathbf{u}', \nu') + p_S(\mathbf{r}, \mathbf{u}, \nu) + p_\nu(\mathbf{r}, \mathbf{u}, \nu). \quad (2.15)$$

Let us now consider each term in the right-hand side of Eqn (2.15) separately. Here Vj is the integral operator of scattering. This is the part of the flux j that is determined by the scattering, at the point \mathbf{r} , of the radiation that appeared or was scattered in the volume earlier. The scattering operator can be obtained by substituting the first term in the right-hand side of Eqn (2.13) into the second term in the right-hand side of Eqn (2.14), which yields

$$Vj(\mathbf{r}', \mathbf{u}', \nu') \equiv \int_{\mathbf{r}'_0}^{\mathbf{r}} d\mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta\nu} d\nu' j(\mathbf{r}', \mathbf{u}', \nu') \times \exp[-t(\mathbf{r}' \rightarrow \mathbf{r}, \nu')] \alpha(\mathbf{r}, \mathbf{u}', \nu') A(\mathbf{r}, \mathbf{u}' \rightarrow \mathbf{u}, \nu' \rightarrow \nu). \quad (2.16)$$

The intensity p_S of the primary scattering of the radiation from the surface at an arbitrary point \mathbf{r} in the volume can be obtained by substituting the second term in the right-hand side of Eqn (2.13) into the second term in the right-hand side of Eqn (2.14). In doing this, it is helpful to change from the integration over angles to the integration over the surface. Suppose the surface S is not concave and let θ be the angle between the direction toward the point \mathbf{r} and the normal to the surface (see Fig. 2). Then $d\mathbf{u}' = d^2\mathbf{r}'_0(\mathbf{r} - \mathbf{r}'_0)^{-2} \cos \theta$, where $d^2\mathbf{r}'_0$ is an element of the surface S . The result for $\nu' = \nu'_0$ can

be written as

$$p_S(\mathbf{r}, \mathbf{u}, \nu) = \int_S d^2\mathbf{r}'_0 \int_{\Delta\nu} d\nu'_0 \cos\theta(\mathbf{r} - \mathbf{r}'_0)^{-2} I_S(\mathbf{r}'_0, \mathbf{u}'_0, \nu'_0) \times \exp[-t(\mathbf{r}'_0 \rightarrow \mathbf{r}, \nu'_0)] \alpha(\mathbf{r}, \mathbf{u}'_0, \nu'_0) A(\mathbf{r}, \mathbf{u}'_0 \rightarrow \mathbf{u}, \nu'_0 \rightarrow \nu). \quad (2.17)$$

The primary sources p_ν in Eqn (2.15) are determined by the primary radiation of the medium and may be arbitrary. Two such sources will be specified below. Here we only note that, if the medium in the volume v is a mixture of various constituents, then the radiation from each of them can be described by its own primary source p_k , and the primary radiation in the volume is determined by the sum

$$p_\nu(\mathbf{r}, \mathbf{u}, \nu) = \sum_k p_k(\mathbf{r}, \mathbf{u}, \nu). \quad (2.18)$$

Equation (2.15) is the basic integral equation for radiation transfer. Once the equation is solved (i.e., the flux j is expressed in terms of the primary sources p), the intensity at an arbitrary point in the medium can be found from Eqns (2.12), (2.13).

Equation (2.15) was obtained for a fairly general case. Indeed, the expressions for the scattering operator V [see Eqn (2.16)] and for the primary source p_S [see Eqn (2.17)] hold for an arbitrary geometry and inhomogeneity of the radiator; the medium may be anisotropic with respect to the absorption and scattering of radiation (α, k, A depend on \mathbf{u}); the scattering may be either elastic or inelastic; the external illumination may be different at various locations at the enclosing surface and may vary with the direction and frequency (I_S depends on $\mathbf{r}'_0, \mathbf{u}'_0, \nu$).

Here we are interested in general relations. However, in solving many practical problems, there is no need for the general equation obtained. From this equation, various specific cases with relatively simple expressions for the scattering operator V and the primary source p_S can be obtained (see, e.g., Ref. [42]).

2.3 Intensity expressed in terms of primary sources

The integral equation of transfer (2.15) is linear in the radiation flux j . Let us first make use of the general properties of linear integral equations. Let a primary radiation source be described by the function $p_m(\mathbf{r}', \mathbf{u}', \nu')$. Any such primary source guarantees the appearance of radiation flux j_m at an arbitrary point in the radiator. The sum of fluxes j_m corresponds to the sum of primary sources. Using Eqns (2.15) and (2.18) we arrive at the following expression for the resulting radiation flux from a mixture of k components at the frequency ν and at the point \mathbf{r}^* , in the direction \mathbf{u} of the ray b (see Fig. 2):

$$j(\mathbf{r}^*, \mathbf{u}, \nu) = \sum_k j_k(\mathbf{r}^*, \mathbf{u}, \nu) + j_S(\mathbf{r}^*, \mathbf{u}, \nu). \quad (2.19)$$

The substitution of Eqn (2.19) into Eqn (2.12) allows the intensity to be written as the sum

$$I(\mathbf{r}, \mathbf{u}, \nu) = \sum_k I_k(\mathbf{r}, \mathbf{u}, \nu) + I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, \nu) + I_S(\mathbf{r}_0, \mathbf{u}, \nu) \exp[-t(\mathbf{r}_0 \rightarrow \mathbf{r}, \nu)]. \quad (2.20)$$

Here, each intensity I_k is determined by the primary radiation of the k th component of the mixture in the entire

volume v of the radiator. The intensity $I_{S, \text{scat}}$ is determined by the passage, at $(\mathbf{r}, \mathbf{u}, \nu)$, of the radiation from the entire surface S after being scattered in the medium. The last term in Eqn (2.20) describes the passage of radiation from the surface S to the same point with the same direction and frequency, but without interaction with the medium — which is possible only from the surface point \mathbf{r}_0 if the initial direction of radiation is \mathbf{u} and its frequency is ν .

In order to express the intensities I_k, I_S in terms of primary sources, it is necessary to express j_m explicitly in terms of p_m , i.e., to solve Eqn (2.15). One way to do this is to find the response function for a unit primary source (Green function) and then to write down the general expression for the required function.

Let $G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}^*, \mathbf{u}, \nu)]$ be the Green function describing the response, in the vicinity of $(\mathbf{r}^*, \mathbf{u}, \nu)$, to a unit source of radiation located in the vicinity of $(\mathbf{r}', \mathbf{u}', \nu')$. The initial and the final point can be chosen arbitrarily. Here, the final point $(\mathbf{r}^*, \mathbf{u}, \nu)$ is taken to be on the ray b for future convenience.

The Green function has a probabilistic meaning, namely

$$G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}^*, \mathbf{u}, \nu)] d^3\mathbf{r}' d\mathbf{u}' d\nu'$$

is the probability that the radiation emitted in the vicinity of $(\mathbf{r}', \mathbf{u}', \nu')$ within a volume element $d^3\mathbf{r}'$, a solid-angle element $d\mathbf{u}'$, and a frequency element $d\nu'$ will arrive, via arbitrary paths, at the point \mathbf{r}^* and will be scattered there by the medium, in the direction \mathbf{u} , with the frequency ν , within unit volume, solid angle, and frequency ranges. No intermediate hits on the surface between $(\mathbf{r}', \mathbf{u}', \nu')$ and $(\mathbf{r}^*, \mathbf{u}, \nu)$ will occur, since in the problem under study all possible reflections of the radiation from the surface S have already been taken into account by the specified intensity of radiation from the surface I_S . The equation for the Green function G can be constructed based on the equation to be solved, i.e., Eqn (2.15) in our case. Let us now write

$$G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}^*, \mathbf{u}, \nu)] = V\{G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}'', \mathbf{u}'', \nu'')]\} + \delta[(\mathbf{r}', \mathbf{u}', \nu') - (\mathbf{r}^*, \mathbf{u}, \nu)] \quad (2.21)$$

and show that this equation is in line with the meaning of the Green function as described above [50, 51].

The passage of radiation from $(\mathbf{r}', \mathbf{u}', \nu')$ through the scattering medium with a final scattering event at \mathbf{r}^* is described by the scattering operator V . The Dirac function accounts for the possibility of the appearance of primary radiation of unit intensity at $(\mathbf{r}', \mathbf{u}', \nu') = (\mathbf{r}^*, \mathbf{u}, \nu)$. The scattering operator V can be written as (see Fig. 2)

$$V\{G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}'', \mathbf{u}'', \nu'')]\} \equiv \int_{\mathbf{r}'_0}^{\mathbf{r}^*} d\mathbf{r}'' \int_{4\pi} d\mathbf{u}'' \int_{\Delta\nu} d\nu'' G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}'', \mathbf{u}'', \nu'')] \times \exp[-t(\mathbf{r}'' \rightarrow \mathbf{r}^*, \nu'')] \alpha(\mathbf{r}^*, \mathbf{u}'', \nu'') A(\mathbf{r}^*, \mathbf{u}'' \rightarrow \mathbf{u}, \nu'' \rightarrow \nu). \quad (2.22)$$

The integrand is the probability that the radiation traveling from $(\mathbf{r}', \mathbf{u}', \nu')$ to $(\mathbf{r}^*, \mathbf{u}, \nu)$ will pass through an arbitrary intermediate point \mathbf{r}'' on the ray b'' , where the radiation has the direction \mathbf{u}'' and the frequency ν'' . This can be seen if we note that $G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}'', \mathbf{u}'', \nu'')]$ is the

probability of passage via arbitrary paths from $(\mathbf{r}', \mathbf{u}', v')$ to $(\mathbf{r}'', \mathbf{u}'', v'')$; $\exp[-t(\mathbf{r}'' \rightarrow \mathbf{r}^*, v'')]$ is the probability of passage from \mathbf{r}'' to \mathbf{r}^* along the ray b'' without interacting with the medium; and $\alpha(\mathbf{r}^*, \mathbf{u}'', v'') A(\mathbf{r}^*, \mathbf{u}'' \rightarrow \mathbf{u}, v'' \rightarrow v)$ is the probability of scattering at the point \mathbf{r}^* in the direction \mathbf{u} with frequency v . These events are independent, so that the probability that all these events occur jointly is given by the product of the probabilities of the individual events.

The radiation we are considering may reach the point \mathbf{r}^* in the above-described way, passing through various points \mathbf{r}'' with various frequencies v'' . These are mutually exclusive events. The probability that all these events occur in combination is determined by the sum of the probabilities of the individual events. Therefore, the integration over the points \mathbf{r}'' on the ray b'' , over all rays passing through \mathbf{r}^* (i.e., over \mathbf{u}''), and over all possible intermediate frequencies v'' takes into account all possible appearances, at $(\mathbf{r}^*, \mathbf{u}, v)$, of the radiation that appeared earlier at $(\mathbf{r}', \mathbf{u}', v')$. Thus, the above equation does indeed agree with the already noted probabilistic interpretation of the Green function.

Given the meaning of the response function, one can write down the solution to Eqn (2.15) for arbitrary primary radiation in the volume $p_m(\mathbf{r}', \mathbf{u}', v')$. In the vicinity of \mathbf{r}^* , in the direction \mathbf{u} , and at the frequency v , we have

$$j_m(\mathbf{r}^*, \mathbf{u}, v) = \int_v d^3\mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta v} dv' p_m(\mathbf{r}', \mathbf{u}', v') \times G[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v)]. \quad (2.23)$$

Here $d^3\mathbf{r}'$ is a volume element around the point \mathbf{r}' . The integration extends over the entire volume v , all possible initial directions \mathbf{u}' at the point \mathbf{r}' and all frequencies v' . The component of the intensity at $(\mathbf{r}, \mathbf{u}, v)$ determined by the primary source p_m can be obtained by the substitution of Eqn (2.23) into the integral in the right-hand side of Eqn (2.12) and can be written in the form

$$I_m(\mathbf{r}, \mathbf{u}, v) = \int_v d^3\mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta v} dv' \times \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r}^* i_m[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)], \quad (2.24)$$

where

$$i_m[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)] \equiv p_m(\mathbf{r}', \mathbf{u}', v') P[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)], \quad (2.25)$$

$$P[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)] \equiv G[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v)] \exp[-t(\mathbf{r}^* \rightarrow \mathbf{r}, v)]. \quad (2.26)$$

The function $i_m[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)]$ represents the contribution to the intensity I_m from the radiation that first appeared in unit volume, solid angle, and frequency elements about $(\mathbf{r}', \mathbf{u}', v')$ and then passed through the scattering medium to the point \mathbf{r}^* , where it underwent its last scattering before reaching the point of observation $(\mathbf{r}, \mathbf{u}, v)$. The integrations in Eqn (2.24) take into account all possibilities of the primary appearance of radiation as well as all possible points of the last scattering on the ray b . The meaning of the function P can be understood from expression (2.25) and the probabilistic interpretation of the Green function; specifically,

$$P[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)]$$

is the probability density for the radiation passage, via arbitrary paths, from $(\mathbf{r}', \mathbf{u}', v')$ to \mathbf{r}^* , scattering at the point \mathbf{r}^* in the direction \mathbf{u} with the frequency v , and arrival at $(\mathbf{r}, \mathbf{u}, v)$ without interacting with the medium again.

Expressions (2.24)–(2.26) also describe the intensities of the scattered external radiation. These can be obtained by substituting Eqn (2.17) into Eqn (2.23) and substituting the result into Eqn (2.12), which yields

$$I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, v) = \int_S d^2\mathbf{r}'_0 \int_v d^3\mathbf{r}' \times \int_{\Delta v} dv'_0 i_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, v)], \quad (2.27)$$

where

$$i_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, v)] \equiv I_S(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) P_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, v)]. \quad (2.28)$$

Here $i_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, v)]$ is the contribution to $I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, v)$ from the primary radiation that originated on the surface S around $(\mathbf{r}'_0, \mathbf{u}'_0, v'_0)$, was scattered first at the point \mathbf{r}' , and then passed through the medium to $(\mathbf{r}, \mathbf{u}, v)$. We omit here the cumbersome expression for P_{scat} and note only that it includes integrations over \mathbf{u}' , v' from Eqn (2.23) and over \mathbf{r}^* from Eqn (2.12). Each first-scattering location \mathbf{r}' has its own corresponding direction \mathbf{u}'_0 of the initial radiation at \mathbf{r}'_0 (see Fig. 2). Therefore, the integrations over \mathbf{r}' in Eqn (2.27) also take into account all initial directions \mathbf{u}'_0 for any given \mathbf{r}'_0 . The probabilistic interpretation of the function P_{scat} is analogous to that of P , with the difference that P_{scat} includes only the probabilities of radiation passing from $(\mathbf{r}'_0, \mathbf{u}'_0, v'_0)$ to $(\mathbf{r}, \mathbf{u}, v)$ after being scattered somewhere inside the volume v .

Thus, the components I_k , $I_{S, \text{scat}}$, i_k , i_{scat} of the total radiation intensity I are now expressed in terms of the primary sources p_k of radiation in the volume and in terms of the external illumination intensity I_S . The intensity components are related to the primary radiation through the probabilities for radiation to pass through its point of origin to the point of observation.

We next write down the expression for the total intensity substituting the expression (2.24) for $m = k$ and Eqn (2.27) into Eqn (2.20). This yields

$$I(\mathbf{r}, \mathbf{u}, v) = \sum_k \int_v d^3\mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta v} dv' \times \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r}^* i_k[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)] + \int_S d^2\mathbf{r}'_0 \int_v d^3\mathbf{r}' \int_{\Delta v} dv'_0 i_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, v)] + I_S(\mathbf{r}_0, \mathbf{u}, v) \exp[-t(\mathbf{r}_0 \rightarrow \mathbf{r}, v)]. \quad (2.29)$$

Similar intensity expressions are obtained in Refs [50, 51] for the case of the refractive index n varying over the medium. Expressions (2.24)–(2.29) simplify in special cases. Let us consider two of them.

(1) In elastic scatterings ($v = v'$), all integrations over v' drop out. This results from the use of Eqn (2.4) with the δ function: after integrating over frequency, only integrands for $v' = v$ survive. (It is recognized that the frequency interval Δv covers all possible frequencies and can be extended down to 0 and up to ∞ .)

(2) If there is no scattering ($\lambda = A = 0$), then, instead of Eqn (2.21) describing the Green function, we obtain

$$G[(\mathbf{r}', \mathbf{u}', \nu) \rightarrow (\mathbf{r}^*, \mathbf{u}, \nu)] = \delta[(\mathbf{r}', \mathbf{u}', \nu) - (\mathbf{r}^*, \mathbf{u}, \nu)]. \quad (2.30)$$

Upon substituting this into Eqn (2.26), substituting the result into Eqns (2.25) and (2.24), and carrying out the integration for $m = k$, we find

$$I_k(\mathbf{r}, \mathbf{u}, \nu) = \int_{r_0}^r p_k(\mathbf{r}^*, \mathbf{u}, \nu) \exp[-t(\mathbf{r}^* \rightarrow \mathbf{r}, \nu)] d\mathbf{r}^*. \quad (2.31)$$

This is a well-known expression, applied in many cases to radiating and absorbing non-scattering media.

3. Relations between radiation intensity components

3.1 Equilibrium relations

Let us discuss briefly the successive steps in the derivation of relations between the intensity components due to different primary radiation sources. First, from the thermodynamic equilibrium conditions for a closed isothermal cavity, equilibrium relations between the thermal radiation components I_k and I_S were obtained for various cases [42, 47–49]. The specific cases differed in the characteristics of the medium and of the radiation–material interaction (we considered elastic and inelastic scattering, isotropic and anisotropic optical characteristics, and different refractive indices for the passage of radiation between scattering events). Next, relations between the partial intensities i_k, i_S were derived along the same lines [50, 51]. Let us explain how the equilibrium relations can be obtained.

Primary sources in the case of thermal radiation are described by the classical Kirchhoff–Clausius law. In the above notation, the rate of the primary emission of radiation by the k th material component can be written as

$$p_k(\mathbf{r}, \mathbf{u}, \nu) = k_k(\mathbf{r}, \mathbf{u}, \nu) n^2(\nu) I^0[T_k(\mathbf{r}), \nu]. \quad (3.1)$$

Recall that, if the temperature variations are smooth enough, such expressions can also be applied locally to a nonisothermal medium. If the primary radiation from an object is determined by its temperature, both the object and radiation are called *thermal*, whether the internal temperature of the object varies or not. For any object containing a radiating, absorbing, and scattering medium, thermal radiation can be described by the expressions which were obtained in Section 2.3 from the equations of transfer. For the primary radiation, Eqn (3.1) should be used. Such a description can be applied, in particular, to the radiation in a closed cavity containing the medium. If the temperature T of all components of the medium and of the cavity surface is everywhere the same, then the total radiation intensity at arbitrary points inside the medium and near the surface is equilibrium and isotropic, and, for a fixed refractive index of the medium n , is determined by the following equalities:

$$I_{\text{eq}}(T, \nu) = I_{S, \text{eq}}(T, \nu) = n^2(\nu) I^0(T, \nu). \quad (3.2)$$

Many important relations (including the Kirchhoff–Clausius law) were derived from the detailed balancing principle, valid under the conditions of thermal equilibrium.

In deriving equilibrium conditions between the components originating from various parts of the radiator, the detailed balancing principle was not employed. In all the above-mentioned studies, only the known equilibrium values of the resulting total intensity (3.2) were used. This approach was used earlier to find equilibrium relations between the individual terms of the integral transfer equation, for which purpose the equilibrium fluxes were introduced into the equation itself. In the theory of radiative heat transfer, the closure conditions were obtained precisely in this way [9, 52, 53]. In Refs [54, 55], the closure conditions serve to relate the resolvents of the integral transfer equations. The known equilibrium value of the thermal radiation intensity was also used in Ref. [56] to relate two equilibrium radiation components — the primary component and that scattered by the medium.

In Refs [42, 47–51], the general solutions of the equation of transfer were equated to the equilibrium intensities in order to obtain equilibrium relations between the intensity components of different nature. Then both sides of the equality were divided by the equilibrium intensity, which yielded equilibrium relations between the relative intensity components (here, we use this procedure only in Section 5.3, in the case where geometrical optics does not work). Since the relative components do not depend directly on temperature but are determined by the optical parameters of the radiation–matter interaction, it was concluded that the equilibrium relations could also be applied to nonisothermal objects provided the temperature changes do not appreciably affect the optical parameters. The range of applicability of the equilibrium relations is limited by the following conditions. First, the relations apply only to thermal radiation. Second, a closed radiating surface must surround the radiator. And finally, the method used to obtain the relations also poses restrictions on their applicability to a nonisothermal radiator.

Following this, a method based on the probabilistic description the stationarity conditions for radiation was proposed to obtain the relations of interest [57, 58]. This approach dramatically extended the range of validity of the resulting relations. The primary radiation might now be arbitrary in its nature and not necessarily thermal. A closed radiating surface around the radiator was no longer needed. The confining surface was allowed to be partially or fully transparent and not necessarily illuminated from the outside. Also the above-mentioned restriction on the variations in the thermal-radiator temperature was removed. This method was used to obtain stationary relations between the thermal-radiation components [57, 58]. In Refs [59, 60], it was even extended to the case of arbitrary primary radiation.

In Section 3, the second method will be used. On this basis, stationary relations will be derived, and it will be shown that the equilibrium relations can be obtained from the stationary ones in some special case. As before, the geometrical-optics approximation will be assumed applicable.

3.2 Radiation disappearance probability

Let us consider a flux of radiation passing through a certain region in a certain direction at a given frequency. Any such flux is produced by the radiation that first originated somewhere and then passed to the region in question. The primary radiation results from the conversion of the energy of the material into radiation energy inside the radiator or, alternatively, enters from the outside. On the other hand,

the radiation flux under study disappears. This may be due to the absorption of energy by matter in various parts of the radiator as well as to the escape of radiation from the object. In a stationary object, the radiation that appears at some rate completely disappears at the same rate. This also refers to the radiation that forms the flux under study, in particular, to its intensity at any place and in any direction. To decide what time periods can be reasonably chosen in the problem, note that the theory of transfer deals with the radiation averaged over times much longer than the radiation oscillation period ν^{-1} and than the time scales for the appearance, disappearance, and propagation of radiation in the object. Consequently, the appearance and disappearance processes can be regarded as instantaneous and simultaneous.

In general, the appearance of radiation in a given part of the object is not compensated by the disappearance of the same amount of radiation in the same part, i.e., there is no detailed balance in the system. The appearance of radiation at a certain place may be compensated by disappearance processes anywhere inside or outside the object.

To obtain stationary relations, we will employ a probabilistic description of radiation. In Section 2.3, by writing the general solutions of the transfer equations in terms of the Green function, we expressed radiation intensity components in terms of the primary sources and the probabilities for radiation passage through the material [see Eqns (2.24)–(2.28)]. On the other hand, from it follows what the above consideration of a stationary radiator that the disappearance of the radiation emerging in a stationary object is a certain event. As known from the probability theory, the probability of such an event is unity [61]. This result can be applied to the probability of disappearance of any radiation flux that appears under stationary conditions, and it is this circumstance that will serve as the basis for the derivation of the relations in question.

Formulating equations for the probability of a process usually requires a detailed account of all events involved in the process and a knowledge of the probability of each of them. In what follows, only two statements of the probability theory will be used: (1) the probability of a combination of independent events equals the product of the probabilities of all these events and (2) the probability of a combination of mutually exclusive events equals the sum of the probabilities of these events. These statements have already been used in interpreting the equations and functions of relevance in Section 2.

We take advantage of the fact that we can consider the disappearance probability for radiation of any direction and any frequency arriving from wherever we wish and at any place within the stationary object. At the point of observation \mathbf{r} , we will consider the radiation that has a frequency ν and whose direction is $(-\mathbf{u})$, opposite to the direction of observation assumed for the expressions for intensity obtained above [Eqns (2.24)–(2.29)]. Let us first discuss the probability of this radiation disappearing due to absorption within a unit volume about an arbitrary point in the medium. This probability is determined by the following set of successive independent events:

(1) Radiation comes from the point \mathbf{r} to an arbitrary point \mathbf{r}^* on the ray b (see Fig. 2) without interacting with the medium.

(2) At the point \mathbf{r}^* , the first scattering event takes place.

(3) After being scattered at \mathbf{r}^* , the radiation comes via an arbitrary path, undergoing scatterings in between, to the

neighborhood of an arbitrary point \mathbf{r}' within a unit cross section, in the direction $(-\mathbf{u}')$ and with a frequency ν' .

(4) In the neighborhood of the point \mathbf{r}' the radiation is absorbed over a unit length.

We next introduce notation for the probabilities of various groups of events in the above set and then derive equations relating them.

Let

$$P_{\text{abs}}[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')]$$

describe the probability of the set of all the events (1)–(4);

$$P[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')]$$

be the same for events (1)–(3);

$$g[(\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')]$$

be the same for events (2) and (3); and finally, let

$$k(\mathbf{r}', -\mathbf{u}', \nu')$$

be the absorption coefficient determining the probability of event (4).

These probabilities should actually be understood as probability densities, P_{abs} referring to unit volume, and P and g , to a unit area about the end point \mathbf{r}' of the path of radiation. We will ignore this refinement for the sake of brevity.

From the above definitions, the relations between the introduced probabilities of independent events follow immediately:

$$\begin{aligned} P_{\text{abs}}[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] \\ = P[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] k(\mathbf{r}', -\mathbf{u}', \nu'), \end{aligned} \quad (3.3)$$

$$\begin{aligned} P[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] \\ = g[(\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] \exp[-t(\mathbf{r} \rightarrow \mathbf{r}^*, \nu)]. \end{aligned} \quad (3.4)$$

We now write down the equation that the function g must satisfy,

$$\begin{aligned} g[(\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] = \\ = V\{g[(\mathbf{r}'', -\mathbf{u}'', \nu'') \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')]\} \\ + \delta[(\mathbf{r}^*, -\mathbf{u}, \nu) - (\mathbf{r}', -\mathbf{u}', \nu')], \end{aligned} \quad (3.5)$$

and explain its meaning.

The probability of radiation passing through the medium after being scattered first at $(\mathbf{r}^*, -\mathbf{u}, \nu)$ is described by the scattering operator V , which can be written in the form

$$\begin{aligned} V\{g[(\mathbf{r}'', -\mathbf{u}'', \nu'') \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')]\} \\ \equiv \int_{\mathbf{r}''_0}^{\mathbf{r}^*} d\mathbf{r}'' \int_{4\pi} d(-\mathbf{u}'') \\ \times \int_{\Delta\nu} d\nu'' g[(\mathbf{r}'', -\mathbf{u}'', \nu'') \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] \\ \times \exp[-t(\mathbf{r}^* \rightarrow \mathbf{r}'', \nu'')] \alpha(\mathbf{r}^*, -\mathbf{u}, \nu) \\ \times A(\mathbf{r}^*, -\mathbf{u} \rightarrow -\mathbf{u}'', \nu \rightarrow \nu''). \end{aligned} \quad (3.6)$$

The multipliers in the integrand determine the probabilities of the following successive independent events (from left to right):

$\alpha(\mathbf{r}^*, -\mathbf{u}, \nu) A(\mathbf{r}^*, -\mathbf{u} \rightarrow -\mathbf{u}'', \nu \rightarrow \nu'')$ is the probability that the first scattering event occurs at \mathbf{r}^* ;

$\exp[-t(\mathbf{r}^* \rightarrow \mathbf{r}'', \nu'')]$ is the probability that radiation passes along the ray b'' from \mathbf{r}^* to \mathbf{r}'' without interacting with the medium;

$g[(\mathbf{r}'', -\mathbf{u}'', \nu'') \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')]$ is the probability that a scattering event occurs at \mathbf{r}'' and is followed by an arbitrary passage from $(\mathbf{r}'', -\mathbf{u}'', \nu'')$ to $(\mathbf{r}', -\mathbf{u}', \nu')$.

The intermediate point \mathbf{r}'' may lie anywhere on the ray b'' , the ray b'' may pass through \mathbf{r}^* in any direction $(-\mathbf{u}'')$, and frequencies ν'' may differ. Since events differing in the values of $\mathbf{r}'', -\mathbf{u}'', \nu''$ are mutually exclusive, the probability of a set of such events should be obtained by integrating over them.

Thus, the scattering operator (3.6) takes into account all possibilities of radiation passing from $(\mathbf{r}^*, -\mathbf{u}, \nu)$ to $(\mathbf{r}', -\mathbf{u}', \nu')$ after being scattered in the medium.

Equation (3.5) takes into account the possibility that the initial $(\mathbf{r}^*, -\mathbf{u}, \nu)$ and final $(\mathbf{r}', -\mathbf{u}', \nu')$ points may coincide. In this case, as described by the scattering operator, radiation may pass through the scattering medium and return to $(\mathbf{r}^*, -\mathbf{u}, \nu)$; however, direct absorption at the initial point, without the random walk of radiation within the medium, is also possible. It is the δ function in Eqn (3.5) that allows for such a possibility. The probabilities of all these mutually exclusive events are add up, thus yielding the right-hand side of Eqn (3.5).

The functions P and g give the probabilities that the radiation under study will reach the vicinity of \mathbf{r}' . The probability of arrival and absorption in a unit volume in this region is given by Eqn (3.3). Absorption by various material components at various locations \mathbf{r}' with different frequencies ν' and directions $(-\mathbf{u}')$, as well as the first scattering at various positions \mathbf{r}^* on the ray b , are mutually exclusive events; therefore, the total probability of radiation from $(\mathbf{r}, -\mathbf{u}, \nu)$ being absorbed by the whole of the medium can be written in the form

$$\begin{aligned}
 P_{v, \text{abs}}(\mathbf{r}, -\mathbf{u}, \nu) &= \sum_k P_{k, \text{abs}}(\mathbf{r}, -\mathbf{u}, \nu) \\
 &= \sum_k \int_v d^3\mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta\nu} d\nu' \\
 &\times \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r}^* P[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}^*, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] \\
 &\times k_k(\mathbf{r}', -\mathbf{u}', \nu'). \tag{3.7}
 \end{aligned}$$

Here, expression (2.10) for the absorption coefficient was used, and the integration variable $(-\mathbf{u}')$ was replaced by \mathbf{u}' , which is allowable because the integration extends over all angles.

In a similar way, we write the expression for the probability that radiation will disappear on the enclosing surface S after being scattered within the volume. Since all the radiation reaching the surface disappears there (the radiation from the surface I_S is specified in the problem), we have

$$\begin{aligned}
 P_{S, \text{scat}}(\mathbf{r}, -\mathbf{u}, \nu) &= \int_S d^2\mathbf{r}'_0 \int_v d^3\mathbf{r}' \\
 &\times \int_{\Delta\nu} d\nu'_0 P_{\text{scat}}[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}'_0, -\mathbf{u}'_0, \nu'_0)]. \tag{3.8}
 \end{aligned}$$

For the sake of brevity, P_{scat} incorporates the probabilities of scattering at the point \mathbf{r}' into various directions with various frequencies as well as the probabilities of the first scatterings occurring at various points \mathbf{r}^* on the ray b .

The complete disappearance of radiation emerging under stationary conditions is a certain event. Its probability is determined by all mutually exclusive disappearances both in the volume and on the surface. Note that radiation may have been either scattered or not in the medium before arriving at the surface. In the case of no scattering, the arrival probability is determined by the exponential of the optical density. Since the probability of a certain event is unity, the condition for the complete disappearance of radiation can be written as

$$P_{v, \text{abs}}(\mathbf{r}, -\mathbf{u}, \nu) + P_{S, \text{scat}}(\mathbf{r}, -\mathbf{u}, \nu) + \exp[-t(\mathbf{r} \rightarrow \mathbf{r}_0, \nu)] = 1. \tag{3.9}$$

This equality will be used below as the basis for deriving stationary relations.

3.3 Stationary relations between intensity components

To obtain a stationary relation, we need to introduce intensity components into the condition (3.9). To do this, we compare the equations for the probabilities of radiation disappearance with those for the radiation intensity. We start by comparing Eqns (3.5) and (3.6), which determine the probability g , with Eqns (2.21) and (2.22), which give the Green function for the transfer equation. It can easily be verified that the equations are completely identical if the reciprocity relation (2.7) or (2.9) is satisfied and if the radiation is equally attenuated when traveling in opposite directions along an arbitrary ray, i.e., the following reversibility conditions are met:

$$\begin{aligned}
 k(\mathbf{r}, \mathbf{u}, \nu) &= k(\mathbf{r}, -\mathbf{u}, \nu), \\
 \alpha(\mathbf{r}, \mathbf{u}, \nu) &= \alpha(\mathbf{r}, -\mathbf{u}, \nu), \tag{3.10} \\
 \exp[-t(\mathbf{r}_1 \rightarrow \mathbf{r}_2, \nu)] &= \exp[-t(\mathbf{r}_2 \rightarrow \mathbf{r}_1, \nu)].
 \end{aligned}$$

These conditions may be violated in highly anisotropic media.

If the equations for the functions G and g are identical, we have

$$g[(\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] = G[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}^*, \mathbf{u}, \nu)]. \tag{3.11}$$

If this equality holds, we find, comparing Eqns (2.26) and (3.4) and using Eqn (3.10) again, that

$$\begin{aligned}
 P[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}^*, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}', -\mathbf{u}', \nu')] \\
 = P[(\mathbf{r}', \mathbf{u}', \nu') \rightarrow (\mathbf{r}^*, \mathbf{u}, \nu) \rightarrow (\mathbf{r}, \mathbf{u}, \nu)]. \tag{3.12}
 \end{aligned}$$

In a similar way, we establish the equality of the probabilities of radiation passing between an arbitrary point on the surface and any point inside the volume. We present this result without derivation:

$$\begin{aligned}
 P_{\text{scat}}[(\mathbf{r}, -\mathbf{u}, \nu) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}'_0, -\mathbf{u}'_0, \nu'_0)] \\
 = P_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, \nu'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, \nu)]. \tag{3.13}
 \end{aligned}$$

Equations (3.12) and (3.13) imply that the probabilities that radiation passes from one arbitrary point in the scattering media to another and back are equal. These equalities follow from the fact that the reciprocity relations

are obeyed in every scattering event in the medium and that the reversibility conditions (3.10) hold.

Using Eqns (3.7), (3.8), equalities (3.12), (3.13), and expressions (2.25), (2.28), we can introduce the partial radiation components i_k , i_{scat} into the stationarity condition (3.9). In doing this, it should be kept in mind that i_{scat} is determined only by that part of the enclosing surface which is illuminated from the outside or radiates. In the general case, the closed surrounding surface can be divided into two parts,

$$S = S_{\text{rad}} + S_{\text{ex}},$$

where S_{rad} supplies radiation to the volume (and $I_S \neq 0$ on it), whereas, from S_{ex} , radiation is not supplied but it only escapes through it from the volume ($I_S = 0$ there). The surface may be either fully absorbing and cold or transparent and not illuminated from the outside. Accordingly, we should divide the region of surface integration in Eqn (3.8) into two parts and then introduce i_{scat} only into that part which radiates or is illuminated from the outside. The result is

$$\begin{aligned} & \sum_k \int_v d^3\mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta v} dv' \\ & \times \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r}^* \frac{i_k[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)] k_k(\mathbf{r}', \mathbf{u}', v')}{p_k(\mathbf{r}', \mathbf{u}', v')} \\ & + \int_{S_{\text{rad}}} d^2\mathbf{r}'_0 \int_v d^3\mathbf{r}' \int_{\Delta v} dv'_0 \frac{i_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}') \rightarrow (\mathbf{r}, \mathbf{u}, v)]}{I_S(\mathbf{r}'_0, \mathbf{u}'_0, v'_0)} \\ & + P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, v) + \exp[-t(\mathbf{r} \rightarrow \mathbf{r}_0, v)] = 1. \end{aligned} \quad (3.14)$$

Here, $P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, v)$ is the probability that radiation arrives at the transmitting surface S_{ex} after being scattered inside the medium. This probability is given by a formula of the form (3.8), but with the integration extending over S_{ex} rather than S . In writing the absorption coefficient k_k , the first of the equalities (3.10) was used here.

Equation (3.14) is the condition of the stationarity of radiation and describes relations between the partial intensity components i_k and i_{scat} in the general case. In Ref. [59], the stationarity condition is also obtained for the special case of the refractive index n varying over the medium.

Let us now discuss the meaning of the quantities involved in Eqn (3.14). First, we note that the emissivity-to-absorptivity ratio p_k/k_k is equal to the intensity of the k th component of the primary radiation that appears in a volume of unit cross section of thickness k_k^{-1} . Hence, $i_k k_k/p_k$ in Eqn (3.14) is the ratio of the intensity component i_k to the intensity of the k th component of the corresponding primary radiation. Further, i_{scat}/I_S is the ratio of the component i_{scat} of the scattered radiation of the surface to the intensity of its primary radiation. The exponential term is the ratio between the intensity component that has passed from \mathbf{r}_0 to the observation point \mathbf{r} without interacting with the medium and the primary radiation from the surface.

Equation (3.14) involves quantities that are the subject of both computational and experimental spectroscopic studies. In experiments, one can directly measure the intensity I_S of the outside radiation, the optical densities t , and the integrated intensities I given by Eqn (2.20). Also the escape probability $P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, v)$ can be measured (see Section 4.4). Such intensity components as i_k and I_S , as well as the ratio p_k/k_k , usually cannot be measured directly, but often should be found based on other measurements. Relation (3.14) may be useful for this purpose.

It is important point to note that Eqn (3.14) can readily be transformed based on its probabilistic interpretation. Only one example will be discussed here. Let a certain part v_{nonrad} of the volume radiate so little that the corresponding intensity component cannot be detected experimentally. On the other hand, the absorption of radiation may be significant in this area. This may occur if some part of the thermal radiator is much cooler than the rest of it. Then, formally, Eqn (3.14) acquires an indefiniteness of the type $i_k/p_k = 0/0$ associated with the volume v_{nonrad} . This indefiniteness can be taken into account in the following way. The integral over the volume v_{nonrad} in relation (3.14) equals the probability $P_{v_{\text{nonrad}}}$ of absorption in this volume. It can be separated out from the total absorption probability excluding v_{nonrad} from the region of integration. Instead, Eqn (3.14) will explicitly involve the probability $P_{v_{\text{nonrad}}}$ along with the term $P_{S_{\text{ex}}}$. This representation may prove to be more suitable for specific problems.

3.4 Special cases of the stationary relation

Relation (3.14) is fairly general and has a wide range of applicability. In applications, however, there is no need for that much generality. For elastic scattering, as in the case of describing the radiation intensity (Section 2.3), all integrations over frequency disappear in Eqn (3.14). For a radiator open on all its sides and not illuminated from the outside, the term containing the integral over the surface S_{rad} vanishes. Finally, the probability of escaping from the object is often zero, $P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, v) = 0$. This in the case, first, if the entire surrounding surface S radiates or is illuminated from the outside. Second, even in the cases in which a nonradiating surface S_{ex} is present, radiation from $(\mathbf{r}, -\mathbf{u})$ can by no means always reach this surface. There are a number of reasons for this. For example, if the optical density of the radiator is high and absorption significant, then radiation may be completely attenuated on its way to the surface S_{ex} . Conversely, if the optical density is low and scattering occurs only at small angles relative to the initial direction $(-\mathbf{u})$, then the radiation under study comes only to that part of the surface which faces the observer. If the nonilluminated surface S_{ex} is outside this part, the radiation will not reach it. In both cases, $P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, v) = 0$.

Let us specify the emissivity-to-absorptivity ratio (p_k/k_k) for two practically important cases. First, in the case of thermal radiators, the primary radiation from each component is determined by the Kirchhoff–Clausius law (3.1). Then

$$\frac{p_k(\mathbf{r}', \mathbf{u}', v')}{k_k(\mathbf{r}', \mathbf{u}', v')} = n^2(v') I^0 [T_k(\mathbf{r}'), v]. \quad (3.15)$$

Second, consider radiation from a gas. The distribution of populations over atomic and molecular energy levels may be either equilibrium or nonequilibrium. The ratio of the emissivity to the true absorptivity can be obtained for each k th gas species through the use of the Einstein relations (see, e.g., Ref. [62]). It is determined by the ratio of the populations (N) of the energy levels between which the radiation–absorption transition takes place. Let l and u be the lower and upper levels of the given transition for the k th component of the gaseous mixture. Then we can write

$$\begin{aligned} p_k(v') &= \frac{a_{u-l} h \nu' N_u}{4\pi}, \\ k_k(v') &= \frac{c^2}{8\pi(v')^2} \frac{g_u}{g_l} a_{u-l} N_l \left(1 - \frac{g_l N_u}{g_u N_l}\right). \end{aligned}$$

Here, a_{u-1} is the probability of a spontaneous radiative transition from the upper to the lower level, and g_u, g_l are the statistical weights of the levels. From this, we obtain

$$\frac{p_k(\mathbf{r}', \mathbf{u}', v')}{k_k(\mathbf{r}', \mathbf{u}', v')} = \frac{2hv'^3}{c^2} \left[\frac{g_u N_l(\mathbf{r}')}{g_l N_u(\mathbf{r}')} - 1 \right]^{-1}. \quad (3.16)$$

For Boltzmann-distributed energy levels, Eqn (3.16) goes over into Eqn (3.15) for $n = 1$ and the corresponding temperature. The temperature that appears in Eqn (3.15) and the population ratio that appears in Eqn (3.16) are often of interest to experimenters. Determining them for scattering media is a quite complex task. Relation (3.14), in which both these quantities are directly involved, may be helpful in solving such problems.

Note that, in both cases discussed, the small frequency changes due to scattering, as well as radiation direction changes, have virtually no effect on the ratio p_k/k_k , although each of the quantities p_k and k_k may strongly depend on the frequency and direction. The reason is that the dependences are almost the same for both quantities. This invariance of the ratio p_k/k_k over frequency and direction changes is a fairly characteristic feature of the problem.

Next we turn to those special cases of the stationary radiation (3.14) which are related to primary radiator characteristics.

(1) Let p_k/k_k be independent of coordinates, i.e., the relative level populations (or the corresponding temperatures) of each k th material component do not vary inside the radiator. Then the ratio p_k/k_k can be taken out of the integral signs in each term in the sum \sum_k . Also, let the surface S_{rad} radiate uniformly and isotropically, and suppose that its radiation depends weakly on frequency, i.e., I_S does not depend on $(\mathbf{r}'_0, \mathbf{u}'_0, v'_0)$. Then I_S can be taken out of the integral signs in the second term on the left. After that, it can easily be verified that the integrals represent the intensities I_k and $I_{S, \text{scat}}$ in accord with Eqns (2.24) and (2.27). In view of this, instead of Eqn (3.14), we obtain

$$\sum_k \left(\frac{p_k}{k_k} \right)^{-1} I_k(\mathbf{r}, \mathbf{u}, v) + \frac{I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, v)}{I_S} + P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, v) + \exp[-t(\mathbf{r} \rightarrow \mathbf{r}_0, v)] = 1. \quad (3.17)$$

Similar relations can also be obtained in this same way if the ratio p_k/k_k or the intensity I_S do not vary inside individual parts of the volume or the surface. We can then separate out the intensities of the corresponding parts in Eqn (3.14). Such a transformation will be used later in Section 4.2.3.

(2) Let the radiator be thermal and surrounded by a closed radiating wall. Then we should use Eqn (3.15) to express the ratio p_k/k_k , and the primary thermal radiation from the surface should be described by the well-known formula

$$I_S(\mathbf{r}_0, \mathbf{u}_0, v) = n^2(v) I^0[T(\mathbf{r}_0, v)]$$

[cf. Eqn (3.2)]. Furthermore, since the surface is closed, we set $P_{S_{\text{ex}}} = 0$ and $S_{\text{rad}} = S$. In this case, Eqn (3.14) immediately yields the component–component relation obtained earlier from the radiation equilibrium condition for a cavity containing a radiating, absorbing, and scattering medium [50, 51].

Instead of Eqn (3.17), we have

$$\sum_k \frac{I_k(\mathbf{r}, \mathbf{u}, v)}{n^2(v) I^0(T_k, v)} + \frac{I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, v)}{n^2(v) I^0(T_S, v)} + \exp[-t(\mathbf{r} \rightarrow \mathbf{r}_0, v)] = 1. \quad (3.18)$$

Such conditions were derived earlier [42, 47–49] for the conditions of thermodynamic equilibrium in a cavity.

(3) In real objects, the primary radiation characteristics p_k/k_k and I_S usually vary over the bulk and surface of the object. However, one is often concerned with and employs the primary radiation characteristics averaged over volume and surface. The observed radiation should remain unchanged upon averaging. This refers to both the total radiation and its components. Using such averages, we can obtain component–component relations from Eqn (3.14). Sometimes, k, α, A — the optical characteristics of the radiation–matter interaction — are also averaged to simplify the problem. Below the relations between the components will be found without this simplification, assuming that the interaction of radiation with the material does not change on averaging. In this case, the probabilities of radiation passing from one location to another, as well as of its passing and being finally absorbed, also remain unchanged.

We now write the radiation intensity for the k th component. We use Eqns (2.24) and (2.25) with $m = k$ and Eqns (3.3) and (3.12), and employ averaging as indicated above, to obtain

$$\begin{aligned} I_k(\mathbf{r}, \mathbf{u}, v) &= \int_v d^3 \mathbf{r}' \int_{4\pi} d\mathbf{u}' \int_{\Delta v} dv' \\ &\times \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r}^* \frac{p_k(\mathbf{r}', \mathbf{u}', v')}{k_k(\mathbf{r}', \mathbf{u}', v')} k_k(\mathbf{r}', \mathbf{u}', v') \\ &\times P[(\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}^*, \mathbf{u}, v) \rightarrow (\mathbf{r}, \mathbf{u}, v)] \\ &= \left(\frac{p_k}{k_k} \right)_{\text{eff}} P_{k, \text{abs}}(\mathbf{r}, -\mathbf{u}, v). \end{aligned} \quad (3.19)$$

Here, $P_{k, \text{abs}}(\mathbf{r}, -\mathbf{u}, v)$ is the probability of the entire k th component absorbing the radiation from $(\mathbf{r}, -\mathbf{u}, v)$, and $(p_k/k_k)_{\text{eff}}$ is the effective (averaged) emissivity-to-absorptivity ratio of the k th component of the medium.

As mentioned earlier, the volume average may be of prime importance, since the emissivity-to-absorptivity ratio normally depends weakly on the direction and, within the range Δv , on the frequency. In the case of thermal radiators, the counterpart of the effective value $(p_k/k_k)_{\text{eff}}$ is the averaged effective temperature of the k th component $T_{k, \text{eff}}$ and, in the case of atomic-line emission, the effective ratio of the populations of the atomic-transition levels, $(N_u/N_l)_{\text{eff}}$.

In a similar way, we can introduce the effective (averaged) radiation intensity of the surrounding surface S with the use of Eqns (2.27), (2.28), (3.8), and (3.13):

$$\begin{aligned} I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, v) &= \int_{S_{\text{rad}}} d^2 \mathbf{r}'_0 \int_v d^3 \mathbf{r}' \int_{\Delta v} dv'_0 I_S(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \\ &\times P_{\text{scat}}[(\mathbf{r}'_0, \mathbf{u}'_0, v'_0) \rightarrow (\mathbf{r}', \mathbf{u}', v') \rightarrow (\mathbf{r}, \mathbf{u}, v)] \\ &= I_{S_{\text{eff}}} \int_S d^2 \mathbf{r}'_0 \int_v d^3 \mathbf{r}' \\ &\times \int_{\Delta v} dv'_0 P_{\text{scat}}[(\mathbf{r}, -\mathbf{u}, v) \rightarrow (\mathbf{r}', -\mathbf{u}', v') \rightarrow (\mathbf{r}'_0, \mathbf{u}'_0, v'_0)] \\ &= I_{S_{\text{eff}}} P_{S, \text{scat}}(\mathbf{r}, -\mathbf{u}, v). \end{aligned} \quad (3.20)$$

The meaning of this averaging is that the real initial radiation from the surface S_{rad} is distributed uniformly over the entire closed surface S in such a way that the intensity of the scattered radiation $I_{S, \text{scat}}$ remains unchanged. We apply Eqns (3.19) and (3.20) to the condition of the full disappea-

rance of radiation (3.9) and found using Eqn (3.7) that

$$\sum_k \left(\frac{p_k}{k_k} \right)_{\text{eff}}^{-1} I_k(\mathbf{r}, \mathbf{u}, \nu) + \frac{I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, \nu)}{I_{S, \text{eff}}} + \exp[-t(\mathbf{r} \rightarrow \mathbf{r}_0, \nu)] = 1. \quad (3.21)$$

The quantity $P_{S_{\text{ex}}}$ does not appear in this relation, as in the equilibrium relation between the thermal-radiation components (3.18). This is the simplest relationship between the radiation components. If the initial radiation were averaged only over the surface S_{rad} , this relation would not differ from Eqn (3.17).

Let us now discuss some properties of the averaged characteristics. We consider first the effective emissivity-to-absorptivity ratio $(p_k/k_k)_{\text{eff}}$. The way of obtaining this quantity is determined by Eqn (3.19), which implies that, for a given observed intensity, the effective ratio depends on the entire set of local quantities p_k/k_k for the radiator and on the probability of radiation passage through the radiator, i.e., on the radiation-material interaction. If the primary radiation varies inside the radiator and if the interaction is frequency dependent, then the effective ratios also can significantly depend on the frequency, although the corresponding local ratios, as indicated above, depend only weakly on it.

As an illustration, we consider the radiation in the vicinity of an atomic spectral line in the case where the intensity observation point \mathbf{r} lies on the periphery of the radiator. The atomic absorption coefficient in the line depends strongly on the frequency, increasing rapidly as the center of the line is approached. Therefore, the closer to the center, the smaller the fraction of radiation that comes to the observer from distant regions of the radiator, or, in other words, the lower the probability P of the arrival of radiation. If the optical density of the radiator is large enough at the center, then the center-of-the-line light comes to the observer only from the radiator periphery closest to him. Let the radiator be thermal. If the temperature at all depths is the same (i.e., p_k/k_k is constant), then frequency changes do not affect the effective ratio in any way. But if the temperature varies, so does the effective ratio $(p_k/k_k)_{\text{eff}}$. To see this, let the temperature in the central part of the radiator be higher than on the periphery, where the observer is located. Then the light observed at the center of the line is that primarily emitted by the cold peripheral regions of the radiator, whereas at a sufficient distance from the line center the light is emitted at all depths in the radiator. Consequently, if the averaging is done according to Eqn (3.19), then $(p_k/k_k)_{\text{eff}}$ for the center-of-the-line frequency is smaller than for off-center frequencies.

The frequency dependence of the effective ratio $(p_k/k_k)_{\text{eff}}$ (i.e., of the effective relative populations or temperatures) can provide useful information about an inhomogeneous radiator. Based on the above-described effect of frequency on the radiation output in a spectral line, a method was developed to study the temperature distributions in thermal radiators in media with [42] and without [63, 64] scattering. In Section 5.5, we will return to the question of determining the effective temperatures at different radiation frequencies in a scattering medium.

We now consider the effective initial surface intensity $I_{S, \text{eff}}$ defined by Eqn (3.20). This quantity was introduced fairly conventionally: suffice it to say that it is related to the intensity of the scattered rather than total observed radiation from the surface. Nevertheless, $I_{S, \text{eff}}$ may be a very useful

quantity, since it is fairly easy to measure experimentally (see Section 4.3) and can be used in Eqn (3.21) to determine other radiation components. Using $I_{S, \text{eff}}$ in this way may emphasize that part of the frequency dependence of the effective initial intensity which reflects the averaging procedure rather than the frequency dependence of the primary radiation, $I_S(\nu'_0)$. We will discuss this question using Eqns (3.20) and (3.13). Let the real initial intensity $I_S(\mathbf{r}'_0, \mathbf{u}'_0, \nu'_0)$ be constant, so that it can be taken out of the integral signs in the first of the equalities (3.20). Then

$$\frac{I_{S, \text{eff}}}{I_{S, \text{scat}}} = \frac{P_{S_{\text{rad}}}(\mathbf{r}, -\mathbf{u}, \nu)}{P_{S_{\text{rad}}}(\mathbf{r}, -\mathbf{u}, \nu) + P_{S_{\text{ex}}}(\mathbf{r}, -\mathbf{u}, \nu)}. \quad (3.22)$$

We use here the fact that the probability of arrival at the closed surface S is the sum of the arrival probabilities for the surfaces S_{rad} and S_{ex} . The above equality implies that the averaging-induced frequency dependence of $I_{S, \text{eff}}$ can be expected if $P_{S_{\text{ex}}} \neq 0$.

4. Application of relations between intensity components to dusty plasmas

4.1 Conditions in a dusty plasma

The relations obtained above are fairly general and may therefore be useful in solving radiation-transfer problems for various objects. In this section, we discuss their application to a dusty plasma, i.e., to a two-phase system consisting of a gas ($k = a$) and dust particles ($k = p$).

Depending on the region of the spectrum, the intrinsic radiation from a dusty plasma is determined by different plasma components [42]. Thus, both the atoms (molecules) of the gaseous phase and dust particles contribute to the radiation in spectral lines. At the centers of sufficiently strong lines, the contribution from the gas predominates. In the continuum, between the lines, the radiation is due to the particles. Dust particles are responsible for the elastic scattering of radiation. Elastic scattering by atoms (molecules) is negligible in comparison with that due to macroparticles. At frequencies within spectral lines, inelastic scattering on atoms (molecules) may generally be important. For the problems of interest here, however, inelastic scattering is of no importance; therefore, only elastic scattering by macroparticles will be considered in what follows. Also, we can put $n = 1$ for gas.

Primary radiation from solid surfaces exposed to a plasma and dust particles is usually thermal and determined by the temperatures T_S and T_p , respectively. The distribution of population over atomic (molecular) energy levels in the gaseous phase can be considered equilibrium only at sufficiently high pressures. Then the radiation of the gaseous phase is also thermal; it is determined by the temperature T_a . The dusty plasma is called thermal in this case. The gas temperature T_a in a thermal dusty plasma usually differs from the particle temperature T_p and the solid-surface temperature T_S . Torches of combustion products at the atmospheric pressure are an example of such a plasma. At sufficiently low pressures, the population distribution over energy levels is usually nonequilibrium, so that the radiation from atoms is not thermal. Such a dusty plasma forms, for example, in a low-pressure discharge.

In experiments with dusty plasmas, one measures the intensity of the total radiation outgoing from the plasma.

The point \mathbf{r} in this case lies either on the boundary of the object ($\mathbf{r} = \mathbf{r}_S$) (see Fig. 2) or outside it. Also, it is usually possible to measure the optical depth of the plasma, $t_0(v) \equiv t(\mathbf{r}_0 \rightarrow \mathbf{r}_S, v)$, between mutually opposite points \mathbf{r}_S and \mathbf{r}_0 . If the plasma is surrounded by opaque walls, small optical holes are usually made in the walls to enable measurements. From Eqn (2.10), for a dusty plasma, the total optical density t_0 within a spectral line can be found to be

$$t_0(v) = t_a(v) + t_p(v). \quad (4.1)$$

In the continuum, $t_0 = t_p$.

4.2 Examples of applications of equilibrium relations to thermal plasmas

In Refs [42, 47–49, 51], possible applications of equilibrium component–component relations to calculations and experiments were discussed. Here, we discuss some typical examples of their application to thermal plasmas. For brevity, we omit the arguments \mathbf{r} and \mathbf{u} at the intensity symbols and we rewrite Eqn (3.18) in the form

$$\frac{I_a(v)}{I^0(T_a, v)} + \frac{I_p(v)}{I^0(T_p, v)} + \frac{I_{S, \text{scat}}(v)}{I^0(T_S, v)} + \exp[-t_a(v) - t_p(v)] = 1. \quad (4.2)$$

4.2.1 Computation error determination. Since the component–component relations are exact and independent of the specifics of the radiating object, they can be used to determine the computation errors in some cases. In solving the transfer equations to obtain the intensity of radiation from a radiant object, a probabilistic method is highly effective. This method, proposed by V V Sobolev [65], starts with calculating, with due account for scattering, the probability of radiation escaping from the object. For this purpose, an integral transfer equation with a specially constructed free term is solved. The outgoing-radiation intensity is expressed in terms of the power of the primary sources and the escape probability. This method has been developed by many workers [2, 42, 56, 66–68] and is currently actively employed in solving a variety of radiation-transfer problems (see, e.g., Ref. [69]).

The probability method has been used to calculate the relative magnitudes of the components of the radiation intensity for a thermal dusty plasma surrounded by opaque radiating walls. The results are presented in Refs [42, 47–49, 68, 70]. The radiator shape was varied, along with a number of parameters, such as the optical densities of atoms and particles [$t_a(v)$, t_p], survival probabilities λ_p , and scattering indicatrices for particles χ . Calculations were performed for both spectral lines and the continuum.

To estimate the computation errors, relation (4.2) was used. The calculated relative intensities and the assumed optical densities were inserted into the left-hand side of Eqn (4.2), and it was the deviation of the result from unity that served as a measure of the error.

4.2.2 Finding individual intensity components. Relation (4.2) involves the relative values of three radiation components — I_a , I_p , and $I_{S, \text{scat}}$. In many cases, one of these is absent. This is the case in the continuum, where $I_a = 0$, and in the case of white, fully scattering particles, for which $I_p = 0$. In such cases, if one of the relative components is known or simple to determine, the second can be found using a component–

component relation. We consider here only one example, radiation in the continuum, assuming that the dust macro-particles are sufficiently large and their scattering indicatrix is strongly forward-elongated, i.e., the scattering angles are small [41]. Then the intensity of the radiation that appears in the bulk of the radiator can be evaluated under the assumption that the forward-scattered light does not interact with the medium. It is admissible to assume in this case that the radiation–material interaction reduces to absorption alone. We then have

$$\frac{I_p(v)}{I^0(T_p, v)} \approx 1 - \exp[-(1 - \lambda)t_p]. \quad (4.3)$$

The well-known transfer approximation [71] yields similar results.

On the other hand, for the other radiation component, $I_{S, \text{scat}}(v)/I^0(T_S, v)$, treating scattering in this crude and simplistic manner is not adequate, since this component is fully determined by scattering. However, it can easily be found from Eqns (4.2) and (4.3) that

$$\frac{I_{S, \text{scat}}(v)}{I^0(T_S, v)} \approx \exp(-t_p) [\exp(\lambda t_p) - 1]. \quad (4.4)$$

The approximate relations (4.3) and (4.4) were employed [48] to obtain the relative intensities $I_p/I^0(T_p)$ and $I_{S, \text{scat}}/I^0(T_S)$ in the continuum for a plane layer. The results were practically identical to those obtained by solving the transfer equation numerically.

4.2.3 Near-electrode region in a heavy-current discharge. To illustrate the use of the component–component relations, we consider here the case in which different regions of the closed surface surrounding a dusty plasma have different temperatures.

Because of electrode erosion, the near-electrode region of a heavy-current discharge often contains macroscopic particles, which emit, absorb, and scatter radiation. The closed surface around the discharge may contain electrodes and opaque walls. The electrodes often have bright, high-temperature spots on their surfaces. Spectroscopic studies of the near-electrode region are performed by measuring the intensity of the radiation coming outward through small holes in the walls, so that the spots (as well as other parts of the surface) are outside the observer's field of view. The observed intensity, however, necessarily includes the scattered radiation from the whole of the surface. In performing experiments, it is helpful to know the contributions of various parts of the radiator to the measured intensity. This information is of diagnostic importance. We use here the near-electrode problem as an example, to discuss one possible way of using the component–component relations.

The surrounding surface may be divided into two parts: $S = S_1 + S_2$, where S_1 is occupied by hot spots and has a temperature T_1 , and S_2 , the rest of the surface, is at a temperature T_2 . According to Ref. [72], these quantities and the optical densities of particles are typically as follows:

$$S_1 = 10^{-2} \text{ cm}^2, \quad S_2 = 20 \text{ cm}^2,$$

$$T_1 = (4000 - 20000) \text{ K}, \quad T_2 \leq 2000 \text{ K},$$

$$\nu = 5 \times 10^{14} \text{ s}^{-1}, \quad t_p = 0.5.$$

Based on these data, a preliminary estimate for the ratio of the scattered-radiation intensities between the spots and the rest of the surface at a given frequency can be obtained using the following very simple formula:

$$\frac{I_{S_1, \text{scat}}}{I_{S_2, \text{scat}}} \approx \frac{I^0(T_1)S_1}{I^0(T_2)S_2}.$$

For $T_1 = 4000$ K,

$$\frac{I_{S_1, \text{scat}}}{I_{S_2, \text{scat}}} \approx 2,$$

and for $T_1 = 20000$ K, we obtain

$$\frac{I_{S_1, \text{scat}}}{I_{S_2, \text{scat}}} \approx 400,$$

i.e., the scattered radiation from a spot may be comparable to or much greater than that from the other surfaces. We will now show that, independent of this ratio, the component–component relations permit the determination of the absolute values of $I_{S_1, \text{scat}}$ and $I_{S_2, \text{scat}}$ for the case of fully scattering particles.

Let us first make use of the general relation (3.14). We start by dividing the surface integration region $S = S_{\text{rad}}$ into two parts, S_1 and S_2 . This gives us a sum of two surface integrals. We then note that the primary radiation from the surfaces is determined by the Planck function for temperatures T_1 and T_2 , respectively. The values of the Planck function can be taken out of each integral sign. The remaining integrals give the intensities of the scattered radiation from the corresponding parts of the surface, $I_{S_1, \text{scat}}$ and $I_{S_2, \text{scat}}$ [see Eqn (2.27)]. The result of this procedure is that the third term in the left-hand side of Eqn (4.2) is replaced by the sum of two ratios each of which is determined by the corresponding surface. Instead of Eqn (4.2) we have, for a continuum,

$$\frac{I_p(\nu)}{I^0(T_p, \nu)} + \frac{I_{S_1, \text{scat}}(\nu)}{I^0(T_1, \nu)} + \frac{I_{S_2, \text{scat}}(\nu)}{I^0(T_2, \nu)} = 1 - \exp[-t_p(\nu)]. \quad (4.5)$$

As can be seen from the above data, the area occupied by the spots is much less than the entire remaining area, i.e., $S_1 \ll S_2$. It is these areas that mainly determine the relative intensities of the scattered radiation from the surfaces in Eqn (4.5). If the observer's field of view is not in the immediate vicinity of the spots, we have

$$\frac{I_{S_1, \text{scat}}(\nu)}{I^0(T_1, \nu)} \ll \frac{I_{S_2, \text{scat}}(\nu)}{I^0(T_2, \nu)},$$

i.e., the relative intensity of the scattered radiation from a spot is negligible and can be dropped in Eqn (4.5). The obtained relation enables the experimental determination of the scattered radiation from both the spots and the rest of the surface. If the particles scatter radiation completely and hence do not radiate themselves ($I_p = 0$), then, instead of Eqn (4.5), we have

$$\frac{I_{S_2, \text{scat}}(\nu)}{I^0(T_2, \nu)} = 1 - \exp[-t_p(\nu)]. \quad (4.6)$$

The quantities that can be measured in experiments are the total scattered radiation from the whole of the surface,

$$I_{S, \text{scat}} = I_{S_1, \text{scat}} + I_{S_2, \text{scat}};$$

the optical density t_p ; and the surface temperature T_2 . Then, from Eqn (4.6), one readily obtains $I_{S_2, \text{scat}}$ for known t_p and T_2 , and from the equality

$$I_{S_1, \text{scat}} = I_{S, \text{scat}} - I_{S_2, \text{scat}}$$

we find $I_{S_1, \text{scat}}$. Thus, using Eqn (4.6) and the results of measurements has led us to the solution of the problem.

Note here that the proposed method does not require a knowledge of the hot-spot-region temperature T_1 , which is very difficult to determine; furthermore, it yields the absolute values for the scattered radiation coming from various parts of the surface. If the macroscopic particles not only scatter but also absorb (emit) radiation, the same method yields a useful upper estimate for the effect of the scattering of radiation by the surfaces.

Let us note an important feature of the above procedure. Although the absolute magnitude of the scattered radiation from the spots may be much larger than that from the rest of the surface, the reverse is true for the relative magnitudes, which appear in the relations we employ. It is this fact that makes it possible to simplify the relations between the relative components in the way discussed above.

This example illustrates the possibilities to modify, simplify, and employ the component–component relations when solving specific experimental problems.

4.3 Experimental studies of spectral-line profiles based on stationary relations

In the spectral diagnostics of plasma, the profiles of atomic spectral lines are the most important source of information. There are many currently available, well-developed diagnostic methods based on the use of profiles for a 'clean' plasma, where scattering on inhomogeneities is not significant. In dusty plasmas, line profiles are affected by macroparticles, and this should be taken into account when applying the existing diagnostic methods. Experimental studies of the effect of particles on the atomic-line profiles are discussed in Refs [42, 73–75]. The experiments were primarily aimed at verifying whether the 'clean' profile of a spectral line can be calculated from its experimentally observed, dust-distorted counterpart. The experiments were performed with the plasma of combustion products, under the atmospheric pressure, so that the atomic radiation could be considered thermal. Specifically, the effect of white scattering particles of aluminum oxide Al_2O_3 on the line profile of sodium (589.0 nm) and cesium (455.5 nm) was studied. The particle size varied between 1 μm and 100 μm . The combustion-product torch was open (Fig. 4).

Intensity measurements were performed both in the presence and in the absence of macroscopic particles, with and without external illumination. The open surface of the torch served as the bounding surface; part of it was illuminated from the outside using a specially designed optical system. The system included an external illumination source 1 (tungsten lamp ribbon) and a small screen 2 (see Fig. 4). The area of the torch surface could be varied, as also could the solid angle within which the external illumination was created. All the illuminated surface was on the side of the torch opposite to the detector Sp. The screen prevented external radiation from directly entering the apparatus, so that this radiation could be detected only after being scattered in the field of view 5 of the spectral device.

To measure the atomic temperature and optical density of the plasma, other optical arrangements (not shown in Fig. 4)

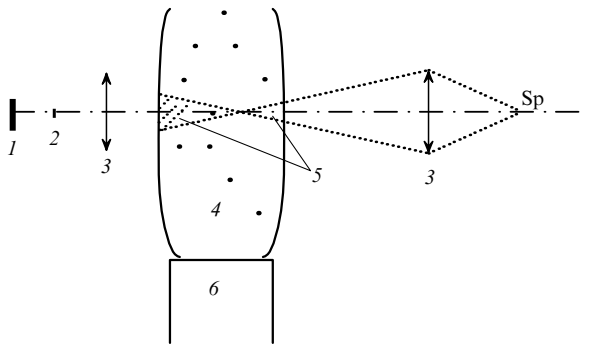


Figure 4. Schematic diagram of an experiment with dusty combustion products: 1, external illumination source; 2, screen; 3, focusing lenses; 4, torch of combustion products with macroparticles; 5, field of view; 6, burner; Sp, entrance to spectral device.

were used. The effective atomic temperature $T_{a, \text{eff}}$ averaged over the observation direction was determined by the generalized reversal method. The optical densities were measured by the attenuation of the external radiation, both in the absence and in the presence of particles. In the former case, the dependence of the optical density of atoms $t_a(\nu)$ on frequency within a spectral line was determined. In the latter case, the total optical density [$t_0(\nu) = t_a(\nu) + t_p$] was measured at various frequencies. The optical densities of the particles were not large ($t_p = 0.1-0.2$) and remained virtually unchanged by frequency variation.

Thus, the experiments described above allowed a comparison to be made between distorted and ‘clean’ profiles of spectral lines.

In our further discussion, the following two experimental results will be used.

(1) The effect of white scattering particles on the atomic-line radiation is noticeable only if external illumination is present. Hence, the line profiles in the presence of particles but without illumination are ‘clean’.

(2) If the particles are of relatively low optical density, their effect is significant at those frequencies for which the atomic optical densities are also not large. This means that, if at the center of a spectral line, the atomic optical density t_a is small, then the particles modify the entire line profile. Note that both the line intensity and the shape of the profile are changed. But if t_a is large at the center of the line then the particles affect only the wings of the line, where the atomic optical density is fairly low. For this reason, atomic temperatures were measured at frequencies in the central part of the powerful resonance line of sodium, where the effect of the particles is not significant.

As an example, figure 5 shows, the results of profile measurements of the cesium 455.5-nm line (in relative units). The optical density of cesium atoms in the line varied from 0.2 at a distance of $\Delta\lambda = 0.017$ nm from the line center to 0.004 at $\Delta\lambda = 0.06$ nm.

To explain the experimental results, numerical solutions to the transfer equation were obtained [73–75]. In the calculations, additional data were used and a number of simplifying assumptions were made. The additional data included particle sizes, approximate values of the complex refractive index of aluminum oxide at high temperatures, and the character of the external illumination. The simplifying assumptions concerned the particles (spherical, uniform in

size, uniformly distributed over the torch), the shape of the plasma body (a plane layer), and the geometry of the external illumination (axial symmetry). Based on the measured, particle-distorted line profiles, the calculations yielded ‘clean’ profiles which allowed comparisons with experimental unperturbed profiles.

In Ref. [76], ‘clean’ profiles were obtained from perturbed ones using the component–component relations for radiation. Let us show that in this case, neither a numerical solution to the transfer equation nor simplifying assumptions are needed to clean a profile from the effect of particles. We start from Eqn (3.21) taking into account, first, that the primary radiation from atoms on the torch surface illuminated from outside can be considered thermal and, second, that no intrinsic radiation of white particles is present ($I_p = 0$). Within a spectral line (where $\nu = \nu_l$), upon suppressing the arguments \mathbf{r} and \mathbf{u} , instead of Eqn (3.21), we have

$$\frac{I_a(\nu_l)}{I^0(T_{a, \text{eff}}, \nu_l)} + \frac{I_{S, \text{scat}}(\nu_l)}{I^0(T_{S, \text{eff}}, \nu_l)} = 1 - \exp[-t_a(\nu_l) - t_p(\nu_l)]. \quad (4.7)$$

Here, $I_a(\nu_l)$ is the required ‘clean’ profile intensity, $I_{S, \text{scat}}(\nu_l)$ is the distorting intensity of the scattered external radiation, and $T_{S, \text{eff}}$ is the effective surface temperature.

For the continuum at $\nu = \nu_{\text{cont}}$, where $I_a = 0$, we obtain

$$\frac{I_{S, \text{scat}}(\nu_{\text{cont}})}{I^0(T_{S, \text{eff}}, \nu_{\text{cont}})} = 1 - \exp[-t_p(\nu_{\text{cont}})]. \quad (4.8)$$

The quantities usually measured directly in experiments are the following: in a line, the total intensity $I(\nu_l) = I_a(\nu_l) + I_{S, \text{scat}}(\nu_l)$ (solid line in Fig. 5) and total optical density $t_a(\nu_l) + t_p$; in the continuum, the intensity of the scattered surface radiation $I_{S, \text{scat}}$ and the optical density of particles t_p . Whereas the optical density of particles virtually does not vary with frequency in a spectral line and in its immediate vicinity (i.e., with the transition from ν_{cont} to ν_l), the scattered intensity changes significantly. Its effect on the observed intensity should be taken into account as follows. First, using $I_{S, \text{scat}}$ and t_p measured in the continuum, $T_{S, \text{eff}}$ can be found from Eqn (4.8). Next, from Eqn (4.7) and the measured total intensities $I(\nu_l)$, one finds the required $I_a(\nu_l)$ and

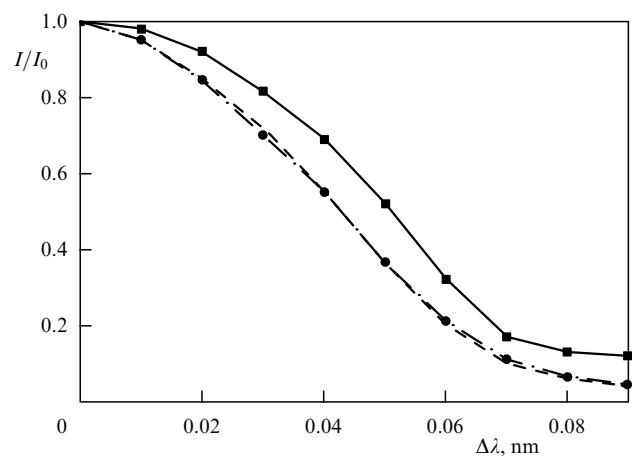


Figure 5. Profile of the 455.5-nm cesium line emitted by the plasma of combustion products with aluminum-oxide particles with an average diameter of 3.6 μm at $t_p = 0.1$. The solid line (squares): the profile measured in the presence of particles and external illumination; dot-dash line (circles), the profile measured without external illumination; dashed line is calculated from the solid one using Eqns (4.7) and (4.8).

$I_{S, \text{scat}}(\nu)$ using the measured optical densities and effective temperature of atoms and the calculated effective temperature $T_{S, \text{eff}}$ of the surface. In Fig. 5, the intensity I_a calculated in this way (dashed line) is compared with the independently measured ‘clean’ profile (dot–dashed line). It is seen that the agreement is quite satisfactory.

In obtaining I_a in this way, it is assumed that averaging over a nonuniformly illuminated surface does not introduce any additional frequency dependences of the effective surface radiation intensity $I_{S, \text{eff}}$ and the effective temperature $T_{S, \text{eff}}$. This assumption is valid provided $P_{S, \text{ex}} = 0$ (see Section 3.4). In the open-torch experiments under consideration, this condition was ensured by externally illuminating the whole torch surface, which might be the escape surface for the radiation whose direction at the observation point was opposite to that of observation. This implies, however, that Eqn (3.17) with $P_{S, \text{ex}} = 0$ could lead to the same result. In this case, Eqns (3.17) and (3.21) are identical. Clearly, the resulting clean profile does not differ from the previously obtained one.

Thus, the above results demonstrate that relations (3.17) and (3.21) are quite effective in processing experimental data. To solve the problem, neither any additional information about the object under study nor complicated calculations proved to be necessary.

4.4 Measurements of relative atomic-level populations using a stationary relation

In studies of dusty plasmas, it is often essential to determine the relative populations of atomic levels. Spectral methods involve measuring the relative intensities of spectral lines and the attenuation of the external radiation. The effect of scattering particles can be treated numerically in either case. To do this, as already mentioned, many characteristics of the object must be specified. On the other hand, the radiation component–component relations we have obtained include the ratios of the emissivities and absorptivities of atoms, which are related to the relative level populations. In what follows, the application of the component–component relations to measuring the relative level populations is analyzed [60].

Let us first discuss the possibility of experimentally determining $P_{S, \text{ex}}(\mathbf{r}, -\mathbf{u}, \nu)$, the probability of radiation escaping through a transparent surface S_{ex} not illuminated externally. (It should be recalled that we mean escaping after being scattered in the medium.) There are at least two different methods to measure this probability experimentally. First, we can illuminate the object at the point \mathbf{r} in the direction $-\mathbf{u}$ and measure the scattered-radiation flux through the surface S_{ex} . Second, we can illuminate the surface S_{ex} from the outside and measure the scattered flux at the point of observation \mathbf{r} in the direction \mathbf{u} . In either case, the ratio of the observed to the illuminating flux equals the required probability.

In such measurements, care must be taken to prevent direct, nonscattered radiation from entering the detector. In the first method, all that is needed is to eliminate the radiation passing through a small surface region around $(\mathbf{r}_0, -\mathbf{u})$. This region is directly opposite to the point of observation. In the second method, this same region should not be illuminated from the outside; otherwise we will measure not the probability $P_{S, \text{ex}}(\mathbf{r}, -\mathbf{u}, \nu)$ but rather the sum $\{P_{S, \text{ex}}(\mathbf{r}, -\mathbf{u}, \nu) + \exp[-t(\mathbf{r} \rightarrow \mathbf{r}_0, \nu)]\}$ (which may also be useful, however). Of course, to obtain $P_{S, \text{ex}}(\mathbf{r}, -\mathbf{u}, \nu)$, the component–component

relations can be employed as well—provided other quantities involved in, for example, Eqn (3.17) have been measured.

We turn now to examples of possible determinations of the relative atomic-level populations. We rewrite the expression (2.20) for the observed intensity and Eqn (3.17), for the case of dusty plasma, as

$$I(\mathbf{r}, \mathbf{u}, \nu) = I_a(\mathbf{r}, \mathbf{u}, \nu) + I_p(\mathbf{r}, \mathbf{u}, \nu) + I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, \nu) + I_S(\mathbf{r}_0, \mathbf{u}, \nu) \exp[-t_0(\mathbf{r}_0 \rightarrow \mathbf{r}, \nu)], \quad (4.9)$$

$$\left(\frac{p_a}{k_a}\right)^{-1} I_a(\mathbf{r}, \mathbf{u}, \nu) + \frac{I_p(\mathbf{r}, \mathbf{u}, \nu)}{I^0(T_p, \nu)} + \frac{I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, \nu)}{I_S} + P_{S, \text{ex}}(\mathbf{r}, -\mathbf{u}, \nu) + \exp[-t_0(\mathbf{r} \rightarrow \mathbf{r}_0, \nu)] = 1. \quad (4.10)$$

Here, p_a/k_a is directly related to the level populations through Eqn (3.16). It is assumed that radiation from dust particles is thermal and is characterized by the temperature T_p . Consider the case where the plasma is open, i.e., the surrounding surface is transparent and not illuminated, but can be illuminated from the outside during the measurements. Then the intensity $I_S \neq 0$ only in the presence of specially arranged external illumination. Let us show how the relative atomic population can be obtained in the case of white particles, when $I_p = 0$. The radiation frequency ν should be chosen so as to lie within a spectral line. To determine the relative populations with the use of Eqns (4.9), (4.10), and (3.16), the following measurements should be performed:

(1) Intensity measurements without external illumination, when $I_S = 0$. From Eqn (4.9), we find $I(\mathbf{r}, \mathbf{u}, \nu) = I_a(\mathbf{r}, \mathbf{u}, \nu)$.

(2) Determination of the optical density $t_0(\mathbf{r} \rightarrow \mathbf{r}_0, \nu)$ from the attenuation of external radiation emitted at the point \mathbf{r}_0 in the direction \mathbf{u} , i.e., for $I_S(\mathbf{r}_0, \mathbf{u}, \nu) \neq 0$.

(3) Measurements of the escape probability $P_{S, \text{ex}}$ using one of the methods indicated above. Either method involves a specially arranged external illumination of the plasma.

(4) Measurement of the intensity I with the same special external illumination that was used to measure $P_{S, \text{ex}}$. (As indicated earlier, this illumination should not enter the detector directly, without scattering). Then, from Eqn (4.9) we obtain

$$I(\mathbf{r}, \mathbf{u}, \nu) = I_a(\mathbf{r}, \mathbf{u}, \nu) + I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, \nu).$$

Since the intensity $I_a(\mathbf{r}, \mathbf{u}, \nu)$ has already been determined at step 1, this equality yields $I_{S, \text{scat}}(\mathbf{r}, \mathbf{u}, \nu)$.

(5) Measurements of the external-radiation intensity I_S .

After performing all these measurements, the required ratio $g_u N_l / (g_l N_u)$ can be determined from Eqns (4.10) and (3.16).

It is clear that, depending on the specific conditions and goals of the experiment, various sets of the required measurements may be employed. For example, in the case described, there is no need to measure the optical density separately (step 2); furthermore, instead of measuring the escape probability $P_{S, \text{ex}}$ (step 3), one can measure the sum $\{P_{S, \text{ex}}(\mathbf{r}, -\mathbf{u}, \nu) + \exp[-t_0(\mathbf{r} \rightarrow \mathbf{r}_0, \nu)]\}$, as described above.

The above examples of using the component–component relations were, for the sake of clarity, chosen so that virtually no additional calculations were needed to solve the experimental problems. Clearly, in many studies, these relations may be combined with other, e.g., computational, methods. Let us demonstrate this for the special example of the determination of the relative atomic-level populations in the case of radiating particles ($I_p \neq 0$). In this case, the particle-

radiation intensity I_p and the Planck function $I^0(T_p)$ (or the particle temperature) have to be found by calculations or additional measurements. If the ratio $I_p(\mathbf{r}, \mathbf{u}, \nu)/I^0(T_p, \nu)$ and the intensity I_p are known, the same measurements as in the previous, white-particle ($I_p = 0$) case yield the relative population $g_u N_1 / (g_1 N_u)$.

If the particles are large enough compared to the radiation wavelength, the ratio $I_p(\mathbf{r}, \mathbf{u}, \nu)/I^0(T_p, \nu)$ can be determined quite simply by calculation (see Section 4.2.2). Then, if the particle temperature T_p is measured by a reversal method, the intensity I_p can also be found.

It is worthwhile to compare the proposed method for determining the relative populations in a dusty plasma with the corresponding method for clean plasmas, where no scattering by macroparticles is present. The main difference is that, in the case of scattering, additional intensity measurements with special indirect (relative to the detector) illumination are needed to determine the escape probabilities $P_{S_{ex}}$ and intensities $I_{S,scat}(\mathbf{r}, \mathbf{u}, \nu)$. These measurements enable one to account for the scattering in the simplest way based on using the component–component relations, e.g., Eqn (4.10).

5. Relations between thermal-radiation components for radiators to which the geometrical-optics approximation does not apply

In the above discussion, all relations between radiation components were obtained under the assumption that the approximation of geometrical optics is applicable to describing the radiation in the radiator. It is of interest to obtain similar relations for scattering media within which geometrical optics does not apply. In this case, one cannot use the classical Kirchhoff law to describe the thermal radiation. Indeed, the very concept of the radiation intensity becomes meaningless.

Solid porous heat-shielding materials are an example of such a medium. These are crystalline structures consisting of a transparent solid substance (e.g., quartz) and air in its pores. Such materials are used, in particular, in orbital spacecraft. Radiation and heat transfer in such materials are of direct relevance to technological progress, which is the reason why these materials are currently the subject of much theoretical and experimental activity [77–80].

On the other hand, in the spectral studies of scattering media, all detecting devices are usually placed outside the medium, in the region where geometrical optics does apply. Studying the optical properties of strongly scattering materials at high temperatures is an example of such a situation [81–83].

In what follows, the relations between the thermal-radiation components will be derived for the case in which radiation is emitted by an elastically scattering medium, where geometrical optics does not apply, but is observed in a region where it does. This derivation will rely on the generalized Kirchhoff law obtained by methods of statistical radio physics [84, 85].

5.1 Thermal radiation characteristics and the generalized Kirchhoff law

Where the geometrical-optics approximation does not work, the basic characteristics of a fluctuating field of thermal radiation are the averaged quadratic and bilinear combinations of the electric and magnetic fields (correlation functions), rather than the intensities employed in the radiation-

transfer theory. With the use of the classical Kirchhoff–Clausius law, the power of the thermal radiation originating at some location in the medium can be expressed in terms of the radiator’s absorptivity and its temperature at this location; see Eqn (3.1). The intensity at every point of observation can then be found by solving the transfer equations (2.24), (2.25). In the generalized Kirchhoff law, the averaged quadratic and bilinear combinations of the thermal electric and magnetic fluctuation fields are already expressed, for any point of observation, in terms of the radiator’s absorption characteristics and temperature. The bilinear combinations depend on those parameters of the medium which determine the passage of radiation through it. The energy characteristics of the thermal radiation can be expressed in terms of the averaged bilinear combinations of the fields.

The generalized Kirchhoff law was obtained [84, 85] based on the use and development of the fluctuation–dissipation theorem of statistical physics [86, 87], which, as its name indicates, relates the occurrence of random thermal fluctuations in a medium to the medium’s ability to dissipate energy, i.e., to convert it to heat. It is thermal fluctuations in the medium that are the source of thermal radiation.

To examine the meaning of the generalized Kirchhoff law, we will briefly outline the main stages of its derivation, as given in Ref. [85]. The starting point is to modify the Maxwell equation by introducing extraneous currents describing the appearance of thermal fluctuations. Then the electric and magnetic fields determined by the resulting equations are thermal fluctuation fields. Along with the thermal fields, the theory also considers auxiliary fields created by unit point dipoles located at the point where thermal radiation is observed. The oscillations of the dipoles are not random but rather fully determined, and they behave harmonically at a chosen frequency. Further, the reciprocity theorem that follows from the Maxwell equations enables the thermal fluctuation field at an arbitrary point of observation to be related to the random extraneous currents with through unit-dipole fields. As the next step, the averaged quadratic and bilinear combinations of thermal fields at the point of observation are expressed in terms of the averaged products of extraneous random currents. The analysis of thermal fields in an isothermal medium and the use of the detailed-balancing principle permit [84] expressing the averaged products of extraneous random currents in terms of the absorption (dissipation) characteristics and temperatures at every point in the medium. It is these expressions that represent the fluctuation–dissipation theorem in the case under consideration. Finally, using the relations following from the reciprocity theorem, the average quadratic combinations of the thermal fluctuating fields at an arbitrary point of observation can be related to the medium’s temperature and to the absorption, by the medium, of the radiation from the unit dipoles located at the same point of observation. These relations express the generalized Kirchhoff law and have a wide range of applicability.

The generalized Kirchhoff law has previously been used [88] to treat some heat-exchange problems using the concept of the averaged Poynting vector for the thermal fluctuation field. Below, the law will be applied to obtain the averaged square of the strength of the thermal-fluctuation electric field.

According to the generalized Kirchhoff law, the averaged square of the absolute strength of a thermal electric field of frequency ν at an arbitrary point of observation \mathbf{r} can be

written, for a given orientation \mathbf{n} (see Fig. 6 below), as [85]

$$\langle |E(\mathbf{r}, \mathbf{n}, \nu)|^2 \rangle = \int_{v_{\text{abs}}} \langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{n}, \nu)|^2 \rangle d^3\mathbf{r}', \quad (5.1)$$

where

$$\langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{n}, \nu)|^2 \rangle = 4\Pi [T(\mathbf{r}'), \nu] q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu). \quad (5.2)$$

Here

$$\Pi(T, \nu) = h\nu \left[\exp\left(\frac{h\nu}{kT}\right) - 1 \right]^{-1} \quad (5.3)$$

is the average energy of a harmonic oscillator of frequency ν at a temperature T (minus the zero energy, which does not participate in the emission process); $q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu)$ is the density of energy losses for the radiation from a unit electrical dipole of frequency ν in the vicinity of \mathbf{r}' , provided that the dipole is located at \mathbf{r} and aligned along \mathbf{n} ; the symbol $\langle \rangle$ signifies an average over random thermal fluctuations.

The absolute magnitude of the moment of a unit electrical dipole of frequency ν is [84]

$$d_{\text{el}} = \frac{1}{2\pi\nu}. \quad (5.4)$$

In scattering media, the passage of radiation from a unit dipole to a point \mathbf{r}' is determined by the interaction of the radiation with the material in the whole volume of the object, so that the energy-loss density of the dipole is also determined by the material characteristics everywhere in the volume.

The expression $\langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{n}, \nu)|^2 \rangle d^3\mathbf{r}'$ in Eqn (5.1) represents contribution of the primary thermal radiation from $d^3\mathbf{r}'$ to the quantity under consideration, the total square of the modulus of the electric-field strength at the point \mathbf{r} . The additivity of the contributions follows from the linearity of the Maxwell equations. Importantly, the primary thermal sources do not correlate with one another. The primary thermal radiation can appear only in those regions where it is absorbed. It is this fact that determines the region of integration v_{abs} in Eqn (5.1).

According to the generalized Kirchhoff law, each contribution is determined, first, by the temperature of the radiator at the point \mathbf{r}' where the primary thermal radiation appears and, second, by the passage of the unit-point-dipole radiation through the medium, from the point of observation \mathbf{r} to the primary radiation point \mathbf{r}' , and by the subsequent dissipation of this radiation in $d^3\mathbf{r}'$. The dipole radiation travels in the medium in the direction opposite to that of the propagation of the observed thermal radiation.

Thus, the generalized Kirchhoff law, Eqns (5.1) and (5.2), enables the description of the fluctuation-averaged square of the thermal electric-field strength. Note that, at any arbitrary point of observation, the contributions from individual primary thermal radiators located in various parts of the object are summed (integrated).

5.2 Experimentally observed radiation as expressed in terms of the primary radiation of the medium

Let us consider the arrangement of the basic elements of an experimental setup for studying thermal radiation. Such a typical arrangement is shown schematically in Fig. 6. Radiation comes from the object v to the detecting apparatus Sp through air as well as through some elements of the optical

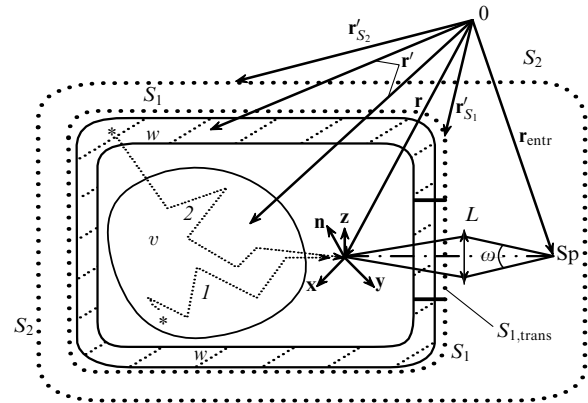


Figure 6. Schematic diagram of an arrangement used to observe an object within which geometrical optics is not applicable: v , object; w , wall; S_1 and S_2 , surrounding surfaces; $S_{1,\text{trans}}$, transparent part of surface S_1 ; L , optical system; Sp, entrance to spectral instrument.

system L . The object v may be surrounded by a transparent or opaque wall w . If the wall is opaque, special holes are used to observe the radiation. Let us assume that the radiation within the object v cannot be described by the methods of geometrical optics. At the same time, they can be applied outside the volume v , at a distance of the order of the correlation length [26]. The correlation lengths in such objects as strongly scattering, randomly inhomogeneous materials are determined by the size of inhomogeneities and by the radiation wavelength. The spectral detecting apparatus can always be focused to an observation point \mathbf{r} at which geometrical optics is applicable.

The propagation of radiation inside the scattering object v determines, first, the outward flow of the own radiation energy of the objects and, second, the passage of external radiation — for example, from the wall w — through the material of the object (note the ‘trajectories’ 1 and 2 in Fig. 6). If the conditions for the applicability of geometrical optics are not fulfilled within the radiator, then the generalized Kirchhoff law can be applied to describe the emission of thermal radiation in the radiator and the emergence of radiation into the geometrical-optics region. The thermal radiation from the walls and other setup elements can be described in the same way, with the region of integration v_{abs} in Eqn (5.1) covering all the thermal radiators listed above. If some parts of the experimental setup neither emit nor absorb radiation but only scatter it, they do not contribute to the region of integration but, of course, affect the passage of radiation and the resulting field at the point observation, i.e., affect the quantity $q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu)$ in Eqn (5.2).

Hereafter, it will be assumed that both the permittivity and magnetic permeability in the region of observation equal unity. Then $U(\mathbf{r}, \nu)$, the averaged spectral-energy density of the radiation at the point of observation \mathbf{r} , at frequency ν , can be written in the form [84]

$$U(\mathbf{r}, \nu) = \frac{\langle |E(\mathbf{r}, \mathbf{x}, \nu)|^2 \rangle + \langle |E(\mathbf{r}, \mathbf{y}, \nu)|^2 \rangle + \langle |E(\mathbf{r}, \mathbf{z}, \nu)|^2 \rangle}{2\pi}. \quad (5.5)$$

Here, $E(\mathbf{r}, \mathbf{x}, \nu)$, $E(\mathbf{r}, \mathbf{y}, \nu)$, $E(\mathbf{r}, \mathbf{z}, \nu)$ are the projections of the spectral density of the electric field at the point \mathbf{r} onto the axes of an arbitrarily oriented orthogonal coordinate system.

Equality (5.5) holds in the wave zone of the radiator, i.e., in the region of applicability of geometrical optics. The average squares of the moduli of the field projections are given by expressions (5.1) and (5.2). Applying these to all the three electric field components in Eqn (5.5) and multiplying and dividing the result by $4\pi v^2/c^3$, allows us to write the average spectral energy density as

$$U(\mathbf{r}, \nu) = \int_{v_{\text{abs}}} u(\mathbf{r}' \rightarrow \mathbf{r}, \nu) d^3\mathbf{r}', \quad (5.6)$$

where

$$u(\mathbf{r}' \rightarrow \mathbf{r}, \nu) = U^0[T(\mathbf{r}'), \nu] P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu) \quad (5.7)$$

is the contribution to the radiation energy at the point \mathbf{r} from the radiation that has originated in a unit volume about \mathbf{r}' and has then passed through the medium to point \mathbf{r} . Here,

$$U^0(T, \nu) = \frac{8\pi\nu^2}{c^3} \Pi(T, \nu) \quad (5.8)$$

is the equilibrium energy of the electromagnetic field at temperature T and frequency ν in the geometric optics approximation, and

$$P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu) = \frac{c^3}{4\pi^2\nu^2} \sum_{\mathbf{n}=\mathbf{x},\mathbf{y},\mathbf{z}} q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu). \quad (5.9)$$

As it follows from the foregoing discussion, the function $P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu)$ depends directly on the object's optical characteristics, which determine the passage of radiation via arbitrary paths from the point of observation \mathbf{r} to the point \mathbf{r}' and the dissipation of radiation there.

Thus, we have expressed the thermal-radiation energy density in terms of the radiator characteristics. Note, however, that in performing optical and, in particular, spectroscopic experiments, one measures the radiation fluxes entering an optical instrument. It is therefore necessary to be able to relate the radiation energy to the fluxes.

According to Ref. [26], we write the relation connecting the intensity and the radiation energy density in the geometrical-optics region as

$$U(\mathbf{r}, \nu) = c^{-1} \int_{4\pi} I(\mathbf{r}, \mathbf{u}, \nu) d\mathbf{u}. \quad (5.10)$$

Here, $I(\mathbf{r}, \mathbf{u}, \nu)$ is the intensity of radiation of frequency ν at the point \mathbf{r} in the direction \mathbf{u} , and $d\mathbf{u}$ is a solid-angle element.

The integral of the intensity over all directions is the total radiation flux F at \mathbf{r} :

$$F(\mathbf{r}, \nu) = \int_{4\pi} I(\mathbf{r}, \mathbf{u}, \nu) d\mathbf{u} = U(\mathbf{r}, \nu)c. \quad (5.11)$$

To obtain the intensity in a given direction from the energy-density expression (5.10), the dependence of the intensity I on the direction \mathbf{u} is needed. If the radiation is isotropic, i.e., I is independent of the direction, we immediately obtain

$$I(\mathbf{r}, \nu) = \frac{cU(\mathbf{r}, \nu)}{4\pi}. \quad (5.12)$$

We note that the expressions for the energy and flux densities we have derived apply to any point in space where

the geometrical-optics approximation can be used. Changing the point and direction of observation changes the fields as well as the dipole-energy-loss density $q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu)$ involved in the generalized Kirchhoff law (5.2). At the same time, all the formulas remain valid independent of the position of the point of observation. Therefore, they can be applied, for example, both to the point of observation with the radius vector \mathbf{r} and to the point \mathbf{r}_{entr} at the entrance to the detector (see Fig. 6). Importantly, the passage of radiation through the focusing optical system (L) directly affects the quantity $q(\mathbf{r}_{\text{entr}} \rightarrow \mathbf{r}', \mathbf{n}, \nu)$ and hence the energy density, intensity, and total radiation flux at the entrance to the instrument (at the point \mathbf{r}_{entr}). This is the point to which the radiation from the object under study is focused.

In view of the linearity of the problem, we can write the intensity I and the total flux F in a form analogous to Eqns (5.6) and (5.7):

$$I(\mathbf{r}, \mathbf{u}, \nu) = \int_{v_{\text{abs}}} i(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu) d^3\mathbf{r}', \quad (5.13)$$

where

$$i(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu) = I^0[T(\mathbf{r}'), \nu] P_I(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu); \quad (5.14)$$

$$F(\mathbf{r}, \nu) = \int_{v_{\text{abs}}} f(\mathbf{r}' \rightarrow \mathbf{r}, \nu) d^3\mathbf{r}', \quad (5.15)$$

where

$$f(\mathbf{r}' \rightarrow \mathbf{r}, \nu) = F^0[T(\mathbf{r}'), \nu] P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu). \quad (5.16)$$

In the above equations, I^0 is given by the Planck formula (2.8), and $F^0 = U^0/c$, with U^0 given by Eqn (5.8). According to Eqns (5.6), (5.7), (5.10), (5.13), and (5.14),

$$P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu) = \int_{4\pi} \frac{P_I(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu)}{4\pi} d\mathbf{u}. \quad (5.17)$$

In the case of isotropic radiation, instead of Eqn (5.17), we have the equality

$$P_U(\mathbf{r}, \nu) = P_I(\mathbf{r}' \rightarrow \mathbf{r}, \nu). \quad (5.18)$$

In the integrals (5.6), (5.13), and (5.15), all the energy, intensity, and flux components (in the integrands) differ in the location where the primary thermal radiation appears. Analogously, in Eqns (5.7), (5.14), and (5.16), the function $P(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu)$ characterizes the emergence of radiation at \mathbf{r}' and its passage to \mathbf{r} . On the other hand, according to the generalized Kirchhoff law (5.2), these radiation components are determined by the passage of radiation from a unit dipole at \mathbf{r} in the opposite direction, to point \mathbf{r}' , and by its dissipation there. This dualism of the radiation components and the functions P is due to the reciprocity relations, which served as the basis for the derivation of the generalized Kirchhoff law. The functions P introduced here are analogous to the functions that appear in the intensity expressions derived in the geometrical-optics approximation [see Eqn (2.25)]. In that case there was also no difference in the propagation of radiation between the cases of mutually opposite directions of propagation, since both the reversibility conditions and reciprocity relations were fulfilled in any scattering event. Note also that the corresponding functions P were the same [see Eqns (3.12) and (3.13)].

The formulas obtained in this section will be used below to derive component–component relations by two different methods.

5.3 Equilibrium relations

We now apply the energy and intensity expressions (5.6) and (5.13) to equilibrium radiation in an isothermal cavity with a scattering medium inside. Let the wall w in Fig. 6 be closed, opaque, and let it have the same temperature as the medium in the cavity has. We use the notation w_c to emphasize that the wall is closed. Both the wall and the medium can emit and absorb thermal radiation. If we assume that absorption occurs within the volume of the wall, the region of integration v_{abs} should be divided into two corresponding parts, v and w_c . Let us observe that, in the region of observation — outside the volume v , where the geometrical optics approximation is applicable, the resulting energy densities and intensities are in this case equal to their equilibrium values U^0 and I^0 given by Eqns (5.8) and (2.8), respectively. Substituting U^0 and I^0 on the left-hand sides of Eqn (5.6) and (5.13) and dividing both sides of each equation by these quantities, we find (assuming for the moment that all the temperatures involved are the same) that

$$\int_v \frac{u(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{U^0[T(\mathbf{r}'), \nu]} d^3\mathbf{r}' + \int_{w_c} \frac{u(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{U^0[T(\mathbf{r}'), \nu]} d^3\mathbf{r}' = 1, \quad (5.19)$$

$$\int_v \frac{i(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{I^0[T(\mathbf{r}'), \nu]} d^3\mathbf{r}' + \int_{w_c} \frac{i(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{I^0[T(\mathbf{r}'), \nu]} d^3\mathbf{r}' = 1. \quad (5.20)$$

The above equations are very simple equilibrium relations between the relative values of the components of the energy density u/U^0 and the intensity i/I^0 . The relation (5.19) between the energy-density components was obtained in Ref. [89], where a relation between the intensity components was also derived under the assumption that radiation inside the cavity is isotropic and hence Eqn (5.12) holds. Note that, in the present work, these relations have been obtained from the equilibrium condition for an isothermal cavity, based on Eqns (5.6) and (5.13), equations which simply express the additivity of the contributions from the primary radiators.

The denominators of the integrands in the last two formulas involve temperatures $T(\mathbf{r}')$ varying inside the object. This suggests that these relations could be applied to nonisothermal objects. Let us explain where this possibility comes from. From Eqns (5.7) and (5.14), it follows that the integrands in Eqns (5.19) and (5.20) involve the functions P_U and P_I , which are determined by the specifics of the radiation–matter interaction and do not depend directly on the temperature. It is for this reason that the last two equalities can be used if the temperature $T(\mathbf{r}')$ varies over the object, provided its variation is small enough to prevent significant changes in the functions P_U and P_I .

The relations we have obtained become simplified if some parts of a non-isothermal radiator have the same temperature. Let the wall w be at one temperature, T_{w_c} , and the substance inside the cavity at another, T_v . Then our relations can be rewritten noting that, after U^0 and I^0 are taken out of the integral signs, what remains there is the energy density and the intensity of the corresponding part of the object. The result is the same if the temperatures vary over the wall and the volume but the radiation can be characterized by averaged (effective) temperatures. The effective temperature of a nonisothermal radiator is the temperature of an isothermal

radiator that has the same optical properties and whose radiation intensity equals that of the actual nonisothermal object. Averaged radiator characteristics of this type have already been discussed in Section 4.2.

Let us write down the relation between the intensity components I_v and I_{w_c} for this case:

$$\frac{I_v(\mathbf{r}, \nu)}{I^0(T_{v, \text{eff}}, \nu)} + \frac{I_{w_c}(\mathbf{r}, \nu)}{I^0(T_{w_c, \text{eff}}, \nu)} = 1. \quad (5.21)$$

This relation is analogous to Eqn (3.18), but was derived for the simplest case of a single-phase medium ($k = 1$), with $n = 1$ in the region of observation. Note also that, unlike Eqn (3.18), in Eqn (5.21) the radiation coming to the point of observation \mathbf{r} directly from the opposite wall is not separated out but is incorporated in the second term.

In Section 5.5 we will show how relation (5.21) can be applied to the analysis of experimental data when determining the effective temperatures of a strongly scattering medium.

5.4 Stationary relations

The purpose of this section is to obtain component–component relations for the radiation in the geometrical-optics region using the stationarity, rather than equilibrium, condition. As in Section 3.2, we will derive again a stationarity condition that should be satisfied by the probabilities of the disappearance of the emerging radiation. However, the method of derivation is different. It makes use of the fact that the generalized Kirchhoff law in the form (5.1), (5.2) involves the energy-loss density of a unit dipole, $q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu)$. The dipole field acts as the Green function in the problem under study [85]. The required relations will be found using the stationarity condition which the radiation of a deterministic unit dipole obeys.

For a unit electric dipole of frequency ν with a moment given by Eqn (5.4), located at the point of observation \mathbf{r} , the radiant energy emitted into ambient space per unit time is

$$\varepsilon(\nu) = \frac{4\pi^2 \nu^2}{3c^3}. \quad (5.22)$$

Let the location \mathbf{r} of an \mathbf{n} -directed dipole be within an imaginary closed surface S . Also, there is a region v_{abs} within the surface where the radiation from the dipole can be absorbed. Further, let the radiation in the object be stationary in the sense that the parameters averaged over a time interval long compared to the oscillation period (ν^{-1}) are time-independent. (It is such radiation that is described by the generalized Kirchhoff law.) Then the dipole-radiation energy disappears in the ambient space at the same rate at which it appears. The energy can disappear due to absorption in the volume and due to outward flow through the surface. This can be written in terms of the volume density of losses and the Poynting vector at the surrounding surface as follows:

$$\varepsilon(\nu) = \int_{v_{\text{abs}}} q(\mathbf{r} \rightarrow \mathbf{r}', \mathbf{n}, \nu) d^3\mathbf{r}' + \int_S \mathbf{S}(\mathbf{r} \rightarrow \mathbf{r}'_S, \mathbf{n}, \nu) d^2\mathbf{r}'_S. \quad (5.23)$$

Here $\mathbf{S}(\mathbf{r} \rightarrow \mathbf{r}'_S, \mathbf{n}, \nu)$ is the Poynting vector of a unit dipole at the point \mathbf{r}'_S of the surface S . This expression will be used as the basis for deriving the required component–component relations for stationary radiation.

Expression (5.23) is valid for any closed surface S provided both the volume v_{abs} and the point of observation \mathbf{r} involved in Eqn (5.23) are within it. The dotted lines in Fig. 6 illustrate two ways in which such a surface can be drawn. First, it may coincide with the outer boundary of the radiating and absorbing walls w and close the open portion of the wall if it exists. This is the surface S_1 . Enclosed within it is the object under study — a radiator with a scattering medium occupying a volume v — as well as the wall w and the point of observation \mathbf{r} . Second, in spectroscopic experiments S can generally be chosen to enclose all experimental devices or their elements that affect the observed radiation. This is precisely the way in which the surface S_2 is drawn in the figure. It encloses the same volume v and the same walls w as well as other parts of the experimental setup, such as the optical system L . The point of observation in this case may lie at the entrance to the detecting instrument and be determined by the radius vector \mathbf{r}_{entr} . In both cases, we can write

$$v_{\text{abs}} = v + w. \quad (5.24)$$

Note, however, that, while in the first case w is the absorbing volume of the wall only, in the second case this is the absorbing volume of all the elements within S_2 , except for v .

We now consider the surface integral in Eqn (5.23) for various cases. This integral vanishes if the radiation flux from the radiator does not reach the surface S . This may be the case, in particular, if the radiator is surrounded from all its sides by opaque, fully absorbing walls w located inside S . In this case, radiation disappears in the bulk, which is completely accounted for by the volume integral. If radiation emerges freely through the entire surface S in all directions, then any region of the surface can contribute to the surface integral. In general, it can be assumed that radiation from the volume may escape through a certain transparent portion of the surface, S_{trans} . Then it is the escape through this portion that contributes to the surface integral. Such a transparent part of the surface S_1 is labeled as $S_{1,\text{trans}}$ in Fig. 6.

Let us now introduce thermal electric fields into equality (5.23) using the relation (5.2) of the generalized Kirchhoff law in the volume integral. We have

$$\begin{aligned} \varepsilon(v) = & \int_{v_{\text{abs}}} \frac{\langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{n}, v)|^2 \rangle}{4\Pi[T(\mathbf{r}'), v]} d^3\mathbf{r}' \\ & + \int_{S_{\text{trans}}} \mathbf{S}(\mathbf{r} \rightarrow \mathbf{r}'_S, \mathbf{n}, v) d^2\mathbf{r}'_S. \end{aligned} \quad (5.25)$$

Equation (5.25) expresses the conservation of energy for a dipole under stationary conditions and includes the squared moduli of the electric field of thermal radiation. Let us employ this equation to relate the radiation-energy-density components.

From Eqn (5.1), (5.5), and (5.6), the energy density at the point \mathbf{r} due to the primary radiation at the point \mathbf{r}' can be rewritten as

$$\begin{aligned} u(\mathbf{r}' \rightarrow \mathbf{r}, v) = & \\ = & \frac{\langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{x}, v)|^2 \rangle + \langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{y}, v)|^2 \rangle + \langle |E(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{z}, v)|^2 \rangle}{2\pi}. \end{aligned} \quad (5.26)$$

As can be seen from Eqns (5.7) and (5.8), the energy density $u(\mathbf{r}' \rightarrow \mathbf{r}, v)$ is proportional to $\Pi[T(\mathbf{r}'), v]$, i.e., depends on the temperature at the point of origin of the radiation.

Let us write Eqn (5.25) for three dipoles located at the point of observation \mathbf{r} and directed along the axes $\mathbf{n} = \mathbf{x}, \mathbf{y}, \mathbf{z}$ of an orthogonal coordinate system ($\mathbf{x}, \mathbf{y}, \mathbf{z}$ are the unit vectors). We then sum the three resulting equations, and use Eqn (5.26) under the sign of the volume integral. Finally, dividing the result by $3\varepsilon(v)$ and using Eqns (5.8) and (5.22), we obtain

$$\int_{v_{\text{abs}}} \frac{u(\mathbf{r}' \rightarrow \mathbf{r}, v)}{U^0[T(\mathbf{r}'), v]} d^3\mathbf{r}' + P_3(\mathbf{r} \rightarrow S_{\text{trans}}, v) = 1. \quad (5.27)$$

Here

$$P_3(\mathbf{r} \rightarrow S_{\text{trans}}, v) = \int_{S_{\text{trans}}} \sum_{\mathbf{n}=\mathbf{x},\mathbf{y},\mathbf{z}} \frac{\mathbf{S}(\mathbf{r} \rightarrow \mathbf{r}'_S, \mathbf{n}, v)}{3\varepsilon(v)} d^2\mathbf{r}'_S \quad (5.28)$$

is the fraction of the energy of the three dipoles that escapes from the volume v through the surface S_{trans} .

Relation (5.27) expresses the fact that the disappearance probability of emerging dipole radiation is unity. The terms in Eqn (5.27) have the following probabilistic meaning. The volume integral is the probability that the radiation from the three dipoles will be absorbed in the volume v_{abs} , and the surface integral $P_3(\mathbf{r} \rightarrow S_{\text{trans}}, v)$ is the probability that the same radiation will escape through the surface S_{trans} . The integrands represent the probability densities for the radiation to be absorbed in a volume around \mathbf{r}' , and for its escaping through a surface around \mathbf{r}'_S , respectively. In view of Eqn (5.7), Eqn (5.27) becomes

$$\int_{v_{\text{abs}}} P_U(\mathbf{r}' \rightarrow \mathbf{r}, v) d^3\mathbf{r}' + P_3(\mathbf{r} \rightarrow S_{\text{trans}}, v) = 1. \quad (5.29)$$

Eqns (5.27), (5.29) are the required stationary relations between the relative values of the components of the average spectral radiation-energy density at the point of observation \mathbf{r} . Indeed, note that the integrands of the volume integrals involve relative density components, each determined by the primary radiation from the corresponding region. Moreover, we can break down the volume integrals — for example, according to Eqn (5.24) — into terms related to the primary radiation from individual parts of the total radiating (absorbing) volume.

If some regions of the radiator can be considered isothermal, we should break down the volume integral in Eqn (5.27) into the corresponding parts and then take the denominators out of the integral sign in each. Assume that the isothermal regions of the object are the radiator v and the wall w , whose temperatures are T_v and T_w , respectively. Then, instead of Eqn (5.27), we find

$$\frac{U_v(\mathbf{r}, v)}{U^0(T_v, v)} + \frac{U_w(\mathbf{r}, v)}{U^0(T_w, v)} + P_3(\mathbf{r} \rightarrow S_{\text{trans}}, v) = 1. \quad (5.30)$$

In the case of nonisothermal radiators, it is useful to introduce, as before, effective temperatures averaged over the entire radiator or over a given region of the volume (or surface). We introduce the effective temperature of an arbitrary radiating volume v_i using Eqns (5.6) and (5.7):

$$\begin{aligned} U_i(\mathbf{r}, v) = & \int_{v_i} u(\mathbf{r}' \rightarrow \mathbf{r}, v) d^3\mathbf{r}' \\ = & \int_{v_i} U^0[T(\mathbf{r}'), v] P_U(\mathbf{r}' \rightarrow \mathbf{r}, v) d^3\mathbf{r}' \\ = & U^0(T_{i,\text{eff}}, v) \int_{v_i} P_U(\mathbf{r}' \rightarrow \mathbf{r}, v) d^3\mathbf{r}'. \end{aligned} \quad (5.31)$$

Therefore,

$$\int_{v_i} P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu) d^3\mathbf{r}' = \int_{v_i} \frac{u(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{U^0[T(\mathbf{r}'), \nu]} d^3\mathbf{r}' = \frac{U_i(\mathbf{r}, \nu)}{U^0(T_{i,\text{eff}}, \nu)}. \quad (5.32)$$

We use these equalities to transform, the volume integrals in Eqns (5.27) and (5.29) into sums of the ratios $U_i(\mathbf{r}, \nu)/U^0[T_{i,\text{eff}}, \nu]$. If Eqn (5.24) is valid, i.e., $i = v, w$, then, instead of Eqns (5.27), (5.29), we obtain a relation quite analogous to Eqn (5.30) but with the T_i replaced by the effective temperatures $T_{i,\text{eff}}$.

Let us consider the case in which $S = S_1$ (see Fig. 6) and the wall w is not closed. In analogy to Section 3.3, we introduce the effective temperature of the entire surrounding surface S or of the wall w . The actual nonuniform radiation that enters the volume v through the surrounding surface is thus replaced by radiation distributed uniformly over the entire closed surface S . In carrying out this procedure, the following two relations are used:

$$\begin{aligned} U_w(\mathbf{r}, \nu) &= \int_w U^0[T(\mathbf{r}'), \nu] P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu) d^3\mathbf{r}' \\ &= U^0(T_{S,\text{eff}}, \nu) \left[\int_w P_U(\mathbf{r}' \rightarrow \mathbf{r}, \nu) d^3\mathbf{r}' + P_3(\mathbf{r} \rightarrow S_{\text{trans}}, \nu) \right]. \end{aligned} \quad (5.33)$$

Here, $P_3(\mathbf{r} \rightarrow S_{\text{trans}})$ is defined by Eqn (5.28).

Instead of Eqn (5.27), we obtain using $T_{i,\text{eff}}$ with $i = v$ and $T_{S,\text{eff}}$:

$$\frac{U_v(\mathbf{r}, \nu)}{U^0(T_{v,\text{eff}}, \nu)} + \frac{U_w(\mathbf{r}, \nu)}{U^0(T_{S,\text{eff}}, \nu)} = 1. \quad (5.34)$$

To pass from the relations between the components of radiation energy to relations between the components of the fluxes of this energy, we use Eqns (5.6), (5.10), (5.11), (5.13), and (5.15). Instead of Eqn (5.27), we obtain

$$\int_{v_{\text{abs}}} d^3\mathbf{r}' \int_{4\pi} d\mathbf{u} \frac{i(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu)}{4\pi I^0[T(\mathbf{r}'), \nu]} + P_3(\mathbf{r} \rightarrow S_{\text{trans}}, \nu) = 1, \quad (5.35)$$

$$\int_{v_{\text{abs}}} \frac{f(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{F^0[T(\mathbf{r}'), \nu]} d^3\mathbf{r}' + P_3(\mathbf{r} \rightarrow S_{\text{trans}}, \nu) = 1. \quad (5.36)$$

The practical application of Eqn (5.35) requires a knowledge of the dependence of the intensity components $i(\mathbf{r}' \rightarrow \mathbf{r}, \mathbf{u}, \nu)$ on the direction \mathbf{u} . The simplest case is isotropic radiation. It was already mentioned (see Section 5.3) that radiation in an isothermal cavity is isotropic. There are other conditions under which radiation can be considered isotropic. In particular, a strongly scattering medium can produce such radiation. Thus, radiation in an unclosed, nonisothermal cavity can also be isotropic if a strongly scattering radiator is placed within it. Then Eqns (5.12) and (5.18) and the equality

$$u(\mathbf{r}' \rightarrow \mathbf{r}, \nu) = \frac{i(\mathbf{r}' \rightarrow \mathbf{r}, \nu)4\pi}{c}$$

hold. The integrands in Eqn (5.27) become

$$\frac{u(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{U^0[T(\mathbf{r}'), \nu]} = \frac{i(\mathbf{r}' \rightarrow \mathbf{r}, \nu)}{I^0[T(\mathbf{r}'), \nu]}.$$

Let us examine one possible application of Eqn (5.36), in which radiation fluxes summed over directions are present. We make use of the fact that at the entrance to the spectral instrument (point \mathbf{r}_{entr} in Fig. 6) the total radiation flux from the object v is localized within the solid angle ω determined by the optical system used. Outside this solid angle, the fluxes from the object are usually negligible at \mathbf{r}_{entr} . Then the integrand in Eqn (5.36) contains the ratio of the recorded flux of the radiation that originated at \mathbf{r}' to the equilibrium flux at the temperature $T(\mathbf{r}')$. If the parts v and w of the object are isothermal, the relations between the fluxes can be obtained directly from Eqns (5.30) and (5.34), simply replacing U by F , U^0 by F^0 , and \mathbf{r} by \mathbf{r}_{entr} .

Thus, in this section, we have obtained a set of relations between the energy, intensity, and flux components of stationary radiation. These relations are represented by Eqns (5.27)–(5.30), (5.34)–(5.36). All relations that do not involve the effective surface temperature $T_{S,\text{eff}}$ do involve the probability $P_3(\nu)$ of the dipole radiation escaping through the surface S_{trans} . This quantity can generally be determined experimentally in a manner similar to that described in Section 4.3, i.e., by properly illuminating the object and measuring the passage of radiation through it. On the other hand, as mentioned above, this quantity may vanish if radiation from the point of observation simply does not reach the transparent portion of the surface S_{trans} . As will be seen from what follows, determining the effective surface temperatures from the relations obtained in this and the previous section is a rather straightforward procedure, so that using relations with effective temperatures may often be preferable. The set of component–component relations is complete enough to allow one to choose the most suitable ones, based on the experimental conditions and requirements.

A comparison of the stationary relations derived here with those obtained earlier shows that the former agree quite well with what was obtained in Section 3.3 for the case where geometrical optics is applicable to a scattering medium. The main difference from the equilibrium relations given in Section 5.3 for thermal-radiation in a closed cavity is that the equilibrium relations do not involve the probability of radiation escaping through a transparent surface. The equilibrium relations are a special case of the stationary ones.

5.5 Component–component relations in application to the spectroscopy of highly scattering materials

The emissivity of highly scattering materials at high temperatures is measured by high-speed-spectrometry methods [81–83]. An experimental setup is schematically shown in Fig. 7. The sample to be studied (4), made of a strongly scattering material, is placed in a furnace (6) — a cylindrical graphite cavity heated by the field of a high-frequency inductor. A horizontal partition divides the cavity into an upper (5) and a lower (3) part. The sample is suspended inside the upper cavity by two thin wires (7) fixed to the sample holder (8). The radiation pyrometer (1) focused onto the furnace floor by an infrared lens (2) serves to measure the furnace hearth temperature T_0 . The top of the furnace is open, allowing the radiation directed by a rotating mirror (9) and an infrared lens (10) to pass to the high-speed spectrometer (12). In performing experiments, the radiation spectrum of the hearth of the heated furnace, $I_0(\nu)$, is measured first. After this, the sample is placed inside the furnace. After the sample has been heated to a stationary temperature, the spectrum of the

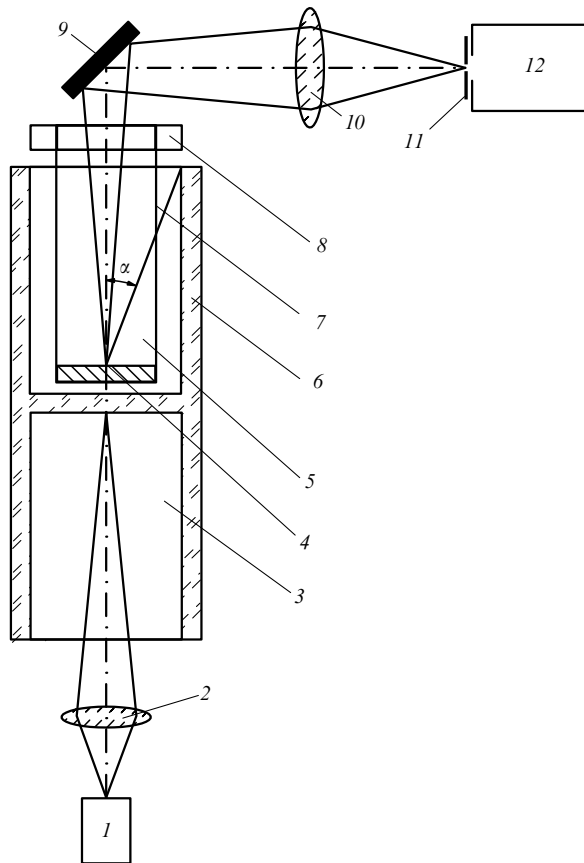


Figure 7. Experimental setup for measuring the emissivity of strongly scattering materials at high temperatures: 1, radiation pyrometer; 2, 10, infrared lenses; 3, lower part of the furnace; 4, sample under study; 5, upper part of the furnace; 6, furnace wall; 7, sample suspension wire; 8, sample holder; 9, rotating mirror; 11 spectrometer's entrance slit; 12, spectrometer.

radiation from the sample in the furnace, $I_{v+w}(\nu)$, is measured. Then the sample-radiation spectrum $I_v(\nu)$ is measured using the method of a falling furnace [90]: the furnace falls down, and its radiation is cut off by a screen. The radiation is recorded immediately after the fall of the furnace, before the sample temperature has significantly changed. The emissivity is determined from the measured radiation spectral-intensity ratio between the sample and the furnace hearth, the furnace being treated as a black body. The black-body approximation is not ideal, however, since the top of the cavity is open and since the walls of the furnace are not quite isothermal (the temperature T_w is variable).

In studying the emissivity, it is necessary to know the temperature of the sample. The temperature of a sample of a strongly scattering and weakly absorbing material is not uniform inside a nonisothermal furnace. In this case, the above-mentioned effective sample temperature $T_{v,\text{eff}}$ is needed. To determine it directly using thermocouples or pyrometers is extremely difficult. Usually, this temperature is determined by numerically solving the heat and radiation transfer equations for the sample. In doing this, the sample's thermal and optical properties as well as thermocouple measurements of the furnace-wall temperatures are used, and many simplifications are used in calculations.

A method for effective-temperature determination based on the component-component relations of Section 5.3 is

proposed in Ref. [91]. The method does not require complex radiation-transfer and heat-conduction calculations, nor are numerous assumptions and much additional information needed.

The method was applied to the study of a fibrous heat-insulating material with a porosity of 93.5%. The samples used were disks 30 mm in diameter and 4 or 9.7 mm in thickness. The spectra were measured in the continuous scanning regime in the wavelength range from 1.5 to 5.0 μm and hearth-temperature range from 800 to 1200 $^\circ\text{C}$. Note that the characteristic fiber diameter did not exceed a few micrometers and the observation area on the surface of the sample was measured in millimeters, which is far larger than the size of the sample inhomogeneities. Consequently, according to Ref. [26], the geometrical-optics approximation could be used in the observation area.

There is an important point to note about these measurements. If a scattering material is present in the furnace, the intensity $I_w(\nu)$ of the furnace-wall radiation that comes to the recording instrument is strongly affected by this material. At the same time, the intensity $I_v(\nu)$ of the radiation from the sample of scattering material is, on the contrary, is virtually independent of whether the sample is inside or outside the furnace, since the inside walls of the graphite furnace absorb the incident radiation almost completely. This means that measuring the sample radiation outside the furnace also yields the component $I_v(\nu)$ due to the primary sample radiation even if the sample is placed in the furnace. Then, from the total intensity $I_{v+w}(\nu)$ of the recorded radiation from the furnace with the sample inside, the second intensity component $I_w(\nu) = I_{v+w}(\nu) - I_v(\nu)$ can be determined. Thus, the described experiments measure the components I_v and I_w of the radiation from the furnace with a sample of scattering material within it.

Now let us employ the relations of Sections 5.3 and 5.4 to determine the effective temperature of the scattering material in the described experiments. First, following Ref. [91], we make use of the equilibrium relation (5.21). First of all, we should note that the equilibrium relations involve the intensities $I_{w_c}(\nu)$ and effective temperatures $T_{w_c,\text{eff}}$ of the walls of the closed cavities, whereas the cavity is partly open in the falling-furnace experiments. The difference in intensities was approximately accounted for by means of the coefficient K defined as, the ratio of the radiation-energy flux onto the top of the sample from the real black-body model to the flux from the closed model. The coefficient K was calculated based on the known geometry of the furnace interior, taking into account the opening angle α (see Fig. 7). It was assumed that $I_{w_c}(\nu) = I_w(\nu)/K$. With the above in mind, Eqn (5.21) becomes

$$\frac{I_v(\nu)}{I^0(T_{v,\text{eff}}, \nu)} + \frac{I_{v+w}(\nu) - I_v(\nu)}{KI^0(T_{w_c,\text{eff}}, \nu)} = 1. \quad (5.37)$$

Generally speaking, this relation is quite sufficient to obtain the effective temperature $T_{v,\text{eff}}$, but, since in Ref. [91] only relative intensities were measured, use was also made of the measured values of $I_0(\nu)$ and T_0 and of the relationship between them,

$$I_0(\nu) = E(T_0, \nu) I^0(T_0, \nu),$$

where $E(T_0, \nu)$ is the effective emissivity of the black-body model, which is calculated by a specifically developed method

[82, 92]. As a result, we have the relation

$$\frac{I_{v+w}(\nu)}{I_0(\nu)} = \frac{KI^0(T_{w_c, \text{eff}}, \nu)}{E(T_0, \nu)I^0(T_0, \nu)} + \frac{I_v(\nu)}{I_0(\nu)} \left[1 - \frac{KI^0(T_{w_c, \text{eff}}, \nu)}{I^0(T_{v, \text{eff}}, \nu)} \right], \quad (5.38)$$

which involves the relative values of the measured intensities and the effective sample and furnace-wall temperatures.

The effective temperatures were found from the above relation in view of the fact that $I_v(\nu) = 0$ in a certain frequency range. The effective surface temperature $T_{w_c, \text{eff}}$ was then obtained from Eqn (5.38) using the Planck function. After that, it was shown by calculations that this temperature is frequency independent over the entire spectral range under study. Then in those spectral regions where $I_v(\nu) \neq 0$, Eqn (5.38) was applied to determine the effective sample temperature as a function of frequency. The results of these measurements are shown in Fig. 8. It can be seen that the effective sample temperature depends on the wavelength, sample thickness, and the furnace-hearth temperature.

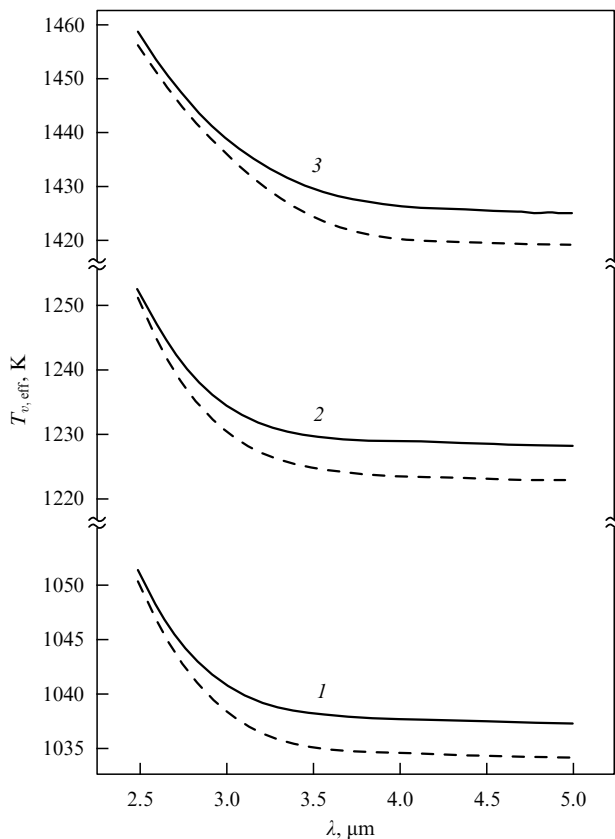


Figure 8. Effective temperature of samples of a quartz heat-insulating material with a thickness of 4 mm (solid lines) and 9.7 mm (dashed lines) for various furnace-hearth temperatures T_0 : 1, 800°C; 2, 1000°C; 3, 1200°C.

Let us see what the stationary relations of Section 5.4 will give in the same case when used instead of the equilibrium relations of Section 5.3. Under the considered experimental conditions, the radiation inside the furnace — the region onto which the spectral apparatus is focused — can be assumed isotropic; then, from Eqns (5.34) and (5.12) we obtain (after noting that, in our case, the surface S coincides with the inner

surface of the furnace):

$$\frac{I_v(\nu)}{I^0(T_{v, \text{eff}}, \nu)} + \frac{I_w(\nu)}{I^0(T_{w, \text{eff}}, \nu)} = 1. \quad (5.39)$$

Unlike the equilibrium relation (5.21) that underlies the processing of experimental data, the radiation intensity and effective temperature involved in this equation refer not to the closed wall w_c but to the partially open wall w . Such a relation is more suitable for open-top furnace experiments. Therefore, it requires no corrections whatsoever for a non-closed wall, i.e., there is no need to employ the approximate coefficient K . Note that the value of $T_{w, \text{eff}}(\nu)$ differs from that found above, which is directly related to the fact that in the stationary case the temperature-averaging procedure is based on other formulas [see Eqn (5.33)] than in the case of a closed cavity.

6. Conclusions

We can summarize our main results as follows:

(1) General relations between the components of the stationary radiation from objects with scattering media have been obtained. Each component is determined by the primary radiation from a certain part of the radiator as well as by the passage of the radiation to the region of observation. The relations do not depend on the specific conditions within the radiator, although the components themselves do.

(2) The radiation component–component relations have been derived both for the case in which geometrical optics can be applied to the scattering medium and for the case in which it cannot. In the former case, the derivation involves general solutions of the transfer equation obtained within the Green function formalism; in the latter, the generalized Kirchhoff law is used to treat thermal radiation from the scattering medium.

(3) The method used to derive the general relations is based on the probabilistic description of radiation. In the case where geometrical optics is applicable, the first step is to write an equation expressing the fact that the probability of disappearance of the radiation emitted in the stationary regime is unity. The introduction of stationary-radiation components into the resulting equations leads to stationary relations between the components. The relations were obtained under the assumption that the reversibility conditions are satisfied in the medium and that the reciprocity relations are obeyed in every scattering event.

(4) If the geometrical optics approximation is not valid, the stationary relations were again obtained by considering a stationary condition that has a probabilistic meaning.

(5) In the special case of a thermal radiator enclosed by a radiating surface, the stationary relations yield equilibrium ones, which can also be derived from the equilibrium-radiation condition for an isothermal cavity with radiating and scattering medium inside.

(6) In the general case, the relations between the radiation fluxes are written in an integral form [see Eqns (3.14), (5.35), (5.36)], while for special cases of practical interest an algebraic form is given [Eqns (3.17), (3.18), (3.21), (5.21), (5.39)].

(7) The quantities involved in the component–component relations are the subject of spectroscopic investigations. The examples given here demonstrate the great potential usefulness of these relations for both experimental and theoretical studies of radiation.

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