Physics – Uspekhi 44 (12) 1211 – 1227 (2001)

REVIEWS OF TOPICAL PROBLEMS

PACS numbers: 11.10.Wx, **12.38.–t**, 12.39.Fe

Chiral effective theory of strong interactions

B L Ioffe

DOI: 10.1070/PU2001v044n12ABEH000972

Contents

Abstract. A review of chiral effective theory (CET) is presented. CET is based on quantum chromodynamics (QCD) and describes strong interaction processes at low energies. It is proved that CET arises as a consequence of the spontaneous violation of chiral symmetry in QCD — the appearance of chiral-symmetry-violating vacuum condensates. The Goldstone theorem is proved for the case of QCD, and the existence of the octet of massless Goldstone bosons (π, K, η) is demonstrated in the limit of massless u, d and s quarks (or the existence of the triplet of massless pions in the limit $m_u, m_d \to 0$). It is shown that the same phenomenon — the appearance of quark condensate in QCD — which is responsible for the Goldstone bosons also gives rise to chiral-symmetry-violating massive baryons. The general form of the CET Lagrangian is derived. Examples of higher order corrections to tree diagrams in CET are considered. The Wess – Zumino term (i.e., the p^4 term in the CET Lagrangian) is given. Low energy sum rules are presented. QCD and CET at finite temperature are discussed. In the CET framework, the T^2 correction to quark condensate in QCD is calculated at finite temperature, and results including higher order temperature corrections are presented. These results indicate on a phase transition to occur at $T \approx 150$ – 200 MeV in QCD. The mixing of current correlators in order of T^2 is proved.

1. Introduction

It is generally accepted — and this fact is indisputable now — that the true theory of strong interactions is quantum chromodynamics (QCD), a nonabelian gauge theory of interacting quarks and gluons. QCD possesses a striking property of asymptotic freedom: the coupling constant $\alpha_s(Q^2)$ decreases logarithmically as a function of the momentum transfer squared Q^2 at large Q^2: $\alpha_s(Q^2) \propto 1/\ln Q^2$ as $Q^2 \to \infty$ [or, which is equivalent, α_s decreases at small distances r, $\alpha_s(r) \propto 1/\ln r$].

This property of QCD allows one to perform reliable theoretical calculation of the processes proceeding at high momentum transfers (at small distances) by using perturbation theory. However, the same property of the theory involves the increase (in the framework of perturbative theory an unlimited increase) of the running coupling constant in QCD at small momentum transfer, i.e. at large distances. Physically, such growth is natural and, furthermore, it is needed, otherwise the theory would not be a theory of *strong* interactions. QCD also possesses another remarkable property, the property of confinement: quarks and gluons cannot leave the region of their strong interaction and cannot be observed as real physical objects. Physical objects, observed experimentally at large distances, are hadrons — mesons and baryons.

These two circumstances — the growth of the coupling constant and the phenomenon of confinement, do not make it possible as a rule, to predict theoretically in QCD the processes at low energies and the properties of physical hadrons. (Some exceptions from this rule are low energy theorems, proved in QCD, and, especially the powerful QCD sum rule method, which, although, starts from small distances, but allows one in many cases to go to rather large ones and to describe the properties of hadrons. The other exceptions are the numerical calculations on lattices.)

However, it became possible to construct a QCD based effective theory, which describes the processes of strong interaction at low energies. The small parameters in the theory are the momenta of interacting particles, more exactly, their ratios to the characteristic hadronic mass scale $M \approx 0.5 - 1$ GeV: $p_i/M \ll 1$. The theory is constructed as a series in powers of p_i/M and is an effective theory, i.e., when

B L Ioffe Institute for Theoretical and Experimental Physics,
B. Cheremushkinskaya ul. 25, 117218 Moscow, Russian Federation
Tel. (7-095) 123 31 93. Fax (7-095) 127 08 33
E-mail: ioffe@vitep5.itep.ru

Received 16 March 2001, revised 24 July 2001
Uspekhi Fizicheskikh Nauk 171 (12) 1273 – 1290 (2001)
Translated by B L Ioffe; edited by S V Semenov

going to the next order terms in p_i/M, in the Lagrangian one must take into account additional terms characterized by new parameters.

The appearance of the effective theory is connected with one more specific property of QCD — the spontaneous breaking of chiral symmetry. The masses of light u, d, and s quarks which enter the QCD Lagrangian, especially the masses of the u and d quarks, from which the usual (nonstrange) hadrons are built, are very small compared with the characteristic mass scale $m_{\mathrm{u}}, m_{\mathrm{d}} < 10$ MeV. Since in QCD the quark interaction proceeds through the exchange of the vector gluonic field, then, if light quark masses are neglected, the QCD Lagrangian (its light quark part) is chirally symmetric, i.e. not only vector, but also axial currents are conserved and the left and right chirality quark fields do not interact with one another.

This chiral symmetry is not realized in the spectrum of hadrons and their low energy interactions. Indeed, in the chirally symmetrical theory the fermion states must be either massless or degenerate in parity. It is evident, that the baryons (particularly, the nucleon) do not possess such properties. This means that the chiral symmetry of the QCD Lagrangian is not realized on the spectrum of physical states and is spontaneously broken. According to the Goldstone theorem, spontaneous breaking of symmetry leads to the appearance of massless particles in the spectrum of physical states — Goldstone bosons.

In QCD Goldstone bosons may be identified with the triplet of π-mesons in the limit $m_{\mathrm{u}}, m_{\mathrm{d}} \to 0$, $m_{\mathrm{s}} \neq 0$ [SU(2)-symmetry] and the octet of pseudoscalar mesons (π, K, η) in the limit $m_{\mathrm{u}}, m_{\mathrm{d}}, m_{\mathrm{s}} \to 0$ [SU(3)-symmetry]. The local $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_A$ symmetry (here V and A mean vector and axial currents, u and d quarks are considered as massless) or $\mathrm{SU}(3)_V \times \mathrm{SU}(3)_A$ symmetry (if m_{s} is also neglected) of the hadronic strong interaction and the existence of massless Goldstone bosons allows one to construct the effective chiral theory of Goldstone bosons and their interactions with baryons, which is valid at small particle momenta.

In the initial version, before QCD, this approach was called the theory of partial conservation of axial current (PCAC). The Lagrangian of the theory represented the nonlinear interaction of pions with themselves and with nucleons and corresponded to the first term in the expansion in powers of momenta in the modern chiral effective theory. (A review of the PCAC theory at this stage was done in Ref. [1].) When QCD had been created, it was proved that the appearance of Goldstone bosons is a consequence of spontaneous breaking of chiral symmetry in QCD vacuum and is tightly connected with the existence of vacuum condensates, violating the chiral symmetry.

It had also been established, that baryon masses are expressed through the same vacuum condensates. Therefore, based on QCD, the mutual interconnection of all set of phenomena under consideration was found. It was possible to formulate the chiral effective theory (CET) of hadrons as a successive expansion in powers of particle momenta and quark (or Goldstone bosons) masses not only in the tree approximation, as in PCAC, but also accounting for loops. (CET is often called chiral perturbation theory — ChPT.)

In this review the foundations, basic ideas and concepts of CET are considered as well as their connection with QCD. The main attention is paid to the general properties of pion interactions. For the pion – nucleon interaction only the general form of the Lagrangian is presented. The physical effects are considered as illustrative examples in a nonsystematic way. In fact, in CET a lot of such effects have been calculated (particularly, for meson – baryon interactions, meson and baryon form-factors etc.) They are very interesting for specialists, but their inclusion in this review would increase its volume drastically. The comparison of the theory with experiment will almost not be discussed. Such a discussion could be the subject of a separate review.

2. The masses of the light quarks

In what follows u, d, s quarks will be called 'light quarks' and all other quarks 'heavy quarks'. The reason is that the masses of the light quarks are small compared with the characteristic mass of the strong interaction $M \approx 0.5 - 1.0$ GeV or m_{p}. This statement is a consequence of the whole set of facts confirming that the symmetry of the strong interaction is $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)$. Here the group generators are the charges corresponding to the left $(V-A)$ and right $(V+A)$ light quark chiral currents and U(1) corresponds to the baryonic charge current. The experiment shows that the accuracy of $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry is of the same order as the accuracy of the SU(3) symmetry: the small parameter characterizing the chiral symmetry violation in strong interactions is generally of order $\sim 1/5 - 1/10$.

The approximate validity of the chiral symmetry means that not only the divergences of the vector currents $\partial_\mu j^q_\mu$ are zero or small, but also of the axial ones $\partial_\mu j^q_{\mu 5}$, where $q = \mathrm{u, d, s}$). (This statement refers to the nonsinglet in flavor axial currents. The divergence of the singlet axial current is determined by the anomaly and is nonzero even for massless quarks — the discussion of this problem is outside the scope of this review.) The divergences of nonsinglet axial currents in QCD are proportional to quark masses. Therefore the existence of chiral symmetry can be understood if the quark masses are small [2, 3]. However, the baryon masses are by no means small: chiral symmetry is not realized in the hadronic mass spectrum in a trivial way by the vanishing of all the fermion masses. This means that the chiral symmetry is broken spontaneously by the physical states spectrum.

According to the Goldstone theorem such a symmetry breaking results in the appearance of massless particles — Goldstone bosons. In the case considered these Goldstone bosons must belong to a pseudoscalar octet. They are massless if the eight quark masses are put to zero. The nonvanishing eight quark masses realize the explicit violation of the chiral symmetry and provide the masses of the pseudoscalar meson octet. For this reason the pseudoscalar meson octet (often called the octet of the Goldstone bosons) plays a special role in QCD.

Heavy quarks are decoupled in the low energy domain (this statement is called the Appelquist – Carazzone theorem) [4]. We ignore them in this review where QCD at low energies is considered.

The QCD Hamiltonian can be split into two pieces

$$H = H_0 + H_1 \,, \tag{1}$$

where

$$H_1 = \int \mathrm{d}^3 x \, (m_{\mathrm{u}} \bar{u} u + m_{\mathrm{d}} \bar{d} d + m_{\mathrm{s}} \bar{s} s) \,. \tag{2}$$

Evidently, because of the vector gluon – quark interaction the first term in Hamiltonian H_0 is $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ invariant and the only source of $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ violation is H_1. The quark masses m_q, $q = \mathrm{u, d, s}$ in Eqn (2) are not renormalization invariant: they are scale dependent. It is possible to write

$$m_q(M) = Z_q\left(\frac{M}{\mu}\right) m_q(\mu)\,, \tag{3}$$

where M characterizes the scale, μ is some fixed normalization point and $Z_q(M/\mu)$ are renormalization factors.

If the light quark masses are small and can be neglected, the renormalization factors are flavor-independent and the ratios

$$\frac{m_{q_1}(M)}{m_{q_2}(M)} = \frac{m_{q_1}(\mu)}{m_{q_2}(\mu)} \tag{4}$$

are scale-independent and have definite physical meaning. (This relation takes place if M is higher than the Goldstone mass m_K: its validity in the domain $M \sim m_\mathrm{K}$ may lead to some doubts.)

In order to find the ratios $m_\mathrm{u}/m_\mathrm{d}$ and $m_\mathrm{s}/m_\mathrm{d}$ consider the axial currents

$$\begin{aligned} j^{-}_{\mu 5} &= \bar{d}\gamma_\mu\gamma_5 u\,,\\ j^{3}_{\mu 5} &= \frac{1}{\sqrt{2}}\left[\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right], \end{aligned} \tag{5}$$

$$j^{\mathrm{s}-}_{\mu 5} = \bar{s}\gamma_\mu\gamma_5 u\,, \qquad j^{\mathrm{s}0}_{\mu 5} = \bar{s}\gamma_\mu\gamma_5 d \tag{6}$$

and their matrix elements between vacuum and π or K meson states

$$\begin{aligned} \langle 0|j^{-}_{\mu 5}|\pi^+\rangle &= \mathrm{i} f_{\pi^+} p_\mu\,,\\ \langle 0|j^{3}_{\mu 5}|\pi^0\rangle &= \mathrm{i} f_{\pi^0} p_\mu\,,\\ \langle 0|j^{\mathrm{s}-}_{\mu 5}|K^+\rangle &= \mathrm{i} f_{\mathrm{K}^+} p_\mu\,,\\ \langle 0|j^{\mathrm{s}0}_{\mu 5}|K^0\rangle &= \mathrm{i} f_{\mathrm{K}^0} p_\mu\,, \end{aligned} \tag{7}$$

where p_μ are π or K momenta.

In the limit of strict SU(3) symmetry all constants in the rhs of (7) are equal: $f_{\pi^+} = f_{\pi^0} = f_{\mathrm{K}^+} = f_{\mathrm{K}^0}$, SU(2) isotopical symmetry results in the equalities $f_{\pi^+} = f_{\pi^0}$, $f_{\mathrm{K}^+} = f_{\mathrm{K}^0}$. The constants $f_{\pi^+} \equiv f_\pi$ and $f_{\mathrm{K}^+} \equiv f_\mathrm{K}$ have the meaning of coupling constants in the decays $\pi^+ \to \mu^+\nu$ and $\mathrm{K}^+ \to \mu^+\nu$. Experimentally they are equal to $f_\pi = 131$ MeV, $f_\mathrm{K} = 160$ MeV. The ratio $f_\mathrm{K}/f_\pi = 1.22$ characterizes the accuracy of SU(3) symmetry.

Multiplying (7) by p_μ and using the equality for the divergence of axial current following from the QCD Lagrangian

$$\partial_\mu\left[\bar{q}_1(x)\gamma_\mu\gamma_5 q_2(x)\right] = \mathrm{i}(m_{q_1} + m_{q_2})\,\bar{q}_1(x)\gamma_5 q_2(x)\,, \tag{8}$$

we get

$$\begin{aligned} &\mathrm{i}(m_\mathrm{u} + m_\mathrm{d})\langle 0|\bar{d}\gamma_5 u|\pi^+\rangle = f_{\pi^+} m^2_{\pi^+}\,,\\ &\frac{\mathrm{i}}{\sqrt{2}}\Big[(m_\mathrm{u} + m_\mathrm{d})\langle 0|\bar{u}\gamma_5 u - \bar{d}\gamma_5 d|\pi^0\rangle\\ &\qquad + (m_\mathrm{u} - m_\mathrm{d})\langle 0|\bar{u}\gamma_5 u + \bar{d}\gamma_5 d|\pi^0\rangle\Big] = f_{\pi^0} m^2_{\pi^0}\,,\\ &\mathrm{i}(m_\mathrm{s} + m_\mathrm{u})\langle 0|\bar{s}\gamma_5 u|K^+\rangle = f_{\mathrm{K}^+} m^2_{\mathrm{K}^+}\,,\\ &\mathrm{i}(m_\mathrm{s} + m_\mathrm{d})\langle 0|\bar{s}\gamma_5 d|K^0\rangle = f_{\mathrm{K}^0} m^2_{\mathrm{K}^0}\,. \end{aligned} \tag{9}$$

We neglect the electromagnetic (and weak) interaction and assume that isotopic invariance may be used for the matrix elements in the lhs of (9). Then

$$\begin{aligned} &\langle 0|\bar{u}\gamma_5 u + \bar{d}\gamma_5 d|\pi^0\rangle = 0\,,\\ &\langle 0|\bar{d}\gamma_5 u|\pi^+\rangle = \frac{1}{\sqrt{2}}\langle 0|\bar{u}\gamma_5 u - \bar{d}\gamma_5 d|\pi^0\rangle\,, \end{aligned} \tag{10}$$

and, as follows from (9), $\pi^\pm$ and π^0 masses are equal in this approximation even when $m_\mathrm{u} \neq m_\mathrm{d}$. Hence the experimentally observed mass difference $\Delta m_\pi = m_{\pi^+} - m_{\pi^0} = 4.6$ MeV is caused by the electromagnetic interaction only. The sign of the K-meson mass difference $\Delta m_\mathrm{K} = m_{\mathrm{K}^+} - m_{\mathrm{K}^0} = -4.0$ MeV is opposite to that of the pion ones. The electromagnetic kaon and pion mass differences in QCD or in the quark model are determined by the same diagrams, and must, at least, be of the same sign. This means, in accord with (9), that $m_\mathrm{d} > m_\mathrm{u}$.

Assuming the SU(3) invariance of matrix elements in (9) and using simple algebra, it is easy to get from (9) and (10)

$$\begin{aligned} \frac{m_\mathrm{u}}{m_\mathrm{d}} &= \frac{\overline{m}^2_\pi - (\overline{m}^2_{\mathrm{K}^0} - \overline{m}^2_{\mathrm{K}^+})}{\overline{m}^2_\pi + (\overline{m}^2_{\mathrm{K}^0} - \overline{m}^2_{\mathrm{K}^+})}\,,\\ \frac{m_\mathrm{s}}{m_\mathrm{d}} &= \frac{\overline{m}^2_{\mathrm{K}^0} + \overline{m}^2_{\mathrm{K}^+} - \overline{m}^2_\pi}{\overline{m}^2_{\mathrm{K}^0} - \overline{m}^2_{\mathrm{K}^+} + \overline{m}^2_\pi}\,. \end{aligned} \tag{11}$$

The bars in (11) mean that the pion and kaon masses here are not the physical ones, but the masses in the limit, when the electromagnetic interaction is switched off.

In order to relate $\overline{m}^2_\pi$ and $\overline{m}^2_\mathrm{K}$ to physical masses, let us again use the SU(3) symmetry. In the SU(3) symmetry the photon is U-singlet and π^+ and K^+ belong to the U doublet[1]. Therefore, the electromagnetic corrections to $m^2_{\pi^+}$ and $m^2_{\mathrm{K}^+}$ are equal:

$$(\delta m^2_{\pi^+})_\mathrm{el} = (\delta m^2_{\mathrm{K}^+})_\mathrm{el}\,. \tag{12}$$

It can also be shown that in the limit $m^2_\pi, m^2_\mathrm{K} \to 0$, the electromagnetic corrections to the π^0 and K^0 masses tend to zero,

$$(\delta m^2_{\pi^0})_\mathrm{el} = (\delta m^2_{\mathrm{K}^0})_\mathrm{el} = 0\,. \tag{13}$$

Equations (12) and (13) may be rewritten in the form of the Dashen relation [6]:

$$(m^2_{\pi^+} - m^2_{\pi^0})_\mathrm{el} = (m^2_{\mathrm{K}^+} - m^2_{\mathrm{K}^0})_\mathrm{el}\,. \tag{14}$$

From Eqns (13), (14) we have

$$\begin{aligned} &\overline{m}^2_\pi = m^2_{\pi^0}\,,\\ &\overline{m}^2_{\mathrm{K}^+} - \overline{m}^2_{\mathrm{K}^0} = m^2_{\mathrm{K}^+} - m^2_{\mathrm{K}^0} - (m^2_{\pi^+} - m^2_{\pi^0})\,. \end{aligned} \tag{15}$$

The substitution of (15) into (11) leads to

$$\begin{aligned} \frac{m_\mathrm{u}}{m_\mathrm{d}} &= \frac{2m^2_{\pi^0} - m^2_{\pi^+} - (m^2_{\mathrm{K}^0} - m^2_{\mathrm{K}^+})}{m^2_{\mathrm{K}^0} - m^2_{\mathrm{K}^+} + m^2_{\pi^+}}\,,\\ \frac{m_\mathrm{s}}{m_\mathrm{d}} &= \frac{m^2_{\mathrm{K}^0} + m^2_{\mathrm{K}^+} - m^2_{\pi^0}}{m^2_{\mathrm{K}^0} - m^2_{\mathrm{K}^+} + m^2_{\pi^+}}\,. \end{aligned} \tag{16}$$

[1] The description of T, U, V subgroups of SU(3) group is given, e.g. in Ref. [5].

Numerically, this gives [7, 8]

$$\frac{m_{\rm u}}{m_{\rm d}} = 0.56\,, \qquad \frac{m_{\rm s}}{m_{\rm d}} = 20.1\,. \tag{17}$$

A strong violation of isotopic invariance, as well as a large difference between u, d and s-quark masses, i.e. the violation of SU(3) flavor symmetry, is evident from Eqns (17). (A more detailed analysis shows that the results (17) only slightly depend on the assumption of the SU(3) symmetry of the corresponding matrix elements used in their derivation.) This seems to be in contradiction with the well established isospin symmetry of the strong interaction, as well as with the approximate SU(3) symmetry. The resolution of this puzzle is that the quark masses are small: the parameter characterizing isospin violation is $(m_{\rm d} - m_{\rm u})/M$ and the parameter characterizing the SU(3) symmetry violation is $m_{\rm s}/M$, where M is the characteristic scale of strong interaction.

An estimate of the absolute value of the quark masses can be obtained in the following way. Suppose that the hadrons which contain strange quarks and which belong to a given unitary multiplet are heavier only because of the strange quark mass. Then from consideration of mass splittings in the baryon octet one can find that $m_{\rm s} \approx 150$ MeV at a scale of about 1 GeV. From Eqns (17) it then follows that

$$m_{\rm u} = 4.2 \text{ MeV}\,, \qquad m_{\rm d} = 7.5 \text{ MeV}\,, \qquad m_{\rm s} = 150 \text{ MeV} \tag{18}$$

at 1 GeV.

Taking these values, one may expect that isospin violation could be of order $(m_{\rm d} - m_{\rm u})/M \sim 10^{-2}$, i.e. of the same order as arising from the electromagnetic interaction. The order of the SU(3) symmetry violation is $\sim m_{\rm s}/M \sim 1/5$. The large $m_{\rm s}/m_{\rm d}$ ratio explains the large mass splitting in the pseudoscalar meson octet. Assuming SU(3) symmetry of the matrix elements in (9), we have

$$\frac{m^2_{\rm K^+}}{m^2_{\pi^+}} = \frac{m_{\rm s} + m_{\rm u}}{m_{\rm d} + m_{\rm u}} = 13\,, \tag{19}$$

in agreement with experiment.

An important consequence of Eqns (9) is that in QCD the mass squares of the pseudoscalar meson octet $m^2_\pi, m^2_{\rm K}, m^2_\eta$ are proportional to the quark masses and vanish when m_q tend to zero: in this limit the octet of pseudoscalar mesons becomes massless.

3. Spontaneous violation of chiral symmetry. Quark condensate

As has already been mentioned, the large baryon masses indicate that chiral symmetry in QCD is broken spontaneously. Indeed, let us consider any process with the participation of a polarized baryon, e.g., any hadron – proton scattering on a longitudinally polarized proton at energies of order 1 GeV. We can treat the initial state of a polarized proton as a state with some fixed quark helicities. Due to the chiral symmetry the helicities are conserved in the course of collision. Therefore, we could expect that proton longitudinal polarization will not change in the collision. However, it is well known — and this is a direct consequence of the Dirac equation for the proton — that the proton mass results in a proton helicity flip with a not small probability. This simple fact — the existence of the proton mass — clearly demonstrates the violation of chiral symmetry in strong interactions at low energies [2].

In all known examples of field theories the spontaneous violation of global symmetry manifests itself in the modification of the properties of the ground state — the vacuum. Let us show that such a phenomenon also takes place in QCD. Consider the matrix element

$$\mathrm{i}q_\mu(m_{\rm u} + m_{\rm d}) \int \mathrm{d}^4 x \, \exp(\mathrm{i}qx) \langle 0 | T\{ j^-_{\mu 5}(x), \bar{u}(0)\gamma_5 d(0)\} |0\rangle \tag{20}$$

in the limit of massless u and d quarks (except for the overall factor $m_{\rm u} + m_{\rm d}$). Put q_μ inside the integral, integrate by parts and use the conservation of the axial current. Then only the term with the equal time commutator will remain:

$$\begin{aligned} &-(m_{\rm u} + m_{\rm d}) \int \mathrm{d}^4 x \, \exp(\mathrm{i}qx) \langle 0 | \delta(x_0) \big[j^-_{05}(x), \bar{u}(0)\gamma_5 d(0) \big] |0\rangle \\ &\quad = (m_{\rm u} + m_{\rm d}) \langle 0 | \bar{u}u + \bar{d}d |0\rangle\,. \end{aligned} \tag{21}$$

Let us now go to the limit $q_\mu \to 0$ in (20) and perform the summation over all intermediate states. The nonvanishing contribution comes only from one pion intermediate state, since in this approximation the pion should be considered as massless. This contribution is equal to

$$q_\mu \langle 0 | j^-_{\mu 5} | \pi^+ \rangle \, \frac{-1}{q^2} \langle \pi^+ | (m_{\rm u} + m_{\rm d}) \bar{u}\gamma_5 d |0\rangle = -f^2_\pi m^2_\pi\,, \tag{22}$$

where (7) and (9) were substituted when going to the right hand side. Putting (22) in the lhs of Eqn (21) we get

$$\langle 0 | \bar{q}q | 0 \rangle = -\frac{1}{2} \, \frac{m^2_\pi f^2_\pi}{m_{\rm u} + m_{\rm d}}\,, \tag{23}$$

where $q = u$ or d and the SU(2) invariance of QCD vacuum were used. Equation (23) is called the Gell-Mann – Oakes – Renner relation [9].

It can also be derived in an other way. Assume the quark masses to be nonzero, but small. Then the pion is massive and (20) tends to zero in the limit $q_\mu \to 0$. However, when we put q_μ inside the integral, in addition to the equal time commutator term (21), the term with the divergence of the axial current will appear. The account of this term, saturated by one pion intermediate state, results in the same Eqn (23).

Numerically, with the quark mass values (18) we have

$$\langle 0 | \bar{q}q | 0 \rangle = -(240 \text{ MeV})^3\,. \tag{24}$$

As follows from Eqn (23), the product $(m_{\rm u} + m_{\rm d})\langle 0 | \bar{q}q | 0 \rangle$ is scale independent, while $\langle 0 | \bar{q}q | 0 \rangle$ depends on the scale and the numerical value (24) refers to 1 GeV. The quantity $\langle 0 | \bar{q}q | 0 \rangle$, called vacuum quark condensate can be also represented as

$$\langle 0 | \bar{q}q | 0 \rangle = \langle 0 | \bar{q}_L q_R + \bar{q}_R q_L | 0 \rangle\,, \tag{25}$$

where q_L and q_R are the left and right quark fields: $q_L = (1/2)(1 + \gamma_5)q$, $q_R = (1/2)(1 - \gamma_5)q$.

[2] In principle, chiral symmetry in baryonic states could be realized in such a way that all baryonic states would be degenerate in parity with a splitting of order $m_{\rm u} + m_{\rm d}$. This is evidently not the case.

It is evident from Eqn (25) that quark condensate violates chiral invariance and its numerical value (24) has a characteristic hadronic scale. The violation of chiral invariance is the violation of global symmetry, because $\langle 0|\bar{q}q|0\rangle$ is noninvariant under global transformations $q \to \exp(\mathrm{i}\alpha\gamma_5)\,q$ with a constant α.

Surely, in perturbative QCD with massless quarks the quark condensate is zero in any order of perturbation theory. Therefore, the nonzero and non-small value of the quark condensate may arise only due to nonperturbative effects. The conclusion is that the nonperturbative field fluctuations, which violate chiral invariance of the Lagrangian, are present and essential in QCD. Quark condensate plays a special role because its lowest dimension, $d = 3$.

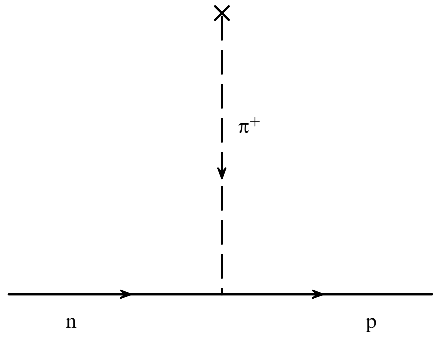

Figure 1. Interaction of nucleon with axial current through an intermediate pion: the solid lines correspond to the nucleon, the dashed line to the pion, the cross indicates the interaction with the external axial current.

4. Goldstone theorem

In Section 3 we presented two arguments in favor of chiral symmetry, approximately valid in QCD because of small u, d, s quark masses, being spontaneously broken. These arguments were: the existence of large baryon masses and the appearance of violating chiral symmetry quark condensate. Let us go to the limit of massless u, d, s quarks and show now that the direct consequence of each of these arguments is the appearance of massless pseudoscalar bosons in the hadronic spectrum.

Consider the matrix element of the axial current $j^+_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 d$ between the neutron and proton states. The general form of this matrix element is

$$\langle p|j^+_{\mu 5}|n\rangle = \bar{v}_{\mathrm{p}}(p_{\mathrm{p}})\left[\gamma_\mu\gamma_5 F_1(q^2) + q_\mu\gamma_5 F_2(q^2)\right]v_{\mathrm{n}}(p_{\mathrm{n}})\,, \quad (26)$$

where p_{n} and p_{p} are neutron and proton momenta, $q = p_{\mathrm{p}} - p_{\mathrm{n}}$, $v_{\mathrm{p}}(p_{\mathrm{p}})$ and $v_{\mathrm{n}}(p_{\mathrm{n}})$ are proton and neutron spinors and $F_1(q^2)$, $F_2(q^2)$ are form-factors. Multiply (26) by q_μ and go to the limit $q^2 \to 0$, but $q_\mu \neq 0$. After multiplication, the lhs of Eqn (26) vanishes owing to axial current conservation. In the rhs, using the Dirac equations for proton and neutron spinors, we have

$$\bar{v}_{\mathrm{p}}(p_{\mathrm{p}})\left[2mg_A + q^2F_2(q^2)\right]\gamma_5 v_{\mathrm{n}}(p_{\mathrm{n}})\,, \quad (27)$$

where $g_A = F_1(0)$ is the neutron β-decay coupling constant, $g_A = 1.26$ and m is the nucleon mass (assumed to be equal for proton and neutron).

The only way to avoid the discrepancy with the vanishing lhs of Eqn (26) is to assume that $F_2(q^2)$ has a pole at $q^2 = 0$:

$$F_2(q^2)_{q^2\to 0} = -2mg_A\,\frac{1}{q^2}\,. \quad (28)$$

The pole in $F_2(q^2)$ corresponds to the appearance of a massless particle with pion quantum numbers. The matrix element in Eqn (26) then has the form (at small q^2):

$$\langle p|j^+_{\mu 5}|n\rangle = g_A\bar{v}_{\mathrm{p}}(p_{\mathrm{p}})\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\gamma_\nu\gamma_5\,v_{\mathrm{n}}(p_{\mathrm{n}})\,, \quad (29)$$

where conservation of the axial current is evident. The second term in the rhs of Eqn (29) can be described by the interaction of the axial current with the nucleon proceeding through an intermediate pion, when the axial current creates a virtual π^+ and then the π^+ is absorbed by a neutron (Fig. 1). The low energy pion-nucleon interaction can be phenomenologically parametrized by the Lagrangian

$$L_{\pi\mathrm{NN}} = \mathrm{i}g_{\pi\mathrm{NN}}\,\bar{v}_{\mathrm{N}}\gamma_5\tau^a v_{\mathrm{N}}\varphi^a\,, \quad (30)$$

where τ^a are the isospin Pauli matrices and $g_{\pi\mathrm{NN}}$ is the πNN coupling constant, $g^2_{\pi\mathrm{NN}}/4\pi \approx 14$.

Using (7) and (30) the second term in Eqn (26) can be represented as

$$-\sqrt{2}\,g_{\pi\mathrm{NN}}f_\pi\bar{v}_{\mathrm{p}}\gamma_5 v_{\mathrm{n}}\,\frac{q_\mu}{q^2}\,. \quad (31)$$

The comparison with (28) gives the Goldberger–Treiman relation [10]:

$$g_{\pi\mathrm{NN}}f_\pi = \sqrt{2}\,mg_A\,. \quad (32)$$

Experimentally, the Goldberger–Treiman relation is satisfied with a 5% accuracy, which strongly supports the hypothesis of spontaneous chiral symmetry violation in QCD. The main modification of Eqn (29) which arises from the nonvanishing pion mass is the replacement of the pion propagator: $q^2 \to q^2 - m^2_\pi$. Then the contribution of the second term vanishes as $q_\mu \to 0$ and becomes very small in the case of neutron β-decay.

Since the only assumption in the consideration above was the conservation of the axial current, this consideration can be generalized to any other component of the isospin 1 axial current, if SU(2) flavor symmetry is supposed, and to any octet axial current in the case of the SU(3) flavor symmetry. In the last case we come to the conclusion that the octet of pseudoscalar mesons is massless in the limit of massless u, d, s quarks.

The massless bosons which arise through spontaneous symmetry breaking are called Goldstone bosons and the theorem which states their appearance is called the Goldstone theorem [11] (see also Ref. [12]). The proof of the Goldstone theorem presented above was based on the existence of massive baryons and on a nonvanishing nucleon β-decay constant g_A. Before proceeding to another proof based on the existence of quark condensate in QCD, let us formulate some general features of spontaneously broken theories.

Let the Hamiltonian of the theory under consideration be invariant under some Lie group G, i.e., the group generators Q_i to commute with the Hamiltonian H:

$$[Q_i, H] = 0\,, \qquad i = 1, \ldots, n\,. \quad (33)$$

The symmetry is spontaneously broken if the ground state is not invariant under G and a subset of Q_l ($l \leqslant m$, $1 \leqslant m \leqslant n$) exists such that

$$Q_l|0\rangle \neq 0\,. \tag{34}$$

Denote: $|B_l\rangle = Q_l|0\rangle$. As follows from (33)

$$H|B_l\rangle = 0 \tag{35}$$

— the states $|B_l\rangle$ have the same energy as vacuum. These states may be considered as massless bosons at rest — Goldstone bosons [3]. The generators Q_j ($j = m+1, \ldots, n$) generate a subgroup $K \subset G$, since from

$$Q_j|0\rangle = 0 \tag{36}$$

it follows that

$$[Q_j, Q_{j'}]|0\rangle = 0\,, \qquad j, j' = m+1, \ldots, n\,. \tag{37}$$

In the case of QCD the group G is $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, which is spontaneously broken to $\mathrm{SU}(3)_V$ — the group where generators are the octet of vector charges. Q_l are the octet of axial charges and $|B_l\rangle$ are the octet of pseudoscalar mesons. [If only u, d quarks are considered massless, all the above may be repeated, but relative to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ group.]

Strictly speaking, the states $|B_l\rangle$ are not well defined, they have an infinite norm. Indeed,

$$\langle B_l|B_l\rangle = \langle 0|Q_l Q_l|0\rangle = \int \mathrm{d}^3 x\, \langle 0|\, j_l(\mathbf{x}, t)\, Q_l(t)\,|0\rangle\,, \tag{38}$$

where $j_l(x)$ is the charge density operator corresponding to the generator Q_l. Extracting the $\mathbf{x}$-dependence of $j_l(\mathbf{x}, t)$ and using the fact that vacuum and intermediate states in (38) have zero momenta, we have

$$\langle B_l|B_l\rangle = \int \mathrm{d}^3 x\, \langle 0|\, j_l(0, t)\, Q_l(t)\,|0\rangle = V\langle |\, j_l(0, t),\, Q_l(t)\,|0\rangle\,, \tag{39}$$

where V is the total volume ($V \to \infty$).

Physically, the infinite norm is well understood, since the massless Goldstone boson with zero momentum is distributed over the whole space. The prescription how to treat the problem is evident — to give a small mass to the boson. In what follows, when the commutators are considered, the problem can be circumvented by first performing the commutation resulting in δ-functions, and then the integration over $\mathrm{d}^3 x$.

Let us now demonstrate how this general theorem works in QCD in a explicit way. Go back to Eqn (21), which at $q_\mu = 0$ may be rewritten as

$$\langle 0|[Q_5^-, \bar{u}\gamma_5 d]|0\rangle = -\langle 0|\bar{u}u + \bar{d}d|0\rangle\,, \tag{40}$$

where

$$Q_5^- = \int \mathrm{d}^3 x\, j_{05}^-(x) \tag{41}$$

is the axial charge generator. It is evident from Eqn (40), that Q_5^- does not annihilate vacuum, i.e. it belongs to the set of generators (34). It is clear that the same property is inherent to all members of the octet of axial charges in SU(3) symmetry (or to members of isovector axial charges in SU(2) symmetry).

Applying the general considerations of Goldstone, Salam and Weinberg [13] to our case, consider the vacuum commutator

$$\langle 0|\big[\, j_{\mu 5}^-(x), \bar{u}(0)\gamma_5 d(0)\big]|0\rangle \tag{42}$$

in coordinate space. Equation (42) can be written via the Lehmann – Källen representation

$$\langle 0|\big[\, j_{\mu 5}^-(x), \bar{u}(0)\gamma_5 d(0)\big]|0\rangle = \frac{\partial}{\partial x_\mu}\int \mathrm{d}\kappa^2\, \Delta(x, \kappa^2)\, \rho^-(\kappa^2)\,, \tag{43}$$

where $\Delta(x, \kappa^2)$ is the Pauli – Jordan (causal) function for a scalar particle with mass κ:

$$(\partial_\mu^2 + \kappa^2)\, \Delta(x, \kappa^2) = 0\,, \tag{44}$$

and $\rho^-(\kappa^2)$ is the spectral function, defined by

$$(2\pi)^{-3} p_\mu\, \theta(p_0)\, \rho^-(p^2) = -\sum_n \delta^4(p - p_n)\, \langle 0|\, j_{\mu 5}^-(0)|n\rangle\langle n|\bar{u}(0)\gamma_5 d(0)|0\rangle\,. \tag{45}$$

The axial current conservation and Eqn (44) imply that

$$\kappa^2 \rho^-(\kappa^2) = 0\,, \tag{46}$$

hence

$$\rho^-(\kappa^2) = N\delta(\kappa^2)\,. \tag{47}$$

The substitution of Eqn (47) into (43) gives

$$\langle 0|\big[\, j_{\mu 5}^-(x), \bar{u}(0)\gamma_5 d(0)\big]|0\rangle = \frac{\partial}{\partial x_\mu}\, D(x) N\,, \tag{48}$$

where $D(x) = \Delta(x, 0)$. Put $\mu = 0$, $t = 0$, integrate (48) over $\mathrm{d}^3 x$ and use the equality $\partial D(x)/\partial t|_{t=0} = -\delta^3(x)$.

The comparison of the result with (21) shows, that N is proportional to the quark condensate and nonzero. This means that the spectrum of physical states contains a massless Goldstone boson which gives a nonzero contribution to ρ^-. Its quantum numbers are those of π^+. It is easy to perform a similar consideration for other members of the pion multiplet in the case of SU(2) symmetry or for the pseudoscalar meson octet in the case of SU(3) symmetry. Obviously, the proof can be repeated for any other operator whose commutator with axial charges has a nonvanishing vacuum average.

A remark here is in order. The Goldstone theorem, which states that "if global symmetry is spontaneously violated then massless particles are present in the spectrum" is a strict mathematical theorem. However, in QCD the two proofs presented above cannot be considered as rigorous, like a mathematical theorem, where the presence of Goldstone bosons in QCD is proved starting from the QCD Lagrangian and by use of the first principles of the theory. Indeed, in the first proof the existence of a massive nucleon was taken as an experimental fact. In the second proof the appearance of nonvanishing quark condensate in QCD was exploited. The

[3] The statement that Q_l are generators of a continuous Lie group is essential — the theorem is not correct for discrete symmetry generators.

latter was proved [see Eqns (20)–(22)] — based on Ward identities, which, as was demonstrated, became selfconsistent only in the case of existence of a massless pion. Therefore, these proofs may be treated as a convincing physical argument, but not a mathematical theorem (cf. Ref. [14]).

5. Nucleon mass and quark condensate

Let us show now, that the two arguments mentioned above in favor of spontaneously broken chiral symmetry in QCD, namely, the existence of large baryon masses and the appearance of violating chiral symmetry quark condensate are in fact deeply interconnected. We demonstrate that baryon masses arise just due to quark condensate. I will use the QCD sum rule method invented by Shifman, Vainshtein and Zakharov [15], in its applications to baryons [16]. (For a review and collection of relevant original papers see Ref. [17]).

The idea of the method is that at virtualities of order $Q^2 \sim 1$ GeV^2 the operator product expansion (OPE) may be used in the consideration of hadronic vacuum correlators. In OPE the nonperturbative effects reduce to the appearance of vacuum condensates and condensates of the lowest dimension play the most important role. The perturbative terms are moderate and do not change the results in an essential way, especially in the cases of chiral symmetry violation, where they can appear as corrections only.

For definiteness consider the proton mass calculation [16, 18]. We introduce the polarization operator

$$\Pi(p) = \mathrm{i} \int \mathrm{d}^4 x \, \exp(\mathrm{i}px) \left\langle 0 \middle| T\eta(x), \bar{\eta}(0) \middle| 0 \right\rangle , \tag{49}$$

where $\eta(x)$ is the quark current with proton quantum numbers and p^2 is chosen to be space-like, $p^2 < 0$, $|p^2| \sim 1$ GeV^2. The current $\eta(x)$ is the colorless product of three quark fields: $\eta = \varepsilon^{abc} q^a q^b q^c$, $q = \mathrm{u}, \mathrm{d}$ (a, b, c are color indices); the form of the current will be specified below. The general structure of $\Pi(p)$ is

$$\Pi(p) = \hat{p} f_1(p) + f_2(p) . \tag{50}$$

The first structure, proportional to $\hat{p}$ conserves chirality, while the second is chirality violating. For each of the functions $f_i(p^2)$, $i = 1, 2$ the OPE can be written as

$$f_i(p^2) = \sum_n C_n^{(i)}(p^2) \, \langle 0 | O_n^{(i)} | 0 \rangle , \tag{51}$$

where $\langle 0 | O_n^{(i)} | 0 \rangle$ are the vacuum expectation values (v.e.v.) of various operators (vacuum condensates), and $C_n^{(i)}$ are coefficient functions calculated in QCD.

For the first, conserving chirality structure function $f_1(p^2)$ OPE starts from dimension zero ($d = 0$) unit operator. Its contribution is described by the diagram of Fig. 2 and

$$\hat{p} f_1(p^2) = C_0 \hat{p} p^4 \ln \frac{\Lambda_\mathrm{u}^2}{-p^2} + \text{polynomial} , \tag{52}$$

where C_0 is a constant, and Λ_u is the ultraviolet cutoff. The OPE for the chirality violating structure $f_2(p^2)$ starts from the $d = 3$ operator, and its contribution is represented by the diagram of Fig. 3:

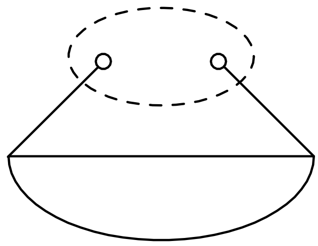

Figure 3. Diagram, corresponding to chirality violating dimension 3 operator (quark condensate). The dots surrounded by circle represent quarks in the condensate phase. All other notation is the same as in Fig. 2.

$$f_2(p^2) = C_1 p^2 \langle 0 | \bar{q} q | 0 \rangle \, \ln \frac{\Lambda_\mathrm{u}^2}{-p^2} + \text{polynomial} . \tag{53}$$

Let us for a moment restrict ourselves to these first order terms of OPE and neglect higher order terms (as well the perturbative corrections).

On the other hand, the polarization operator (49) may be expressed via the characteristics of physical states using the dispersion relations

$$f_i(s) = \frac{1}{\pi} \int \frac{\mathrm{Im} f_i(s')}{s' + s} \, \mathrm{d}s' + \text{polynomial} , \qquad s = -p^2 . \tag{54}$$

The proton contribution to $\mathrm{Im}\,\Pi(p)$ is equal to

$$\begin{aligned} \mathrm{Im}\,\Pi(p) &= \pi \langle 0 | \eta | p \rangle \, \langle p | \bar{\eta} | 0 \rangle \, \delta(p^2 - m^2) \\ &= \pi \lambda_N^2 (\hat{p} + m) \, \delta(p^2 - m^2) , \end{aligned} \tag{55}$$

where

$$\langle 0 | \eta | p \rangle = \lambda_\mathrm{N} v(p) , \tag{56}$$

λ_N is a constant, $v(p)$ is the proton spinor and m is the proton mass.

Still restricting ourselves to this rough approximation, we may take the calculated expression for $\Pi(p)$ in QCD [Eqns (52), (53)] equal to its phenomenological representation Eqn (55). The best way to get rid of the unknown polynomial is to apply the Borel transformation to both sides of the equality, defined as

$$\begin{aligned} \mathcal{B}_{M^2} f(s) &= \lim_{\substack{n \to \infty,\, s \to \infty \\ s/n = M^2 = \mathrm{const}}} \frac{s^{n+1}}{n!} \left(-\frac{\mathrm{d}}{\mathrm{d}s} \right)^n f(s) \\ &= \frac{1}{\pi} \int_0^\infty \mathrm{d}s \, \mathrm{Im} f(s) \, \exp\left(-\frac{s}{M^2} \right) , \end{aligned} \tag{57}$$

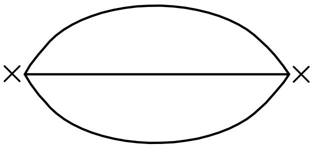

Figure 2. Bare loop diagram, contributing to the chirality conserving function $f_1(p^2)$: solid lines correspond to quark propagators, crosses indicate the interaction with external currents.

if $f(s)$ is given by dispersion relation (54). [Sometimes relation (57) is called the Laplace transform.] Note that

$$\mathcal{B}_{M^2} \frac{1}{s^n} = \frac{1}{(n-1)!(M^2)^{n-1}} \,. \tag{58}$$

Owing to the factor $1/(n-1)!$ in Eqn (58) the Borel transformation suppresses the contributions of high order terms in OPE.

We now specify the quark current $\eta(x)$. It is clear from (55) that the proton contribution will dominate in some region of the Borel parameter $M^2 \sim m^2$ only in the case when both calculated QCD functions f_1 and f_2 are of the same order. This requirement, together with the requirements of absence of derivatives and of renormcovariance fixes the form of the current in unique way (for more details see Refs [16, 19]):

$$\eta(x) = \varepsilon^{abc}(u^a C\gamma_\mu u^b)\gamma_\mu\gamma_5 d^c \,, \tag{59}$$

where C is the charge conjugation matrix. With the current $\eta(x)$ (59) the calculations of the diagrams of Figs 2 and 3 can be easily performed, the constants C_0 and C_1 are determined and after Borel transformation two equations (sum rules) arise (on the phenomenological sides of the sum rules only proton state is accounted):

$$M^6 = \tilde{\lambda}_{\mathrm{N}}^2 \exp\left(-\frac{m^2}{M^2}\right), \tag{60}$$

$$-2(2\pi)^2 \langle 0|\bar{q}q|0\rangle\, M^4 = m\tilde{\lambda}_{\mathrm{N}}^2 \exp\left(-\frac{m^2}{M^2}\right), \tag{61}$$

$$\tilde{\lambda}_{\mathrm{N}}^2 = 32\pi^4 \lambda_{\mathrm{N}}^2 \,.$$

It can be shown that this rough approximation is valid at $M \approx m$. Using this value of M and dividing (60) by (61) we get a simple formula for the proton mass [16]:

$$m = \left[-2(2\pi)^2 \langle 0|\hat{q}q|0\rangle\right]^{1/3} . \tag{62}$$

This formula demonstrates the fundamental fact that the appearance of the proton mass is caused by spontaneous violation of chiral invariance: the presence of quark condensate. [Numerically, (62) gives the experimental value of proton mass with an accuracy better than 5%].

A more refined treatment of the problem of the proton mass calculation was performed: high order terms of OPE were accounted for, as well as excited states in the phenomenological sides of the sum rules and the stability of the Borel mass dependence was checked. In the same way, the hyperons, isobar and some resonances masses were calculated, all in good agreement with experiment [20–22]. I will not dwell on these results. The main conclusion is: the origin of baryon masses is in spontaneous violation of chiral invariance — the existence of quark condensate in QCD. Therefore, three phenomena: baryon masses, quark condensate and the appearance of Goldstone bosons are tightly connected.

6. Chiral effective theory at low energies

An effective chiral theory based on QCD and exploiting the existence and properties of the Goldstone bosons may be formulated. This theory is an effective low energy theory, which means that the theory is selfconsistent, but only in terms of expansion in powers of particle momenta (or in the derivatives of fields in the coordinate space). The Lagrangian is represented as a series of terms with increasing powers of momenta. The theory breaks down at sufficiently high momenta, the characteristic parameters are $|\mathbf{p}_i|/M$, where $\mathbf{p}_i$ are the spatial momenta of the Goldstone bosons entering the process under consideration and M is the characteristic scale of strong interaction. (Since $\mathbf{p}_i$ depend on the reference frame, some care must be taken when choosing the most suitable frame in each particular case.)

The physical basis of the theory is the fact that in the limit of vanishing (or small enough) quark masses the spectrum of Goldstone bosons is separated by the gap from the spectrum of other hadrons. The CET working in the domain $|\mathbf{p}_i|/M \ll 1$, is a selfconsistent theory and not a model. Such a theory can be formulated based on the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry with pions as (quasi) Goldstone bosons. Then one may expect the accuracy of the theory to be of the order of that of the isospin theory, i.e. of a few percent. Or the theory may be based on the $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry with an octet of pseudoscalar bosons π, K, η as (quasi) Goldstone bosons. In this case the accuracy of the theory is of the order of the violation of the SU(3) symmetry, i.e., of order $m_s/M \approx 10-20\%$. For definiteness, the main part of this section deals with the case of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.

The heuristic arguments for the formulation of the chiral theory are the following. In the limit of quark and pion mass going to zero, (7) can be replaced by the field equation

$$j^i_{\mu 5} = -\frac{1}{\sqrt{2}} f_\pi\, \partial_\mu \varphi^i_\pi \,, \tag{63}$$

$$j^i_{\mu 5} = \frac{1}{2}\, \bar{q}\gamma_\mu\gamma_5\tau^i q, \qquad q = u, d\,, \tag{64}$$

where φ^i_π is the pion field, τ^i are the Pauli matrices and $i = 1, 2, 3$ is the isospin index. [The normalization of the current $j^i_{\mu 5}$ is changed compared with (7) in order to have the standard commutation relations of current algebra.] Taking the divergence from Eqn (63) we have

$$\partial_\mu j^i_{\mu 5} = \frac{1}{\sqrt{2}} f_\pi m^2_\pi \varphi^i_\pi \,. \tag{65}$$

Equations (63) and (65) are correct near the pion mass shell. Since the pion state is separated by the gap from the other massive states in the channel with pion quantum numbers these equations can be treated as field equations valid in the low energy region (usually they are called the equations of partial conservation of axial current PCAC).

A direct consequence of Eqn (65) is the Adler selfconsistency condition [23]. Consider the amplitude of the process $A \to B + \pi$, where A and B are arbitrary hadronic states in the limit of vanishing pion momentum p. The matrix element of this process can be written as

$$M_i(2\pi)^4\, \delta^4(p_A - p - p_B) = \int \mathrm{d}^4 x\, \exp(\mathrm{i}px)(\partial^2_\mu + m^2_\pi)\langle B|\varphi^i_\pi|A\rangle \,. \tag{66}$$

The substitution of Eqn (65) gives

$$M_i = \frac{\mathrm{i}(p^2 - m^2_\pi)}{(1/\sqrt{2})f_\pi m^2_\pi}\, p_\mu \langle B|j^i_{\mu 5}(0)|A\rangle \,. \tag{67}$$

Going to the limit $p_\mu \to 0$ we get

$$M(A \to B\pi)_{p\to 0} \to 0\,, \tag{68}$$

which is the Adler condition. When deriving (68) it was implicitly assumed that the matrix element $\langle B|j^i_{\mu 5}|A\rangle$ does not contain pole terms, where the axial current interacts with an external line. Generally, the Adler theorem does not work in such cases.

The chiral theory is based on the following principles:

(1) The pion field transforms under some representation of the group $G = SU(2)_L \times SU(2)_R$.

(2) The action is invariant under these transformations.

(3) After breaking the transformations reduce to SU(2) — the transformations which are generated by the isovector vector current.

(4) In the lowest order the field equations (63), (65) are fulfilled.

The pion field may be represented by the 2×2 unitary matrix $U(x)$: $U^{-1} = U^+(x)$, depending on $\varphi^i_\pi(x)$. The condition $\det U = 1$ is imposed on $U(x)$. Therefore the number of degrees of freedom of matrix U is equal to that of three pionic fields $\varphi^i_\pi(x)$. The transformation law under the group G transformations is given by

$$U'(x) = V_L\, U(x)\, V_R^+\,, \tag{69}$$

where V_L and V_R are unitary matrices of $SU(2)_L$ and $SU(2)_R$ transformations. Relation (69) satisfies the necessary condition, that after breaking, when G reduces to SU(2) and $V_L = V_R = V$, the transformation law reduces to

$$U' = V\, U(x)\, V^{-1}\,, \tag{70}$$

— the transformation induced by the vector current.

It can be shown that the general form of the lowest order effective Lagrangian, where only the terms up to p^2 are kept and the breaking arising from the pion mass is neglected, is [24–27]

$$L_{\text{eff}} = k\,\mathrm{Tr}\,(\partial_\mu U \partial_\mu U^+)\,, \tag{71}$$

where k is some constant.

The conserved vector and the axial currents (Noether currents), corresponding to Lagrangian (71) can be found by applying to (71) the transformations (69) with

$$V_L = V_R = 1 + \frac{\mathrm{i}}{2}\,\boldsymbol{\varepsilon\tau} \tag{72}$$

in the case of vector current and

$$V_L = V_R^+ = 1 + \frac{\mathrm{i}}{2}\,\boldsymbol{\varepsilon\tau} \tag{73}$$

in the case of axial current. (Here $\boldsymbol{\varepsilon}$ is an infinitesimal isovector.) The results are

$$\begin{aligned} j^i_\mu &= \mathrm{i}k\,\mathrm{Tr}\left(\tau_i\,[\partial_\mu U, U^+]\right),\\ j^i_{\mu 5} &= \mathrm{i}k\,\mathrm{Tr}\left(\tau_i\,\{\partial_\mu U, U^+\}\right). \end{aligned} \tag{74}$$

One may use various realizations of the matrix field $U(x)$ in terms of pionic fields $\varphi^i_\pi(x)$. All of them are equivalent and lead to the same physical consequences [28, 29]. Mathematically, this is provided by the statement that one realization differs from the other by a unitary (nonlinear) transformation (69). One of the useful realizations is

$$U(x) = \exp\left(\mathrm{i}\alpha\boldsymbol{\tau\varphi}_\pi(x)\right), \tag{75}$$

where α is a constant. Substitution of Eqn (75) into (71) and expansion in powers of the pionic field up to the 4th power gives

$$L_{\text{eff}} = 2k\alpha^2(\partial_\mu\boldsymbol{\varphi}_\pi)^2 + \frac{2}{3}\,k\alpha^4\left[(\boldsymbol{\varphi}_\pi\,\partial_\mu\boldsymbol{\varphi}_\pi)^2 - \boldsymbol{\varphi}_\pi^2(\partial_\mu\boldsymbol{\varphi}_\pi)^2\right] + \ldots \tag{76}$$

From the requirement that the first, kinetic energy, term in (76) has the standard form, we have

$$k\alpha^2 = \frac{1}{4}\,. \tag{77}$$

Substitution of Eqn (75) into (74) in the first nonvanishing order in pionic field and taking into account of (77) results in

$$\begin{aligned} j^i_\mu &= \varepsilon_{ikl}\,\varphi^k_\pi\,\frac{\partial\varphi^l_\pi}{\partial x_\mu}\,,\\ j^i_{\mu 5} &= -2\sqrt{k}\,\frac{\partial\varphi^i_\pi}{\partial x_\mu}\,. \end{aligned} \tag{78}$$

The formula for the vector current — the first in Eqns (78) — is the standard formula for the pion isovector current.The comparison of the second equation (78) with (63) finally fixes the constant k and, because of (77), α:

$$k = \frac{1}{8}\,f_\pi^2\,, \qquad \alpha = \frac{\sqrt{2}}{f_\pi}\,. \tag{79}$$

Therefore, the effective Lagrangian (71) as well as $U(x)$ are expressed through one parameter — the pion decay constant f_π, which plays the role of the coupling constant in the theory. From dimensional grounds it is then clear that the expansions in powers of the momenta or in powers of the pionic field are in fact the expansions in p^2/f_π^2 and φ^2/f_π^2. Particularly, the expansion of the effective Lagrangian (71) takes the form

$$L_{\text{eff}} = \frac{1}{2}(\partial_\mu\boldsymbol{\varphi}_\pi)^2 + \frac{1}{3}\frac{1}{f_\pi^2}\left[(\boldsymbol{\varphi}_\pi\partial_\mu\boldsymbol{\varphi}_\pi)^2 - \boldsymbol{\varphi}_\pi^2(\partial_\mu\boldsymbol{\varphi}_\pi)^2\right] + \ldots \tag{80}$$

We turn now to the symmetry breaking term in the CET Lagrangian. This term is proportional to the quark mass matrix

$$\mathcal{M} = \begin{pmatrix} m_{\mathrm{u}} & 0\\ 0 & m_{\mathrm{d}} \end{pmatrix}. \tag{81}$$

In the QCD Lagrangian the corresponding term transforms under $SU(2)_L \times SU(2)_R$ transformations according to the representation 1/2, 1/2. This statement may be transferred to chiral theory by the requirement that in chiral theory the mass matrix (81) transforms according to

$$\mathcal{M}' = V_R \mathcal{M} V_L^+\,. \tag{82}$$

The term in the Lagrangian linear in $\mathcal{M}$ and of the lowest (zero) order in pion momenta, invariant under

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ transformation has the form

$$L' = \frac{f_\pi^2}{4}\left\{B\,\mathrm{Tr}\,(\mathcal{M}U^+) + B^*\,\mathrm{Tr}\,(\mathcal{M}U)\right\}, \tag{83}$$

where B is a constant and the factor f_π^2 is introduced for convenience. We impose the requirement of T-invariance to the Lagrangian (83). The pion field is odd under $T(\varphi_\pi^i) = -\varphi_\pi^i$, so $TU = U^+$ and, as a consequence, $B = B^*$ and

$$L' = \frac{f_\pi^2}{4}\,B\,\mathrm{Tr}\left[\mathcal{M}(U + U^*)\right]. \tag{84}$$

In the lowest order of the expansion in pionic fields (84) reduces to

$$L' = \frac{1}{2}\,B(m_\mathrm{u} + m_\mathrm{d})\left[f_\pi^2 - \boldsymbol{\varphi}_\pi^2 + \frac{1}{6f_\pi^2}(\boldsymbol{\varphi}_\pi^2)^2\right]. \tag{85}$$

The first term in the square bracket gives a shift in the vacuum energy resulting from symmetry breaking, the second corresponds to the pion mass term in the Lagrangian $-(m_\pi^2/2)\boldsymbol{\varphi}_\pi^2$. With this identification we can determine the constant B:

$$B = \frac{m_\pi^2}{m_\mathrm{u} + m_\mathrm{d}} = -\frac{2}{f_\pi^2}\,\langle 0|\bar{q}q|0\rangle, \tag{86}$$

where the Gell-Mann – Oakes – Renner relation (23) was used.

Relation (86) can be also obtained in another way. We have from the QCD Lagrangian

$$\frac{\partial}{\partial m_\mathrm{u}}\,\langle 0|L|0\rangle = -\langle 0|\bar{u}u|0\rangle. \tag{87}$$

Differentiating (85) we get

$$\frac{1}{2}\,Bf_\pi^2 = -\langle 0|\bar{u}u|0\rangle, \tag{88}$$

which coincides with (86).

As a simple application of the effective Lagrangians (80), (85), we calculate the pion-pion scattering amplitude in the first order in $1/f_\pi^2$. The results are [30]

$$T = \delta^{ik}\delta^{lm}A(s,t,u) + \delta^{il}\delta^{km}A(t,s,u) + \delta^{im}\delta^{kl}A(u,t,s), \tag{89}$$

$$A(s,t,u) = \frac{2}{f_\pi^2}(s - m_\pi^2), \tag{90}$$

$$s = (p_1 + p_2)^2, \qquad t = (p_1 - p_3)^2, \qquad u = (p_1 - p_4)^2. \tag{91}$$

Here p_1, p_2 are the initial and p_3, p_4 the final pion momenta. The isospin indices i, k refer to initial pions, l, m — to final ones. For example, for the $\pi^+\pi^0 \to \pi^+\pi^0$ scattering amplitude we get [30]

$$T = \frac{2}{f_\pi^2}(t - m_\pi^2), \tag{92}$$

where T is related to the c.m. scattering amplitude $f_\mathrm{c.m.}$ by

$$f_\mathrm{c.m.} = \frac{1}{16\pi}\,\frac{1}{E}\,T, \tag{93}$$

and E is the energy of π^+ in the c.m. system.

The other often used realization, instead of Eqn (75), is

$$U(x) = \frac{\sqrt{2}}{f_\pi}\left[\sigma(x) + \mathrm{i}\boldsymbol{\tau}\boldsymbol{\varphi}_\pi(x)\right] \tag{94}$$

supplemented by the constraint

$$\sigma^2 + \boldsymbol{\varphi}^2 = \frac{1}{2}\,f_\pi^2. \tag{95}$$

It can be shown by direct calculations that the expressions for effective Lagrangians up to φ^4 obtained in this realization coincide with (80), (85) on the pion mass shell. In higher orders (φ^6, φ^8, etc.) the expressions for effective Lagrangians in these two realizations are different even on the mass shell. But, according to general arguments of Coleman, Wess and Zumino [28], the physical amplitudes become equal after adding one-particle reducible tree diagrams. Since the SU(2) group is isomorphic to O(3) the realization (94) is equivalent to that where the O(4) real four-vector $U_i(x)$, which satisfies the constraint $U_i U_i^T = 1$, $i = 1, 2, 3, 4$, is used instead of the 2×2 matrix $U(x)$ [26].

The chiral effective Lagrangian (71) is the leading term in the expansion in pion momenta. The next term of order of p^4, consistent with Lorentz and chiral invariance, parity and G-parity symmetry, has the general form [26]

$$L_{2\,\mathrm{eff}} = l_1\left[\mathrm{Tr}\,(\partial_\mu U\partial_\mu U^+)\right]^2 + l_2\,\mathrm{Tr}\,(\partial_\mu U\partial_\nu U^+)\,\mathrm{Tr}\,(\partial_\mu U\partial_\nu U^+), \tag{96}$$

where l_1 and l_2 are constants. The term of the second order in quark masses is added to Eqn (96). If the spatial momenta of pions in the process under consideration are close to zero $|\mathbf{p}| \ll m_\pi$, the contribution of this term is of the same order as (96), since $p^2 = m_\pi^2 \sim (m_\mathrm{u} + m_\mathrm{d})$. Its general form is [26]

$$L'_{2\,\mathrm{eff}} = l_4\,\mathrm{Tr}\,(\partial_\mu U\partial_\mu U^+)\,\mathrm{Tr}\left[\chi(U + U^+)\right] + l_6\left\{\mathrm{Tr}\left[\chi(U + U^+)\right]\right\}^2 + l_7\left\{\mathrm{Tr}\left[\mathrm{i}\chi(U - U^+)\right]\right\}^2, \tag{97}$$

$$\chi = 2B\mathcal{M}. \tag{98}$$

In order to perform the next to leading order calculations in CET it is necessary, besides the (96), (97) contribution, to go beyond the tree approximation in the leading order Lagrangians and to calculate the one-loop contributions arising from Eqns (71), (84). As can be seen, the parameter of the expansion is $(1/\pi f_\pi)^2 \sim (1/500\ \mathrm{MeV})^2$ and, as a rule, small numerical coefficients also arise. Therefore, the n-loops contribution is suppressed compared to the leading order tree approximation by the factor $[p^2/(\pi f_\pi)^2]^n$.

Loop integrals are divergent and require renormalization. Renormalization can be performed in any scheme which preserves the symmetry of the theory. This can be dimensional regularization or a method where finite imaginary parts of the scattering amplitudes are calculated and the whole amplitudes are reconstructed using dispersion relations (an example of such a calculation is given below) or any others. The subtraction terms arising in loop calculations (pole contributions as $d \to 4$ in dimensional regularization or subtraction constants in the dispersion relation approach) are absorbed by the coupling constants of the next order effective Lagrangian, like l_1 and l_2 in Eqn (96). Theoretically the unknown constants l_i are determined by comparison with the experimental data.

As a result of loop calculations and of the account of higher order terms in the effective Lagrangian the coupling constant f_π entering (71), (84) acquires some contributions and is no more equal to the physical pion decay constant defined by (84). For this reason the coupling constant f_π in (71), (84) should be considered as a bare one, f_π^0 which will coincide with the physical f_π after accounting for all higher order corrections.

A similar statement refers to the connection between m_π^2 and $m_u + m_d$ in Eqn (86). If B is considered as a constant parameter of the theory, then the relation (86) is modified by high order terms. Particularly, in the next to leading order [31]

$$m_\pi^2 = \widetilde{m}_\pi^2 \left[1 + c(\mu)\, \frac{m_\pi^2}{f_\pi^2} + \frac{m_\pi^2}{16\pi^2 f_\pi^2} \ln \frac{m_\pi^2}{\mu^2} \right], \tag{99}$$

where

$$\widetilde{m}_\pi^2 = B(m_u + m_d)\,, \tag{100}$$

μ is the normalization point and $c(\mu)$ is the μ-depending renormalized coupling constant expressed through l_i. (The total correction is μ-independent.)

The appearance of the nonanalytic in m_π^2 (or m_q) term $\sim m_\pi^2 \ln m_\pi^2$ — the so called 'chiral logarithm' — is a specific feature of the chiral perturbation theory. The origin of their appearance are infrared singularities of the corresponding loop integrals. f_π also contains the chiral logarithm [31]:

$$f_\pi = f_\pi^0 \left[1 + c_1(\mu)\, \frac{m_\pi^2}{f_\pi^2} - \frac{m_\pi^2}{8\pi^2 f_\pi^2} \ln \frac{m_\pi^2}{\mu^2} \right]. \tag{101}$$

Let us present two examples of loop calculations.

1. Find the nonanalytical correction to the pion electric radius, proportional to $\ln m_\pi^2$ [32, 33]. The one-loop contribution to the pion form-factor comes from the $\pi\pi$ interaction term in the Lagrangian given by Eqn (80) and is equal to

$$\frac{\mathrm{i}}{f_\pi^2} \int \frac{\mathrm{d}^4 k_1\, \mathrm{d}^4 k_2}{(2\pi)^4}\, \delta^4(q + k_1 - k_2)(k_1 + k_2)_\mu \times \frac{1}{k_1^2 - m_\pi^2}\, \frac{1}{k_2^2 - m_\pi^2} (p_1 + p_2)(k_1 + k_2)\,. \tag{102}$$

Here p_1 and p_2 are the initial and final pion momenta and q is the momentum transfer ($q^2 < 0, p_1 + q = p_2$).

The integral in (102) can be calculated in the following way. Consider the integral

$$\begin{aligned} \mathrm{i} \int & \frac{\mathrm{d}^4 k_1\, \mathrm{d}^4 k_2}{(2\pi)^4} (k_1 + k_2)_\mu (k_1 + k_2)_\nu \\ & \times \frac{1}{k_1^2 - m_\pi^2}\, \frac{1}{k_2^2 - m_\pi^2}\, \delta^4(q + k_1 - k_2) \\ & = A(q^2)\, (\delta_{\mu\nu} q^2 - q_\mu q_\nu)\,. \end{aligned} \tag{103}$$

The form of the rhs of Eqn (103) follows from gauge invariance. Calculate the imaginary part of $A(q^2)$ at $q^2 > 0$. We have

$$\begin{aligned} & \mathrm{Im}\, A(q^2)\, (\delta_{\mu\nu} q^2 - q_\mu q_\nu) \\ & = -\frac{1}{8\pi^2} \int \mathrm{d}^4 k\, (2k - q)_\mu (2k - q)_\nu\, \delta\big[(q - k)^2\big] \\ & = \frac{1}{48\pi} (\delta_{\mu\nu}\, q^2 - q_\mu q_\nu)\,. \end{aligned} \tag{104}$$

(The pion mass can be neglected in our approximation.)

$A(q^2)$ is determined by the dispersion relation:

$$A(q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{M^2} \frac{\mathrm{d}s}{s - q^2}\, \mathrm{Im}\, A(q^2) = \frac{1}{48\pi^2} \ln \frac{M^2}{4m_\pi^2 - q^2}\,. \tag{105}$$

(The subtraction term is omitted, M^2 is a cutoff.) Substitution of Eqns (103) and (104) into (102) gives for the correction to the $\gamma\pi\pi$ vertex:

$$(p_1 + p_2)_\mu \left[F(q^2) - 1 \right] = (p_1 + p_2)_\mu\, \frac{q^2}{48\pi^2 f_\pi^2} \ln \frac{M^2}{4m_\pi^2 - q^2}\,, \tag{106}$$

where $F(q^2)$ is the pion form-factor. The pion electric radius is defined by

$$r_\pi^2 = 6\, \frac{\mathrm{d}F(q^2)}{\mathrm{d}q^2}\,, \tag{107}$$

and the part nonanalytical in m_π^2 is equal to

$$r_\pi^2 = -\frac{1}{8\pi^2 f_\pi^2} \ln m_\pi^2\,. \tag{108}$$

2. The quark condensate also becomes nonanalytical, proportional to $m_\pi^2 \ln m_\pi^2$ correction [34]. Using (87) and (85) we get

$$\langle 0|\bar{u}u|0\rangle = -\frac{1}{2} f_\pi^2 B \left\langle 0 \middle| 1 - \frac{\varphi_i^2}{f_\pi^2} \middle| 0 \right\rangle. \tag{109}$$

The mean vacuum value of φ_i^2 is given by

$$\begin{aligned} \lim_{x \to 0} \langle 0 | T\varphi_i(x), \varphi_i(0) | 0 \rangle &= \frac{3\mathrm{i}}{(2\pi)^4} \lim_{x \to 0} \int \mathrm{d}^4 k\, \frac{\exp(\mathrm{i}kx)}{k^2 - m_\pi^2} \\ &= A m_\pi^2 + C m_\pi^2 \ln m_\pi^2 + \ldots \end{aligned} \tag{110}$$

In order to find C we differentiate (110) over m_π^2. We have

$$\frac{3\mathrm{i}}{(2\pi)^4} \int \mathrm{d}^4 k\, \frac{1}{(k^2 - m_\pi^2)^2} = -\frac{3\pi^2}{(2\pi)^4} \ln \frac{M^2}{m_\pi^2}\,. \tag{111}$$

Substitution of Eqn (111) into (109) with the account of (86) gives

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{u}u|0\rangle_0 \left(1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2} \ln \frac{M^2}{m_\pi^2} + A m_\pi^2 \right). \tag{112}$$

Generalization for the three massless quarks case, when the s-quark is also considered as massless and the symmetry of the Lagrangian is $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ is straightforward. The matrix $U(x)$ is a 3×3 unitary matrix, and the leading order Lagrangian has the same forms (71) and (84) with the evident difference that the quark mass matrix $\mathcal{M}$ is now 3×3 matrix. In the formulae for axial and vector currents (64) and (74) τ_i should be substituted by the Gell-Mann matrices λ_n ($n = 1, \ldots, 8$) and the same substitution must be done in the exponential realization of $U(x)$:

$$U(x) = \exp\left(\frac{\sqrt{2}\,\mathrm{i}}{f_\pi} \sum_n \lambda_n\, \varphi_n(x) \right), \tag{113}$$

where $\varphi_n(x)$ is the octet of pseudoscalar mesonic fields. Because the algebra of λ_n matrices differs from that of τ_i

and, particularly, the anticommutator λ_n, λ_m does not reduce to δ_{nm}, a linear realization as simple as (94) is impossible in this case.

The symmetry breaking Lagrangian (84) in the order of φ_n^2 — the mass term in the pseudoscalar meson Lagrangian — is nondiagonal in mesonic fields: the effective Lagrangian contains the term proportional to $(m_{\rm u} - m_{\rm d})A\varphi_3\varphi_8$. The presence of this term means that the eigenstates of the Hamiltonian ($|\pi^0\rangle$ and $|\eta\rangle$ states) are not eigenstates of Q^3 and Q^8 generators of $\mathrm{SU}(3)_V$: in η there is an admixture of the isospin 1 state (the pion) and vice versa [33, 36].

In general we can write:

$$H = \frac{1}{2}\,\widetilde{m}_\pi^2\varphi_3^2 + \frac{1}{3}\,\widetilde{m}_\eta^2\varphi_8^2 + A(m_{\rm u} - m_{\rm d})\,\varphi_3\varphi_8 + \text{kinetic terms}\,. \tag{114}$$

The physical π and η states arise after orthogonalization of the Hamiltonian (114):

$$|\pi\rangle = \cos\theta\,|\varphi_3\rangle - \sin\theta\,|\varphi_8\rangle\,, \qquad |\eta\rangle = \sin\theta\,|\varphi_3\rangle + \cos\theta\,|\varphi_8\rangle\,. \tag{115}$$

It can be shown [35, 36, 27] that the constant A in (114) is equal to

$$A = \frac{1}{\sqrt{3}}\,\frac{m_\pi^2}{m_{\rm u} + m_{\rm d}}\,, \tag{116}$$

and the mixing angle is given by (at small θ)

$$\theta = \frac{1}{\sqrt{3}}\,\frac{m_\pi^2}{m_\eta^2 - m_\pi^2}\,\frac{m_{\rm u} - m_{\rm d}}{m_{\rm u} + m_{\rm d}}\,. \tag{117}$$

This result is used in consideration of many problems, where isospin is violated, e.g. the decay rate $\psi' \to \mathrm{J}/\psi\pi^0$ [37], and the amplitude of $\eta \to \pi^+\pi^-\pi^0$ decay. The violating isospin amplitude $\eta \to \pi^+\pi^-\pi^0$ is found to be [38, 39] (in its derivation Eqn (117) was exploited):

$$T(\eta \to \pi^+\pi^-\pi^0) = \frac{\sqrt{3}}{2f_\pi^2}\,\frac{m_{\rm u} - m_{\rm d}}{m_{\rm s} - (m_{\rm u} + m_{\rm d})/2}\left(s - \frac{4}{3}\,m_\pi^2\right), \tag{118}$$

where $s = (p_\eta - p_{\pi^0})^2$.

In the three flavor case the next to leading Lagrangian contains a few additional terms in comparison with Eqns (96), (97) [25, 27]:

$$L'_{2\,\rm eff} = l_3\,\mathrm{Tr}\,(\partial_\mu U\,\partial_\mu U^+\,\partial_\nu U\,\partial_\nu U^+) + l_5\,\mathrm{Tr}\left[\partial_\mu U\,\partial_\mu U^+\,\chi(U + U^+)\right] + l_8\,\mathrm{Tr}\,(\chi U\chi U^+ + U\chi^+ U\chi)\,. \tag{119}$$

In the case of three flavors in the order of p^4, a term of different origin proportional to the totally antisymmetric tensor $\varepsilon_{\mu\nu\lambda\sigma}$ arises. As was pointed out by Wess and Zumino [40], its occurrence is due to anomalous Ward identities for vector and axial nonsinglet currents. Witten [41] had presented the following euristic argument in the favor of this term. The leading and next to leading Lagrangians (71), (84), (96), (97), (119) are invariant under discrete symmetries $U(x) \to U^+(x)$, $U(\mathbf{x}, t) \to U(-\mathbf{x}, t)$. According to Eqn (75) this is equivalent to $\varphi_i(x) \to -\varphi_i(x)$.

In the case of pions this operation coincides with G-parity, but for the octet of pseudoscalar mesons this is not the case. Particularly, such symmetry forbids the processes $\mathrm{K}^+\mathrm{K}^- \to \pi^+\pi^-\pi^0$ and $\eta\pi^0 = \pi^+\pi^-\pi^0$, which are allowed in QCD. In QCD the symmetry under the sign change of pseudoscalar meson fields is valid only if supplemented by space reflection, i.e. $\varphi_i(-\mathbf{x}, t) \to -\varphi_i(\mathbf{x}, t)$. Therefore, one may add a term to the chiral Lagrangian, which is invariant under the latter operation, but violates separately $\mathbf{x} \to -\mathbf{x}$ and $\varphi_i(x) \to -\varphi_i(x)$. Evidently, such a term is proportional to $\varepsilon_{\mu\nu\lambda\sigma}$.

The general form of the term added to the equation of motion is unique:

$$\frac{1}{8}\,f_\pi^2(-\partial_\mu^2 U^+ + U^+\,\partial_\mu^2 U\cdot U^+) + \lambda\varepsilon_{\mu\nu\lambda\sigma}\{U^+\,\partial_\mu U\cdot U^+\,\partial_\nu U\cdot U^+\,\partial_\lambda U\cdot U^+\,\partial_\sigma U\} = 0\,, \tag{120}$$

where λ is a constant. (Other nonleading terms are omitted.) Equation (120) is noninvariant under $U^+ \to U$ and $\mathbf{x} \to -\mathbf{x}$ separately, but conserves parity. However, Eqn (120) cannot derived from the local Lagrangian in four dimensional space-time, because the trace of the second term in the lhs of Eqn (120) vanishes. Witten [41] had shown that the Lagrangian can be represented formally as an integral over some five-dimensional manifold, where the Lagrangian density is local. The integral over this manifold reduces to its boundary, which is precisely a 4-dimensional space – time.

In the first nonvanishing order in mesonic fields the contribution to the Lagrangian (the so called Wess – Zumino term) is equal to [40 – 42]

$$L_{\rm WZ}(U) = \frac{n}{15\pi^2 f_\pi^2}\int \mathrm{d}^4x\ \varepsilon_{\mu\nu\lambda\sigma}\,\mathrm{Tr}\,(\Phi\,\partial_\mu\Phi\,\partial_\nu\Phi\,\partial_\lambda\Phi\,\partial_\sigma\Phi)\,, \tag{121}$$

$$\Phi = \sum \lambda_m\,\varphi_m\,.$$

The coefficient n in Eqn (121) is an integer number [41]. This statement follows from the properties of mapping of 4-dimensional space – time into the SU(3) manifold produced by the field U. It is clear from (121) that $L_{\rm WZ} = 0$ in the case of two flavors: the only antisymmetrical tensor in the flavor indices is ε^{ikl} and it is impossible to construct an antisymmetrical in coordinates expression from the derivatives of pionic fields.

In order to find the value of n it is instructive to consider the interaction with the electromagnetic field. In this case the Wess – Zumino Lagrangian is supplemented by terms which form a gauge invariant Lagrangian together with (121) [41]:

$$L_{\rm WZ}(U, A_\mu) = L_{\rm WZ}(U) - en\int \mathrm{d}^4x\ A_\mu\,J_\mu + \frac{\mathrm{i}}{24\pi^2}\,e^2 n\int \mathrm{d}^4x\ \varepsilon_{\mu\nu\lambda\sigma}(\partial_\mu A_\nu)A_\lambda \times \mathrm{Tr}\left[e_q^2(\partial_\sigma U)U^+ e_q^2 U^+(\partial_\sigma U) + e_q U\,e_q U^+(\partial_\sigma U)U^+\right]. \tag{122}$$

Here

$$J_\mu = \frac{1}{48\pi^2}\,\varepsilon_{\mu\nu\lambda\sigma}\,\mathrm{Tr}\left[e_q(\partial_\nu U\cdot U^+)(\partial_\lambda U\cdot U^+)(\partial_\sigma U\cdot U^+) + e_q(U^+\,\partial_\nu U)(U^+\,\partial_\lambda U)(U^+\,\partial_\sigma U)\right], \tag{123}$$

$e_q = \mathrm{diag}\,(2/3, -1/3, -1/3)$ is the matrix of quark charges, and e is the proton charge. The amplitude of $\pi^0 \to \gamma\gamma$ decay can be found from the last term in Eqn (122). It is given by

$$T(\pi^0 \to \gamma\gamma) = \frac{ne^2}{48\sqrt{2}\,\pi^2 f_\pi}\,\varepsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}\,. \tag{124}$$

On the other side, the same amplitude is determined in QCD by anomaly. Use the anomaly condition [43–45]

$$\partial_\mu j^3_{\mu 5} = \frac{\alpha}{2\pi}\,N_c(e_u^2 - e_d^2)\,F_{\mu\nu}\widetilde{F}_{\mu\nu} = \frac{\alpha}{12\pi}\,N_c\,\varepsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}\,, \tag{125}$$

where N_c is the number of colors and e_u, e_d are u and d quark charges. For the amplitude $T(\pi^0 \to \gamma\gamma)$ we have, exploiting the PCAC condition (65):

$$T(\pi^0 \to \gamma\gamma) = \frac{e^2}{48\sqrt{2}\,\pi^2 f_\pi}\,N_c\,\varepsilon_{\mu\nu\lambda\sigma}F_{\mu\nu}F_{\lambda\sigma}\,. \tag{126}$$

Equation (124) coincides with (126), if $n = N_c$ [41].

The other physically interesting object, the $\gamma\pi^+\pi^-\pi^0$ vertex is determined by the second term in the rhs of Eqn (122) and is equal to

$$\Gamma(\gamma\pi^+\pi^-\pi^0) = -\frac{i}{3}\,\frac{en}{\sqrt{2}\,\pi^2 f_\pi^3}\,\varepsilon_{\mu\nu\lambda\sigma}A_\mu\,\partial_\nu\pi^+\,\partial_\lambda\pi^-\,\partial_\sigma\pi^0\,. \tag{127}$$

Again, if $n = N_c$, this result agrees with QCD calculations based on VAA anomaly or with the phenomenological approach, where the anomaly was taken as granted [46–48].

The CET is also valid for the pion-baryon low energy interaction, where a lot of results have been obtained. We restrict ourselves here to presenting the effective pion–nucleon interaction Lagrangian in the leading order (see, e.g., Ref. [1], good reviews where high order terms are considered are in Refs [49, 50]):

$$L_{\pi N} = -\frac{g_A}{\sqrt{2}\,f_\pi}\,\bar{\psi}_N\gamma_\mu\gamma_5\boldsymbol{\tau}\,\partial_\mu\boldsymbol{\varphi}\psi_N - \frac{1}{2f_\pi^2}\,\bar{\psi}_N\gamma_\mu\boldsymbol{\tau}\,[\boldsymbol{\varphi}\partial_\mu\boldsymbol{\varphi}]\,\psi_N\,, \tag{128}$$

where ψ_N are nucleon spinors and g_A is the axial neutron β-decay constant ($g_A = 1.26$). The first term in Eqn (128) is a standard pion–nucleon interaction with pseudovector coupling, the second one represents the contact $\pi\pi N\bar{N}$ interaction. Using the Goldberger–Treiman relation [32] it is easy to convince oneself that in the first order in pionic field (128) coincides with the standard form of πN interaction Lagrangian (30).

7. Low energy sum rules in CET

Using the CET technique important low energy sum rules can be derived, which of course are also valid in QCD. The most interesting, which have been tested by experiment, refer to the difference of the polarization operators of vector and axial currents. Let us define

$$\Pi^U_{\mu\nu}(q) = i\int d^4x\,\exp(iqx)\langle 0|T\{U_\mu(x), U_\nu^+(0)\}|0\rangle$$
$$= (q_\mu q_\nu - q^2\delta_{\mu\nu})\,\Pi_U^{(1)}(q^2) + q_\mu q_\nu\,\Pi_U^{(0)}(q^2)\,, \tag{129}$$

$$U = V, A\,,\qquad V_\mu = \bar{u}\gamma_\mu d\,,\qquad A_\mu = \bar{u}\gamma_\mu\gamma_5 d\,, \tag{130}$$

where V_μ and A_μ are vector and axial quark currents.

The imaginary parts of the correlators are the so-called spectral functions ($s = q^2$):

$$v_1(s)/a_i(s) = 2\pi\,\mathrm{Im}\,\Pi^{(1)}_{V/A}(s)\,,\qquad a_0(s) = 2m\,\mathrm{Im}\,\Pi^{(0)}_A(s)\,, \tag{131}$$

which are measured in τ-decay. (The spectral function isotopically related to v_1 is measured in e^+e^--annihilation.) The spin 0 axial spectral function $a_0(s)$ which is mainly saturated by one pion state will not be interesting for us now. $\Pi_V^{(1)}(s)$ and $\Pi_A^{(1)}(s)$ are analytical functions of s in the complex s-plane with a cut along the right semiaxes, starting from the threshold of the lowest hadronic state: $4m_\pi^2$ for $\Pi_V^{(1)}$ and $9m_\pi^2$ for $\Pi_A^{(1)}$.

Besides the cut, $\Pi_A^{(1)}(q^2)$ has a kinematical pole at $q^2 = 0$. This is a specific feature of QCD and CET, which follows from the chiral symmetry in the limit of massless u- and d-quarks and its spontaneous violation. In this limit the axial current is conserved and a massless pion exists. Its contribution to the axial polarization operator is given by

$$\Pi^A_{\mu\nu,\pi}(q) = f_\pi^2\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right). \tag{132}$$

When the quark masses are taken into account, then in the first order of quark masses, or, which is equivalent — in m_π^2, Eqn (132) is modified to

$$\Pi^A_{\mu\nu,\pi}(q) = f_\pi^2\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2 - m_\pi^2}\right). \tag{133}$$

We decompose Eqn (133) into the tensor structures of (129):

$$\Pi^A_{\mu\nu,\pi}(q) = -\frac{f_\pi^2}{q^2}(q_\mu q_\nu - \delta_{\mu\nu}q^2) - \frac{m_\pi^2}{q^2}\,q_\mu q_\nu\,\frac{f_\pi^2}{q^2 - m_\pi^2}\,. \tag{134}$$

The pole in $\Pi_1^A(q^2)$ at $q^2 = 0$ is evident. Let us write the dispersion relation for $\Pi_1^V(s) - \Pi_1^A(s)$. This may be a nonsubtracted dispersion relation, since perturbative terms (besides the small contribution from u and d quarks mass square) cancel in the difference, and the OPE terms decrease with $q^2 = s$ at least as s^{-2} (the term $\sim m_q\langle 0|\bar{q}q|0\rangle$ in OPE). We have

$$\Pi_1^V(s) - \Pi_1^A(s) = \frac{1}{2\pi^2}\int_0^\infty ds'\,\frac{v_1(s') - a_1(s')}{s' - s} + \frac{f_\pi^2}{s}\,. \tag{135}$$

The last term in the rhs of Eqn (135) represents the kinematical pole contribution.

Let us go to $s \to \infty$ in Eqn (135). Since $\Pi_1^V(s) - \Pi_1^A(s) \to s^{-2}$ in this limit we get the sum rule (the first Weinberg sum rule [51]):

$$\frac{1}{4\pi^2}\int_0^\infty ds\,\left[v_1(s) - a_1(s)\right] = \frac{1}{2}\,f_\pi^2\,. \tag{136}$$

The accuracy of this sum rule is of the order of the chiral symmetry violation in QCD, or next order terms in CET, i.e. $\sim m_\pi^2/M^2$ (e.g. a subtraction term). If the term $\sim m_q\langle 0|\bar{q}q|0\rangle \sim f_\pi^2 m_\pi^2$ in OPE may be neglected, then, performing in Eqn (135) the expansion up to s^{-2}, we get the second Weinberg sum rule:

$$\frac{1}{4\pi^2}\int_0^\infty ds\,s\left[v_1(s) - a_1(s)\right] = O(m_\pi^2)\,. \tag{137}$$

(For other derivations of these sum rules — see Ref. [52].)

I present here one more sum rule derived in CET (in its earlier version — PCAC), the Das – Mathur – Okubo sum rule [53]:

$$\frac{1}{4\pi^2}\int_0^\infty \mathrm{d}s\ \frac{1}{s}\left[v_1(s)-a_1(s)\right]=\frac{1}{6}f_\pi^2\langle r_\pi^2\rangle-F_A\,, \tag{138}$$

where $\langle r_\pi^2\rangle$ is the mean pion electromagnetic radius and F_A is the pion axial vector form-factor in the decay $\pi^- \to e^-\nu_\mu\gamma$ (in fact, F_A is a constant with high accuracy).

The comparison of the sum rules (136) – (138) with the results of the recent measurements of $v_1(s)-a_1(s)$ in τ-decay by the ALEPH collaboration [54] are presented in Fig. 4 versus the upper limit of integration.

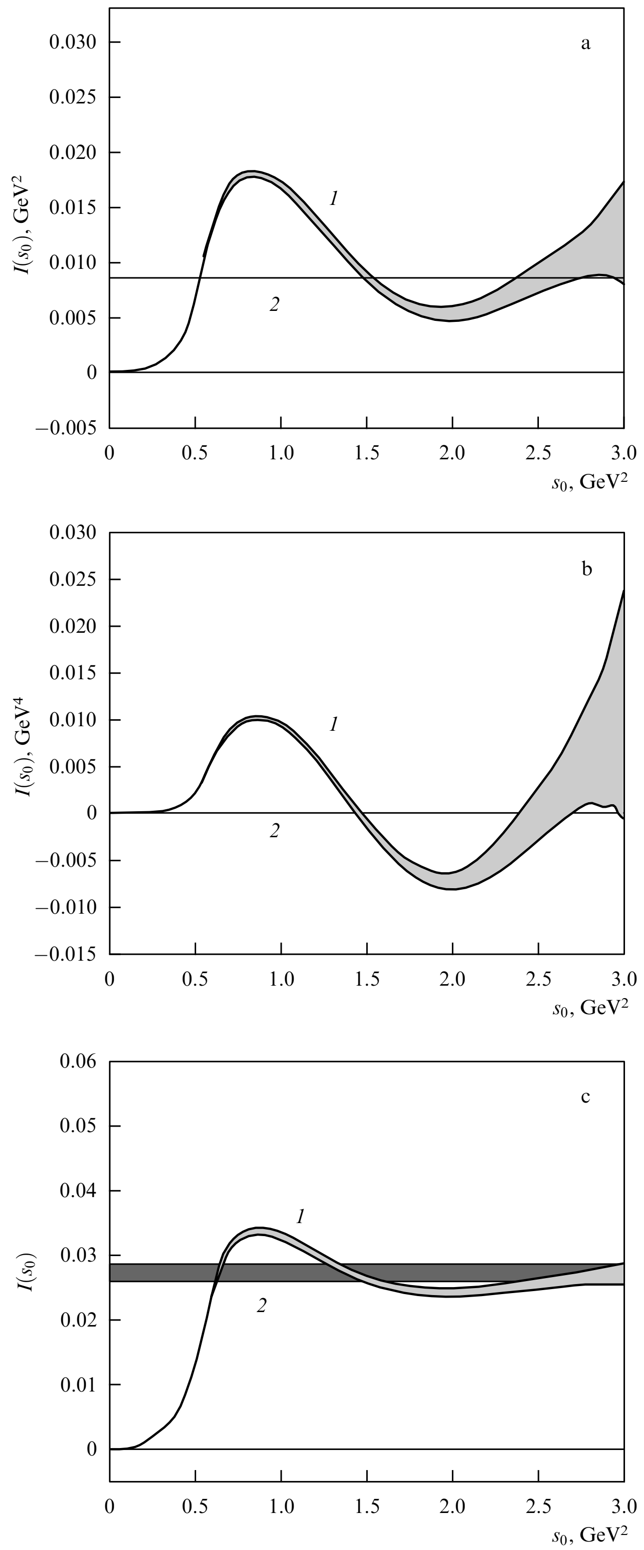

Figure 4. Sum rules (136) – (138) [(a) – (c) correspondingly] as functions of upper limit of integration s_0: (*1*) data of ALEPH [54]; (*2*) — CET predictions. $I(s_0)$ mean the lhs's of Eqns (136) – (138), where the upper limit of integration is put to s_0. The shaded areas represent the experimental errors.

8. QCD and CET at finite temperature

CET is a useful tool for studying QCD at finite temperature. It is a common belief that with temperature increase any hadronic system undergoes a phase transition with the restoration of chiral symmetry and liberation of color — deconfinement (for reviews see Refs [55 – 57]). These two phenomena can proceed in a single phase transition or may be separated. An estimate of the critical temperature(s) T_c was found from lattice calculations, from studies of suitable correlation functions, in the framework of models and from the study of temperature dependence of condensate in the framework of CET. All of this indicates that $T_c \approx 150-250$ MeV.

I present here the simple argument [58], based on the consideration of any hadronic correlator $P(x)$ at large space-like distances $\mathbf{x}$. One may expect that

$$P(x) \sim \exp\left[-\mu(T)\,|\mathbf{x}|\right], \tag{139}$$

where $\mu(T)$ is a temperature dependent screening parameter.

Equation (139) is valid if: (1) $\mu|\mathbf{x}| \gg 1$; (2) $|\mathbf{x}| \lesssim (\alpha_s(T)\,T)^{-1}$, because at such $|\mathbf{x}|$ the infrared divergence arises in the theory [59]. At high temperature $\mu(T)$ is given by Matsubara frequency:

$$\begin{aligned}&\mu = 2\pi T \ \text{ for bosons (two quarks)}\,,\\ &\mu = 3\pi T \ \text{ for baryons (three quarks)}\,.\end{aligned} \tag{140}$$

At low T, $\mu(T)$ is equal to the mass of the corresponding hadron (except for the pion, where the conditions (1) and (2) cannot be satisfied simultaneously). Taking, as examples, ρ- and a_1-mesons, we find that the matching of two regimes occurs at 150 – 200 MeV.

Quark condensate may be considered as an order parameter in QCD. Its vanishing at some critical temperature $T = T_c$ would indicate the phase transition — the restoration of chiral symmetry at $T = T_c$. Bearing this in mind, we calculate T^2 correction to $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle$ quark condensate in the limit of massless u- and d-quarks [60, 61]. The mean value of any operator O at finite temperature is given by

$$\langle O\rangle_T = \sum_n \left\langle n \left| O\,\frac{1}{\exp(H/T)\pm 1}\right| n\right\rangle \rho_n\,, \tag{141}$$

where the '±' signs refer to Fermi and Bose systems, ρ_n is the density of the state $|n\rangle$. At low T and massless u- and d-quarks the main contribution comes from the states of massless pions. Contributions of all other particles are exponentially suppressed by factors $\exp(-m/T)$, where m is the particle mass. (Summation over n should be performed over a Hilbert space of physical particles, since at small T the system is in the confinement phase and the problem is characterized by large distances.)

In the order of T^2 it is enough to account for only one pion state in Eqn (141). This gives

$$\Delta_T\langle\bar{u}u\rangle = 3\int\frac{d^3p}{(2\pi)^3\cdot 2E}\,\langle\pi^+|\bar{u}u|\pi^+\rangle\,\frac{1}{\exp(E/T)-1}\,, \quad (142)$$

where Δ_T means the temperature correction and factor 3 comes from three pion states — π^+, π^-, π^0. It is clear that the one-pion phase space factor results in the required power T^2, two-pion states give T^4, etc. From the QCD Lagrangian we have

$$\langle\pi^+|\bar{u}u|\pi^+\rangle = -\frac{\partial}{\partial m_u}\langle\pi^+|L|\pi^+\rangle\,. \quad (143)$$

Substitution of the chiral effective Lagrangian (85) into (143) instead of the QCD Lagrangian leads to

$$\langle\pi^+|\bar{u}u|\pi^+\rangle = \frac{1}{2}\,B\langle\pi^+|2\varphi^+\varphi|\pi^+\rangle = B = -\frac{2}{f_\pi^2}\,\langle 0|\bar{u}u|0\rangle\,. \quad (144)$$

Therefore,

$$\Delta_T\langle\bar{u}u\rangle = -\frac{6}{(2\pi)^3 f_\pi^2}\,\langle 0|\bar{u}u|0\rangle\int\frac{d^3p}{(2\pi)^3\cdot 2E}\,\frac{1}{\exp(E/T)-1}$$
$$= -\frac{T^2}{4f_\pi^2}\,\langle 0|\bar{u}u|0\rangle\,. \quad (145)$$

Quark condensate decreases with increasing of temperature. If such linear with T^2 behavior continued up to $T = 2f_\pi \approx 250$ MeV, the quark condensate would vanish at this temperature and chiral symmetry would be restored. In fact, the calculation of higher order terms in T^2 (up to T^6) gives [62, 63]

$$\langle\bar{q}q\rangle_T = \langle 0|\bar{q}q|0\rangle\left[1 - \frac{N_f^2-1}{N_f}\,\frac{T^2}{6f_\pi^2} - \frac{N_f^2-1}{2N_f^2}\left(\frac{T^2}{6f_\pi^2}\right)^2 - N_f(N_f^2-1)\left(\frac{T^2}{6f_\pi^2}\right)^3\ln\frac{M}{T}\right], \quad (146)$$

where N_f is the number of flavors ($N_f = 2$ for u and d massless quarks) and M is a cutoff. All three terms in the expansion have the same sign which indicates a lowering phase transition temperature, up to $T_c \approx 150$ MeV. The quark condensate temperature dependence at low T is shown in Fig. 5.

For gluonic condensate the situation is more subtle. The operator $G_{\mu\nu}G_{\mu\nu}$ is proportional to the trace of the energy-momentum tensor $\theta_{\mu\nu}$ and the latter is the generator of conform transformation. However, the massless non-interacting pion gas is conformally invariant. (Pions are non-interacting at low T because of the Adler theorem.) For this reason the low temperature expansion for gluonic condensate starts from $\sim T^8$ term [62].

Consider finally T^2 corrections to the correlators of vector and axial currents in the limit of massless quarks [64].

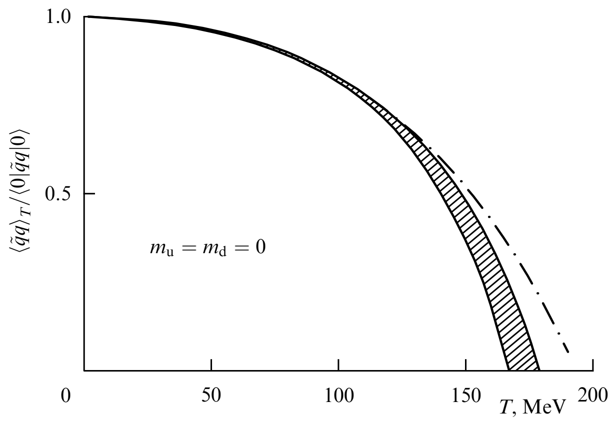

Figure 5. Temperature dependence of quark condensate up to 3 loops (at $m_u = m_d = 0$). Equation (146) — dot-dashed line. The shaded area is the same with the model accounting for massive states [62, 63].

At finite T the correlators are defined as $(q^2 = -Q^2 < 0)$

$$\Pi^U_{\mu\nu}(q,T) = i\int d^4x\,\exp(iqx) \times \sum_n\left\langle n\left|T\left\{U^a_\mu(x),\,U^a_\nu(0)\exp\frac{\Omega-H}{T}\right\}\right|n\right\rangle\,, \quad (147)$$

$$U = V, A\,,\qquad V^a_\mu = \bar{q}\gamma_\mu\,\frac{\tau^a}{2}\,q\,,\qquad A^a_\mu = \bar{q}\gamma_\mu\gamma_5\,\frac{\tau^a}{2}\,q\,. \quad (148)$$

Here

$$\exp\left(-\frac{\Omega}{T}\right) = \sum_n\left\langle n\left|\exp\left(-\frac{H}{T}\right)\right|n\right\rangle\,.$$

To evaluate $\Pi^U_{\mu\nu}(q,T)$ at low temperature, $T^2 \ll Q^2$ in the sum over $|n\rangle$ in Eqn (147) only the vacuum and pion states should be taken into account. The matrix elements

$$\langle\pi|T\{U^a_\mu(x),\,U^a_\nu(0)\}|\pi\rangle \quad (149)$$

are easily evaluated applying reduction formulas to pions and using Eqn (65). The equal time commutators which arise are calculated by current algebra relations [or can be derived from Eqn (78)]. The integration over pionic phase space (relativistic Bose gas) can be done with the help of the formula (for massless pions):

$$\int\frac{d^3q}{(2\pi)^3\cdot 2q}\,\frac{1}{\exp(q/T)-1} = \frac{T^2}{24}\,. \quad (150)$$

The result is:

$$\Pi^V_{\mu\nu}(q,T) = (1-\varepsilon)\,\Pi^V_{\mu\nu}(q,0) + \varepsilon\Pi^A_{\mu\nu}(q,0)\,,$$
$$\Pi^A_{\mu\nu}(q,T) = (1-\varepsilon)\,\Pi^A_{\mu\nu}(q,0) + \varepsilon\Pi^V_{\mu\nu}(q,0)\,, \quad (151)$$

where $\varepsilon = T^2/3f_\pi^2$.

If $\Pi^{V/A}_{\mu\nu}(q,0)$ are represented through dispersion relations by the contributions of physical states in V and A channels, say ρ, a_1, π, etc. poles, then according to (151) in the correlators $\Pi^{V,A}(q,T)$ the poles do not shift in order T^2 and appear at the same position as at $T = 0$. The consequence of (151) is that, at $T \neq 0$ in transverse vector channel apart from the poles corresponding to vector particle, there arise poles corresponding to axial particles and vice versa. In the same way a pion pole appears in the longitudinal part of the vector channel. The same phenomenon of parity mixing (and, in some cases also isospin mixing) appears at finite T also in other channels, including baryonic channels [65].

9. Conclusion

The goal of this review is to convince the reader that the CET of strong interactions is on the one hand a direct consequence of QCD, of the chiral symmetry of QCD and its spontaneous violation; and on the other hand, a very effective tool with high predictive power for solving the problems of strong interactions at low energies.

It was demonstrated that in QCD the masses of light quarks (u, d and, in some extent, also s) are small and to a good approximation, when these masses are neglected, QCD is chirally symmetric. However, the physical spectrum of the real world (including the vacuum state) does not possesses this symmetry: there is nonvanishing (in the limit $m_{\mathrm{u}}, m_{\mathrm{d}} \to 0$) symmetry violating quark condensate, and the baryon masses are by no means small, in contradiction with chiral symmetry.

It was shown that these two facts — the large baryon masses and the appearance of quark condensate are tightly interconnected: the first can be expressed through the second. The violation of chiral symmetry on the physical spectrum means that chiral symmetry is broken spontaneously. A direct consequence of this fact is the appearance of massless Goldstone bosons in the spectrum (the pion in case of SU(2) symmetry, where u- and d-quarks are considered as massless and s-quark as massive). The known symmetry of the theory and the existence of massless Goldstone bosons allows one to construct CET, valid at low energies. CET is an effective theory, which means that when going to the next approximation — higher powers of particle momenta — new additional terms in the theory Lagrangian appear.

In the review, the CET Lagrangian in the first and second orders in momenta was explicitly constructed and its main features were discussed. Using a few examples it was demonstrated that CET is very powerful in the consideration of low energy interactions of pions. Low energy sum rules, which are the subject of direct experimental tests, were presented. It was demonstrated that CET is a very suitable tool for the study of QCD at finite temperature. The indications for phase transitions in QCD were obtained from this study.

I am very thankful to H Leutwyler for enlighting discussion of various aspects of CET. I am also very indebted to him for his hospitality at Bern. I am also thankful to J Speth for his hospitality at Juelich Forschungszentrum, where this work was finished and to the A von Humboldt Foundation for financial support of this visit.

This work was made possible in part by Award No. RP2-2247 of the U.S. Civilian Research and Development Foundation for Independent States of the Former Soviet Union (CRDF), by the Russian Fund for Basic Research grant 00-02-17808, and an INTAS Call 2000 Grant (Project 587).

References

1. Vaĭnshteĭn A I, Zakharov V I *Usp. Fiz. Nauk* **100** 225 (1970) [*Sov. Phys. Usp.* **13** 73 (1970)]
2. Gasser J, Leutwyler H *Nucl. Phys. B* **94** 269 (1975)
3. Weinberg S, in *A Festschrift for I.I. Rabi* (Trans. New York Acad. Sci., Ser. 2, Vol. 38, Ed. L Motz) (New York: New York Acad. of Sciences, 1977) p. 185
4. Appelquist T, Carazzone J *Phys. Rev. D* **11** 2856 (1975)
5. Berestetskiĭ V B *Usp. Fiz. Nauk* **85** 393 (1965) [*Sov. Phys. Usp.* **8** 147 (1965)]
6. Dashen R *Phys. Rev.* **183** 1245 (1969)
7. Gasser J, Leutwyler H *Phys. Rep.* **87** 77 (1982)
8. Leutwyler H *J. Moscow Phys. Soc.* (6) 1 (1996)
9. Gell-Mann M, Oakes R J, Renner B *Phys. Rev.* **175** 2195 (1968)
10. Goldberger M L, Treiman S B *Phys. Rev.* **110** 1178 (1958)
11. Goldstone J *Nuovo Cimento* **19** 154 (1961)
12. Nambu Y, Jona-Lasinio G *Phys. Rev.* **122** 345 (1961)
13. Goldstone J, Salam A, Weinberg S *Phys. Rev.* **127** 965 (1962)
14. Coleman S "Laws of hadronic matter", in *Proc. 11 Course of the "Ettore Maiorana" Intern. School of Subnuclear Physics* (Ed. A Zichichi) (New York: Academic Press, 1975)
15. Shifman M A, Vainshtein A I, Zakharov V I *Nucl. Phys. B* **147** 385, 448 (1979)
16. Ioffe B L *Nucl. Phys. B* **188** 317 (1981); **192** 591 (1982)
17. Shifman M A (Ed.) *Vacuum Structure and QCD Sum Rules* (Current Physics – Sources and Comments, Vol. 10) (Amsterdam: North-Holland, 1992)
18. Chung Y et al. *Nucl. Phys. B* **197** 57 (1982)
19. Ioffe B L *Z. Phys. C* **18** 67 (1983)
20. Belyaev V M, Ioffe B L *Zh. Eksp. Teor. Fiz.* **83** 876 (1982) [*Sov. Phys. JETP* **56** 493 (1982)]
21. Belyaev V M, Ioffe B L *Zh. Eksp. Teor. Fiz.* **84** 1236 (1983) [*Sov. Phys. JETP* **57** 716 (1983)]
22. Ioffe B L *Acta Phys. Pol. B* **16** 543 (1985)
23. Adler S L *Phys. Rev.* **137** B1022 (1965); **139** B1638 (1965)
24. Weinberg S *Physica A* **96** 327 (1979)
25. Leutwyler H, in *Lectures at the XXX Intern. Universitätswochen für Kernphysik* (*Schladming, Austria, Feb. 1991*); Preprint BUTP-91/26 (1991)
26. Gasser J, Leutwyler H *Ann. Phys.* (New York) **158** 142 (1984)
27. Gasser J, Leutwyler H *Nucl. Phys. B* **250** 465 (1985)
28. Coleman S, Wess J, Zumino B *Phys. Rev.* **177** 2239 (1969)
29. Callan S G (Jr.) et al. *Phys. Rev.* **177** 2247 (1969)
30. Weinberg S *Phys. Rev. Lett.* **17** 616 (1966)
31. Langacker P, Pagels H *Phys. Rev. D* **8** 4595 (1973)
32. Bég M A B, Zepeda A *Phys. Rev. D* **6** 2912 (1972)
33. Volkov M K, Pervushin V N *Yad. Fiz.* **20** 762 (1974) [*Sov. J. Nucl. Phys.* **20** 408 (1975)]
34. Novikov V A et al. *Nucl. Phys. B* **191** 301 (1981)
35. Ioffe B L *Yad. Fiz.* **29** 1611 (1979) [*Sov. J. Nucl. Phys.* **20** 827 (1979)]
36. Gross D J, Treiman S B, Wilczek F *Phys. Rev. D* **19** 2188 (1979)
37. Ioffe B L, Shifman M A *Phys. Lett. B* **95** 99 (1980)
38. Osborn H, Wallace D R *Nucl. Phys. B* **20** 23 (1970)
39. Gasser J, Leutwyler H *Nucl. Phys. B* **250** 539 (1985)
40. Wess J, Zumino B *Phys. Lett. B* **37** 95 (1971)
41. Witten E *Nucl. Phys. B* **223** 422 (1983)
42. Zahed I, Brown G E *Phys. Rep.* **142** 1 (1986)
43. Adler S L *Phys. Rev.* **177** 2426 (1969)
44. Bell J S, Jackiw R *Nuovo Cimento* **60** 1517 (1969)
45. Bardeen W A *Phys. Rev.* **184** 1848 (1969)
46. Terent'ev M V *Pis'ma Zh. Eksp. Teor. Fiz.* **14** 140 (1971) [*JETP Lett.* **14** 94 (1971)]
47. Adler S L et al. *Phys. Rev. D* **4** 3497 (1971)
48. Terent'ev M V *Usp. Fiz. Nauk* **112** 37 (1974) [*Sov. Phys. Usp.* **17** 20 (1974)]
49. Meißner U-G "Chiral nucleon dynamics", in *Lectures at the 12th Annual Hampton Univ. Graduate Studies at CEBAF, Newport News, June 1997*; Preprint KFA-IKP(TH)-97-20 (1997); hep-ph/9711365
50. Leutwyler H *PiN Newslett.* **15** 1 (1999); hep-ph/0008123; Becher T, Leutwyler H *JHEP* **0106** 017 (2001); hep-ph/0103263
51. Weinberg S *Phys. Rev. Lett.* **18** 188, 507 (1967)

52. Ioffe B L, Khoze V A, Lipatov L N *Hard Processes* (Amsterdam: North-Holland, 1984)
53. Das T, Mathur V S, Okubo S *Phys. Rev. Lett.* **19** 859 (1967)
54. Barate R et al. (ALEPH Collab.) *Eur. J. Phys. C* **4** 409 (1998)
55. Shuryak E V *Phys. Rep.* **61** 71 (1980)
56. Gross D J, Pisarski R D, Yaffe L G *Rev. Mod. Phys.* **53** 43 (1981)
57. McLerran L *Rev. Mod. Phys.* **58** 1021 (1986)
58. Eletsky V L, Ioffe B L *Yad. Fiz.* **48** 602 (1988) [*Sov. J. Nucl. Phys.* **48** 384 (1988)]
59. Linde A D *Rep. Prog. Phys.* **42** 389 (1979)
60. Benétruy P, Gaillard M K *Phys. Rev. D* **32** 931 (1985)
61. Gasser J, Leutwyler H *Phys. Lett. B* **184** 83 (1987); **188** 477 (1987)
62. Leutwyler H *Nucl. Phys. B* (*Proc. Suppl.*) **4** 248 (1988)
63. Gerber P, Leutwyler H *Nucl. Phys. B* **321** 387 (1989)
64. Dey M, Eletsky V L, Ioffe B L *Phys. Lett. B* **252** 620 (1990)
65. Eletsky V L, Ioffe B L *Phys. Rev. D* **47** 3083 (1993)