However, in the case of the teleportation of a light field distributed in space and time, such a definition runs into a fundamental difficulty. This difficulty is associated with the nature of the field as a system with a large number of degrees of freedom, and can be understood from the following simple consideration. Let the fidelity of the quantum teleportation of the state of the field confined within a certain space – time region be less than unity by a small but finite increment. If we now consider a system with the parallel teleportation of many volumes of the field in the same state, then the fidelity defined according to (14) equals a high power of the fidelity found earlier — that is, in the limit of a large system, it will always be zero. In this way, in the case of quantum holographic teleportation, one should use not the fidelity defined by (14) (which may be referred to as 'global fidelity'), but rather the reduced fidelity defined for the reduced set of degrees of freedom that is of interest for us.

If we assume that, in our example, the relevant degrees of freedom correspond to $n = 1, 2, \ldots$ volumes of averaging (pixels) as defined above, then, for the important special case of the input field $A_{in}(\rho, t)$ in the multimode Glauber coherent state, quantum calculations yield [11]

$$F_n = \frac{1}{\det\left\{\delta_{i,j} + (1/2)\mathcal{C}(i, j)\right\}} \, . \tag{15}$$

Hence we see that the fidelity of teleportation of the quantum-field state for n pixels depends on both their number and the correlations of the noise field on the pixels of interest. The correlations lose their importance if the area of averaging S is much larger than the coherence area S_c and the correlation matrix becomes diagonal. Then, as in the case of a spatially single-mode field, quantum holographic teleportation exhibits high fidelity with efficient squeezing and deep EPR correlations in the quantum channel.

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Action of optical radiation on the boundary of a rarefied resonant medium. New possibilities and problems

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The reflection of resonant optical radiation from the boundary of a gaseous medium has long been considered a trivial illustration of the laws of physical optics [1]. The use of this effect for studying the interactions of atoms with one another and with the solid surface became possible only after the experimental discovery [2] and theoretical interpretation [2, 3] of narrow resonances in reflection spectra, free of Doppler broadening. First of all, the resonant collisional broadening in the centers of atomic lines, inaccessible to the standard absorption spectroscopy because of self-absorption effects in optically dense atomic vapors, was determined [4]. The constants of the Van der Waals interaction between a solid surface and a resonantly excited atom, for which other measurement techniques are not known, were also measured [5]. At the same time, inadequacies of the theoretical treatment gave rise to certain problems and paradoxes with interpreting experimental data without proper account for such attendant circumstances as the nonexponential absorption in the medium, deviation of the local field from the mean field, the saturation of atomic transitions, and the interplay of various factors causing shifts, broadening, and deformation in the spectral contours of the lines of selective mirror reflection.

This communication deals with solving certain theoretical problems that impede the more extensive use of the diagnostic capabilities of the reflection spectroscopy of rarefied gaseous media. Special attention will be paid to the nonlinear optical processes associated with the reflection of resonant optical radiation from the boundary of a rarefied gaseous medium [6] and effects of the second order with respect to the optical density of the medium.

A specific feature of rarefied resonant gas as a dispersive optical medium is that, even though the proper geometric dimensions of the atom are small compared to the radiation wavelength, the response of an atom is essentially nonlocal if the free path of the atom without changes in polarization exceeds the radiation wavelength. This condition is equivalent to requiring that the Doppler broadening of the resonant transition in the atom be greater than its homogeneous width. The latter is the sum of the radiation and the collisional width. The radiation width of an allowed optical transition is about 10 MHz, which is much less than the characteristic magnitude of the Doppler broadening (500 MHz). Therefore, the Doppler broadening exceeds the homogeneous width right up to concentrations of order $10^{14}\,\mathrm{cm}^{-3}$. For higher concentrations, the resonant transmission of excitation upon the collision of an excited atom with a nonexcited one reduces the free path of the atom without changes in polarization to a value that is less than the wavelength of the radiation resonant to the atomic transition. In this case, the polarization of the atom is mainly determined by the field acting upon the atom at the point where the atom is currently located. If, however, the concentration is less than $10^{14}\,\mathrm{cm}^{-3}$, the optical response of the resonant gas is essentially nonlocal.

The description of homogeneous media with a nonlocal response mainly consists in taking into account the spatial dispersion of the dielectric permittivity. Then the propagation of additional light waves becomes possible in the medium. The ordinary boundary conditions are not sufficient to calculate the coefficients of reflection and transmission at the boundary of such a medium, and it becomes necessary to formulate additional boundary conditions. A similar situation has been considered in detail in the optics of crystals [8]. In the case of a rarefied resonant gas, the additional solutions of the dispersion equation correspond not to propagating waves but rather to heavily damped waves whose contributions can be neglected. The spatial structure of the main wave is then very complicated and does not reduce to a simple exponential dependence on the coordinates. The additional boundary conditions that should be specified to determine this structure have the simple microscopic meaning of the initial polarization of atoms bounced off the wall that confines the gas, but they can hardly be formulated in macroscopic terms.

If the field inside the medium does not vary exponentially, the concept of the refractive index becomes meaningless. Nevertheless, the coefficient of reflection for normal incidence can be calculated using the conventional Fresnel formulas with the refractive index replaced by the surface admittance of the gas defined by analogy with the anomalous skin effect in metals, as the logarithmic derivative of the field at the boundary divided by the wave number and the imaginary unit. The surface admittance can be calculated using the Maxwell–Bloch set of equations [9]. In the conventional theory of dispersion, the coordinate dependence of the field is assumed to be exponential, and the motion of atoms is taken into account by inclusion of the Doppler shift of the resonant frequency in the expression for the steady-state polarization of the atom. If the Doppler broadening is strong, the polarization of the majority of the atoms that have bounced off the boundary reaches a steady state at distances from the boundary that are much larger than the wavelength. The replacement of the actual behavior of the polarization in the surface layer of such a considerable thickness with the steady-state polarization leads to essentially wrong results. The proper approach consists in solving the set of Maxwell–Bloch equations with due account for the boundary conditions for the density matrix of the atoms that bounce off the boundary. In most cases, we can assume that a collision with the surface completely quenches the electron excitation [10], so that the atom recedes from the boundary in the electron ground state. The boundary condition for atoms approaching the boundary consists in delimiting the polarization at large distances from the boundary. In the first order with respect to the optical density of the gas, this condition is equivalent to taking into account only the steady-state polarization of the atoms approaching the boundary. In Fig. 1, the shape of the spectral line of the selective mirror reflection from the boundary of a rarefied resonant gas (curve *1*) is compared with the results of the conventional theory of dispersion, obtained without taking into account the transient process of the establishment of the steady-state polarization of the reflected particles (curve *2*). In the region of anomalous dispersion of vapor, instead of a odd contour corresponding to the variation of the refraction index in a Doppler-broadened gas, we can see a narrow peak of the reflection coefficient free of Doppler broadening. The reason for such a sharp change in the shape of the reflection line is
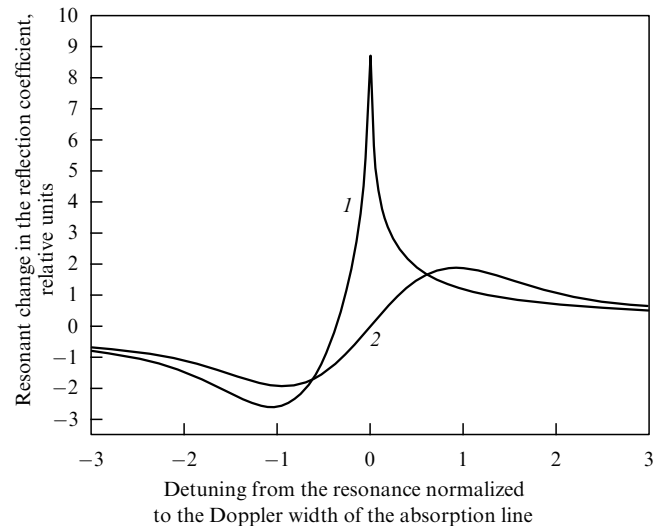
Figure 1. Spectral contour of a resonance, free of Doppler broadening, in the reflection spectrum from the glass – rarefied gas interface in the region of anomalous dispersion. The Doppler width is 100 times the homogeneous linewidth (curve *1*). For comparison, curve *2* represents the result of the conventional theory of dispersion neglecting the transient process of the establishment of the steady-state polarization of the particles that bounce off the interface.

that, on account of the transient process of the establishment of the steady-state polarization, the receding particles do not give a resonant contribution to the blue wing of the line, where it could be expected in accordance with the sign of the Doppler frequency shift. At the same time, in the red wing of the line, where the polarization amplitude of the reflected particles does not exhibit any resonance-specific features, the contribution of the particles that bounce off the boundary to the coefficient of reflection exhibits a resonant increase because the spatial structure of the transient polarization precisely matches the spatial structure of the reflected wave. The steady-state polarization of the atoms approaching the boundary exhibits a resonant increase in the red wing of the absorption line in complete agreement with the sign of the Doppler frequency shift. Thus, the receding atoms that are seeking a steady polarization state give exactly the same contribution to the coefficient of reflection as the atoms in the steady state that approach the boundary with an equal and oppositely directed velocity. As a result, averaging over velocities effectively reduces to the integration over one half of the Maxwell distribution, corresponding to the velocities of only the approaching particles. Such an abrupt termination of the velocity distribution at zero velocity gives rise to a narrow peak in the reflection coefficient.

If the resonant gas forms a thin layer between two dielectric media, we encounter two new circumstances at once. Firstly, the polarization of the atoms approaching the front boundary does not have time to reach a steady value after being bounced off the rear boundary, and secondly, the collision with the rear boundary leads to the quenching of polarization for atoms with velocities of either sign. The shape of the spectral reflection line depends in this case on the thickness of the gas layer. If the thickness of the layer is one and a half times the wavelength, we see a even spectral contour, and the amplitude of the field reflected from the thin layer is four times larger than that of the field reflected from a thick layer [11].

If the saturation effects are taken into account, the atoms moving in opposite directions give different contributions to the coefficient of reflection. If the longitudinal and the transverse relaxation time differ considerably, then the power of saturation is considerably different for atoms approaching the boundary and atoms receding from the boundary [12]. The relatively low power of radiation at which the saturation effect becomes noticeable for the reflection of resonant radiation from the boundary of rarefied vapor, facilitates the use of such a boundary [13] and a narrow layer of vapor [14] for nonlinear optical control of light beams, and for image processing. The transient process of the establishment of the steady-state polarization and population difference for the energy levels of the atoms receding from the boundary also leads to the appearance of longitudinal periodic structures near the reflecting surface. Of special interest is the possibility of controlling the period and depth of oscillations in the population difference by adjusting the intensity or frequency of radiation [15].

In the first order with respect to the optical density of the vapor, the maximum of the narrow Doppler-free contour of the selective mirror reflection coincides with the frequency for the transition in an isolated atom. However, a more efficient use of the diagnostic capabilities of selective reflection requires calculating the position of the maximum at least to terms quadratic with respect to the density of the gas. Such a calculation poses considerable mathematical difficulties because the boundary conditions for the atoms moving in opposite directions are posed in different regions. The exact solution obtained in Ref. [3] using the Wiener–Hopf method is very cumbersome, and can hardly be extended to more complicated cases. We succeeded in developing a simpler method that allows the calculation of the coefficient of reflection in the second order with respect to the optical density of the medium. The explicit analytical expression for the shift of the reflection maximum [16] implies that this shift is proportional to the product of the densities of the atoms moving in opposite directions. This circumstance, as well as the above-mentioned easy saturation of the particles approaching the boundary, explains the experimentally noted absence of a strong blue shift of the reflection maximum predicted in Ref. [3]. The same method was used to express the analytical correction to the position of the reflection maximum due to the deviation of the local field from the mean field [17]. The magnitude of this correction greatly exceeds the estimates derived from the intuitive physical assumption of the smallness of the Lorentz–Lorenz correction in a nonuniformly broadened medium [18], and helps to understand the earlier numerical results [19].

The narrow resonances of selective mirror reflection will certainly find wide practical application, and will be used to study various physical processes on the gas–solid interface, to which they are extremely sensitive. In this context, the further development of the theory, which will help in extracting much information about the processes on and near the surface and also give an adequate description of devices based on the effect of selective reflection from rarefied resonant vapor, seems very expedient [20].

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