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Cosmic vacuum

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Contents

Abstract. Recent observational studies of distant supernovae have suggested the existence of cosmic vacuum whose energy density exceeds the total density of all the other energy components in the Universe. The vacuum produces the field of antigravity that causes the cosmological expansion to accelerate. It is this accelerated expansion that has been discovered in the observations. The discovery of cosmic vacuum radically changes our current understanding of the present state of the Universe. It also poses new challenges to both cosmology and fundamental physics. Why is the density of vacuum what it is? Why do the densities of the cosmic energy components differ in exact value but agree in order of magnitude? On the other hand, the discovery made at large cosmological distances of hundreds and thousands Mpc provides new insights into the dynamics of the nearby Universe, the motions of galaxies in the local volume of 10 – 20 Mpc where the cosmological expansion was originally discovered.

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1. Introduction

Many specialists in cosmology are inclined to consider the discoveries made over the last two-three years in modern cosmology as a revolution. The actual scales of changes have become more and more clear with time and it seems that such a definition of what happens should sooner or later be accepted. Just have a look at only three statements of the modern revolution in cosmology:

(a) vacuum dominates in the Universe. Its energy density exceeds all 'conventional' forms of cosmic matter taken together;

(b) antigravity drives the dynamics of expansion;

(c) cosmological expansion is accelerated, while the four-dimensional space-time of the world remains static.

Most remarkably, these are not theories or hypotheses to be tested experimentally, but a direct consequence of firm observational data. The revolution has been made by astronomer-observers, who have studied remote supernova explosions [1]. Due to their exceptional brightness, supernovae can be observed from very large, really cosmological distances. Omitting other details, we just recall that data on special type Ia supernovae have been used. These supernovae are thought to be a kind of 'standard candle', since their proper luminosity indeed lies within rather narrow limits (meantime, the supernova experts continue to argue within which exactly). This allows one to determine how the visible, registered brightness of the supernovae depends upon the distance. Of course, at small distances this is simply the classical inverse square law, but at a very large distance from the sources cosmological effects become significant (the corresponding basic formula were already prepared long ago in the *The Classical Theory of Fields* by Landau and

Lifshitz [2]), and then the character of the dependence allows us in principle to know something new about the Universe as a whole.

The first group of observers [1], who reported their results in 1998, had at their disposal data on only a few supernovae of the needed type at needed distances, but even this was sufficient to recover a cosmological effect from the law of the visible brightness decrease with distance. More precisely, it is better to consider not the distance, but the redshift of the source, as is usually done for remote sources. It turned out that the brightness decreased on average much faster than expected from a cosmological theory considered standard three years ago. Such an additional fading suggests that at a given redshift there is some effective addition to the distance. But this is possible when (and, as anyone now thinks, only when) the recession velocity of a source does not decrease, but increases with time.

This discovery changes, at first, our understanding of the modern stage of cosmological evolution, today's state of the Universe. It had been thought before that the entire history of the cosmological expansion was the history of its braking after the original 'Big Bang'. Now it turns out that in our very epoch the dynamics of expansion has passed from the stage of deceleration to the stage of acceleration. This is a really important change in the picture of the world, and it strikes the cosmologists, physicists, and astronomers close to cosmology. One can hear or read sometimes that the new world picture refutes the usual Friedmann cosmological theory. But this is, of course, not the case. The theory developed by A A Friedmann in Petrograd in 1922 – 1924 has a rich physical content that actually provides for, as a feasible possibility, the transition from deceleration of the cosmological expansion to its acceleration. Mathematically, this is provided by Friedmann's theory comprising Einstein's cosmological constant. It is this constant that is capable of creating (more precisely, of describing in the solution) an antigravity, which induces an accelerated expansion.

The physical interpretation of the cosmological constant, introduced by Einstein into General Relativity in a somewhat formal way, gradually developed decade by decade, starting from studies by de Sitter, Lemaitre, Tolman, and Bondi. It is generally recognized that the cosmological constant describes a cosmic vacuum, i.e. such a state of cosmic energy that has a space density constant in time and equal everywhere, in any reference frame. Although this vacuum is called cosmic, it is present everywhere and appears in atomic physics and microphysics, where it represents the lowest energy state of quantum fields. This is the very vacuum in which interactions of elementary particles take place and which directly manifests itself in experiments, for example as the Lamb shift of atomic spectral lines and the Casimir effect. Undoubtedly, vacuum is present in these experiments, however the value of its density escapes from being measured. The problem of the vacuum density is believed to be the most complicated problem of modern fundamental physics [3, 4].

In the *The Classical Theory of Fields* [2], the Friedmann solution is presented in the form of a variant without vacuum, without a cosmological constant. In the original papers by Friedmann 1922 – 1924 (they were first published in German; a Russian translation can be found in *Usp. Fiz. Nauk* 1963 [5]), this constant may take both zero and non-zero values. When *The Classical Theory of Fields* was being written, nobody cast doubts that with the discovery of the cosmological expansion all grounds for introducing a cosmological constant into General Relativity fully and finally disappeared. That was also the opinion of Einstein himself who once called (in a conversation with G A Gamow in Princeton) the idea of the cosmological constant his biggest blunder in science. According to L D Landau, cosmology frequently makes errors and never doubts.

The discovery of cosmological acceleration in direct astronomical observations poses some new problems in cosmology, physics, and astronomy. Perhaps the most severe of the newest problems is: Why does the cosmic vacuum density have exactly the value that is discovered in observations? Adjacent to it is the problem of cosmic coincidences: Why do the densities of different components of the cosmic medium have close, in order of magnitude, observed values? This is one of the main points discussed below. On the other hand, the discovery made at very large cosmological distances (hundreds and thousands of Megaparsecs) apparently sheds a new light on what happens in our closeby surroundings in the Universe, in the Local Volume with a radius of 10 – 20 Mpc where, basically, the cosmological expansion was first discovered. This is another principal point of the paper. But it worth beginning with the story of how the 'standard' cosmological model looks today.

2. Density of vacuum

Vacuum was introduced into cosmology together with Einstein's cosmological constant Λ, and its density is expressed through the value of this constant

$$\rho_{\mathrm{V}} = \frac{\Lambda}{8\pi G} \,. \tag{1}$$

Here and below a system of units is used in which the velocity of light $c = 1$; G is Newton's gravity constant.

From the very beginning the role of the cosmological constant was to create, or more precisely, to describe antigravity. Einstein assumed that in this way one could balance the gravity of matter in the Universe to provide stationarity of matter and then of the Universe itself. Following the ancient tradition, coming from the roots of classical science, Einstein believed that the Universe as a whole must be eternal and unchanging. Interest in Einstein's model, in the de Sitter model containing no matter at all and in which only vacuum is present, in cosmic vacuum and the cosmological constant alternately disappeared completely or appeared from time to time for some reasons, and this story is extensively described in the literature, including well known monographs and textbooks [6 – 10]. We shall not repeat all these points, which have been presented many times with great completeness and tiny details, and to the cited books we just add a reference to the pioneering papers by É B Gliner [11], which, probably, are not very broadly known. These papers were (not, however, from the very beginning) highly estimated by Ya B Zel'dovich. The ideas first put forward in Refs [11], laid the foundation to the highly popular until now model of inflation in the very early Universe. But even irrespective of this concrete model, the ideas [11] serve as the first and as yet the only reasonable hypothesis on the physical reason for cosmological expansion. According to Gliner, the expansion is due to antigravity of cosmic vacuum, and the matter itself appeared as a result of quantum fluctuations of this vacuum. A D Sakharov and L E Gurevich paid due attention to these concepts.

Vacuum not only has a certain energy density, but also a pressure. If the density of vacuum is positive, its pressure is negative. The relationship between density and pressure, i.e. the equation of state, for vacuum takes the form $p_V = -\rho_V$. This and only this equation of state can be reconciled with the definition of vacuum as a form of energy with eternally and everywhere constant density, independently of the reference frame.

The vacuum equation of state can be straightforwardly derived in quantum field theory [12]. But its density value has not so far been obtained in this way. As we noted above, this is an outstanding problem, and we shall return to this point at the end of this paper.

According to the Friedmann theory, not only the density of matter induces gravity, but also its pressure in the combination $\rho + 3p$. Vacuum induces antigravity namely because its gravitating energy, $\rho_G = \rho_V + 3p_V = -2\rho_V$, is negative for a positive density.

The supernova observational data [1] mentioned above imply that the vacuum density exceeds the total density of any other sort of cosmic energy. The value of the densities can be conveniently expressed in units of the critical density $\rho_c = 3H^2/8\pi G = (0.6 \pm 0.1) \times 10^{-29}$ g cm^{-3}, where $H = 65 \pm 15$ km s^{-1} Mpc^{-1} is the Hubble constant. Then the relative vacuum density is

$$\Omega_V = \frac{\rho_V}{\rho_c} = 0.7 \pm 0.1\,. \tag{2}$$

As inferred from supernovae, this vacuum density value is justified by the bulk of data on the age of the oldest stars in the galaxy, on the large-scale structure formation, and especially on the cosmic microwave background anisotropy in combination with data on the dynamics of rich galaxy clusters (see [13–15] and references therein).

Hidden mass, or as it is now usually called, dark matter, only slightly concedes vacuum in density:

$$\Omega_D = \frac{\rho_D}{\rho_c} = 0.3 \pm 0.1\,. \tag{3}$$

The density is likely to be the best-known quantity of this component of cosmic matter. Dark matter emits neither visible light nor other electromagnetic waves, and in general does not interact practically at all with electromagnetic radiation. Our Galaxy comprises about 10 times as much dark matter as glowing matter in the stars. It forms an extended corona, or halo, around the stellar disk of the Milky Way. Such dark halos apparently exist around all sufficiently massive isolated galaxies. Dark matter is also confined in groups of galaxies and in the biggest cosmic systems, clusters and superclusters of galaxies. Like in our own Galaxy, dark matter amounts to 90%, and sometimes even more, of the total mass of these systems. It manifests itself only by the gravitational attraction it creates, and only due to its gravitational effect was it found (more precisely, suspected) as early as in 1930s by F Zwicky, who studied the kinematics and dynamics of a rich cluster of galaxies in the Coma constellation (or Coma Berenices). Galaxies in this cluster move with velocities of about a thousand kilometers per second, and they could remain bound with such velocities within the observed volume of the cluster only provided that the total mass of the cluster were about ten times as high as the sum of the individual galaxy masses it comprises. As cited in book [8] based on Ya B Zel'dovich's lectures at Moscow State University, "it seems quite astonishing that more than 90% of the mass of the Universe consists of a form of matter unknown to us. However, this conclusion seems to be unavoidable". The importance of the dark matter problem is evident, but not less evident is its extreme complexity. The physical nature of the constituents of dark matter is so far unknown. A very broad range of possibilities is being discussed, from elementary particles with a small mass (less than that of the electron) to dwarf stars and massive (more than Solar mass) black holes, etc. The masses of the dark matter candidates thus differ by a good 60 orders of magnitude — this is the actual numerical assessment of the current uncertainty in this point!

The luminous matter of stars and galaxies follows next to dark matter. In line with what has already been said, its cosmic density (averaged over the entire world volume) is an order of magnitude smaller than the dark matter density:

$$\Omega_B = \frac{\rho_B}{\rho_c} = 0.02 \pm 0.01\,. \tag{4}$$

Finally, the fourth component of the cosmological medium is radiation, or ultrarelativistic matter, with a density of

$$\Omega_R = \frac{\rho_R}{\rho_c} = 0.8 \times 10^{-5}\alpha\,, \tag{5}$$

where the constant factor $1 < \alpha < 10-30$ takes into account the contribution from neutrinos, gravitons, and other possible ultrarelativistic particles and fields of cosmological origin, in addition to the well measured contribution from the cosmic microwave radiation. Clearly, there is a significant uncertainty in the value of this contribution.

These are the modern data on energy densities which apparently satisfy all observational bounds existing today. With the value of the Hubble constant given above, these data are consistent with both open and flat [13, 15] and, generally speaking, closed cosmological models. The flat model corresponds to $\Omega = \Omega_V + \Omega_D + \Omega_B + \Omega_R = 1$; in the open model this sum of the relative densities is less than unity, and in the closed model is larger than unity.

Sometimes in the literature, especially in popular literature, not speaking about advertising press-releases, interviews, etc., one can meet statements that the flat model is fully proven either theoretically (with a reference, say, to the inflation model), or finally justified by some or other superprecision measurements. For example, in one of the seminars at the Sternberg Astronomical Institute of Moscow State University, a guest speaker reported as fresh news from some international meeting that the flat model is now undoubtedly proven, since the measured Ω is unity. Somebody from the audience asked the speaker with what accuracy it was measured to be unity. As the speaker hesitated with an answer, R A Syunyaev reported that as far as he knew, the accuracy was ± 0.2 in that particular case.

It is worth noticing that the values (2)–(5) by themselves have met unprecedented thus far general agreement, which, due to the uniqueness of the phenomenon, has specially been named 'cosmic concordance' [15]. The concordance is not full in one point only: some believe that it is vacuum that has been discovered, while others prefer another interpretation by assuming that the cosmological expansion is induced not by vacuum but by an unknown thus far and fully hypothetical quintessence. The latter is thought to be [16] a special form of cosmic energy with the equation of state $p = q\rho$, where q is a constant parameter, $-1 < q < -1/3$. It is easy to see that the

effective gravitating density is negative for this type of energy. This means that, not being a vacuum, the quintessence is also capable of inducing antigravity and hence cosmological expansion acceleration. In the classical ancient picture of the world, the quintessence is the fifth element in addition to earth, water, air, and fire; this is the cosmic substance which celestial bodies were thought to be made of. The word is nice; the hypothesis is popular. This is clear: it creates a new degree of freedom in cosmology, so let us wait for new fascinating results in this direction.

At this point it is worth making a note with regard to cosmological terminology which partially changed after and because of the discovery of cosmic vacuum. In addition to notions of concordance and quintessence, there also appeared other new terms which have been widely used over the last 2–3 years, though their meaning has not yet been fully accepted. Different components of the cosmological medium are now frequently referred to as forms of cosmic energy, which includes vacuum as one of the forms. Hidden masses, which are thought to be cold, i.e. non-relativistic, are named both dark energy and dark matter. In some publications, dark energy is used to designate vacuum and quintessence together (this seems to be not successfully coined). 'Ordinary' matter is called baryons, although it comprises, of course, electrons, but only protons and neutrons are more often assumed. The cosmic microwave background, gravitons, and all other ultrarelativistic particles and fields of cosmological origin are called relativistic energy.

3. Accelerated expansion

The paper by Zel'dovich [17], published in the above mentioned Friedmann issue of *Physics – Uspekhi* in 1963 (the fortieth anniversary of the theory of an expanding Universe was celebrated at that time), explains how the dynamics of cosmological expansion can obviously be illustrated using the language of Newtonian mechanics. There is a way of considering, suggested by E A Milne and W G MacCree in the beginning of the 1930s, which allows one to avoid all (more precisely, nearly all) paradoxes of Newtonian gravity that appear when one attempts to apply classical mechanics to an infinite spatial distribution of gravitating masses. This way allows one to obtain a result which exactly coincides with what follows from the relativistic Friedmann theory. It turns out that one can forget about infinity by considering a sphere of a finite size, which is mentally separated from the total homogeneous matter distribution. The outer layers have no effect on the dynamics of the sphere due to spherical symmetry, and the mass inside the sphere influences a point on its surface as if all the mass were concentrated at the sphere's center. Then the inverse square law gives the equation of motion for a particle on the surface of the sphere:

$$\ddot{R} = -\frac{GM}{R^2}\,. \tag{6}$$

Here R is the radius of the sphere, M is its total gravitating mass:

$$M = \rho_{\mathrm{G}}\,\frac{4\pi}{3}\,R^3\,. \tag{7}$$

Let us use this example to show the role of vacuum in the dynamics of cosmological expansion. If the total gravitating density of the sphere ρ_{G} includes all the four above mentioned components of cosmic medium, we obtain

$$\rho_{\mathrm{G}} = -2\rho_{\mathrm{V}} + \rho_{\mathrm{D}} + \rho_{\mathrm{B}} + 2\rho_{\mathrm{R}}\,, \tag{8}$$

where the gravitation effect of pressure (which is absent in the Newtonian gravity but should, of course, be taken into account in our consideration) is accounted for both vacuum and radiation with its equation of state $p_{\mathrm{R}} = \rho_{\mathrm{R}}/3$.

Only vacuum and non-relativistic matter with density ρ_{M} are present in Einstein's cosmological model; so in that model $\rho_{\mathrm{G}} = -2\rho_{\mathrm{V}} + \rho_{\mathrm{M}}$. The Einstein world is static since the effective gravitating density ρ_{G} in this model is assumed to be zero. The condition $\rho_{\mathrm{G}} = 0$ entails the relationship between the densities $\rho_{\mathrm{M}} = 2\rho_{\mathrm{V}}$, which describes the balance between the gravity of matter and antigravity of vacuum. In this case the force and acceleration in the equation of state (6) for the sphere are zero, and for its radius to be unchanged, the velocity of particles of the sphere must be zero as well. In Friedmann's model these conditions are not obligatory, hence the possibility of dynamics and evolution: the sphere can, generally speaking, contract or expand.

For adiabatic contraction or expansion of a homogeneous sphere, the relation between the density change and pressure is given by the equation

$$\mathrm{d}\rho = -3(\rho + p)\,\mathrm{d}\ln R \tag{9}$$

for any component of the medium provided no energy exchange is possible between the components. As is easy to check, this relation follows from the thermodynamic identity $\mathrm{d}E = T\,\mathrm{d}S - p\,\mathrm{d}V$ (where, as usual, E, T, S are the total internal energy (including the rest-mass energy), its temperature and entropy in the volume V, respectively), assuming $\mathrm{d}S = 0$. It is easy to find from Eqn (9) how the densities of matter and radiation change with time when the sphere contracts or expands:

$$\rho_{\mathrm{D}} = \frac{C_{\mathrm{D}}}{R^3}\,, \qquad \rho_{\mathrm{B}} = \frac{C_{\mathrm{B}}}{R^3}\,, \qquad \rho_{\mathrm{R}} = \frac{C_{\mathrm{R}}}{R^4}\,. \tag{10}$$

Here the three values C are arbitrary constants of integration. The same thermodynamic equation (9) indicates once again that vacuum with its equation of state $p_{\mathrm{V}} = -\rho_{\mathrm{V}}$ should have a constant density in time: $\rho_{\mathrm{V}} = C_{\mathrm{V}}$.

Substituting Eqn (8), (10) into equation of motion (6) and integrating over time yields:

$$\frac{1}{2}\,\dot{R}^2 = A_{\mathrm{V}}^{-2}R^2 + A_{\mathrm{D}}R^{-1} + A_{\mathrm{B}}R^{-1} + \frac{1}{2}\,A_{\mathrm{R}}^2R^{-2} + E\,. \tag{11}$$

Here E is an integration constant; more precisely, its value is time-independent but is a function of the total mass of the sphere. The sphere radius R itself, obviously, depends on the total mass; the radius plays the role of a Eulerian coordinate for a particle on the surface of the sphere, and the mass of baryons inside the sphere, which does not change with time for a given particle, serves as a Lagrangian coordinate. The constants A in the above equation are given by the general relationship

$$A = \left[\left(\frac{1+3w}{2}\right)^2 \kappa C\right]^{1/(1+3w)} = \left[\left(\frac{1+3w}{2}\right)^2 \kappa\rho R^{3(1+w)}\right]^{1/(1+3w)}, \tag{12}$$

in which $w = p/\rho$ for every component of the cosmic medium. For vacuum $w = -1$, for dark matter and baryons $w = 0$, for

radiation $w = 1/3$; $\kappa = 8\pi G/3$. The constants A can be found if the corresponding densities are known for some value of R. These integrals thus provide initial conditions in Friedmann's theory. As is obvious from Eqn (12), the integrals A for different equations of state has the same dimensionality (of length). Their numerical values are close to each other in order of magnitude and are $10^{28} - 10^{26}$ cm (see Sections 11, 13 below).

The integral A for matter without pressure emerged (and was so designated) in the first cosmological paper by Friedmann [5] [see Eqn (5) in this classical paper]. We shall call integrals (12) for different forms of cosmic energy 'Friedmann's integrals'.

As usual, the first integral of any equation of motion is energy, and the quantity E in Eqn (11) is the total specific mechanical energy of a particle. The kinetic energy stays on the LHS of Eqn (12), and the potential energy (both these energies also relate to unitary mass) represents the sum of the first four terms on the RHS of this equation, taken with the opposite sign. The total energy E can be positive, negative or zero; the corresponding types of motion are usually called hyperbolic, elliptic, and parabolic.

Remarkably, in Friedmann's cosmology the dynamics of an expanding Universe is described by precisely the same type of equation as the Newtonian energy conservation law (11)

$$\frac{1}{2}\,\dot{a}^2 = A_{\mathrm{V}}^{-2} a^2 + A_{\mathrm{D}} a^{-1} + A_{\mathrm{B}} a^{-1} + \frac{1}{2}\,A_{\mathrm{R}}^2 a^{-2} - \frac{1}{2}\,k\,. \quad (13)$$

In Friedmann's theory $a(t)$ is a scale factor to which all distances change proportionally in an expanding world; for models with non-zero spatial curvatures this quantity is also the curvature radius of the three-dimensional space. The sign of the curvature in Eqn (13), $k = 1, 0, -1$ (for closed, flat, and open models, respectively), is opposite to the sign of the total energy E in a Newtonian analog to the Friedmann equation. So there is one-to-one relation between the space curvature and the dynamical type of the cosmological expansion. In the subsequent cosmological formulas we shall make use of the value $a(t)$ of Friedmann's theory instead of $R(t)$ of the Newtonian theory; in particular, in Eqn (12), a has the meaning of R.

The exact similarity of the relativistic and Newtonian equations is not a simple coincidence. In this case this is an obvious manifestation of one of the main principles of theoretical physics, the correspondence principle. According to this principle, any new more general theory must include as a limiting or particular case the old theory within its applicability area. One may think that the Newtonian equations for a homogeneous sphere can be applied provided that the expansion velocity of the sphere $\dot{R}$ be much smaller than the speed of light and the gravitational potential on the sphere's surface be much less than the square of the speed of light. These conditions are certainly met for a sufficiently small sphere. However, in Friedmann's world all distances, including small ones, change proportionally to the scale factor $a(t)$; consequently, for a small sphere $R \propto a$, too. This entails the need for the exact similarity of equations for a and R as a function of time.

(It should be noted that the Newtonian description of cosmological expansion does have some paradoxes. Indeed, the equation of motion (6) is written, as one might think, in an inertial frame. In this frame a particle at the center of the sphere considered is at rest; one can put the origin of coordinates there. However, in a homogeneous world all particles are equivalent and then the same equation of motion can also be written in a frame connected, for example, with a particle on the surface of the sphere. However a particle on the surface does not move with a constant velocity with respect to the sphere's center; instead, by Eqn (6), it has a non-zero acceleration. So both frames cannot be inertial simultaneously. This paradox is absent in General Relativity, in which all freely falling frames, that is those connected with physical bodies freely moving in a gravity field, are equivalent.)

As is obvious from Eqns (11) and (13), the dynamical role of vacuum is different in different stages of cosmological expansion. At early stages, at small R or $a(t)$ (formally, when $R \propto a \to 0$), the term on the RHS of both equations that describes vacuum must be smaller than the four other terms ($\rho_{\mathrm{V}} R^2 \to 0$). Then the effect of vacuum at these stages of expansion is insignificant. In that case one can neglect vacuum and integrate Eqns (11), (13) (see, for example, Ref. [18]) and thus solve the problem under the condition of matter and radiation domination. Since the gravity of normal matter and radiation produces a negative acceleration, $\ddot{R} \propto \ddot{a} < 0$, the cosmological expansion brakes in these early evolutionary stages of the world.

At a later time the role of vacuum becomes significant and, as follows from Eqns (11), (13), sooner or later vacuum becomes dynamically dominant, when the vacuum term on the RHS of these equations largely exceeds the three other terms on the RHS that describe non-vacuum components of the cosmological medium. In this limiting case of large time intervals (formally when $R \propto a \to \infty$) the gravity of the non-vacuum components can be neglected and the solution of Eqns (11), (13) has the form

$$a(t) = A_{\mathrm{V}} f(t)\,, \qquad f(t) = \sinh\frac{t}{A_{\mathrm{V}}}\,, \qquad \exp\frac{t}{A_{\mathrm{V}}}\,, \qquad \cosh\frac{t}{A_{\mathrm{V}}}\,, \quad (14)$$

for $k = -1, 0, +1$, respectively. Here as above $A_{\mathrm{V}} = (\kappa\rho_{\mathrm{V}})^{-1/2} \sim 10^{28}$ cm is the Friedmann integral for vacuum, which is obtained from the general relation (12) for $w = -1$. The integral turns out to be a constant coefficient of the solutions for $k \neq 0$; it is also natural to choose it for normalization of the scale factor at $k = 0$.

Since vacuum with a positive density induces effective antigravity (its effective gravitating density $\rho_{\mathrm{V}} + 3p_{\mathrm{V}} < 0$, as we already noted several times), the acceleration $\ddot{R} \propto \ddot{a}$ proves to be positive for a dynamically dominating vacuum and solution (14) describes a cosmological expansion accelerating in time. For all three possible variants of Friedmann's model, corresponding to the three dynamical types, the cosmological expansion proceeds for an infinitely long time, according to Eqn (14). In the limit of large time intervals the expansion follows an exponential law for all three variants.

The change of deceleration by acceleration and transition to the vacuum dominance in the cosmological expansion dynamics corresponds to the equation of densities $\rho_{\mathrm{D}} + 2\rho_{\mathrm{B}} + 2\rho_{\mathrm{R}} = 2\rho_{\mathrm{V}}$, which obviously has the same meaning as in the static Einstein model. But in the Friedmann model this equality is possible only at one moment of time, and only at this instant $t = t_{\mathrm{V}}$ does the acceleration $\ddot{R} \propto \ddot{a}$ vanish. The corresponding redshift is

$$z(t_{\mathrm{V}}) = \frac{a(t_0)}{a(t_{\mathrm{V}})} - 1 \simeq \left(\frac{2\rho_{\mathrm{V}}}{\rho_{\mathrm{D}}}\right)^{1/3} - 1 \simeq 0.7\,. \quad (15)$$

Here t_0 is the present age of the Universe; the numerical estimate of this redshift is based on the observed densities (2) – (5).

As we mentioned above, the effect of cosmological acceleration discovered in supernova observations appears as a redshift dependence of the visible supernova brightness for large redshifts, large but not exceeding $z(t_V)$, as it must be, since at earlier times the expansion was not accelerated but decelerated.

In cosmological solution (14) the Hubble parameter is $\dot{a}/a = H \sim A_V^{-1}$ practically for any k soon after the transition to vacuum dominance. The Hubble constant in the stage of total vacuum prevalence does not depend on time and is determined solely by the cosmic vacuum density. It is easy to check that this relationship between the Hubble constant and vacuum density is in agreement (to within measurement errors) with observational values (see data in Section 2). It is essential that here two independent types of cosmological measurements are used.

Now we are in a position to write down the solution to Friedmann's cosmological equation (13) for any time:

$$\int (A_V^{-2} a^2 + 2A_D a^{-1} + 2A_B a^{-1} + A_R^2 a^{-2} - k)^{-1/2} \, \mathrm{d}a = t \,. \tag{16}$$

Here we conventionally write a plus sign before the square root as expansion, not contraction, of the cosmic medium is considered. The time is counted from the moment when $a = 0$. Solution (16) is illustrated in Fig. 1, which is plotted making use of the data on the Hubble constant and densities of dark matter, baryons, and radiation [Eqns (2) – (5)]. Cosmological expansion starts accelerated at $t > t_V \simeq 7-8$ billion years; the present age of the universe is $t_0 \simeq 15$ billion years. At present $a(t_0) \sim A_V \sim 10^{28}$ cm. The last approximate relation means the coincidence of the growing in time value of $a(t)$ with the constant length A_V; it is one of the cosmic coincidences that characterize the present epoch, and as we shall show below, it is also essential to understand other cosmic coincidences (see Sections 11, 13).

Interestingly, as is clear from Eqn (16), in both limiting cases, when $a \to 0$ and $a \to \infty$, the dynamics of cosmological expansion does not depend on the sign of the total energy E or the sign of the space curvature k. For all three variants of dynamics and curvature, the expansion begins in a parabolic regime; then, during a finite time interval, the possibility of deviation of the expansion dynamics from this regime may appear, and after that the expansion comes to the parabolic regime again which lasts for an infinitely long time.

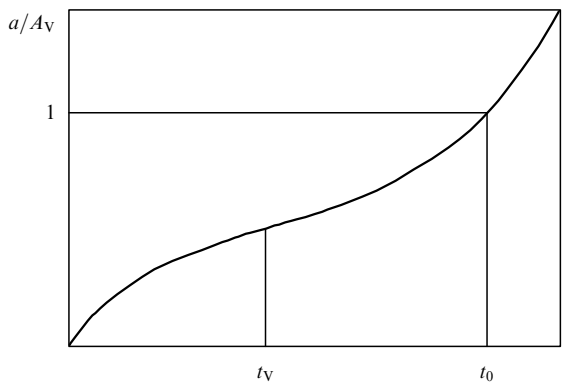

Figure 1. Friedmann model: the dependence of the scale factor on the age of the universe.

4. Four-dimensional world today

Let us pass from the dynamics of the universe to its geometry. This is the second important aspect of the modern standard cosmological model. The geometry of the four-dimensional Friedmann world is described by the metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2 F(\chi)^2 \, \mathrm{d}\Omega^2 - a^2 \, \mathrm{d}\chi^2 \,, \tag{17}$$

where $\mathrm{d}s$ is an infinitesimal distance between two close points-events in the four-dimensional space – time. Here t, χ designate a proper time (i.e. the time as measured in the co-moving frame) and a Lagrangian coordinate, respectively; the factor before the angular part of the interval $F = \sin\chi, \chi, \sinh\chi$ for $k = 1, 0, -1$, respectively; $\mathrm{d}\Omega^2 = \sin^2\theta \, \mathrm{d}\phi^2 + \mathrm{d}\theta^2$.

Together with solution (16), which gives the functional dependence of a on time, metric (17) comprises the complete theoretical information about the world that cosmology supplies. According to Eqn (17), the four-dimensional world has an isotropic three-dimensional space in the frame co-moving with matter. All lengths in the space increase proportionally to the scale factor $a(t)$, so galaxies, or more precisely, their systems, move away from each other, so the observer notices and measures their motions making use of redshifts in the spectrum of light emitted by them. The light propagates along nul-geodesics, for which interval (17) vanishes. This entails — in the limit of not too large redshifts — a Hubble law, i.e. a linear dependence of the recession velocity of an object on its distance. It is Friedmann's theory with the dynamics given by Eqn (16) and geometry given by interval (17), taken together with observed cosmic densities (2) – (5) and the Hubble constant, that constitutes the standard cosmological model of our day.

Of primary interest for us now are those features of the new picture of the world that are related to the presence of cosmic vacuum in the Universe. Assuming $a(t)$ to follow the exponential time dependence corresponding, as we have just shown, to vacuum dominance, the Friedmann solution takes the form of the famous de Sitter solution obtained in 1917, even before the solutions found by Friedmann (who, incidentally, highly estimated de Sitter and made accurate references to his papers). Hubble considered the de Sitter solution as a plausible theoretical model for the cosmological expansion he discovered. In papers [11], this solution was used to describe the original acceleration of cosmic matter; according to inflation theory, this stage lasts for not more than a tiny fraction of a second.

De Sitter's solution is a particular case of Friedmann's solution, corresponding to the total absence of non-vacuum forms of energy. For $a(t) = A_V \exp(t/A_V)$ metric (17) represents this solution in the frame of flying apart (with acceleration) test particles. In this limiting case, all non-vacuum components of the cosmic medium become test particles (i.e. non-gravitating).

The most important feature of de Sitter's solution is that the space-time described by this solution is static: it has no time-dependent 4-invariants. This implies that the de Sitter metric can be reduced to a form with no expansion at all. Indeed, a world containing vacuum energy with constant and time- and space-independent density must be time-indepen-

dent and spatially homogeneous. All events, i.e. four-dimensional points, in such a world are indistinguishable, which means that at any point of this world nothing happens, so this world is eternal, unchangeable, and has ideally symmetric geometrical properties.

In such conditions a reference frame should exist in which the de Sitter metric element is time-independent. To be specific, we provide one possible static form of de Sitter's solution:

$$\mathrm{d}s^2 = S(r)\,\mathrm{d}\tau^2 - r^2\,\mathrm{d}\Omega^2 - S(r)^{-1}\,\mathrm{d}r^2\,. \tag{18}$$

Here τ, r are new time and space coordinates; $S(r) = 1 - (r/A_{\mathrm{V}})^2$. Metrics (18) can be deduced from metric (17) by simple coordinate transformation from τ, χ to τ, r, if $a(t) \propto \exp(t/A_{\mathrm{V}})$ is substituted into Eqn (17).

There are other forms of this solution in which three-dimensional space looks different. But in all cases the vacuum that fills any of these spaces has an everywhere homogeneous and always constant density ρ_{V}. The vacuum density takes one and the same value in all these cases, so by fixing this value on one of the co-moving frames, as has really been done in supernova measurements, we are sure that it is exactly the same in any other measurements in any reference frame.

It is important that in any of the admissible forms for the de Sitter interval, the differential geometry of the four-dimensional world given by this metric is exactly the same as in form (18). This is geometry of a four-dimensional world with constant curvature radius. The four-dimensional curvature of the world K is directly related to Friedmann's integral for vacuum: $K = A_{\mathrm{V}}^{-2}$. This curvature (Riemann) is positive for a positive vacuum density.

It is well known that a sphere is a two-dimensional space of constant positive curvature. The three-dimensional space of positive curvature, which appears in Einstein's model and in one of the three (closed) Friedmann models, is called a hypersphere. A four-dimensional space of constant positive curvature is also some 'sphere', though strongly different (mostly by the metric signature) from its two- and three-dimensional analogs.

The four-dimensional curvature K vanishes only if the vacuum density is set to zero; then metric (18) transforms into a Galilean (or, which is the same, Lorentzian) metric of special relativity. Formally speaking, a world with zero four-dimensional curvature contains nothing, not even vacuum. Essentially this means that special relativity can not be applicable to cosmological problems on space-time scales approaching or exceeding Friedmann's integral A_{V}.

The non-zero curvature of the real four-dimensional world, $K = A_{\mathrm{V}}^{-2} \sim 10^{-56}\ \mathrm{cm}^{-2}$ is apparently one of the main constants of nature. In any case, this is the basic constant in cosmology, of course, in addition to the vacuum density, which it is directly connected with. Friedmann's integral $A_{\mathrm{V}} = K^{-1/2} \sim 10^{28}$ cm is the basic cosmological length. The time corresponding to this length is $A_{\mathrm{V}}/c \sim 10$ billion years. The corresponding mass is $\sim \rho_{\mathrm{V}} A_{\mathrm{V}}^3 \sim 10^{55}$ g. Making use of the characteristic length, time, and mass, one can compose a 'cosmic' system of units. As is obvious from figures quoted at the end of the previous section, in the present epoch $a \sim t \sim H^{-1} \sim 1$, should they be written in 'cosmic' units.

Let us return to the evolutionary history of the world. As becomes clear from above, the transition from the normal matter-dominating stage to the vacuum-dominating one in an expanding Universe means a gradual disappearance of dynamics in the four-dimensional world. One may say that the space-time carcass of the world gets frozen, it stops changing with time and, as a result, turns out to be fixed forever. But in matter embedded in this eternal and unchangeable world many processes do take place and will occur (for example, there are supernovae explosions giving us knowledge about accelerated expansion). However, these events, processes, transformations have virtually no effect on the four-dimensional world metric, and will have an even smaller impact in the future. On can say that the stronger the acceleration of cosmological expansion induced by antigravitating vacuum, the closer our four-dimensional world is to absolute static, unchangingness, and complete rest. This is the most important dynamical and geometrical effect of vacuum in cosmology.

Clearly, in these new circumstances, the traditional question as to whether the real cosmological model is open, closed or flat becomes less severe and principal. And there were so many hot debates on this subject; so many expensive observational programs, including space ones, were aimed at precise determination of cosmological parameters and thus at solving the greatest, as was thought, problem of natural science! Now it is obvious that the fate of the world, i.e. whether it will expand eternally or the observed expansion will be changed by contraction in future, does not depend on the discrimination between three possible geometries of the three-dimensional world. According to solution (16), cosmological expansion continues unlimitedly in all three models. These models differ only by the way their three-dimensional spaces are singled out from one and the same unique four-dimensional space-time. Depending on this way, these three-dimensional volumes can have either zero or non-zero curvature. Of course, it is of interest to know what specific sign the curvature of 'our' specific isotropic three-dimensional space has, in which the observed cosmological expansion proceeds homogeneously and isotropically. However, much more important is the remarkable fact that the geometry of the four-dimensional space-time became known even irrespective of this. And everything is known — precisely everything (!) — that one can infer in principle from General Relativity: it is a static space–time of constant positive curvature, and the numerical value of this curvature has been rather accurately measured.

But what cannot be expected from General Relativity is a description of the topology of the four-dimensional world, of its geometrical structure as a whole. In a postal polemic with Einstein, Friedmann pointed out that General Relativity is based on differential geometry (Friedmann gave courses on differential geometry in Petrograd University) which has no topology. Differential geometry provides only some general bounds on topology, but does not determine it. Einstein, in turn, believed that the static cosmological solution with a positive three-dimensional space curvature that he found describes a Universe, the three-dimensional space of which is closed as a whole, like a sphere. But essentially this was an arbitrary additional hypothesis, which did not follow from General Relativity itself. Friedmann's models, open and closed, are thus named only by tradition; these names should not mislead as to the possible topology of three-dimensional spaces in these models — nothing is really known about it.

The latter fully relates, of course, to the de Sitter world. Its four-dimensional differential geometry is known — it is completely described by interval (18), but its topology, its geometry as a whole, remains a fully open issue. There are

many interesting and various mathematically admissible variants of the global structure of such a world (see some of them in lectures [8]). But will it be possible some time in future to test these variants in observations and to choose one of them? Anyway, the world's topology becomes the most principal, hardly the only really important question of geometrical character in modern cosmology.

Returning from geometry to the dynamics of the Universe, we recall that cosmological evolution appears to have started in a vacuum-dominating static space – time [10, 11]. Most likely (and in the spirit of the glorious cosmological tradition, which Landau talked about, it should be said 'undoubtedly' at this point), this evolution will proceed eternally in the static vacuum world, too. It is between these two static vacuum states that cosmological evolution proceeds.

It should be noted that the initial value of vacuum density at very small times and the modern value of this density are quite different, and the former is believed to be much larger than the latter. The initial vacuum is considered by inflationary model; an attractive — as well as original — description of this model can be found in book [10]. In this elegant model, the initial vacuum state of the world is imitated by some scalar fields specially introduced for this purpose. They provide an effective vacuum equation of state of the cosmic medium. It is essential that the scalar fields evolve in the early Universe and, in particular, change the effective vacuum density. Ordinary matter is created by these changes, so the Universe comes to a physical state that can be described by Friedmann's theory already in that early stage of its existence. Inflation is a very special, large and interesting field, which is beyond the scope of the present discussion; a lot of papers have been devoted to it over the last 10 – 15 years. This is a remarkable hypothesis, which involved in cosmology the most brilliant ideas of elementary particle physics thereby very much enriching modern cosmology. But one has to agree, nevertheless, that inflation was not able to predict the presently measured value of the vacuum density. It does not provide quantitative estimates on the relation between the vacuum density actually observed and the initial value of this density during the inflation epoch. How and why the originally high density of vacuum, or inflation scalar field, fell to exactly its present value is the most difficult question in inflation theory. And yet without answering this question much of what inflation promises to explain remains somewhat up in the air. But what is really important is that inflation convincingly demonstrated the applicability of Friedmann's theory to describe the history of the world starting from at least its first second (actually, from small fractions of second, see Sections 12 and 13) and showed how, at least in principle, cosmological expansion could initially appear and then pass to the Friedmann regime of evolution.

It us remains to notice that in another (not about topology) dispute between Einstein and Friedmann, the debate on whether the world is static or evolving (see about this dispute in Refs [19, 20]), both classics prove to be ultimately right, but each on its own. Einstein's idea about an eternal and unchangeable Universe is realized, although in not in its static model, but instead in the ideally symmetric de Sitter solution. And Friedmann's concept of a nonstationary Universe is manifest in its general theory of evolution of the Universe and in the observed phenomenon of Hubble recession of galaxies. Friedmann's concept also incorporates, in particular, Einstein's idea as a limiting case.

5. Hubble's enigma

When Einstein published his static model Universe in 1917, cosmological expansion had already in fact been discovered by US astronomer Vesto Slipher, who reported this in a paper published in the same 1917, remarkable for cosmology. Though, by reporting on cosmic nebulae's recession, Slipher did not recognize himself what precisely he had discovered; neither the distances to the nebulae, nor their true nature was then known. Obviously, his paper did not contain any words about cosmology. But this science itself in its present understanding did not exist before the theory of general relativity (1916) and the first cosmological paper by Einstein in 1917. Seven years later, in 1924, Friedmann discussed Slipher's discovery at one of his seminars in Petrograd University and, according to D D Ivanenko who was present at this seminar, considered this discovery in a cosmological context, quite correctly, as direct observational evidence of an expanding Universe. Slipher's discovery was also reported by the popular journal *Mirovedenie*, which was published at that time in the USSR.

Friedmann's theory assumes that matter is distributed homogeneously in the Universe, and this is indeed the case. The large-scale distribution of galaxies is statistically homogeneous on scales larger than 100 – 300 Mpc, and the isotropy of the Universe is most precisely confirmed by the cosmic microwave background radiation, which is isotropic to an accuracy of at least a hundredth of a percent. Friedmann's theory predicts that cosmological expansion in a homogeneous and isotropic world must proceed linearly: at any given moment of the world history, the recession velocity of an object at a distance R away is proportional to this distance: $V = HR$, where H is a constant coefficient which does not depend on either the distance to the object or its position on the sky. This relationship is a direct consequence of the homogeneity and isotropy of the Universe; any observer anywhere in the Universe will see such a cosmological expansion. This relationship was discovered by Hubble in observations carried out in 1927 – 1929, and the constant H has fairly borne his name since then.

Hubble already knew, from his own studies, that the Universe is the world of galaxies. He plotted a diagram of the relationship of V to R for two dozen galaxies, whose velocities were measured and distances were estimated by him. The original Hubble diagram is reproduced in our Fig. 2.

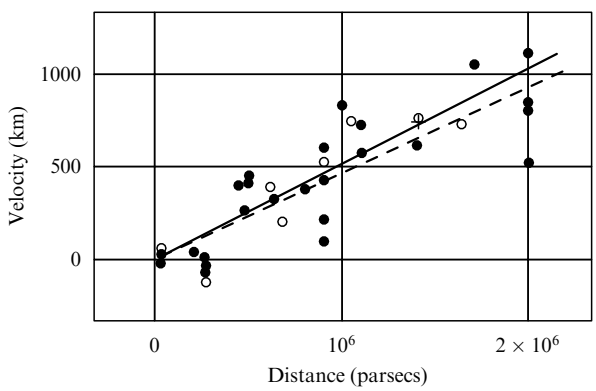

Figure 2. Hubble's original diagram of 1929. The velocity in the ordinate axis legend should be in km s^{-1}, not in km. Heliocentric reference frame.

The velocities measured by Hubble were just slightly larger than a thousand km s^{-1}. Using the presently accepted value of the Hubble constant $H = 50-75$ km s^{-1} Mpc^{-1}, it is easy to see that the distance limit of his observations was about 20 Mpc. Hubble himself then thought 2 Mpc; such was his systematic error in the distance scale, almost ten times.

But within the 20 Mpc limits in the Universe there is no homogeneity and isotropy; as we have just noted above, the Universe acquires these properties only at scales larger than 100 – 300 Mpc. There is a broad literature on this subject: we just refer to books [6 – 10] and recent publications [21 – 30]. In contrast, the distribution of matter in the near volume is very inhomogeneous: there are groups of galaxies with a size of 1 Mpc and larger, all of them belong to the big Virgo cluster of galaxies, the center of which lies at about the same 20 Mpc in the direction of the Virgo constellation, etc. How in such conditions is a regular cosmic flow with linear dependence of velocity on distance made possible?

The smallest recession velocities measured by Hubble were about 1 – 2 hundred km s^{-1}, which implies that the Hubble flow has its origin very close to us, at distances of only a few Mpc. But this is a catastrophically different, not cosmological space scale.

So it remains to ask whether Hubble's discovery has anything to do with cosmology.

70 years after the first cosmological publication by Hubble, in 1999, the question on the nature of the local (up to 20 Mpc) Hubble flow was clearly formulated in a paper by A Sandage [31] (see also his publications in 1972 and 1986 [32, 33]). To date, galaxies with recession velocities of hundreds of thousands of km s^{-1} have become available to observations, which corresponds to distances of thousands of Mpc. These are undoubtedly cosmological scales. At such scales, the linear expansion law is certainly and reliably established, in full agreement with the theoretical expectations in the spirit of isotropic Friedmann's cosmological models. But most striking is that for such global cosmological scales the Hubble constant has virtually the same numerical value as in the local volume up to distances of only 10 – 20 Mpc. According to Sandage [31], cosmological expansion is traced out down to 1.5 – 2 Mpc, and "the local rate is similar to the global one, if not exactly equal to it, at the $\sim 10\%$ level". In other words, the general picture of expansion looks as if the global cosmological flow actually starts just near to us and, extending further almost to the world horizon, kept its kinematic identity everywhere. But this is absolutely impossible!

This is Hubble's enigma. Sandage, one of the greatest observational cosmologists, directly writes in his 1999 paper: "We are still with this mystery forever" [31].

6. Local flow

Now we would like to discuss the most recent observational data on galactic motions in the Local Volume, which have been obtained by I D Karachentsev's group at SAO RAN. The principal result is as follows: based on a much larger observational data set than was accessible to Hubble in 1929, the existence of a regular expansion flow with a linear velocity dependence for distances from 2 to 8 Mpc is securely demonstrated and confirmed.

Karachentsev's group has recently presented [34] two variants of the Hubble diagram (Figs 3 and 4). The first of them (see Fig. 3) includes data on the motion of 145 galaxies

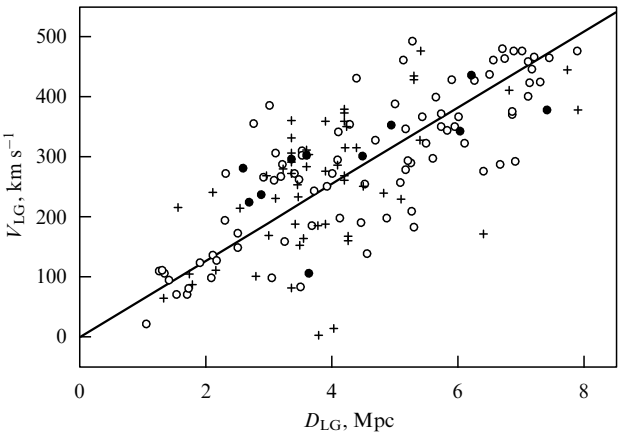

Figure 3. Hubble diagram for 145 galaxies from the Local Volume [34]. Local Group barycenter reference frame.

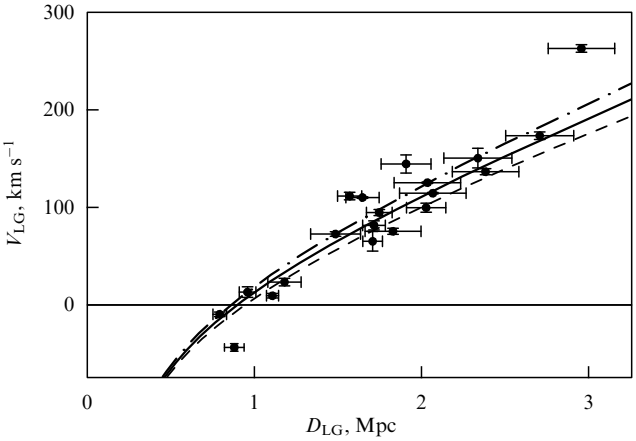

Figure 4. Hubble diagram for 20 galaxies from the Local Volume using high-precision distances [34]. Local Group barycenter reference frame.

up to distances of 8 Mpc (these are mostly original observations). The velocities are measured with an accuracy of better than 5 km s^{-1}, the distance errors are 20%. The velocities are given in the Local Group mass center frame. This group comprises our Galaxy and the giant Andromeda galaxy, which is comparable in mass with ours and lies 700 kpc away; these two galaxies make up the main mass of the group. In addition, the group comprises another 2 – 3 tens of less massive galaxies, mostly small dwarf galaxies. The full size of the group is about 1 Mpc. The vast majority of other galaxies up to distances of 8 Mpc outside the Local Group (the total number of galaxies there is not less than two hundred) are also dwarfs. The Hubble constant derived from the motion of these 145 galaxies is 64 ± 10 km s^{-1} Mpc^{-1}, which practically coincides with its 'global' value as measured on the scales of thousands of Mpc.

The velocity dispersion of 145 galaxies is 74 km s^{-1}. One part of this value is due to observational errors. Another part is due to galactic motions in groups; the latter, however, may also reflect the real anisotropy of recession of all galaxies in this volume. From the viewpoint of the problem under consideration, it is important that this dispersion is comparable with the regular linear expansion velocity up to distances of about 2 Mpc and is appreciably smaller than the regular velocity for distances from 3 to 8 Mpc. The mean amplitude of deviations from the linear dependence calculated for the entire volume does not exceed 10%.

Of most interest for our goals is the diagram (see Fig. 4) for 20 galaxies with high-precision (better than 7%) distance measurements [34]. These are dwarf galaxies lying within 3 Mpc distance and, with rare exceptions, not entering groups; their kinematics reflects in a most clear way the dynamics in our nearby surroundings. Not suffering from motions inside groups, the velocities of these galaxies exhibit a strikingly small dispersion around the linear relationship, of only 25 km s^{-1}. This the first reliable assessment of chaotic motions of the Local Volume galaxies, which well exceeds observational errors (about 15 km s^{-1}). And as we see, the stream of these galaxies is even more cool and quiet than the already not too violent general flow in the entire volume up to 8 Mpc. Dynamically, these galaxies serve as a very good tool for measuring the velocity field, and hence, gravity field in the Local Volume.

The high degree of regularity of motions of these nearby galaxies is in a sharp contrast with their extremely irregular, inhomogeneous space distribution. According to Teerikorpi and Paturel's group [24, 35], the strongest irregularities and inhomogeneities are observed exactly within 2 – 3 Mpc, and then they gradually smoothen farther away (on average in increasing volumes) remaining, however, far from weak there too, up to distances of 200 Mpc.

This general observational picture suggests the following corollary: the kinematics of galaxies in the Local Volume has a weak relation to their space distribution. The kinematics is highly regular, whereas the spatial distribution is strongly inhomogeneous both inside this volume and even within more extended volumes around us. The observed kinematics of galaxies is driven by dynamics, which, consequently, does not relate in fact with the galactic mass distribution.

But if not the masses of the galaxies, what then drives the motion of these bodies in the Local Volume? Such is the new physical setting of the problem on the nature of the local Hubble flow.

In the spirit of the newest discoveries in cosmology, one may suppose that the Hubble flow is driven by cosmological vacuum. This answer [36, 37] assumes cosmological acceleration due to the cosmological constant, i.e. vacuum (an alternative interpretation in the spirit of the hypothetical quintessence was mentioned above, and we will return to it again below).

7. Vacuum in the Local Volume

According to observations, most non-vacuum matter in the nearby 3 – 5 Mpc volume is concentrated in the Local Group. The mass of the Local Group is actually the sum of the baryonic mass of our Galaxy and the Andromeda galaxy plus the dark matter that 10 times exceeds the baryonic mass. The total non-vacuum mass of the Local Group is $M_{\mathrm{LG}} \simeq (1-3) \times 10^{12} M_{\odot}$ [7, 34] and is concentrated in a volume of approximately 1 Mpc in size.

To assess the dynamical picture on scales from 1 to 8 – 10 Mpc, we compare two quantities: the repulsion force induced by vacuum at some distance R from the Local Group barycenter, and the attraction force induced by the total mass of the Local Group at the same distance. The repulsion force is

$$F_1 = -\frac{GM_1}{R^2} = \frac{4\pi}{3} G 2\rho_{\mathrm{V}} R \simeq 7 \times 10^{-13} R \ \mathrm{cm\ s^{-2}}\,. \qquad (19)$$

Here M_1 is the total effective vacuum mass in a sphere of radius R; it equals the effective gravitating vacuum density times the volume of this sphere. The force is written per unit mass, i.e. this is acceleration. The distance R at the end of expression (19) is in Mpc.

To make a rough estimate of the attraction force induced by the Local Group mass, we neglect non-sphericity in the total mass distribution; in fact this is, of course, not a sphere, rather a dumbbell, but the difference in the estimates for ultimate distances is not too big, as is easy to check. We shall not add the masses of other galaxies inside the volume with radius R to the Local Group mass — this also would not change the result significantly. Then in this first and main approximation

$$F_2 = -\frac{GM_2}{R^2} \simeq -3 \times 10^{-11} R^{-2} \ \mathrm{cm\ s^{-2}}\,. \qquad (20)$$

Here $M_2 \simeq M_{\mathrm{LG}}$; at the end of the above formula the radius is measured in Mpc. The two forces (19) and (20) become comparable at $R \simeq 3$ Mpc, and vacuum antigravity prevails at larger distances.

Our estimate, of course, is rather rough, but it is obvious, reliable and stable in essence. It is not difficult to make a more refined calculation; for example, one can construct a family of equal-acceleration surfaces in the Local Volume and find that at which the radial acceleration vanishes. This surface in fact does not differ significantly from the 2 – 3 Mpc sphere. The surfaces of larger sizes become virtually spherical, so starting from distances 4 – 5 Mpc an almost strictly (to within an accuracy at lest 20 – 30%) spherically symmetric acceleration is established. Vacuum fully dominates at such distances.

In a regular spherically symmetric acceleration field at distances 3 – 4 Mpc and beyond, it is natural to expect regular motion of test bodies, which are the low-mass galaxies from Fig. 4 and, in general, all galaxies from the Local Volume. But this is exactly how the Local Flow appears in observations: it is indeed very regular starting from distances as small as 1.5 – 2 Mpc — this is so according to Sandage [31] and to the three variants of the Hubble diagram shown here. There is no paradox here if vacuum with its ideally homogeneous density drives the Local Volume dynamics. Thus, the contradiction between the regular kinematics of galaxies in the Local Volume and the highly irregular distribution of non-vacuum forms of energy within this volume disappears. Starting from distances above a few Mpc, galaxies of the Hubble flow move as test particles in the ideal vacuum background, which provides their acceleration (more precisely, pushes them additionally).

With the discovery of vacuum the Universe as a whole gets more homogeneous than was thought earlier based only on the distribution of galaxies in it. It is homogeneous not only at the proper cosmological scales of larger than 100 – 300 Mpc. Dynamically, it already appears homogeneous beyond several of Mpc. One can say that cosmology now starts not from hundreds of Mpc away, but from several Mpc from us. It almost approaches us, the Milky Way. And all this is due to the dynamical dominance of vacuum in both the Universe as a whole and in its small volumes, like the Local Volume. (Notice that in places with a higher concentration of non-vacuum energy, vacuum starts dominating at larger scales than in the Local Volume; for example, in rich clusters of galaxies, like the Coma or Virgo clusters, the zero acceleration radius can be as high as 10 – 30 Mpc.)

A critical point of this consideration is the kinematic identity of the Hubble flow at all scales from several Mpc to the largest galactic distances. This question, which until recently seemed to be non-resolvable for thinking astronomer-observers, now appears to have been clarified. Indeed, as far as at all these scales vacuum with its everywhere constant density dominates, the expansion rate which is characterized by the Hubble constant must be the same everywhere, since the Hubble constant is determined by the vacuum density only. According to solution (14), which is valid at every point of a vacuum-dominating region, and also to Eqn (19) at distances > 3 Mpc, the Hubble constant in these conditions is $H = \dot{a}/a = \dot{R}/R \simeq 1/A_{\mathrm{V}}$.

This is, as one may believe [36, 37], the solution of the 'mystery' which has existed from the times of Hubble's work in 1929 and of which Sandage wrote quite recently.

8. Vacuum and Local Volume bulk motion

There is yet one more interesting aspect in the dynamics of the nearby Universe. As we said above, the diagrams in Figs 2–4 show the recession velocities of galaxies in the Local Group barycenter; it lies on the line between the centers of main galaxies of the group, closer to the Andromeda galaxy, because it is somewhat more massive than ours. But there is a special frame in cosmology; it makes use of the cosmic microwave background radiation and comprises the entire Universe. This is a genuine global frame, and it is in this frame where the three-dimensional space of Friedmann's model is homogeneous and isotropic. The Sun and Earth move with respect to the cosmic microwave background radiation with a speed of about 300 km s^{-1}. But it turns out that the Local Group's barycenter is not at rest relative to the cosmic microwave background radiation either it moves with a relative velocity of about 600 km s^{-1} [26]. Observations indicate that the Local Volume moves with an appreciable general bulk velocity of 500–600 km s^{-1}. The Local Volume moves virtually as a whole, being a part of a much more extended region up to 100–150 Mpc in diameter (see recent paper [27] and references therein). All galaxies in this big volume move together in the direction of the Big Attractor, as this direction in space is sometimes called.

Notice that the bulk velocity is larger than the regular expansion rate up to 10 Mpc distances. It is 10–20 times as large as the velocity dispersion within these distances. So this is by far not a weak perturbation but a strong kinematic effect for the Local Volume scales.

The point that has very actively been discussed thus far is how to bring into consistency all three observed properties of the Local Volume: (1) the strong inhomogeneity of matter density; (2) the bulk motion; and (3) the regular Hubble expansion inside the Local Volume. Such a picture seems impossible in dynamics driven by gravity of galaxies themselves. But the three Local Volume properties turn out to be quite consistent if the galaxy dynamics in the Local Volume is driven not by their self-gravity but by cosmic vacuum.

This astronomical phenomenon reveals the main mechanical feature of the vacuum, according to which it can not be used as a reference frame. Rest and motion relative to vacuum are indistinguishable since vacuum appears the same everywhere, in any reference frame. The relic radiation is also almost ideally homogeneous and isotropic, but only in its own frame. A radiometer moving with respect to the relic background radiation will indicate that the radiation has a dipole anisotropy, which can be immediately used to determine the radiometer's relative velocity. But there is not and cannot be such a device that could measure its own velocity relative to vacuum because this quantity has no physical meaning. More precisely, it is identically equal to zero: in any reference frame vacuum looks absolutely identical and any frame is co-moving for vacuum. Or in other words: two frames can move with respect to each other with any velocity, but vacuum will be co-moving for each of them.

There is one more feature specific only for vacuum: inducing antigravity on each body, it experiences no back gravitational effect. Newton's third law 'the action is equal to the reaction' is not applicable to vacuum. It can be said that vacuum has a non-zero (and negative) active gravitating mass, while its passive gravitating mass and inertial mass are zero. (Note only that all this relates to physical conditions where gravity fields and all other fields are weak; in strong fields, polarization of vacuum and some other effects become possible, in which the local vacuum properties change under the strong external action.)

In the Local Volume, vacuum induces antigravity on the galaxies such as if it itself were moving with the Local Group barycenter. By its main mechanical properties it co-moves with the Local Volume and controls its dynamics exactly in the same way as it drives the global dynamics of cosmological expansion. Since it does not suffer any back reaction from galaxies, its dynamical effect is independent of the galaxies themselves, both of their spatial distribution, and of their motion. This allows the Local Volume to expand as a small separate Universe inside the entire large Universe with its global Hubble flow. But of importance is the fact that both these streams, global and local, have the same kinematics and the same expansion rate determined by the Hubble constant, since vacuum is the same for both the whole Universe and the Local Volume, which additionally has a bulk motion.

It is clear that the global expansion flow on scales exceeding 100–150 Mpc must appear anisotropic when observed from the Local Group. In this flow, an additional anisotropy emerges due to the Local Group's relative motion. But this anisotropy is small and hidden by observational errors: its value is determined by the ratio of the bulk velocity of the Local Group (600 km s^{-1}) to the regular Hubble expansion velocity on these scales (above 6000 km s^{-1}), i.e. smaller than 10%.

Such is the possible answer to the question about the consistency of the regular expansion of the Local Volume with its strong inhomogeneity and general bulk motion [36]. The above considerations are based on exact solutions of Newtonian hydrodynamics [40] for a one-dimensional flow superimposed on isotropic Hubble expansion. This is a generalization of Zel'dovich's solution [41] with the same symmetry in the case where cosmic vacuum is present. The solution allows a quantitative description of the above three properties of the Local Volume; it also provides a description of the dynamical prehistory of this nearby volume [39] and demonstrates which 'initial conditions' led to the observed value of the bulk velocity.

In conclusion, one can say that a true understanding of the physical sense of the discovery made by Hubble in the Local Volume of the Universe became clear only after the discovery of cosmic vacuum. In fact, Hubble discovered the phenomenon of global expansion of the Universe from deep inside the homogeneity cell of galactic distribution. In essence he thus also revealed the presence of cosmic vacuum. And the

reference to de Sitter's paper he cites in his work exactly hits the target.

It is noteworthy that quintessence, not being a vacuum, is unable to reproduce the observed kinematics of galaxies in the Local Volume. Unlike vacuum, its density varies in space and time so it can not play the role of vacuum in driving galactic kinematics. So one can say that the bulk velocity of the inhomogeneous Local Volume with a regular Hubble flow inside it observationally favors vacuum and disfavors quintessence.

9. Cosmic intermittency?

What still remains unclear in the Local Hubble flow is the observed value of the velocity dispersion in it, 25 km s^{-1} according to Ref. [34] (see also the most recent paper [42] which gives 38 km s^{-1}). Obviously, the flow must be fairly regular, but why with such a high precision? Where does nature take this particular value 25 (or 38) km s^{-1} from?

Let us use the well known general result from Landau and Lifshitz's textbook [2]. The result is that in an expanding world, chaotic motions that do not change the general gravity field can only decay. This occurs according to the law: $Pa = \text{const}$, where P is the momentum of random motion, and $a(t)$ is the scale factor of the cosmological model, as above. This is a very general result: for example, it entails the law of decrease of temperature of any known forms of energy during cosmological expansion, redshift of light, etc. From it also ensues the law of decay of irregular motions in the Hubble flow. In particular, at the vacuum-dominating stage of cosmological expansion, when the gravity of non-vacuum forms of energy can be neglected in comparison with the antigravity of vacuum, random deviations from the regular Hubble velocity decay according to Landau and Lifshitz. Then the velocity dispersion of test bodies, such as dwarf galaxies from the Local Volume, falls with time as $\Delta v \propto a^{-1}$.

During the period of vacuum dominance in the dynamics of the Universe, the value of a increases approximately by 2 times, 1.7 more precisely, as follows from Eqn (15). So irregular velocities, and hence, the velocity dispersion in the Hubble flow, drop twice, too. But this is hardly sufficient to get the required 25 km s^{-1}. Indeed, by the moment when this cooling started, some random motions could remain in the Hubble flow, whose velocities could be comparable with the regular Hubble velocity on some or other scale. For example, on the scale which is now 10 Mpc, the Hubble velocity is approximately (again now) 500 km s^{-1}. Regular velocities grow as a with exponential expansion, so at the moment of the cooling start the velocity was two times as small as now, i.e. about 250 km s^{-1}. If there was a chaotic velocity of 250 km s^{-1} at the same time on the same scale, it must decrease twice due to the general cooling of the Hubble flow and should be now 125 km s^{-1}. If we expand such an estimate to smaller scales down to, say, 3 Mpc, the mean irregular velocity in the Local Volume would be near 80 km s^{-1}. This is not too far from the value given by Sandage for the velocity dispersions in the Hubble flow (60 km s^{-1}), but is three times larger than the value given by Karachentsev [34]. Such a characteristic divergence of figures can hardly be ignored.

Numerical modeling of general evolution of the large-scale structure of the Universe with high spatial resolution for large space volumes poses the same severe problem. Such modeling well reproduces the observed features of the large-scale structure of the Universe (see, for example, Refs [43, 44]). One could expect smaller-scale structures, such as the Local Volume to appear in such models, the spatial resolution being high enough. And they are indeed seen, but a dedicated analysis [45] indicates that the model velocity dispersion in such small volumes turns out to be very high, 150 – 300 km s^{-1}. Analysis [45] did not actually take into account the cooling action of vacuum; this effect can twice decrease the velocity dispersion, i.e. down to 75 – 150 km s^{-1}, but hardly smaller, in volumes 3 – 10 Mpc in size. So the discrepancy of the model's results with Karachentsev's data is fairly significant.

Thus, a very low velocity dispersion in the observed Hubble flow seems to the present author a problem, which cannot be resolved by 'vacuum cooling'. In looking for a solution to this issue, it is useful to address the basic assumptions used in cosmological modeling [45], which yields an unacceptably large velocity dispersion. Possibly, the setting of the problem in this case is somewhat simplified. In particular, the statistics of protogalactic perturbations is determined by practically only one quantity, the power law index of such perturbations. Apparently, it is sufficient to model the largest structures, but small volumes suggest that the statistics of real perturbations is most probably more complicated and rich. As one can suppose [36], the statistics of real perturbations could reflect such a universal property of non-linear dynamic systems as intermittency. This feature is observed in a very wide range of natural phenomena, from turbulent motions in the ocean or laboratory plasma [46] to dynamical chaos in the three-body problem [47].

The intermittency phenomenon consists in a random sequence of relatively quiet and more excited states of a system in space and/or in time. For example, in the ocean there can be local regions of almost laminar flows in the strong turbulent background. Something similar can be found in cosmology; quiet and cool local regions of the general Hubble flow can coexist and randomly alternate with regions of high velocity dispersion, and this could occur everywhere in the Universe. The corresponding spatial structure can be both quasi-periodic and fully chaotic, as in experimentally known examples of intermittency in hydrodynamic systems.

If so, the Local Volume could be considered as one of the regions of relatively quiet flow with a velocity dispersion of 30 – 40 km s^{-1} in a background of very irregular flows with a velocity dispersion of 75 – 150 km s^{-1} on the same volume scales. It is not excluded that intermittency appears on the largest cosmic scales, on which one finds [48] an enigmatic quasi-periodic structure in the general distribution of clusters and superclusters with a characteristic size $\sim$ 100 Mpc.

Specialized satellites MAP (Microwave Anisotropy Probe) to be launched in 2001 and PLANCK, which is scheduled for launch in 6 years, could trace the cosmic intermittency features, if it really exists, in the anisotropy of the cosmic microwave background. MAP must have an angular resolution of up to 0.2 – 0.3°, so the Local Volume scale could be available. The intermittency could cause additional spots in the observed microwave background anisotropy; the spots could have sizes from the resolution limit to, say, ten degrees, which corresponds to the size of the quasi-periodicity cell [48] in the observed distribution of galaxies. As for the amplitude of deviations from the mean background temperature, it could be as high as at the MAP sensitivity level (20 μK) and higher, especially on small scales. A distinct feature of these imprints of cosmic intermittency in the relic background is that their statistics cannot be

described by only one simple Zel'dovich – Harrison spectrum. In addition, radiation in the intermittency spots should not be polarized, as opposite to the expected effect from primordial gravitational waves of cosmological origin (about the latter, see review [49]).

Notice that theory of turbulence also started from a universal spectrum and then evolved to an understanding of the richness, variety, and real complexity of turbulence, and the detection and recognition of the intermittency phenomenon proved to be the key stage in this evolution.

10. Problem of cosmic coincidences

The vacuum density, as we have seen, exceeds the three other densities of cosmic energy forms taken together. Adding a little bit of Anti-During ("Who is that?", the reader of happy post-graduate age would ask), one can say that the world contains nothing but moving matter and vacuum at rest; vacuum even prevails. But strikingly, the difference in densities is not very large, especially between vacuum and dark matter [see again data (2) – (5)]. The latter suggests that the effective gravitating density of vacuum, which is constant, has started dominating dark matter only rather recently, at a redshift $z < 0.7$, according to formula (15). So the present epoch of evolution of the Universe is the epoch of continuing transition from dark matter dominance to vacuum dominance.

As is clear from Eqns (2) – (5), not only these two densities, but all four densities have close values to an order of magnitude. The latter can be explained simply by the fact that we are measuring them at the given transitional epoch. Indeed, why, for example, is the density of baryons close to that of ultrarelativistic energy? These densities change with time according to different laws, $\rho_{\mathrm{B}} \propto a^{-3}$ and $\rho_{\mathrm{R}} \propto a^{-4}$, respectively [see Eqn (10)]. Together with a more general question on the coincidence of the four observed cosmic densities, this question is the essence of a big issue in modern cosmology called the cosmic coincidence problem. Presently, this problem is likely to be the most severe and principal for the entire physics of the Universe [13, 15]. Undoubtedly, the most important and fundamental aspect of the problem is linked to the specific value of the cosmic vacuum density: why does this density take the value measured in observations?

It is not the first time that cosmology has met the problem of numerical coincidences: this topic has a long history, going back to the coincidence of 'big numbers' found by Dirac (see textbooks [6 – 9] about this problem). But the coincidence of cosmic densities opens a new page in this story, though in this variant the problem intersects with what was discussed about the coincidences before.

The aforementioned hypothesis of quintessence appeared as a reaction to the problem of cosmic coincidences [16]. It turned out that one can construct a variant of the quintessence model such that the density of this hypothetical energy would indeed be close to that of dark matter at present or even at all times. This, however, cannot be done without invoking very special assumptions, which have not been justified thus far. Besides, it is unclear what then to do with the relativistic energy whose density falls off with expansion more rapidly than that of dark matter. For this reason, here we shall not dwell upon the details of such an approach to the density coincidence problem, especially considering that, as discussed slightly above, the observed kinematics of the Local Volume apparently speaks against quintessence and favors cosmological vacuum.

Below (as in the above sections of the article) we shall assume that the observed cosmological acceleration is completely due to cosmological vacuum and shall discuss how one can try to resolve the cosmic coincidence problem within such an approach. First we shall formulate the problem anew using integrals of Friedmann's cosmological equations (12), and then try to discuss the physics which could underlay the observed density coincidences.

11. Friedmann's integrals

As one can see above in Section 3, Friedmann's integrals represent the basic constants of cosmology. For non-relativistic matter and radiation the integrals imply conservation of the total number of particles of each given sort in the co-moving volume. It is interesting that the corresponding integral exists for vacuum, with its value calculated according to the general formula (12), although the interpretation of vacuum in terms of particles is incorrect. By the essence of Friedmann's equations, the general and main feature of the integrals is that they follow from the adiabaticity condition, i.e. in this case from the condition of the internal energy transformations from one form to another. All four forms of energy, including vacuum, satisfy this condition after the processes related to vacuum transformations (phase transitions) have been completed in the very early Universe (see books [6 – 10] about these processes).

Being arbitrary constants of integration, Friedmann's integrals are not bound by any *a priori* constraints other than trivial ones and are fully independent of each other. For example, in a cold Universe (unrealistic but a formally not excluded beforehand variant), the radiation integral would be zero, while the non-relativistic particle integral would be non-zero. For the dynamics described by Friedmann's solution (60), the integrals serve as 'initial conditions'. From the viewpoint of physics, they are indeed determined by real physical conditions in the early Universe at the stage of generation of the observed forms of cosmic energy.

As was noted soon after the discovery of the cosmic microwave background radiation [50], Friedmann's integrals, as calculated using the known values of matter and radiation densities, are close to each other in order of magnitude. In the spirit of Gamow's ideas on cosmic coincidences [51], a suggestion was put forward [50] that the coincidence of integrals could be more fundamental than the closeness of densities of two cosmic energies. Indeed, the relation $A_{\mathrm{B}} \sim A_{\mathrm{R}} \sim 10^{26}$ cm found in [50], contains in a compact form the entire complex of different physical relationships in cosmology. It is easy to check that these two equalities are sufficient to provide a quantitative formulation of the charge asymmetry of the Universe, of the cosmological entropy per baryon, of the helium production in the primordial nucleosynthesis, of the duration of the radiation dominating epoch, etc.

As we shall now see, two other Friedmann integrals, for vacuum and dark energy, are also close to each other and to the two aforementioned integrals. Making use of observational data (2) – (5), the Friedmann integrals for dark energy, baryons, and ultra-relativistic particles can be found using additionally the present value of the curvature radius $a(t_0) \sim A_{\mathrm{V}}$ (or a normalized scale factor) — this approxima-

tion was given in Section 3. In this way we find:

$$
\begin{aligned}
A_{\mathrm{V}} &= (\kappa\rho_{\mathrm{V}})^{-1/2} \sim 10^{61} M_{\mathrm{Pl}}^{-1}\,, \\
A_{\mathrm{D}} &= \frac{1}{4}\,\kappa\rho_{\mathrm{D}} a^3 \sim 10^{60} M_{\mathrm{Pl}}^{-1}\,, \\
A_{\mathrm{B}} &= \frac{1}{4}\,\kappa\rho_{\mathrm{B}} a^3 \sim 10^{59} M_{\mathrm{Pl}}^{-1}\,, \\
A_{\mathrm{R}} &= (\kappa\rho_{\mathrm{R}})^{1/2} a^2 \sim 10^{59} M_{\mathrm{Pl}}^{-1}\,.
\end{aligned}
\tag{21}
$$

Here the numbers are given in units $k = c = \hbar = 1$; in these units the gravitation constant $G = M_{\mathrm{Pl}}^{-2}$. The Planck mass is $M_{\mathrm{Pl}} = 1.2 \times 10^{19}$ GeV.

Obviously, all four numbers are close to each other in order of magnitude [52], and the result can be presented in the compact form:

$$
A_w \sim 10^{60\pm1} M_{\mathrm{Pl}}^{-1}\,, \qquad w = \left[-1, 0, 0, \frac{1}{3}\right]. \tag{22}
$$

Since $w = 0$ both for dark energy and baryons, the approximate equality of the corresponding integrals arises simply due to the nearness of the observed densities of these forms of energy. However, the structure of Eqn (12) demands that for these two integrals to be close to the vacuum integral, not only must all three density values be close to each other, but also the current value of the three-dimensional space curvature radius should be close to the vacuum integral value. This is a special feature of the epoch of observations. The same applies to the approximate equality of radiation and vacuum integrals.

The numerical equality of four values in (22) is, of course, approximate, but the degree of coincidence is sufficiently high: the dispersion of the integral values does not exceed a few percent in the relative logarithmic scale: $\lg(A_{\mathrm{V}}/A_{\mathrm{B}})/\lg A_{\mathrm{V}} \simeq 0.03$.

Relations (22) extend the initial variant of Friedmann's integral coincidences in an open Universe [50], by adding the relation of these integrals to integrals for other forms of cosmic energy to the equality of A_{B} and A_{R}, i.e. vacuum and dark energy. Unless this is a pure arithmetic accident, the coincidence of the four Friedmann's integrals represents one of the essential, and in addition unchangeable, features of the evolving world. Anyway, empirical relations (22) are valid over as long a time interval as these very forms of cosmic energy exist in nature.

The cosmic coincidence problem, which was formulated in terms of densities, now appears in a different light. As we see, the coincidence of densities of four cosmic energy forms in fact represents the coincidence of four constant numbers. Since one of the densities is constant in time (the vacuum density) and the three others decrease with expansion, the density coincidence is, obviously, a temporal phenomenon, an accidental episode, which takes place only in the present epoch. For example, in the early Universe at the age ~ 1 s, the densities of baryons and relativistic particles were $\sim 10^{30}$ and $\sim 10^{40}$ times as large as that of vacuum, correspondingly, having a difference between themselves of ten orders of magnitude, too. But even at that time the approximate equality of integrals (22) held and remained conserved during the subsequent history of the Universe. This equality will also hold in the future, until the epoch of decay of protons and/or dark matter particles. Over the proton life time $> 10^{31} - 10^{32}$ years, cosmic densities and their relations change by many orders of orders of magnitude (!), whereas equality (22) remains valid, being the equality between time-independent quantities.

From the point of view of relations (22), the observed closeness of vacuum and dark energy density is due to only the modern epoch being the epoch of transition from dark energy dominance to vacuum dominance, as was already noted above. Another question is why we should live in exactly this transition epoch; here other arguments connected with anthropic principles appear [53, 54] and we shall not dwell on them. But the question why all four densities proved to be coincident now becomes clear: this is the direct consequence of equalities (22) if we apply them as the 'primary' relationships to calculate densities at an epoch when temporarily (and hence accidentally) the curvature radius of an open world has coincided with the value of A_{V}.

Note here one somewhat technical but still interesting point. If in addition to open or closed Friedmann models one considers the flat model, the coincidence of the four integrals at $E = k = 0$ assumes a certain normalization of the scale factor, and in Eqn (14) $a(t_0) \sim A_{\mathrm{V}}$ for $E = k = 0$ was assumed. As is clear from Eqn (12), the integral for vacuum is calculated irrespective of the normalization; its definition does not contain the scale factor. The integrals for the three other energy forms do depend on the normalization as they contain the scale factor. As a result, not only the numerical value of the integrals depend on the normalization, but also the very possibility of their coincidence. This is clear from the relationship

$$
\frac{A_{\mathrm{D}}}{A_{\mathrm{V}}} \sim \frac{A_{\mathrm{B}}}{A_{\mathrm{V}}} \sim \left(\frac{A_{\mathrm{R}}}{A_{\mathrm{V}}}\right)^{3/2} \sim \left[\frac{a(t_0)}{A_{\mathrm{V}}}\right]^3. \tag{23}
$$

Here the near equality of all energy densities is taken into account and the dependence of the integrals on the normalization, i.e. on the ratio $a(t_0)/A_{\mathrm{V}}$, is explicitly shown. Only two first equalities here have a normalization-independent sense. Indeed, $A_{\mathrm{D}}/A_{\mathrm{V}}$, $A_{\mathrm{B}}/A_{\mathrm{V}}$, and $(A_{\mathrm{R}}/A_{\mathrm{V}})^{3/2}$ depend on the scale factor in the same way so their approximate equality holds for any normalization (in the case of close densities). Equalities (23) also hold, naturally, in both open and closed models, but in the case of warped space the very definition of the integrals contains not simply the scale factor, but the curvature radius, which gives the additional relation $a(t_0) \sim A_{\mathrm{V}}$ leading ultimately to (22). The normalization used in (14) for $E = k = 0$ contradicts nothing at all, but there are not and cannot be any independent external grounds for it (apart from naturalness). On the other hand, it is interesting in itself that the flat model admits such a normalization (i.e. choosing one co-moving volume for all energy forms) that Friedmann's integrals for ordinary energy forms calculated over this volume prove to be equal both each other and the vacuum integral. One may say that the natural normalization allows one to find such a relation in the real world that would escape the observer who accidentally used another normalization. The special normalization, nevertheless adequate to the task under consideration, thus serves as a mean to discover a quite objective physical fact.

12. On the origin of species

Now let us turn from the empirical analysis of recent observations in terms of Friedmann's integrals to the

possible physical nature of the approximate equality of these integrals. Does this fact follow from the known fundamental physical laws? Does it reflect some deeper relationships in nature?

To answer these questions one needs to address the physics of the early Universe, the processes, which, as one believes [6 – 10], could lead to the generation of the observed cosmic energy forms, to the 'origin of species' in cosmology, as one sometimes says. In addition to what has been already said about this in Section 2, we present here a short list of modern data on the origin and physical nature of the four basic components of cosmic medium.

Among all the ingredients of the cosmic medium, only the origin of the relic microwave radiation can be considered as simple and fully understood. These are electromagnetic waves, or photons, which were in thermodynamic equilibrium with hot cosmic plasma until the temperature fell during expansion to $\simeq 3000$ K at an age of the Universe $\simeq 1$ million years. At that moment recombination of electrons with protons occurred after which radiation stopped interacting efficiently with matter any more. Since then this photon gas has continued to expand by remaining virtually homogeneous, isotropic and in thermodynamic equilibrium. Its present temperature has been measured with a very high accuracy (for cosmology) to be 2.736 ± 0.003 K. Each cubic centimeter of space of the Universe contains approximately $n_\gamma \simeq 400$ relic photons.

The relativistic component of the cosmic medium includes, in addition to the relic radiation, massless and/or light neutrinos and antineutrinos (with equal numbers). Their total number amounts to about the same value as for photons, about 300 per cubic centimeter. The total contribution of neutrinos to the density of the relativistic component is slightly less than from the relict radiation. Neutrinos and antineutrinos were also initially in thermodynamic equilibrium with the cosmic plasma, but due to the small cross-section of interaction with electrons they separated from plasma much earlier than photons; this occurred at a temperature $\simeq 10^{10}$ K $\simeq 1$ MeV at a Universe age $\simeq 1$ s. Neutrinos weakly interact with each other, so neutrinos and antineutrinos have not annihilated and remain up to now as one more relic of the initial hot state of the world. Their present temperature is about 2 K.

Much less is known about other possible relativistic particles and fields of cosmological origin. Primordial gravitons must almost certainly be present among them [49]; the inflation model also predicts their existence. It is not excluded that the quantity of gravitons and possible other particles of which little or nothing is presently known may be much larger than of relic photons; the relative contribution from neutrinos, gravitons, etc. is taken into account by the factor α (not well-determined) in Eqn (5).

The 'usual' matter of the Universe, of which the Earth (and everything on it), other planets and stars are made, consists of baryons (protons, neutrons) and electrons, whose number is equal to that of protons. As for the corresponding antiparticles, i.e. anti-baryons (anti-protons, anti-neutrons) and positrons, a tiny quantity of them are present due to secondary processes of particle and antiparticle creation during high-energy particle collisions, for example in cosmic rays or in powerful accelerators.

The numerical measure of such a charge asymmetry can be determined using modern values of the baryon density and the relic radiation energy density. The point is that in the hot early Universe, at a high temperature exceeding the rest mass of particles, baryons and anti-baryons (more precisely, the quarks and anti-quarks they consists of) had to be present in almost equal amounts; the concentration (number density per unit volume) of both would have to nearly exactly coincide with that of relic photons. Later on, after baryons and anti-baryons annihilated, the excess of particles over antiparticles survived and yielded the baryon density presently observed. As the number of relic photons in the co-moving volume has not (virtually) changed since then, the present ratio of baryonic to relic photon concentrations gives the ratio of the excess number of baryons to their initial total number. At the present baryon density $\rho_B \simeq 2 \times 10^{-31}$ g cm^{-3} their number density is $n_B \sim 10^{-7}$ per cubic centimeter. In this way we obtain the quantity $B = n_B/n_\gamma \sim 10^{-10}$ called the cosmic baryon number, which measures both the present and original charge asymmetry of the world with respect to baryons and anti-baryons. It is this small dimensionless cosmic number that ensued surviving of the usual matter in the early evolving Universe and its existence in the present world.

Two treatments of such an overwhelming excess of particles over antiparticles are principally viable. This excess can be assumed to exist from the very beginning, i.e. that the Universe was born already being strongly asymmetric relative to particles and antiparticles. This point of view was actively debated in the beginning of the 1960s, and sometimes one assumed that the primordial number excess of particles over antiparticles could be one of the fundamental constants of nature with the same status as, say, the Planck constant.

Another approach to the charge asymmetry problem assumes that the value B characterizing this asymmetry has not such a fundamental nature and in fact must be 'derived' from more general physical laws. It is assumed in this case that the Universe could be strictly symmetric in baryons and anti-baryons from the very beginning, and the baryonic excess appeared in the early Universe in some evolutionary way. Such a viewpoint was first put forward more than 30 years ago in papers [55, 56]. The appearance of the baryonic charge in the Universe, or, as one says, baryogenesis, requires, according to Ref. [55], some conditions to be fulfilled, the principal of which being the instability of the proton. C- and CP-invariance (i.e. the symmetry between particles and antiparticles C and combined charge C and parity P symmetry) should also be broken. In addition, the non-stationarity of the world, a rapid cosmological expansion, should be provided in order that the interactions of primordial particles of the cosmic medium occurred in non-equilibrium conditions. While the latter condition seemed natural in the early Universe, the former, the proton non-stability, appeared to be an extremely brave (at that time) hypothesis. But it was this hypothesis that made, as we now know, an evolutionary approach to this problem possible and fruitful.

By the end of the 1970s it became known that proton decay is one of the consequences of the Grand Unification notion, which assumes the unified nature of strong, weak, and electromagnetic interactions (as is well known, Einstein's dream was to elaborate a unified theory of all four interactions, including gravitation). We already noted in the previous paragraph the characteristic proton decay time; it must be extremely large, not less than $10^{31} - 10^{32}$ years. As for C- and CP-invariance breaking, this phenomenon is directly observed in decays of K^0 and anti-K^0 mesons. So the

combination of the most recent ideas and experimental facts of elementary particle physics with cosmology definitely indicates that all three necessary conditions for baryogenesis in the early Universe can really be satisfied.

Specific versions of the cosmological baryogenesis discussed in the last two decades studied the possibility of this process in a very early epoch, when the cosmic matter temperature was close to the characteristic Grand Unification temperature $M_{\rm GUT} \sim 10^{14}$ GeV $\sim 10^{-5} M_{\rm Pl}$ and the age of the world was $t \sim 10^{-34}$ s. Another variant suggests the baryogenesis at much lower temperatures, typical for electroweak processes $M_{\rm EW} \sim 10^3$ GeV $\sim 10^{-16} M_{\rm Pl}$, when the age of the Universe was $t \sim 10^{-12}$ s (see Refs [57–60] for recent review).

Here we would like to cite book [9]: "The scenario of baryogenesis is one of the great triumphs of unification of particle physics with cosmology..." (p. 158). We should only add that 'deriving' the cosmic baryonic number $B \sim 10^{-10}$ from fundamental theory still remains an unsolved problem. It has been successfully shown that it is possible in principle and the number itself lies within natural limits for this theory; but one obtains it only using special models and not from first principles of physics. But maybe this number is in fact not so basic to take a certain value in the unique Universe we know?

Now some words about dark matter. Its nature, as was said in Section 2, is now totally unknown, and the spectrum of discussed possibilities extends from hypothetical elementary particles to dwarf stars or massive black holes. It is by itself so much an obvious gap in our present knowledge of the Universe that nothing is likely to be added. But we do say something positive in this regard. In the spirit of ideas similar to the baryogenesis picture, one may assume that the nature of dark matter is somehow linked to non-equilibrium processes in the cosmic medium at the early stages of expansion of the Universe. In the just-cited book [9] and books [6, 8] as well, for example, the kinetics of freezing of particles and antiparticles in the cosmic medium is described in detail. Let the mean annihilation cross-section be σ, the mean number density of particles be n and their velocity be v. Then the characteristic annihilation time is

$$\tau = (\sigma v n)^{-1} . \tag{24}$$

The time τ (not to be mixed with time in the de Sitter solution above, which is noted by the same letter!) from the very beginning could be small in comparison with the cosmological age t, and then this time could become larger than the cosmological age t. Then particle-antiparticle annihilation would stop and they would be preserved in the Universe.

This was exactly the case with the neutrino and antineutrino (see above), but it is unlikely that neutrinos play the role of dark matter particles due to the smallness of their masses. However, the same could occur with sufficiently massive dark matter particles, assuming a dark matter consisting of some elementary particles and antiparticles. Such particles should not emit and interact with electromagnetic radiation. Like neutrinos, they could only participate in weak (and of course gravitational) interactions. Given appropriate masses, they could provide the observed density of dark matter. One such model was discussed in a recent paper [61], in which the assumption was made that the freezing of dark matter particles occurred at temperatures close to the characteristic electroweak temperature $M_{\rm EW} \sim 1$ TeV. Paper [61] emphasizes the key role of electroweak processes both in the physics of elementary particles and in the cosmology of the early Universe (see also the aforementioned reviews [57–60]).

Finally, let us address (although with not so much detail as this point actually deserves) the nature of cosmic vacuum and first of all the origin of its observed density. A small but non-zero value of the vacuum density has always been considered as a principal difficulty of any fundamental physical theory [3, 4, 60]. Expressed through the Planck mass, the vacuum density found in astronomical observations is

$$\rho_{\rm V} \simeq 2 \times 10^{-123} M_{\rm Pl}^4 . \tag{25}$$

Here we are dealing not only with vacuum in cosmology, but also with vacuum in the microworld — as we already said, this is one and the same physical object. In early days of relativistic quantum theory, when the question of the nature of Dirac's sea, of its energy infinity first arose, G A Gamow said that Dirac's vacuum must manifest itself via gravity [8]. In 1960–1970s, at seminars and especially in the corridors of the theoretical department of Leningrad Physico-Technical Institute, this was one of the permanent themes of lively discussions, frequently with references to Gamow, who was once a participant of such discussions inside the same walls. Is the vacuum density infinite? But then the space curvature must be infinitely large. Setting the space curvature radius above the distance to the horizon yields an upper limit to the vacuum density. And the joke following Ya I Pomeranchuk was: Vacuum is not empty, it is filled with a deep physical meaning...

Meantime, beyond the ocean the question on the numerical value of the vacuum density was dubbed a 'problem of naturalness in theoretical physics' [4]. It was thought that such a basic quantity like vacuum density should be expected to take some very pronounced value and two variants were discussed: either zero, or the Planck density $\rho_{\rm Pl} \sim M_{\rm Pl}^4$. But with the huge Planckian density the space curvature should be extraordinary large; this was inadmissible. And should the vacuum density be zero, this fact would never be proven experimentally.

The actual density measured due to the gravitational effect of vacuum (what Gamow had in mind, though apparently he never spoke of antigravity and accelerated expansion) is now known. It is in a good agreement with considerations on the upper limit that follows from the lower bounds of the space curvature radius. And in the spirit of the previous arguments of naturalness, this density should appear unnaturally small compared with the Planckian density — one hundred twenty three orders of magnitude smaller. And infinitely large with respect to zero.

What other fundamental quantities could be compared with vacuum density? If, instead of the Planck mass, one takes the mass corresponding to the Grand Unification, $M_{\rm GUT} \sim 10^{-5} M_{\rm Pl}$, the density $\rho_{\rm GUT} \sim M_{\rm GUT}^4 \sim 10^{-20} M_{\rm Pl}^4$ is obtained. This value is one hundred orders of magnitude higher than the observed vacuum density value. If, further, one accepts the energy scale of weak interactions, $M_{\rm EW} \sim 10^{-16} M_{\rm Pl}$, the density would be $\rho_{\rm EW} \sim M_{\rm EW}^4 \sim 10^{-64} M_{\rm Pl}^4$; the gap is again very large. Of fundamental energy scales the quark–hadron scale $M_{\rm QH} \sim 10^{-19} M_{\rm Pl}$ remains, but it, too, gives an unacceptably high density $\rho_{\rm QH} \sim M_{\rm QH}^4 \sim 10^{-76} M_{\rm Pl}^4$. Apparently, microphysics suggests no appropriate scale.

But let us address the principal aspect of the problem. Where does vacuum energy come from at all? According to

one of the basic results of quantum mechanics that follows from Heisenberg's uncertainty principle, the lowest energy of a quantum oscillator is not zero, it is $\omega/2$. These are the so-called 'zero oscillations' that provide the non-zero energy of the lowest energy state of quantum fields. Such is the principal answer to the question [8]. But quantum theory does not allow one to really calculate the corresponding total energy density associated with zero oscillations. If one considers an ensemble of quantum oscillators as a model for a physical field, then summing up the energy of all zero oscillations over all available frequencies up to infinity would yield an infinite energy and an infinite vacuum energy density.

To avoid such divergences, one usually bounds the frequency range from above at some limiting value. For example, the limiting frequency can be taken to be corresponding to the Planck energy $M_{\rm Pl}$, so that $\omega_{\max} \sim M_{\rm Pl}$. Such a choice is favored by the reliable fact that at energies higher than the Planckian one the conventional notions of physics, including the very notion of frequency, lose their usual sense. But the vacuum density obtained in this way (as one may see, for example, from dimensionality considerations) is of the order of the fourth power of frequency and, hence, should be equal to $\sim \rho_{\rm Pl}$, which, as we already noted, differs from the reality by more than one hundred orders of magnitude.

Given the actual value of the vacuum density from cosmology, one needs to choose the limiting frequency and the corresponding energy scale $M_{\rm V}$ at the level

$$\omega_{\rm V} \sim M_{\rm V} \sim 10^{-31} M_{\rm Pl}\,. \tag{26}$$

The quantity $M_{\rm V}$ is 12 orders of magnitude smaller than the quark – hadron energy $M_{\rm QH}$, which implies that the physics of cosmic vacuum is determined not by ultra-high but instead by ultra-low energy processes. The limiting frequency (26) corresponds to the wavelength $\lambda_{\rm V} \sim M_{\rm V}^{-1}$, which is approximately 1 mm. This length is very large compared with the characteristic microphysics scales $\sim 10^{-13} - 10^{-15}$ cm and very small compared with Friedmann's cosmic integrals $\sim 10^{28} - 10^{26}$ cm (but, strange to say, is very close to our usual human scales!).

To find some commensurable scale for frequency (26) (and with it the characteristic length $\lambda_{\rm V}$), let us address the physics of the early Universe. The cosmological expansion rate $\sim t^{-1}$ (which has the same dimensionality as frequency) is given by the standard relation $t^{-1} \sim (G\rho)^{1/2}$, where the density should be taken as the relativistic energy density which dominates in the Universe during the first million years of its expansion: $\rho = \rho_{\rm R} \sim T^4$. Then $t^{-1} \sim T^2/M_{\rm Pl}$ GeV (here we used the well-known relation between the Newtonian gravity constant and the Planck mass in 'microphysics' units; T is the temperature of the medium). The expansion rate matches the characteristic limiting frequency at $T \simeq 3 \times 10^{-16} M_{\rm Pl} \sim 10^3$ GeV. But the latter value is very near the energy scale of weak interactions $M_{\rm EW}$. This then entails that the limiting frequency $\omega_{\rm V}$ is numerically close to the value of combination $M_{\rm EW}^2/M_{\rm Pl}$. If so, the vacuum energy density is

$$\rho_{\rm V} \sim \omega_{\rm V}^4 \sim \left(\frac{M_{\rm EW}}{M_{\rm Pl}}\right)^8 M_{\rm Pl}^4\,. \tag{27}$$

The numerical value of this combination of two fundamental energy scales, naturally, is in order of magnitude close to the observed value (25).

It is interesting that in the already mentioned paper [61] expression (27) was obtained from a complicated field-theoretic model (though not without invoking rather arbitrary and very strong additional assumptions). But if relation (27) indeed has a deep essential sense, the nature of vacuum must be somehow linked to the physics of electroweak processes in the early Universe at an age of $t \sim 10^{-12}$ s. At that time the current event horizon was close the characteristic wavelength $\lambda_{\rm V} \sim 1$ mm (which again is amazingly coincident, for example, with the thickness of the sheet of paper on which the present text is printed). Perhaps, it is in this epoch, when cosmic temperature has dropped to electroweak energies, that the last time jump in the initial vacuum state (phase transition) occurred, which determined the present vacuum density value.

We stress once again that relation (27), though it would seem attractive, can, of course, neither be taken as rigorously proved, nor as ultimate. Undoubtedly, the possibilities of other approaches to the problem are far from being exhausted. For example, there is a point of view according to which a vacuum of fermions and a vacuum of bosons have opposite signs of energy, so that the total value of vacuum density could be non-zero, provided that there is a strict symmetry between fermionic and bosonic states (called supersymmetry). And this symmetry being not precise would result in a tiny, but limited value of the vacuum energy density (see Ref. [8]). The existence of a scalar field imitating a vacuum equation of state with negative energy [62] could also diminish the vacuum density from Planckian values to the actually observed one. Yet this idea, too, is thought not to lead to a consistent and direct solution of the problem. The possible reasons for difficulties and failures in this way are discussed in Refs [4, 60]. Searches for new approaches are continuing, and of the most recent papers we just note model [63] involving three contributions to the vacuum density: from zero oscillations, from space curvature (low energies), and from neutral boson condensate (high energies). Many attempts have also been made to obtain a reasonable value of the vacuum density in the framework of the brane theory (see [64] and references therein).

13. Freeze-out model

As seen from above, the origin of both vacuum and non-vacuum forms of energy remains in fact unknown. Under such circumstances the revealing the nature of Friedmann's integrals (22) must appear hopeless. This is indeed the case. But nevertheless one can try to say something positive about the case, not pretending to much. Certainly, in this way we cannot avoid invoking some additional assumptions, more or less likely or at least not contradicting each other and reliably established facts; the derivation of the integrals from the first principles remains an important task for the future, most likely, very remote.

Let us speak briefly about one model [52], which is capable of showing how — at least in principle — the coincidence of the integrals in the early Universe could arise. Note from the very beginning that this model is essentially incomplete; it is by no means concerned with the difficult question on the nature of baryonic charge, so Friedmann's integral for baryons in this particular model is absent altogether. As for dark matter, the model assumes that it consists of elementary particles with a non-zero rest mass m, which participate only in weak interactions and, of course, gravitate. Admitting the

possibility mentioned in the previous section, we assume that these particles exist now together with corresponding antiparticles, and the annihilation does not occur due to weakness of the particle-antiparticle interactions and their low density at present.

In fact, the annihilation could stop at some very early time when the cosmic matter temperature dropped to $T \sim m$ (we use, as above, the units in which $k = c = \hbar = 1$) and the characteristic annihilation time scale [Eqn (24) with $v \sim c = 1$] became longer than the age of the Universe, i.e. the annihilation rate became lower than or at least comparable with the cosmological expansion rate:

$$n\sigma \sim \frac{1}{t}\,. \tag{28}$$

Here n, as well as above, is the number density of particles, $\sigma \sim m^{-2}$ is their annihilation cross-section, t is the age of the Universe at that early time. As this was undoubtedly the radiation-dominated epoch, one can use the standard cosmological formula $t \sim (G\rho_{\mathrm{R}})^{-1/2}$ for early times. Besides, we use the definition of Friedmann's integrals (12) to introduce integrals A_{D} (with density $\rho_{\mathrm{D}} \sim mn$) and A_{R} into the model's equations. Then Eqn. (28) takes the form:

$$A_{\mathrm{D}} \sim am^3 A_{\mathrm{R}} M_{\mathrm{Pl}}^{-2}\,. \tag{29}$$

Also we take into account that $\rho_{\mathrm{R}} \sim T^4 \sim m^4$ at that time; this allows us to write

$$A_{\mathrm{R}} \sim a^2 m^2 M_{\mathrm{Pl}}^{-1}\,. \tag{30}$$

In Eqns (28), (29) $a \simeq A_{\mathrm{V}}(1+z)^{-1}$ is the curvature radius [or the normalized scale factor, as in Eqn (14)] at the time t; z is the corresponding redshift.

The kinetics described by these phenomenological equations must be supplemented with the physics responsible for the interaction of cosmic matter particles at the present time. In the spirit of the previous paragraph, we shall consider that the physics of electroweak interactions with the energy scale $M_{\mathrm{EW}} \sim 1$ TeV must play the central role in the process of freezing. Moreover, we accept correspondingly that the vacuum energy density is given by Eqn. (27) that contains two fundamental energy scales, M_{Pl} and M_{EW}.

According to general formula (12), the integral for vacuum with density (27) is

$$A_{\mathrm{V}} \sim \left(\frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\right)^4 M_{\mathrm{Pl}}^{-1}\,. \tag{31}$$

Making similar considerations, we need to assume that the redshift at the present time must be expressed in this case through a simple combination of the same two fundamental constants:

$$z \sim \frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\,. \tag{32}$$

In addition, to be consecutive, we need to assume that only these two fundamental constants M_{Pl} and M_{EW} enter the problem, so that in the equations of the kinetic model, the mass of a particle m must be identified with M_{EW}. Then we shall have

$$A_{\mathrm{M}} \sim A_{\mathrm{V}} A_{\mathrm{R}} \left(\frac{M_{\mathrm{EW}}}{M_{\mathrm{Pl}}}\right)^4 M_{\mathrm{Pl}}\,, \tag{33}$$

$$A_{\mathrm{R}} \sim A_{\mathrm{V}}^2 \left(\frac{M_{\mathrm{EW}}}{M_{\mathrm{Pl}}}\right)^4 M_{\mathrm{Pl}}\,. \tag{34}$$

Basically, this is already a solution to the problem since A_{V} is known and we only need to write the answer explicitly:

$$A_{\mathrm{D}} \sim A_{\mathrm{R}} \sim A_{\mathrm{V}} \sim \left(\frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\right)^4 M_{\mathrm{Pl}}^{-1}\,. \tag{35}$$

Thus the approximate equality of integrals turns out to be a corollary of physical processes that took place in the early Universe at the electroweak interaction energy scale.

To find the numerical value of integrals in (35), one detail is worth taking into account. Namely, in paper [61] and another recent paper [65], a 'gravitational' scale, or 'reduced' Planck scale, $\bar{M}_{\mathrm{Pl}}$, is introduced such that $\bar{M}_{\mathrm{Pl}} \simeq gM_{\mathrm{Pl}}$, where $g \simeq 0.1-0.3$. The factor g accounts for the fact that the Newtonian gravity constant $G = M_{\mathrm{Pl}}^{-2}$ enters most cosmological relations in combinations like $8\pi G$, $6\pi G$, or $32\pi G/3$. (in the same way one may take into account the fact that formulas of the kinetic model should actually have such dimensionless factors as the number of degrees of freedom etc.; see again textbooks [6–9]). If we follow this example, result (35) can be represented as

$$A_w \sim g^4 \left(\frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\right)^4 M_{\mathrm{Pl}}^{-1} \sim 10^{61\pm1} M_{\mathrm{Pl}}^{-1}\,, \qquad w = \left[-1, 0, \frac{1}{3}\right]. \tag{36}$$

The numerical agreement with the empirically found relationship (22) is quite good. With the same degree of accuracy

$$\rho_{\mathrm{V}} \sim g^8 \left(\frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\right)^8 M_{\mathrm{Pl}}^4 \sim 10^{-122\pm2} M_{\mathrm{Pl}}^4\,. \tag{37}$$

This estimate should be compared with the observed value (25).

Finally, we note that the corresponding formula for the redshift takes the form $z \sim gM_{\mathrm{G}}/M_{\mathrm{EW}} \sim 10^{15}$. At such a z the temperature in the stage considered should be $T \sim 1$ TeV $\sim M_{\mathrm{EW}}$, which once again directly points to the crucial role of electroweak interactions in the process.

We recall that Friedmann's integral for baryons remains outside the scope of this model. Meanwhile its nature apparently also might be related to the physics of weak interactions in the early Universe. As we mentioned in the previous section, the necessary conditions for baryogenesis can be provided by the physics of weak interactions. These conditions can be recovered during decays of original massive bosons, if they do not respect the baryonic charge conservation. Expressed via cosmic baryon number B, Friedmann's integral for baryons must then have the form:

$$A_{\mathrm{B}} \sim B\,\frac{M_{\mathrm{QH}}}{M_{\mathrm{Pl}}}\,A_{\mathrm{R}}(A_{\mathrm{R}}M_{\mathrm{Pl}})^{1/2} \simeq g^6 B\,\frac{M_{\mathrm{QH}}}{M_{\mathrm{Pl}}}\left(\frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\right)^6 M_{\mathrm{Pl}}^{-4}\,. \tag{38}$$

The latter quantity differs from the empirical value A_{B} in Eqn (22) by 4–6 orders of magnitude (which is less than 10% in logarithmic scale).

14. Symmetry of cosmic energies

Let us return to the kinetic model described above for three Friedmann integrals and pay attention to the fact that

equations (29), (30) with account of (32) can be represented in a symmetric form:

$$\frac{A_{\mathrm{D}}}{A_{\mathrm{R}}} \sim \frac{A_{\mathrm{R}}}{A_{\mathrm{V}}} \sim \frac{A_{\mathrm{V}}}{A_0}\,, \tag{39}$$

where

$$A_0 \sim \left(\frac{M_{\mathrm{Pl}}}{M_{\mathrm{EW}}}\right)^4 M_{\mathrm{Pl}}^{-1}\,. \tag{40}$$

We emphasize that in the new formulation (39) the value of A_0 is not introduced into the model from the outside; this combination of two basic energetic scales appears in the model on its own provided that, as one assumes, only these two fundamental scales are present in the problem. The system of Eqns (39), (40) represents two equations for three unknowns, which are the three Friedmann integrals. For the system to have a unique solution, it must be fully determined, and namely for this purpose an additional relationship (27) taken from Section 12 was used.

But the same system of equations can be added in another way, for example, by assuming, with reference to empirical relationship (22), that $A_{\mathrm{R}} \sim A_{\mathrm{D}}$. Then an expression for the vacuum density can be derived from the model, which exactly coincides with Eqn (27). Thus, the empirical knowledge of the values of two Friedmann integrals allows us to prove — in the framework of the present model — the equality of all three integrals and to find their value as a combination of fundamental energy scales. In other terms, in the framework of the kinetic model with electroweak processes, the cosmic vacuum density is 'derived' in this way from observational data on dark matter and relativistic energy.

These considerations, but mainly and most importantly original empiric data (22), permit us to see the place of vacuum in the entire range of cosmic energies in a new light. When we spoke before about the proximity of the density of vacuum to that of other energy forms, we had to compare a constant (vacuum density) with variable values, which are the densities of non-vacuum energy forms. Clearly, the coincidence of values that differently depend on time or are time-independent altogether, can be only temporary and, in this sense, accidental. And if one takes not the densities but the corresponding Friedmann integrals, three new constants of the same cosmological origin appear in addition to the vacuum constant, and all four values are given by the same formula (12). In this way the possibility emerges to compare vacuum with other forms of cosmic energy using the language of constant quantities of one mathematical and physical nature. And then all four constants turn out to be numerically close to each other.

In the context of Friedmann's integrals, the observed value of vacuum density proves to be quite natural (recall the naturalness problem in theoretical physics [4]) since it is this value that ensues the approximate equality of the vacuum integral to that for other energy forms. Should this density be zero, the vacuum integral would be infinite; if the density were about the Planckian value, the integral would be extraordinarily small. And this would really seem unnatural.

The close coincidence of Friedmann's integrals suggests a certain kind of symmetry unifying the known forms of cosmic energy and putting them in correspondence with each other. This symmetry has not a temporary or an accidental character. It is linked to the very physical nature of cosmic energies, to their origin, and persists as long as these energy forms are present in the Universe.

Therefore, the two sharp problems of cosmology and fundamental physics — the problem of cosmic coincidences and the problem of naturalness — appear to be tightly related to each other and are reduced together to a more general problem of symmetry of cosmic energies: Why is $A_{\mathrm{V}} \sim A_{\mathrm{D}} \sim A_{\mathrm{D}} \sim A_{\mathrm{R}}$? The freeze-out model considered in the previous section sketches a preliminary possible approach to looking for the answer to this question.

Revealing a new type of symmetry in cosmology, the symmetry between vacuum and non-vacuum forms of cosmic energy in the space of Friedmann's integrals, is one of the consequences of the newest revolution in the science of the Universe.

15. Conclusion

The cosmological revolution of our day is a big jump in the accumulation of specific knowledge about nature, in understanding its physical laws. It has led to drastic changes in the physical picture of the world, to a quite novel understanding of what the contemporary Universe is. Cosmic vacuum with its antigravity turned out to prevail in the Universe, which makes the galaxies recede with an acceleration from each other. But neither the galaxies, nor their own antigravity nor even time itself are incapable of affecting the present cosmic vacuum, it remains absolutely at rest, it is unchangeable and permanent. We suddenly realized that we inhabit a four-dimensional space-time that has recently completed its cosmic evolution and has now virtually attained an ideal, regular, geometrically symmetric state which will continue forever.

The world of cosmic vacuum comes near to the close surroundings of our living area in the Universe — to the Local Group of galaxies, where our Galaxy, the Milky Way, neighbors on the Andromeda Nebula. Controlled by cosmic vacuum antigravity, the Hubble flow of receding galaxies starts from distances of only a few Megaparsecs away from us and continues towards the horizon of observations at distances of thousands of Megaparsecs, everywhere keeping its kinematic identity, the same rate of motion, which is given by vacuum with its constant density in space and time.

The expansion flow is likely to be due to the primordial vacuum of the Universe, whose density was extremely high; it exceeded the presently observed value by many orders of magnitude. Due to this fact vacuum itself was able to change, evolve, by generating in this way the 'ordinary' matter, non-vacuum forms of cosmic energy. This evolution of the primordial vacuum was completed during the first small fractions of a second of the existence of the Universe, leaving after itself a multi-component cosmic medium. In this medium, vacuum and non-vacuum cosmic energies ultimately proved to be consistent, symmetric with respect to each other. Their time-independent genuine physical characteristics, Friedmann's integrals, have since then been numerically close to each other and will remain close while these energy forms themselves remain in nature.

For many years, now perhaps since the discovery of the cosmic microwave background in 1965, cosmological science has not had such a fresh renewal, and its relation with fundamental physics has not been revealed in such a direct and obvious way. And again, as then, the new way of development was initiated by a remarkable discovery due to

the long-term systematic efforts and experimental skill of astronomer-observers.

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