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Chernogolovka 2000: Mesoscopic and strongly correlated electron systems [1]

The current state of quantum mesoscopics

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1. Introduction

The first International Conference on "Mesoscopic and Strongly Correlated Electron Systems" was held in Chernogolovka in the summer of 1997. The proceedings of this conference were published in the special issue of *Physics – Uspekhi*, February 1998 [1] (http://www.ufn.ru). The idea to bring together experts in such different (originally) branches of physics as semiconductors, disordered metals and superconductors, was found to be extremely timely and productive, and it was decided to continue a regular chain of such conferences, with a 3-year interval. The second 'Meso' conference in Chernogolovka was held in July of 2000, and its proceedings were published in the supplement to *Usp. Fiz. Nauk* [2]. The general features that characterize both the newly emerging field of physics called 'Quantum mesoscopics', and the style of the conference, were described in "Mesoscopic unification", an introductory article to the *Chernogolovka 97* proceedings [1]. Below we provide a review of the main branches of quantum mesoscopics presented at *Chernogolovka 2000*.

For a start, we briefly outline the domain of quantum mesoscopics. Consider a submicron-size system of $10^3 - 10^9$ electrons, appearing in the form of a metal, semiconductor or even insulator. Such a system cannot be treated by usual means of quantum mechanics of a few particles: although the Schrödinger equation for the total multiparticle wave function can be written, it is of little use usually, since there is no way to solve it, even numerically. On the other hand, the powerful methods of many-body statistical mechanics are also not appropriate for such a system, since the fluctuations of its global quantities may be comparable with the expectation values. In other terms, quantum mesoscopics deals with systems which are quite large in comparison with atoms and most molecules, but still too small to neglect the specifically quantum properties of each of its electrons, as is commonly done in 'macroscopic' solid state physics. Apart from their purely academic interest, such systems appear as a possible bridge between nano-electronics for computations and telecommunications, and molecular biology (DNA and proteins are typically mesoscopic objects), with the most far-reaching of the foreseen applications being the quantum computer. Below we provide a brief account of the results presented at *Meso-2000*.

[1] Proceedings of this conference are available at: http://www.turpion.org/specials/pu/proceedings/2001/10/index.phtml

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2. Nanoscale devices for quantum computing

We begin our review from the last in the list of main subjects of the conference, which has shown the most remarkable progress during recent years. It is suffice to say that till summer 1997 not even a purely theoretical scheme for the nanoscale implementation of qubits had been published. The upsurge of interest started one year later, when several theoretical papers appeared, with proposals to use single-electron transistors (the Coulomb blockade effect) for qubit implementation [3], as well as different types of frustrated Josephson arrays [4, 5], electron spins of quantum dots [6] and donor impurities in semiconductors [7] (for an earlier state-of-the-art report see *Physics – Uspekhi* [8]). Experimental progress followed immediately, at least along the first two directions mentioned: the first experimental demonstration of coherent quantum manipulation with a solid-state device was reported by Nakamura, Pashkin and Tsai [9] from NEC Research (Tsukuba), who used a single-Cooper-pair box (or superconductive single-electron transistor). The next two quantum-coherent devices belonged to the Josephson-array family and came from Stony Brook [10] and Delft [11].

The results of the last mentioned experiment [11] were presented at *Chernogolovka 2000* by F K Wilhelm from the Delft Technical University (published in the proceedings of *Chernogolovka 2000* [2], p. 117). The idea of this experiment was to use a loop of three small Josepshon junctions with the Josephson coupling energy E_J being large compared to the charging energy $E_C = e^2/2C$ (where C is the junction's capacitance); at the same time the junctions are weak in the sense that the self-induced magnetic flux in the loop is very small, $2\pi\mathcal{L}I_c \sim 10^{-3}\Phi_0$. Being put into an external magnetic field producing flux $\Phi \approx \Phi_0/2$, the loop appears to be a bistable nearly degenerate system, the two stationary states corresponding to opposite directions of circular current flow. The difference of energies between these two states is $\delta E \sim I_c/c\,(\Phi - \Phi_0/2)$. The charging energy, although small, still produces a nonzero amplitude Δ of *quantum tunneling* between the above two classical states of opposite current directions, $\Delta \sim \sqrt{E_J E_C}\exp(-a\sqrt{E_J/E_C})$, with $a \sim 1$. The main outcome of the reported experiment was the demonstration of resonant microwave absorption by the quantum 3-junction loop at the flux-modulated frequency $f_0(\Phi)$. The behavior of $f_0(\Phi)$ follows theoretical predictions and allows one to extract the tunneling frequency $\Delta/\hbar \sim 600$ MHz.

The general approach to the problem of quantum measurement of a qubit state, with particular emphasis on the implementation of a qubit using a single-charge box device, was presented in the report by Yu G Makhlin (University of Karlsruhe and the L D Landau Institute for Theoretical Physics, Moscow). The interaction between the qubit and the measuring device is treated in detail by means of a master equation for the reduced density matrix. Two kinds of detectors are considered: a single-electron transistor (SET) and a quantum point contact (QPC), both capacitively coupled to the qubit. The measurement process is characterized by three time scales. On the shortest scale, defined by the dephasing time τ_φ, the phase coherence between the two eigenstates of the qubit is lost, while their occupation probabilities remain unchanged. Later measurement-induced transitions mix the eigenstates, changing their occupation probabilities on a time scale $t_{\mathrm{mix}} > \tau_\varphi$ and erasing information on the initial state of the qubit. The origin of the mixing is that the charge operator (the measured quantity) and the qubit's Hamiltonian do not commute. The third scale t_{meas} provides an estimate of the minimal time needed for the given measurement to discriminate with a high probability between two possible states of the qubit. In the setup considered, a quantity which is measured straightforwardly is the number m of electrons passed through the SET or QPC during a given time interval t. The measurement is effective if $t_{\mathrm{meas}} \ll t_{\mathrm{mix}}$, i.e. the occupation of the qubit's states does not change during the measurement procedure. Makhlin presented the results of calculations of t_{meas} and t_{mix}, as well as for the probability distribution of transmitted charge $P(m, t)$ for both measuring setups. The results of the reported studies are published in Ref. [12].

The review of theoretical developments in the field of 'spintronics' (manipulations with electron spins) by the Basel group was presented by E V Sukhorukov and G Burkard, see Ref. [2], p. 126. First, the architecture of a quantum computer with qubits as electron spins in quantum dots (QD) was described. The main idea is to use the sensitivity of electron exchange amplitudes between two neighboring QD to the value of the electric gate potentials, in order to manipulate the spin-exchange Hamiltonian. Next, the measuring problems were addressed. It was shown that spin-entangled electron states (singlet and triplet) can be identified via different values of the Fano factor (noise-to-current ratio): the noise is increased for singlets and decreased for triplets, in comparison with its magnitude for uncorrelated electrons. The sensitivity of the above results to the presence of Fermi-liquid interaction inside QD was studied; it was shown that the fidelity of entangled single and triplet states is reduced by z_F^2, where z_F is the Fermi-liquid renormalization factor.

The report by A V Shytov [13] (L D Landau Institute) about the solution of the Landau – Zener transition problem in the presence of an interaction with environment is evidently related to any nanoscale realization of qubits. It deals with a generic problem of nonadiabatic transitions in a two-level system described by a time-dependent Hamiltonian and coupled to uncontrollable microscopic excitations from the thermal bath. It was shown that at zero temperature the coupling to the environment always suppresses the Landau – Zener probability P_{LZ}, whereas at relatively high temperatures the effect of coupling is non-monotonic: relatively weak coupling enhances P_{LZ}, but strong coupling suppresses it. A specific example of mesoscopic two-level system to be used as a quantum interferometer was studied by Yu M Gal'perin (University of Oslo and A F Ioffe Physical-Technical Institute, St. Petersburg) et al., who considered a single-channel superconductive quantum-point contact (SQPC) subject to superconductive phase difference ϕ and a microwave e.-m. field. There are two current-carrying Andreev levels localized in the vicinity of the SQPC, with the interlevel spacing proportional to $[1 - D\sin^2\phi/2]^{1/2}$ (where D is the contact's transparency). The present report was devoted to the problem of decoherence of Andreev states due to (i) voltage noise and (ii) time-dependent variations of transparency D. These results are presented in Ref. [2], p. 121.

A completely new approach to the implementation of decoherence-free quantum computations with the use of unpaired Majorana fermions in quantum wires was presented by A Yu Kitaev (Microsoft Research and L D Landau Institute) and published in Ref. [2], p. 131. The very idea of physically fault-tolerant quantum computation is due to Kitaev [14], who proposes to look for some strongly correlated quantum system which could encode two (or more) quantum-mechanically degenerate states in a completely delocalized way, so that any physical perturbation acting on a small part of the system would not be able to perturb such an 'encoded qubit'. To realize this idea, one needs to find a system with a very peculiar ground state. It should not have any usual local order parameter (i.e. all standard symmetries of the Hamiltonian should not be broken in the ground state), but it should possess a spectral gap and so-called 'topological degeneracy' — that is, degeneracy related to global topological properties of the system. A physical example of such a system exists: it is Laughlin's fractional quantum Hall ground state. It was shown by X Wen and Q Niu [15] that Laughlin's state with filling factor $\nu = 1/2m + 1$ defined *on the surface of torus* is indeed $1/\nu$-fold degenerate. This property is robust with respect to any local perturbation of the Hamiltonian up to an accuracy of order $\exp(-L/L_{\mathrm{H}})$ where L is the system size and $L_{\mathrm{H}} = \sqrt{\Phi_0/H}$ is the magnetic length. Unfortunately, it is very difficult to fabricate a two-dimensional (2D) electron gas on a torus and then put it in a quantizing magnetic field. One of the most intriguing problems both for theory and for experiment is to find some realisable 2D example of topological degeneracy. A promising candidate could be a strongly frustrated quantum antiferromagnet in the resonant valence bond (RVB) state, proposed long ago by P W Anderson [16], and demonstrated by recent numerical studies [17]. However, Kitaev's report (see Ref. [2], p. 131) dealt with another approach to the realization of the same basic idea — now he proposes a model of a one-dimensional electron system able to split the low-energy degree of freedom of a physical electron into two real Majorana fermions, localized at opposite ends of the system. Once again, since any physical perturbation ought to be local in space, the probability that it affects the quantum state of such a 'Majorana pair' is very low. A physical realization of Kitaev's proposal could possibly be achieved with the use of superconductors with triplet pairing.

3. Mesoscopic superconductivity

This subject is a little bit older than the previous one; it was originated (experimentally) less than 10 years ago. Studies in this branch of mesoscopic physics provide a very important part of fundamental background for the engineering of nanoscale quantum-coherent devices. An example of such an interrelation is provided by at least three Conference reports. The first of them, the theoretical talk by G B Lesovik (L D Landau Institute), is devoted to the problem of spin entanglement of electrons in a normal metal near the NS interface: the mere fact that Cooper pairs are spin singlets leads to quantum-mechanical correlations (entanglement) between the spins of two electrons which left the superconductor but still retain phase coherence. Moreover, it was shown [18] that spin entanglement should lead to a novel drag effect (coined 'proximity drag'): if the S terminal is connected to two N terminals via spin selective normal wires (e.g. made of ferromagnets with opposite spin polarizations), passing current through one of these leads should lead to a current of the same direction in the other, even in the absence of any voltage drop between the second N terminal and the superconductor. In addition, a general discussion of the problems of measurement and of wave function collapse was given (see Ref. [2], p. 74).

The behavior of superconductor–ferromagnet (SF) hybrid structure was also at the heart of an experimental talk by V V Ryazanov (Institute of Solid State Physics, Chernogolovka), although in a completely different sense: the first realizations of SFS Josephson π-junctions were reported. Although theoretical prediction of the possible existence and behavior of SFS Josephson junctions was made quite long ago [19], all attempts to fabricate them failed till 1999, when the first experiments by Ryazanov's group were carried out. The key idea was to use diluted ferromagnet CuNi with a very low Curie temperature (about 20 K). The relatively weak exchange field in such an F interlayer made it possible, first of all, to obtain measurable Josephson critical currents through the F layer of thickness $d_{\rm F}$ about 20–30 nm. Detailed studies [20] confirmed expectations that in some interval of $d_{\rm F}$ the lowest-energy state of the junction is reached when the phase difference between the S terminals is equal to π (thus the term π-junction). Moreover, the same junction was shown to be able to switch from normal (0) to π-behavior as a function of temperature. The most direct proof of that was provided by the measurement of the critical current $I_{\rm c}(\Phi)$ through an intrinsically frustrated array of five π-junctions as a function of external magnetic flux Φ (see Ref. [2], p. 81).

The third report of this session, also related to attempts to realize quantum computing, was given by D A Ivanov (Eidgenössische Technische Hochschule, Zürich) [21]. He considered electron states localized inside the vortices of a p-wave superconductor. Contrary to the case of normal s-wave vortices, where lowest-energy localized states have low but nonzero energies, $\epsilon_n \sim (n+1/2)\Delta^2/E_{\rm F}$; a vortex in an ideal p-wave superconductor contains an electron level at exactly zero energy. In the specific realization of p-wave pairing with an order parameter of the same type as in the A-phase of ^{3}He, the situation is even more strange: a zero-energy state inside the vortex is *real*, i.e. it is represented by a Majorana fermion instead of a normal electron (cf. the subject of Kitaev's talk in Ref. [2], p. 131). It appears, further, that the ground state of a superconductor which contains some even number ($2k$) of such vortices is 2^k-fold degenerate! Vortex braiding generates a set of unitary transformations acting on this 2^k-fold subspace. Thus the system considered in this report can be seen as another approach to the realization of Kitaev's idea of fault-tolerant quantum computation. Unfortunately, superconductors with paring symmetry of the A-phase need to be discovered still; moreover, the physical realization of vortex braiding is not quite clear. Nevertheless, the very fact that topological stability can be realized in a relatively well-understood condensed-matter ground state such as the A-phase of ^{3}He seems to be very promising.

Strong experimental indications for the existence of intrinsic superconductivity in carbon nanotubes were presented in the report by A Yu Kasumov (Institute of Microelectronics Technology, Chernogolovka, and University of Orsay). Measurements of $I-V$ characteristics were performed on several devices consisting of single-wall nanotubes (SWNT) and ropes made of about 100 SWNTs suspended between Au–Pt bilayer electrodes. Out of several devices studied, the one with the longest nanotube showed a decrease of resistance by a factor of 100 at lowest temperatures (the effect was magnetic-field sensitive and disappeared at sufficiently high field). In the samples with shorter nanotubes the same effect was much less developed, which seems to be in agreement with proximity-effect suppression of intrinsic superconductivity of nanotubes by the normal-metal electrodes. These findings (see Ref. [2], p. 69, and Ref. [22]) are in agreement with earlier results of the same group, where an anomalously high critical Josephson current was measured in similar devices with superconductive Au–Ta electrodes: the measured values of $I_{\rm c}$ were a factor 40 higher than would follow from the standard Ambegaokar–Baratoff relation (even assuming the energy gap of the bilayer to be equal to its value for pure Ta). Needless to say, intrinsic superconductivity in a really one-dimensional quantum wire like SWNT is an extremely interesting object both theoretically and in terms of possible applications. The theory of NS transport via quantum point contacts of superconductors with 'unusual' conductors like gated 2D electron gas (2DEG) structures (or, possibly, SWNT as well) was presented by N M Chtchelkachev [23] (L D Landau Institute). General results for the critical current $I_{\rm c}$ dependence upon the gate potential and SN interface resistance were found for the case of ballistic transport across the N region. It was shown, in particular, that the Josepshon current $I(\phi)$ has a maximum for a phase difference π across such a contact; as a result, the value of $I_{\rm c}$ can be found from the consideration of the lowest positive Andreev level at $\phi = \pi$.

While it is generally accepted that electron transport via a normal metal–superconductor junction is due to the Andreev reflection process, and thus quantized in integers of $2e$, direct experimental proof of this fact was lacking till recently. In one of the first experiments of this kind (presented by A A Kozhevnikov, Yale University), the current and its fluctuations were measured on a mesoscopic structure made of diffusive normal metal and superconductor in series, in the presence of high-frequency rf radiation. Sharp features seen in the noise intensity at bias voltage

$V = hv/2e$ confirmed that the effective charge is $2e$, cf. Ref. [24]. The same kind of measurements were performed on the more complicated NS structure of Andreev interferometer type; here both current and noise were found to be modulated by a magnetic flux with periodicity $h/2e$ corresponding to the charge quantum $2e$.

Spatially resolved STM spectroscopy is known to be a powerful tool of microscopic study of inhomogeneous superconductive states. Two experimental reports at the conference dealt with this technique. In the first of them, given by I Maggio-Aprile (University of Geneva), a study of the vortex core in YBCO and BiSCCO high-temperature superconductors was presented [25]. Contrary to the case of the 'normal' superconductor $NbSe_2$ studied previously by the same technique, the number of localized electron levels inside the core was found to be just one in YBCO and even zero in BiSCCO, which is in sharp disagreement with the prediction based on any kind of BCS-like theory. The effective size of the vortex core in BiSCCO extracted from these data is considerably shorter than the usually accepted value of about 2 nm for the superconductive coherence length in this material. An immediate consequence of this unexpected observation is that the strength of vortex pinning due to small-scale compositional disorder should be much higher (at low temperatures) than was expected previously. The second talk, given by C Chapelier (CEA-Grenoble), dealt with local density of states (DOS) measurements in Au – Nb nanostructures prepared by UV lithography (see Ref. [2], p. 71, and Ref. [26]). The majority of the obtained data is in good agreement with the theory based upon the Usadel equation for inhomogeneous dirty superconductivity with a phenomenologically introduced electron decoherence time $t_{\rm dc}$. A fit to the data results in the rather short value of $t_{\rm dc} \approx 20$ ps for gold (which is actually in agreement with the outcome of the completely different experiment by the Saclay group [27]). The 'theoretical partner' of this last report was given by P M Ostrovsky (L D Landau Institute), who presented calculations of the single-particle density of states in the normal part of SN and SNS nanostructures *beyond* the standard Usadel-equation approach (which predicts the existence of a sharp energy gap $E_{\rm g}$ of order of the Thouless energy $E_{\rm Th}$). Namely, it was demonstrated by the field-theory instanton technique, that mesoscopic fluctuations lead to a finite (although exponentially decaying) DOS at energies $E < E_{\rm g}$. Physically, this is due to the presence of 'quasi-localized' [29] states in the N part of the structure.

General problems of *thermal* fluctuations in two-dimensional superconductors were discussed in the talk presented by A A Varlamov from INFM, Rome, and the Moscow Institute of Steel and Alloys (see Ref. [2], p. 94), where special attention was given to the 'ultra-clean' limit for superconductors. Dealing with superconductor electrodynamics in fluctuation regime it is necessary to remember that in the vicinity of the critical temperature the role of effective size of the fluctuational Cooper pair is played by the Ginzburg – Landau coherence length $\xi_{\rm GL}(T) = \xi_0\sqrt{T_{\rm c}/T - T_{\rm c}}$. Thus clean superconductors ($\ell \gg \xi_0$) in the vicinity of the transition could be formally subdivided into clean, which is still local ($\xi_0 \ll \ell \ll \xi_{\rm GL}(T)$), and ultra-clean nonlocal ($\xi_{\rm GL}(T) \ll \ell$) limits. The ultra-clean limit might be relevant, in particular, to high-temperature superconductors. As is well known, the first order fluctuation corrections to conductivity in the vicinity of the superconducting transition are presented by the Aslamazov – Larkin (AL), Maki – Thompson (MT) and (DOS) contributions. First one has the simple physical meaning of direct charge transfer by the fluctuation pairs themselves which can be easily derived from the phenomenological time-dependent Ginzburg – Landau equation, while the Maki – Thompson and DOS contributions have a purely quantum origin and can be calculated in the framework of the microscopic approach only. In the work presented by Varlamov, the theory of fluctuation conductivity was reexamined for an arbitrary impurity concentration including the ultra-clean limit. It was demonstrated that the formal divergency of the fluctuation DOS contribution obtained previously for the clean case is removed by the correct treatment of the nonlocal ballistic electron scattering: in the ultra-clean limit the density-of-states quantum corrections are canceled by the Maki – Thompson term and only classical paraconductivity remains.

The terra incognita of superconductive pairing effects in inhomogeneous systems and hybrid structures in presence of strong electron – electron interaction and/or strong disorder was approached in two theoretical and two experimental talks of this conference. A Frydman (Hebrew University of Jerusalem) presented very unusual experimental results on low-bias excess conductance in normal metal – Anderson insulator – superconductor junctions (specifically, the junctions Au – InO_x – Pb were studied). It was found [30] that the low-bias Andreev conductance $G_{\rm A}$ may greatly exceed *double* the value of the normal-state conductance $G_{\rm N}$ of the same structure (obtained by magnetic-field suppression of superconductivity in Pb). Theoretically, the maximum ratio $G_{\rm A}/G_{\rm N} = 2$ is attained in the case of ballistic transport in the N region; for a diffusive normal metal without interactions at low T and V, $G_{\rm A} = G_{\rm N}$. Among the data reported by Frydman, the majority of the 80 samples studied gave $G_{\rm A}/G_{\rm N}$ in the range between 2 and 4, while in a few cases even the ratio of 5 was exceeded. No theory of the superconductive proximity effect in Anderson insulators is presently available, and the results of the reported experiment present a very interesting challenge for theorists.

Much less surprise could be expected to come from the studies of well-defined SNS junctions reported by T I Baturina from the Institute of Semiconductor Physics, Novosibirsk (see Ref. [2], p. 91). A novel idea employed in these experiments was to use superconductive and normal regions made of the same material — a PtSi film of relatively low sheet resistance close to 100 Ω. The point is that the film of PtSi is superconductive with $T_{\rm c} \approx 0.56$ K, whereas narrow (submicron) constrictions (made of the same film with the use of electron lithography and subsequent plasma etching) were shown to be normal at all temperatures. This original technology made it possible to prepare nearly ideal SN junctions with very low interface resistance. The $I(V)$ characteristics of both individual SNS junctions (with the normal region length $L_{\rm N}$ much longer than thermal coherence length $\xi_{\rm T} = \sqrt{\hbar D/T}$) and of 2D arrays of few hundreds junctions were studied. Due to the condition $L_{\rm N} \gg \xi_{\rm T}$ no Josepshon current was measured; nevertheless, some completely unexpected coherence effects were manifestly found by comparison of the low-bias $I(V)$ anomalies in single junctions and arrays. Namely, single junctions demonstrate a relatively broad zero-bias resistance minimum (with a width of about 140 μV), whereas the same minimum is only 10 μV in a 2D array of the same kind of junctions. In addition, the 2D array demonstrated a very rich subharmonic energy gap structure, with sharp resistance minima at $eV = \pm 2\Delta/n$, for integer

$n = 2, 4, 5, 6, 8, 10, 16$. There is no theoretical explanation for these observations, in spite of the fact that everything seems to have been done in the validity range of standard 'dirty superconductivity' theory.

Meanwhile, the theoretical development reported by M A Skvortsov (L D Landau Institute) dealt with the effects of the repulsive electron–electron interaction upon conductance and noise in 2D diffusive hybrid systems with superconductors connected to normal metal via tunnel barriers (SIN structures). The developed approach is based on the synthesis of the Keldysh formalism [32] with the functional-integral representation of electron dynamics in terms of effective Q-matrix field theory [33, 34]. This approach was recently started in Ref. [35] and generalized for SIN structures with low interface conductance $G_{\rm T}$ in Ref. [36]. The reported new development (see Ref. [2], p. 76, and Ref. [31]) allows one to treat SIN structures with an arbitrary relation between the interface conductance $G_{\rm T}$ and the diffusive conductance $G_{\rm N}$. Technically, the Keldysh effective action is represented as a functional of the spatially local matrix fields $Q(t, t'|\mathbf{r})$ whose average values coincide with the normal and anomalous electron Green functions. Subsequent integration over diffusive modes by means of the functional renormalization group method allows one to represent the action of the full system in terms of Q-matrices of superconductive and normal reservoirs, $Q_{\rm S}$ and $Q_{\rm N}$. This functional, coined the 'proximity action', allows the complete determination of the whole statistics of charge transfer in the SIN structure in the presence of the Cooper interaction in a diffusive N metal. The 'proximity action' method can be considered as a generalization of the scattering-matrix approach [37, 38] for the problems where the electron–electron interaction should be taken into account. Actual results were presented for the conductance and noise in 2D SIN structures, where the interaction corrections were shown to scale as $g^{-1/2}\ln(L/d)$. These corrections lead to a nonmonotonic dependence of both the conductance and the Fano factor upon the temperature, voltage and magnetic field.

Another dimension of the same general problem was discussed in the talk given by Fei Zhou (Princeton University and NEC Research Institute), who considered the effects of strong mesoscopic fluctuations in disordered superconductive films. The standard theory of dirty SC films assumes, following Abrikosov–Gor'kov [39] and Anderson [40], that a random static impurity potential can be treated as a single-electron problem, i.e. that the pairing amplitude Δ is uniform in the space. Fei Zhou analyzed the specific case when the above assumption is severely violated, namely, a disordered SC film in a parallel magnetic field H close to the Chandrasekar–Clogston paramagnetic limit. In the limit of strong spin-orbit scattering, $\tau_{\rm so}\Delta \ll \hbar$, the breakdown of superconductivity due to the paramagnetic depairing effect proceeds via a second-order phase transition. In the vicinity of this transition two important effects (which are far beyond the standard Abrikosov–Gor'kov–Anderson approximation) come together: (i) the superconductive pairing amplitude becomes strongly inhomogeneous in space (i.e. the formation of isolated islands with local superconductivity is predicted); (ii) the Josepshon-like interaction between different islands appears to be of random sign, leading to the formation of a superconductive glass state with a spontaneous breakdown of time-inversion symmetry (see Ref. [2], p. 87).

4. General theory of mesoscopic networks

Two theoretical reports given at the conference were devoted to the development of general methods able to treat different mesoscopic effects within the same calculational scheme. A very general approach to the description of all kinds of coherent effects in mesoscopic structures was presented by Yu V Nazarov (Delft Technical University). It is based on the method proposed in Ref. [36] (and further developed in the work presented by Skvortsov, as discussed above). It is suggested that apparently different phenomena such as Coulomb blockade, electric noise, mesoscopic conductance fluctuations, the counting statistics of current, spin injection, etc. can be treated within the same general theory dealing with an 8×8 matrix field $Q(t, t'|\mathbf{r})$, whose eight indices comprise the spin, Gor'kov–Nambu and Keldysh indices of underlying electrons. A discretized finite-element version of this theory (developing a very useful idea presented originally in Ref. [41]) was schematically presented. Upon discretization, the system is presented as a collection of nodes, connectors and terminals. Matrix fields $Q(t, t'|j)$ are fixed at terminals $j_{\rm t}$ and fluctuate on nodes $j_{\rm n}$; the properties of connectors determine the form of the action in terms of $Q(t, t'|j)$.

The talk given by G Montambaux (Orsay University) was focused on mesoscopic networks (also called 'graphs') made of quasi-one-dimensional (1D) diffusive conductors. First of all, a number of different physical quantities (a weak-localization correction to the conductivity $\Delta\sigma_{\rm wl}$, the variance of conductance fluctuations $\langle\delta\sigma^2\rangle$, the typical magnetization of the network

$$M_{\rm typ} = \left[\langle M^2\rangle - \langle M\rangle^2\right]^{1/2}$$

and the interaction-induced correction $M_{\rm int}$ to the average magnetization $\langle M\rangle$) were expressed in terms of the spectral determinant $S_{\rm d}(\gamma) = \prod_n(\gamma + E_n)$ of the diffusion equation on the system in question. This step is general for any mesoscopic structure; it does not rely upon the quasi-1D nature of graph-like structures: the latter becomes important in a second step, when the expression for $S_{\rm d}(\gamma)$ in terms of $N \times N$ matrix M is derived. Here N is the number of nodes in the graph, and the matrix M is defined via the lengths of the graph links $l_{\alpha\beta}$, the corresponding phases

$$\theta_{\alpha\beta} = \frac{2\pi}{\Phi_0}\int_\alpha^\beta \mathbf{A}\,\mathrm{d}\mathbf{l}\,,$$

and the phase-breaking length L_ϕ. As a physical application, a relation was found between persistent current magnetizations of an array of connected rings and that of collection of the same rings being isolated (see Ref. [2], p. 65).

5. Superconductor–metal–insulator transitions

This subject is in some sense a part of the field of mesoscopic superconductivity, although a very special part. The essence of the problem is in the complicated interplay between Cooper pairing, Anderson localization and the disorder-enhanced Coulomb interaction. The presence of (at least) three different types of interactions which are all of the same order of magnitude makes the competition between them rather hard, and no quantitatively reliable theoretical prediction has yet been made. In this sense the subject is rather different from the 'bulk' of the field of mesoscopic superconductivity (but close to subjects of the last four reports of that session). Below we give a brief account of the state of the art in the field and then proceed to the discussion of new results presented at the conference.

In thin films superconductivity usually disappears when the sheet resistance of the film $R_\square$ becomes comparable with $R_Q = h/(2e)^2 \approx 6.5$ kΩ. There are two basic theoretical scenarios of superconductivity destruction upon increasing $R_\square$: 'bosonic' [42, 43] and 'fermionic' [44]. The first one assumes that the film is effectively inhomogeneous, either due to its granularity, or due to strong fluctuations of the superconductive pairing interaction or of the electron mean free path. Thus, the first stage of the superconductive transition is the formation of local SC order in some isolated regions (grains, islands, etc.) which are coupled via a weak Josepshon-like interaction. Phase correlations induced by Josepshon coupling compete with charge correlations due to Coulomb repulsion between Cooper pairs. Competition between Josephson coupling E_J and charging energy E_C is known [42, 45] to be the driving mechanism of zero-temperature phase transitions between the superconductive and insulating states in artificial arrays [46, 47], granular films [48] and bulk materials [49]. In such systems there are no free electrons at very low temperatures due to Cooper pairing, but pairs may become localized due to Coulomb repulsion. This is the 'bosonic' mechanism of superconductivity suppression. Evidently, on the non-SC side the insulating ground state should be observed. The effective charging energy E_C^{eff} for Cooper pairs coincides with its bare value $E_C = (2e)^2/2C$ only in the limit of highly resistive junctions between grains, $R_j \geqslant R_Q$. At lower R_j the effective capacitance is upscale renormalized and E_C^{eff} decreases as $-\ln E_C^{\mathrm{eff}}/E_C \approx \pi^2 R_Q/8R_j$ (a similar result for normal junctions can be found in [50, 51]). At the same time, the bare E_C in systems demonstrating a superconductor–insulator (S–I) transition is frequently found in the range of a few Kelvin, i.e. of the same order of magnitude as the typical superconductive gap Δ which also determines $E_J = \Delta R_Q/2R_j$. Therefore, E_J usually becomes comparable to E_C^{eff} at $R_j \sim R_Q$. In thin films an individual junction's resistance R_j is of the same order of magnitude as the sheet resistance $R_\square$, thus the above-mentioned relation $R_\square \sim R_Q$ follows. Observation of this numerical fact in many experiments lead M P A Fisher [43] to an attempt to formulate a general theory based upon the idea of duality between vortices and Cooper pairs. If this duality were an exact property of the Hamiltonian, it would lead to $R_\square = R_Q$ at the S–I transition, i.e. the critical value of the dimensionless sheet resistance $g = \hbar/e^2 R_\square$ would be equal to $2/\pi$. However, there is no theoretical reason to think of this duality as an exact symmetry. Therefore, it is not surprising that in many experiments $g_c \neq 2/\pi$. Moreover, in some of them [53] the critical value of $R_\square$ differs from R_Q very strongly.

Homogeneously disordered superconductive films [55–57, 60] present another group of systems where quantum fluctuations lead to a loss of superconductivity. The theory of T_c suppression in such films was developed in Ref. [44]. The qualitative idea behind this theory is that the disorder-enhanced Coulomb repulsion leads to a decrease of Cooper attraction and thus to a decrease in T_c. The superconductive transition temperature vanishes [34, 44] when g decreases to $g_{\mathrm{Fin}} = [(2\pi)^{-1}\ln(1/T_{c0}\tau_{tr})]^2$, where T_{c0} is the BCS transition temperature of the material (measured e.g. in thick-film samples) and τ_{tr} is the elastic scattering time. At smaller $g < g_{\mathrm{Fin}}$ the metallic state is expected, at least down to not very low temperatures where weak localization corrections $g(T) = g_0 - (1/\pi^2)\ln[1/(T\tau)]$ are indeed weak. This second ('fermionic') mechanism of superconductivity suppression is clearly different from the 'bosonic' one [42, 43] since its basic feature lies in the disappearance of Cooper pairs altogether. Experimental data supporting the fermionic mechanism are reviewed in Ref. [52]. A drawback of this theory is that it neglects quantum fluctuations of phase in the bosonic (order-parameter) field $\Delta(x, t)$, (i.e., it can be considered as a kind of BCS theory with a renormalized attraction constant).

The 'bosonic' and 'fermionic' approaches to superconductor–metal–insulator transitions have been developing almost independently over a number of years, since it has usually been assumed that they are applicable, correspondingly, to two different types of systems. Recent years of experimental and theoretical studies have shown, that the relation between the two approaches is much more intricate. First, the theoretical discussion of relations between bosonic and fermionic mechanisms of quantum phase transition out of SC state in uniformly disordered system was given by A I Larkin in Ref. [62]. It was argued that the fermionic mechanism governs the transition in thin films if the critical value g_F appears to be larger than $2/\pi$, i.e. when $\ln(1/T_{c0}\tau) \geqslant 5$. At larger values of $T_{c0}\tau$ Finkel'stein's corrections to the Cooper pairing strength were argued to be less important than localization effects and the direct Coulomb repulsion, i.e. the bosonic mechanism prevails.

Experimentally, quantum transition out of the superconductive state is frequently studied as a function of magnetic field B. If the fermionic mechanism is effective, a superconductor–metal phase transition is expected on increasing B, in the vicinity of $T = 0$ transition superconductive energy gap should be strongly suppressed, and weak negative magnetoresistance (MR) should be seen for a perpendicular field orientation due to the orbital effect of the magnetic field upon weak localization corrections (in the case of strong spin-orbital effects positive MR should be observable). On the other hand, the bosonic mechanism should manifest itself above the critical field B_c via stronger negative MR due to suppression by the field of the excitation gap induced by local pairing correlations. In this case negative MR is expected to be present both for perpendicular and transverse directions of $\mathbf{B}$, although the value of the critical field B_c (leading to suppression of long-range superconductive coherence) certainly depends upon the orientation of $\mathbf{B}$. The theory of negative MR in granular superconductors at low temperatures was developed in Ref. [61] for the asymptotic region of high magnetic fields, where pairing effects are almost suppressed, so the whole effect is weak. This theory is most directly applicable to the experiments [54] on thick (200 nm) granular Al–Ge film with the size of Al grains close to 10 nm. For the sample that was superconducting at $B < B_c < 1$ kOe, the maximum resistance value $R(B_{\mathrm{peak}} \sim 2.5\ \mathrm{T})$ measured at $T = 0.3$ K was about twice the resistance at the $B \to \infty$ limit. The state of the system at $B > B_c$ was called a 'Bose insulator'. The same kind of behavior is seen in Ref. [54] on another sample which never becomes globally superconducting, but reveals giant negative MR at high fields.

Now we turn to a review of the conference talks on the subject. All three experimental reports (presented by V F Gantmakher (Institute of Solid State Physics, Chernogolovka), J Valles (University of Providence) and S Okuma

(Tokyo Institute of Technology)) dealt with homogeneously disordered films of different kinds. Valles reported resistive and tunneling measurements on very thin Bi and PbBi films evaporated on an amorphous layer of Sb or Ge (see Ref. [2], p. 104). The films with normal-state resistance below about 10 kΩ are superconducting at low temperature, while more resistive films become insulating. A study of the tunneling density of states on the superconductive side of the transition revealed that suppression of the quasi-particle gap (i.e. of the order parameter absolute value) becomes increasingly important as the film resistance approaches its critical value. This points out the importance of the fermionic mechanism of superconductivity suppression in the films studied. On the other hand, the pure BCS theory with reduced gap is not sufficient to describe the data for nearly-critical films: (i) the tunneling $I(V)$ deviates rather strongly from the rescaled BCS form, and (ii) the reduced width $\Delta T/T_c$ of the thermal superconductive transition grows with decreasing T_c, and becomes of order 1 for $T_c \approx 0.1$ K. Both these features show that some quantum fluctuations not included in the renormalized BCS theory of Finkel'stein should be important here.

The discussion of 'bosonic' phenomena in unconventional superconductors was the main subject of the talk given by Gantmakher. As for the first example, results obtained for moderately thick (20 nm) films of InO_x were reported [58]. A unique feature of these films is that the actual oxygen content (and thus the carrier density) can be varied continuously and reversibly over a very broad window of R_N. In a number of samples, the superconductive state formed at zero magnetic field was transformed to an insulating state (with nearly Arrhenius growth of resistance at low T) by application of a magnetic field in the few Tesla range. On further increasing the field, the resistivity passes through a maximum and tends to saturation at a weakly T-dependent level in the limit of very high fields. The most surprising feature of these data is the very strong negative MR effect on the high-field side, of the kind discussed above in relation to experiment [54] on a granular system. In spite of the absence of any morphological grain structure in InO_x films, the ratio R_{max}/R_{14T} of the maximum (at $T = 30$ mK) resistivity to its nearly asymptotic value at $B = 14$ T was found to be about 5, which is the highest negative MR ratio ever reported in superconductors. The qualitative features of MR resistance are similar for perpendicular and parallel orientations of the field, although B_c and B_{peak} values differ by a factor of about 2. Note also that the critical (at $B = B_c$) resistance of the above-described sample was rather high, close to 8 kΩ. This very pronounced Bose-insulator behavior seen in nominally homogeneously disordered film requires the development of new concepts like Cooper pairing of (nearly) localized electrons, which results in the formation of localized Cooper pairs. The simplest example of this kind can be found in a very old 'pre-BCS' paper [63], where the formation of bosons out of strongly bound pairs of electrons was proposed as a mechanism of superconductivity. However, any real theory of this kind encounters a problem: even in systems close to the S – I transition, like InO_x, the effective number of electrons $n_e \xi^3 \Delta/E_F$ in the correlation volume is always large compared to unity (although many orders of magnitude less than in normal superconductors), so the Cooper pairs should strongly overlap each other. It is not yet clear how to describe Cooper pairs composed of disorder-localized but still strongly overlapping electron states. The second part of Gantmakher's talk addressed the behavior of the low-temperature layered cuprate superconductor NdCeCuO, see Ref. [59]. In a magnetic field it demonstrates a shift of resistive curves $R(T,H)$ without considerable broadening, which allows standard determination of the upper critical field $H_{c2}(T)$ like in normal low-T_c materials. Unlike in those, however, anomalous H_{c2} behavior near the critical temperature, $H_{c2}(T) \propto (1 - T/T_c)^{3/2}$ was found. The same power law was derived previously for a model of Bose condensation of a weakly nonideal Bose gas in a magnetic field, therefore the data on NdCeCuO were interpreted in favor of the existence of 'prelocalized' bosons in this material.

In the talk given by S Okuma (see Ref. [2], p. 108), a comparison and distinction was made between superconductive properties of 'thick' (more than 100 nm) and 'thin' (about 5 nm) films of amorphous Mo_3Si. Thick films demonstrated 'vortex glass' behavior with vanishing linear resistivity at relatively low magnetic fields. At higher fields $B > B_0$ a transition to the 'vortex liquid' state with a finite linear resistivity $\rho(T)$ takes place. It is important that this resistivity is nonzero in the zero-temperature limit; in other terms, upon loss of superconductivity a metallic ground state is formed. Since the critical field B_0 is found to be considerably below the usual H_{c2} field, this metallic state is not just a normal Fermi liquid. Very qualitatively, it can be understood as a quantum liquid of vortex lines. On the contrary, thin films show typical 2D superconductive behavior at low fields, with a thermally assisted flux flow, whose activation energy decreases with B as $U(B) \propto \ln(B_0/B)$. At $B > B_0 \approx 2.35$ T a resistivity behavior 'schematically' similar to that obtained by Gantmakher et al. on InO_x is observed: a maximum of $R(B)$ at some $B_p > B_0$ and a region of negative MR are seen. However, the total magnitude of the effect in this case is almost two orders of magnitude lower than in InO_x; thus it is not clear if the same physics of the 'Bose insulator' is relevant here. If it really is, the origin of huge quantitative difference in the values of MR could be traced to the difference in sheet resistances of the samples studied: the high-B resistance of the thin film from Okuma's work was close to 2 kΩ, i.e. at least a factor of 5 lower than in the InO_x films measured by Gantmakher et al. In this respect it might be worth recalling the above-mentioned exponential dependence of renormalized capacitance on the contact resistance (see also Refs [50, 51]). On a different note, we would like to comment that the relatively smooth activated behavior of resistivity of Mo_3Si at low fields (with a logarithmic dependence U on B) does not necessarily continue down to arbitrary low fields: a very interesting counterexample is provided by the results of recent measurements [64] on rather similar Mo – Ge films with $R_\square \sim 1$ kΩ.

Trying to summarize all three experimental reports reviewed above, one sees a big variety of interesting phenomena in apparently similar systems, which are still awaiting a reliable theoretical description. Another very interesting experimental puzzle [47, 53, 64 – 66] is the apparent existence of a 'Bose metal' — the state with constant nonzero resistivity in 2D at very low temperatures — in a parameter window between the superconductive and insulating states. However, as frequently happens, a theoretical talk on the same subject (given by M V Feigel'man, L D Landau Institute) was addressing yet another model of superconductor – metal transition, which has not yet been realized experimentally. This new model (see Ref. [2], p. 99, and Ref. [67]) consists of an array of small superconductive islands sitting on a dirty metal (or 2DEG) film. The interface conductances between the islands and film are large, as well as the sheet conductance of the film, $g = \hbar/e^2 R_\square \gg 1$. Clearly, this model is designed to incorporate the basic features of both the 'bosonic' and 'fermionic' approaches discussed above: it contains both quantum fluctuations of the phases θ_j on all SC islands, and the effects due to the presence of the Fermi sea of unpaired electrons, which diffuse in a random potential in the presence of Coulomb repulsion. The question is considered: under which circumstances will the whole film be macroscopically superconducting at $T = 0$? The usual answer to this question would follow from the well-known Schmid model [68] of resistively shunted Josepshon junction (RSJJ): superconductivity is stable as long as the normal-state resistance R_n between neighboring islands is smaller than R_Q. The main message of Feigel'man's report is that the above answer is incorrect. In fact, superconductivity is lost *earlier*, already at $R_n/R_Q \sim 1/\sqrt{g}$. This result comes about when quantization of electron transport in a diffusive medium is taken into account properly (contrary to the standard RSJJ model, where quantum dissipation is treated in the linear approximation). Calculations presented in this talk determine the effective Coulomb energy of an SC island in good contact with the metal reservoir, as well as the proximity-induced Josephson coupling between the islands. A comparison between the two energies determines the SC–M transition line. An alternative interpretation of the same calculation is that it provides an overall Cooper interaction constant λ_{eff} for the whole array (film + islands); the region of parameters, where $\lambda_{\text{eff}} < 0$, corresponds to a macroscopically superconductive state. Unfortunately, the effects of a magnetic field and pairing fluctuations upon the total system's resistance have not been studied yet.

6. Single-electron transistors

In the above section of this review we analyzed the fundamental consequences of the Coulomb blockade phenomenon as the origin of quantum phase transitions; from the practical viewpoint, it offers the possibility of extremely precise measurement of electric charge. The devices able to perform such measurements are known as single-electron transistors (SETs), and two conference talks were devoted to their engineering and investigation. The Coulomb blockade of conductivity which is at the basis of this mesoscopic electronic device operation, occurs until the charging energy $E_C = e^2/2C$ is more than the temperature. The operating temperature is, typically, in the sub-Kelvin range for tunnel junction sizes more than $0.1 \times 0.1\ \mu\text{m}^2$ and $C > 0.1$ fF. Now the efforts of many experimentalists are focusing on raising the device operating temperature, which is one of the main drawbacks of the implementation of serial single-electron devices. A talk by Yu A Pashkin et al. (NEC Research Institute, Tsukuba) presented a lithographically made Al single-electron transistor that shows a perceptible modulation of working characteristics at room temperature [72]. The charge-equivalent noise at 300 K is measured to be $4 \times 10^{-2}\ e$ $\text{Hz}^{-1/2}$ at 1 Hz and is expected to be 1000 times lower in the white-noise regime at higher frequencies, that makes its application as a sensitive electrometer possible. The room-temperature SET was fabricated by standard e-beam lithography and commonly used angle evaporation. The size of the evaporated structures is determined by the size of the window in the mask, whose lower reproducible limit is about 10 nm, and is about window size if an island of a transistor is evaporated as a first layer. However, the authors of the talk could go beyond this limit by evaporation of the island as a second layer. The charging energy of the device was estimated to be remarkably high, about 115 meV.

A new type of single-electron transistor with highly resistive Cr film strips instead of the traditional tunnel junction was presented by V A Krupenin et al. (Moscow State University and Physikalisch-Technische Bundesanstalt, Braunschweig) (see Ref. [2], p. 113). As is discussed in Section 5, disorder-enhanced Coulomb interaction in films with sheet resistance larger than $R_Q \equiv h/4e^2 \cong 6.5$ kΩ can also lead to blockade of conductivity. This 'resistive' SET with a total asymptotic resistance of 110 kΩ has shown a very sharp Coulomb blockade and reproducible, deep and strictly e-periodic gate modulation over wide ranges of bias currents and gate voltages. The noise figure of the new SET was found to be similar to that of typical $Al/AlO_x/Al$ single-electron transistors, viz. $\delta Q \approx 5 \times 10^{-4} e\ \text{Hz}^{-1/2}$ at 10 Hz. In the Coulomb blockade state the $I-V$ curve exhibits a large region with vanishingly small current. The current decay in the region of the blockade corner is substantially steeper ($\alpha > 10$) than the dependence $I \propto V^3$ which is typical of tunnel barrier transistors at low T [69] because of the co-tunneling effect, i.e coherent tunneling of two electrons across two junctions simultaneously [70]. This dependence indicates that the co-tunelling process, i.e. arrival of an electron at an island and simultaneous escape of another electron from the island, is strongly suppressed. Such suppression of co-tunneling occurs in 1D arrays of tunnel junctions with a large number of junctions [69] or in SETs supplied with miniature on-chip resistors, the so-called R-SETs [71], due to the effect of the dissipative electromagnetic environment [73] created by these resistors. The latter system has the advantage of efficiently controlling the transport current by a gate. It is interesting that — although the proposed new resistive device is based on another junction type — its behavior is similar to that of R-SET, i.e. it combines the property of deep modulation of a single electron current and considerable suppression of the co-tunneling current. The other advantage of the new SET device is that it is relatively easy to fabricate. The main difficulty consists in evaporating a Cr film with a well-defined sheet resistance.

7. Localization and quantum chaos

The notion of quantum localization of non-interacting electrons placed in a random potential, predicted long ago by P W Anderson [74], became one of the cornerstones of quantum mesoscopics soon after the seminal paper by L P Gor'kov, A I Larkin, and D E Khmel'nitskiĭ [75] on the *weak localization* theory. This theory layed a firm foundation for the studies of different effects in quantum transport; their general feature is that they are due to quantum interference of electron waves, and thus cannot be described in terms of the standard semiclassical kinetic equation approach. Almost simultaneously, another very important direction of studies was opened by B L Al'tshuler and A G Aronov [76], who demonstrated the increasing importance of the electron–electron interaction due to the slow diffusion of electrons in a random potential caused by impurities. The opposite limit of strongly localized electron states in doped semiconductors was extensively studied by A L Efros and B I Shklovskiĭ [77] who introduced, in particular, the notion of a *Coulomb gap* in the density of states of strongly localized interacting electrons. A multitude of subsequent studies were devoted to attempts to solve two basic problems: (i) to describe the Anderson transition between localized and delocalized states of non-interacting electrons, and (ii) to understand metal–insulator transitions in real materials, where electrons interact inevitably. Even the first of these problems still is not solved in terms of an analytical theory, although the basic features of the Anderson localization transition are now well-understood, in a large part due to extensive numerical studies. However, the attempts to solve this problem led to a number of very interesting 'side' results. In particular, the deep relations between type of statistics of energy levels and wave functions on one hand, and the problem of regular versus chaotic dynamics on the other hand, were understood and gave birth to the new branch of studies called 'quantum chaos'.

Several conference reports were devoted to the subjects in this field. T Fromhold from the University of Nottingham presented (see Ref. [2], p. 24) numerical results on the semiclassical motion of electrons confined to the lowest miniband of a heterostructure superlattice in a strong magnetic field. Tilting the magnetic field away from the superstructure axis induces a transition from stable regular motion to chaotic dynamics. This transition is argued to be of purely quantum-mechanical origin associated with the dispersion relation of the heterostructure miniband. The onset of chaos delocalizes the semiclassical orbits and discrete eigenvalues related to them. An argument is presented for the sharp increase of electric conductivity due to chaos-induced delocalization.

The talk given by A D Mirlin (University of Karlsruhe) concentrated on a problem of eigenfunction statistics near and at the point of Anderson transition. Specifically, the probability distribution of the inverse participation ratio (IPR) $P_q = \int \mathrm{d}^d r |\psi(\mathbf{r})|^{2q}$ was studied numerically for the power-law random banded matrix model (PRBM). The critical PRBM is defined as a random matrix whose off-diagonal elements are distributed independently, with the variance $\langle |H_{ij}|^2 \rangle$ which scales as $[b/(i-j)]^2$ until $|i-j|$ becomes comparable to the matrix rank. It was shown previously that such a model is exactly at the Anderson critical point for any value of b. PRBM was found to be a very useful 'laboratory' for studies of the Anderson transition, since different analytical methods can be applied to it; at large b the problem was studied [78] by a renormalization-group (RG) method within the supermatrix sigma-model approach, whereas at $b \ll 1$ a completely different RG method proposed originally by Levitov [86] was employed [79]. In the work discussed the distribution function $\mathcal{P}(p_q)$ of the normalized (to its typical value) IPRs $p_q = P_q / P_q^{\mathrm{typ}}$ was obtained numerically over the broad interval of b. The main conclusions of this study are as follows: (i) $\mathcal{P}(P_q)$ is scale-invariant, (ii) the scaling of P_q^{typ} with the system size defines a set of fractal exponents D_q which are well-defined non-fluctuating quantities.

Whereas the theory of mesoscopic fluctuations (i.e. irregular dependences of conductance and other physical quantities upon slight modification of impurity potential, magnetic or electric field, etc.) is well-developed [80] for non-interacting diffusive electrons, its extension to the case of an interacting system is rather nontrivial. An interesting step in this direction was made by B Z Spivak (University of Washington), who presented a theory of mesoscopic sensitivity of speckles in nonlinear disordered media [81]. It was shown that an arbitrarily weak non-linearity introduced into the Schrödinger equation with a random potential leads to unbound growth (exponential with the size of the system) of the sensitivity of the solution to an initial condition. Very roughly, this phenomenon reminds one of the situation with the exponential spreading of trajectories in classically chaotic dynamical problems.

The understanding of behavior of *interacting and nearly localized* electrons has been relatively scarce till now, although it was generally believed (at least until very recently) that one basic fact is well-documented: any 2D electron system becomes insulating in the zero-temperature limit (unless it becomes superconductive, which is discussed above). Also, the crossover to insulating behavior (with the resistivity being some exponential function of temperature) usually becomes sharp when the sheet resistance increases to the von Klitzing value $R_{\mathrm{K}} = h/e^2 \approx 27$ kΩ. However, both these 'common-sense points' have been questioned in recent years, and some related works were presented at the conference. We start this subject from the description of the very unusual results presented by Z D Kvon et al. (Institute of Semiconductor Physics, Novosibirsk): a 2D antidot lattice fabricated from a GaAs/AlGaAs heterostructure demonstrated a power-law temperature dependence of resistivity, $R(T) \propto T^{-\alpha}$, even in the range $R \geqslant 10 R_{\mathrm{K}}$. The exponent α was found to rise smoothly with R, but it did not exceed 0.7 even for the largest values of R. Moreover, after application of a magnetic field $B \geqslant 1$ T, even this moderate trend to insulating behavior disappeared and the structure demonstrated almost T-independent conductance with $R \gg R_{\mathrm{K}}$ (note some similarity with the 2D metal state discussed in relation with the problem of superconductor–metal–insulator transitions). These findings (see Ref. [2], p. 20) are in apparent disagreement with major statements of the theory of quantum transport; the origin of the discrepancy is not understood yet.

The major source of discussions during last years about the validity of accepted theory of quantum transport in 2D was due to an experimental finding by Pudalov et al. on metallic behavior in inversion layers of Si [82, 83] (similar results were later obtained in other 2DEG systems). The anomalous (delocalizing) sign of the resistivity dependence on temperature, seen in these data, led to a number of speculations about the limits of the validity of weak localization theory in the presence of interactions, and on the possible existence of the $T = 0$ metal state in 2D. This idea looks rather intriguing. However, it has apparently lost its main original motivation: new analysis of the data on magnetoresistance in Si-MOSFETs presented by V M Pudalov (P N Lebedev Physical Institute, Moscow) indicates that major anomalous trends in the $R(T, H)$ behavior at moderately low temperatures can be attributed to semiclassical effects (the finite ratio of actual temperature to the Fermi energy of 2DEG). Thus the 'current conclusion' given by Pudalov is: existing data over the range $R \ll R_{\mathrm{K}}$ can be explained [84] in terms of the temperature-dependent Drude formula in combination with conventional weak-localization theory.

A different point of view on the origin of the strange nonmonotonic behavior of $R(T, H)$ in ultra-clean Si-MOSFETs was presented by A M Finkel'stein (Weizmann Institute of Science, Rehovot). He has shown that straightforward generalization of his renormalization group theory [34] for an interacting disordered Fermi liquid for the case of a two-valley semiconductor explains the nonmonotonic $R(T)$ behavior at $H = 0$ with a five-fold drop at low T (and suppression of this effect by a parallel magnetic field) without any adjustable parameters! The key idea is that the inclusion of an additional (with respect to the spin one) degeneracy due to the valleys strongly increases the antilocalizing term in the logarithmic contribution to conductivity, already found in [34] for the case of disordered 2DEG with a spin-dependent interaction. In the usual single-valley case the amplitude of the spin-dependent interaction Γ_2 never becomes so strong that antilocalization prevails. Finkel'stein has now shown that the same is not true if a degenerate two-valley system is considered: with Γ_2 in a reasonable range, the antilocalizing sign of the overall correction to conductivity exists within the experimental range of temperatures. Although a two-valley structure is natural for absolutely clean Si-MOSFET, disorder scattering leads to mixing of the valleys, and sufficiently strong mixing reduces the problem back to the standard one. Since impurity scattering is certainly sample-dependent, it may explain the non-universality of the observed $R(T)$ nonmonotonic behavior [85].

Four conference reports were devoted to different aspects of the same fundamental problem — the role of interactions between electrons localized by disorder. L S Levitov (Massachusetts Institute of Technology) presented a development of his theory [86] of wave function delocalization in the presence of strong diagonal disorder and long-range random hopping amplitudes which scale as $|t_{ij}| = V_{ij}|\mathbf{r}_i - \mathbf{r}_j|^{-d}$. Here $V_{ij} = \mathbf{a}_i\mathbf{a}_j$, d is space dimensionality, and $\mathbf{a}_i$ are random uncorrelated n-dimensional vectors. The power-law decay of hopping amplitudes with the power equal to the space dimensionality leads to marginal behavior of this model (for even slower decay of hopping amplitudes, the localization is absent, whereas for faster decaying amplitudes all states are localized). The characteristics of a marginal system change logarithmically slowly with increasing space scale L, so the use of RG methods is natural. Levitov constructed an analytical real-space RG for the distribution function $f(\mathbf{a})$ of renormalized random parameters $\mathbf{a}$. The RG flow leads to fixed-point solutions for any $n > 1$; the corresponding RG analysis of wave functions evolution shows that the inverse participation ratio P_2 scales with L with a single exponent D_2 — in agreement with the results presented by Mirlin (see above) for the case of an independent random V_{ij}. However, for $n = 1$ no stable fixed-point solution of the RG equation exists; correspondingly, the statistics of P_2 is multifractal, i.e. described by a continuous set of scaling exponents only. So far we have reviewed the properties of the non-interacting model introduced by Levitov in Ref. [86]. Physically, random hopping of the form studied appears most naturally in the problem of interacting two-level systems in glasses, where long-range interaction of either dipole – dipole or elastic strain origin is present. In such a case the local degrees of freedom are represented by effective spin-1/2 variables σ_i which introduce strong local nonlinearities. The random variables V_{ij} become matrices in the pseudospin space: $V_{ij}^{ab}\sigma_i^a\sigma_j^b$, whereas the diagonal disorder is of the form $\epsilon_i\sigma_i^z$. Keeping the V^{zz} component only, one would reduce the problem to the classical Coulomb gap of Efros and Shklovskiĭ [77], who show that the local density of states is suppressed due to interactions. Levitov considered [87] another reduction of the full problem: he kept the V^{xx}, V^{yy} and V^{xy} components, i.e. he dealt with the XY exchange model. Using a pseudofermion representation for spins 1/2, he again constructed a real-space RG and showed that its flow leads to a singular *upturn* of the DOS at low energies — a result opposite to that of the classical Coulomb gap problem. Physically, this means that the thermal conductivity in structural glasses can be significant in spite of the absence of delocalized elementary excitations. The same statement should apply to *electron glasses*, studied experimentally by Vaknin et al. (see below).

The theory of spectrum statistics in the situation of a shallow Coulomb gap was discussed by Ya M Blanter (Delft University of Technology). Usually the level statistics is considered to be Poissonian in the insulating regime. However, this is correct in the case of non-interacting electrons only, since electron – electron interactions produce level correlations. This effect was studied [88] in the case of relatively short-range (due to screening by external gates) electron interactions. In this case interaction-induced corrections to the DOS are small, $\delta\nu(E) \ll \nu_0$, and amenable to a detailed analytic treatment. It was shown that $\delta\nu(E)$ is negative and singular at $T = 0$ even for a short-range interaction, whereas at relatively high temperature $\delta\nu(E)$ is a nonmonotonic function of E/T. Level correlations $\langle\delta\nu(E_1)\delta\nu(E_2)\rangle$ are shown to be repulsive for $E_1E_2 < 0$ (two states on different sides of the Fermi surface), but attractive for $E_1E_2 > 0$.

The experimental work reported by A Vaknin (Hebrew University of Jerusalem) made a very interesting link between the historically distant problems of electron transport in weak insulators and ageing phenomena in glasses and spin glasses. The time-resolved conductance of insulating In_2O_3 films was measured as a function of time t spent after cycling of the gate potential V_g (which controls the equilibrium density of electrons) during the 'waiting time' t_w. It was found [89] that the shape of the t-dependence of the induced modification of conductance $\Delta G(t)$ scales very accurately with the waiting time t_w, whereas the overall magnitude of the effect depends on V_g and T as well. The strong dependence on t_w indicates the presence of very slow processes of collective electron motion, roughly similar to the processes of spins rearrangement in spin glasses [90]. The redistribution of electrons between localized states can be considered in terms of the model of interacting two-level systems. Since the Coulomb energy variation due to single hopping event varies in a dipole fashion, this model resembles the one studied by Levitov. Moreover, the fact that the temperature affects only the magnitude of the observed $\Delta G(t)$, but not its t-dependence, suggests the importance of quantum processes. The theory of quantum ageing in electron glass is not available at present; it seems plausible that it can be constructed starting from Levitov's approach.

M E Gershenson (Rutgers University) presented results of experimental studies of non-ohmic transport in the 2D electron layer formed in a Si-doped GaAs structure, both in the weak localization (WL) and strong localization (SL) regimes [91]. It was shown that all the data are very well described by a 'hot electron' model, which assumes that the major non-ohmic effect is due to electron overheating with respect to the phonon bath temperature, $T_e > T$. The measured resistance depends upon electron temperature T_e and thus upon the power W dissipated in the sample. Two important conclusions were made: (i) the electron – phonon interaction is roughly the same in WL and SL regimes; (ii) in the SL regime the resistance depends on T_e only. This means that hopping transport in the localized regime (traditionally considered to be the phonon-assisted process) is actually governed by electron – electron interactions, at least in the 2DEG structure studied.

To close the description of this section, we mention the theoretical report by I M Suslov (P L Kapitza Institute for Physical Problems, Moscow) who presented detailed first-principles calculations of the density of states behavior $\nu(E)$ for the Schrödinger equation with a random potential, at energies E near the Anderson's mobility edge E_g. Although $\nu(E)$ is regular at E_g, its actual calculation is highly nontrivial due to the absence of any simple small parameter in the problem. Nevertheless, Suslov was able to develop a reliable calculation method (see Ref. [2], p. 31, and Ref. [92]), based upon a combination of a diagram technique and instanton analysis, which allows one to calculate $\nu(E)$ at all energies, at least for a model with nearly 4 space dimensions, $4 - d \ll 1$.

8. Quantum Hall effect

It was found during the last decade that the fractional quantum Hall effect (FQHE) and other exotic behaviors of electrons confined in two dimensions can be understood in terms of composite particles. Indeed, what are the true particles of a 2D-electron system subjected to a strong transverse magnetic field at low enough temperature? It happens that these electrons effectively attach all or a substantial fraction of the external magnetic field, thus transforming themselves into particles called *composite fermions*. The outcome is that the strongly interacting electrons in a magnetic field B transform into weakly interacting composite fermions in a weaker magnetic field B^*, given by $B^* = B - 2pN\Phi_0$, where $2p$ is an even integer, the flux quantum $\Phi_0 = h/e$, and N is the 2D-electron density. Equivalently, one can say that the electrons at filling factor ν convert into composite fermions with filling factor $\nu^* = N\Phi_0/|B^*|$, given by $\nu = \nu^*/(2p\nu^* \pm 1)$. So, the attachment to each electron of an infinitely thin, massless magnetic solenoid carrying $2p$ flux quanta pointing antiparallel to B, turns it into composite fermion. Such a conversion preserves the minus sign associated with the exchange of two fermions, just because the bound state of an electron and an even number of flux quanta is itself a Fermi quasi-particle. It also leaves the Aharonov–Bohm phase factors associated with all closed paths unchanged, because the additional phase factor due to a flux $\Phi = 2p\Phi_0$ is $\exp(2\pi i\Phi/\Phi_0) = 1$. Thus the FQHE for 2D electrons is interpreted as an integer quantum Hall effect of composite fermions — in effect, an observation of composite-fermion Landau levels. This rather simple explanation of the FQHE not only explains all the observed fractional plateaus in a single step, but it also unifies the fractional and the integer quantum Hall effects. The observation of composite fermions in the region around the filling factor $\nu = 1/2$, where there is neither a QHE nor any other sort of excitation energy gap, was a watershed for the composite-fermion concept. It was an explicit demonstration that the composite fermion is more general than its manifestation in the FQHE, where it forms Landau levels. The observation of the composite-fermions Fermi sea explicitly verifies its Fermi statistics, and the measurements of its cyclotron radius confirms that it carries a charge $(-e)$. The concept of composite fermions is built up by adding the Chern–Simons statistical gauge fields to the theory. In the report by M A Baranov et al. (Kurchatov Institute, Moscow, and Amsterdam University) the main ingredients of a unifying theory for abelian quantum Hall states in two dimensions were summarized. The theory presented (see Ref. [2], p. 44) combines Finkel'stein's approach to localization and the Coulomb interaction effects with the topological concept of an instanton vacuum as well as Chern–Simons gauge invariant theory. The authors elaborate on the meaning of a new symmetry (F invariance) for systems with an infinite-range interaction potential. The main results of the theory were presented in terms of a scaling diagram of conductances.

Numerous properties of these composite fermions and the quantum fluids they form have been established within the last few years. In experiments a Fermi sea, Shubnikov–de Haas oscillations, and cyclotron orbits of them have been observed. The particles' charge, spin, statistics, mass, and magnetic moment were measured. In mesoscopic experiments, the composite fermions behave like billiard balls. I V Kukushkin et al. (Institute of Solid State Physics, Chernogolovka, and the Max Planck Institute, Stuttgart) presented magnetooptical experimental studies of composite fermions of 2D-electron gas in a GaAs-based heterojunction (see Ref. [2], p. 36). The measurement of circularly polarized photoluminescence was used as an efficient tool for analysis of spin polarization of composite fermion states at different fractional filling factors. Particularly the Fermi energy and the Zeeman splitting of CFs were measured from the temperature dependence of the electron spin polarization at $\nu = 1/2$. It was found that the Zeeman splitting of CFs was enhanced by a factor of 2.5 due to the interaction between composite particles. This interaction is very sensitive to the finite width of the 2D channel. On the other hand, the spin polarization at $\nu = 1/3$ and $\nu = 2/3$ exhibits activated behavior and the derived spin-wave gaps are close to the simultaneously measured transport values.

Magnetocapacitance experimental studies of the quantum Hall effect states in the presence of long-range potential fluctuations were presented in the talk by M O Dorokhova and S I Dorozhkin (Institute of Solid State Physics, Chernogolovka). It was found that the capacitance minima widths are independent of the magnetic field and are the same for even, odd and fractional QHE-states being measured as a function of the average electron density (see Ref. [2], p. 39). This result indicates that the width of the capacitance minima in the investigated samples is governed by the long-range charge density fluctuations. The magnitudes of energy gaps found at even filling factors are very close to the cyclotron gaps. At odd filling factors $\nu = 1$, 3 and 5, the energy gaps appear to be enhanced in comparison with the Zeeman splitting and this enhancement decreases with magnetic level number.

2D-electron systems at $\nu = 1$ and certain other filling factors in the quantum Hall regime exhibit spontaneous magnetic order. This constitutes a very peculiar kind of ferromagnetism: the electrons in this state are free to move around as in metals like, for instance, iron, and yet it exhibits a charge excitation gap that manifests itself by precisely quantized Hall conductivity and the vanishing of the dissipative longitudinal conductivity σ_{xx}. Under $\nu = 1$ the lowest spin state of the lowest Landau level is completely filled and the exact ground state (neglecting Landau-level mixing) is rather simple: it is a single Slater determinant precisely represented by Laughlin's wave function. In contrast to iron, this 2D ferromagnet is 100% polarized, because the kinetic energy has been frozen into discrete Landau levels and polarizing the electron gas costs no kinetic energy. But the low-energy spin degrees of freedom of this unusual ferromagnet have some novel properties. A few years ago

S Sondhi and collaborators made a notable discovery: in the vicinity of $\nu = 1$, because the exchange energy is large and prefers locally parallel spins, the Zeeman energy being small, it is energetically cheaper to form a topological spin texture by partially turning over some of the spins. Such a topological object was called a 'skyrmion' (in analogy with the Skyrme model in nuclear physics). Since the system is an itinerant magnet with a quantized Hall conductivity, it turns out that the skyrmion texture accommodates precisely one extra unit of charge. Various transport and optical measurements, the NMR Knight shift and other experiments have confirmed the prediction that each charge added to or removed from the state flips over a handful of spins.

The phase diagram and skyrmion energy gap for a bilayered heterostructure at integer filling factors have been studied by S V Iordansky et al. (L D Landau Institute for Theoretical Physics and Lancaster University) using the hidden symmetry method (see Ref. [2], p. 49). As a result, three phases have been found: the ferromagnetic, canted antiferromagnetic and the spin-singlet phase. Each phase violates the SU(4) hidden symmetry and is stabilized by anisotropy interactions. The authors have identified charged excitations of the bilayer with skyrmions or topological excitations making use of the nonlinear sigma model. It was found that the skyrmion energy gap varies dramatically over the bilayer parameter space and exhibits a sharp dip in the canted antiferromagnetic phase of the bilayer. The calculated skyrmion energy gap was compared with the diagonal activated conductivity energy measured by Devyatov et al.

E V Devyatov et al. (Institute of Solid State Physics, Chernogolovka, and the Universities of Glasgow and Hamburg) presented tunneling measurements of the Coulomb pseudogap 2D-electron bilayer system in a quantizing magnetic field at filling factor $\nu = 1$, see Ref. [93]. The tunnel current–voltage characteristics show that for the double maximum observed in the tunnel resistance in the vicinity of $\nu = 1$ the magnitude of the pseudogap is linear in energy with a slope that depends on the filling factor, magnetic field and temperature. At the same time, at $\nu = 1/3$ and $2/3$, a completely different behavior of the current–voltage curves was found and has been interpreted as a manifestation of fractional gaps.

Anomalous transport properties of 2D electrons and composite fermions in the presence of long-range disorder were discussed by D G Polyakov et al. (Karlsruhe University). It was found (see Ref. [2], p. 27) that the low-ω behavior of ac conductivity is governed by memory effects associated with return processes that are neglected in Boltzmann transport theory, namely the return-induced correction to $\mathrm{Re}\,\sigma$ exhibits a kink proportional to $|\omega|$. This effect is of quasi-classical origin and dominates (in comparison with quantum corrections proportional to $\ln\omega$) over a wide frequency range, where $k_{\mathrm{F}} d \gg 1$ (d — scale of disorder). The quasi-classical zero-ω anomaly is shown to be strongly enhanced in a high enough magnetic field, within the framework of the composite fermion theory at the half-filled Landau level. It was demonstrated that non-Markovian quasi-classical kinetics leads to a strong magnetoresistance $\Delta\rho_{xx}$. The quasi-classical memory effect accounts for the positive $\Delta\rho_{xx}$ observed at small deviations from half-filling. At larger deviation, the positive magnetoresistance is followed by a sharp falloff of ρ_{xx}.

From elementary particle physics it is known that the electron is the quintessential example of an elementary particle, namely, high energy experiments to date have revealed no evidence of any internal structure, no evidence that an electron is made up of some other, more fundamental constituents. But in the case of condensed matter physics studying the behavior of carriers in a semiconductor at low temperatures, electrons can play by a different set of rules. The bright example is the fractional quantum Hall effect, which can be explained by invoking quasi-particles, which behave like distinct particles that each carry a fraction (!) of an electron charge. One should bear in mind that on more fundamental ground, quasi-particles in the FQHE are collective excitations of interacting electrons. In the paper by the Weizmann Institute group, presented by M Reznikov, experimental observation of the shot noise produced by quasi-particles in a FQHE system was discussed. An ordinary electrical current has tiny fluctuations, or noise, because the current is carried by discrete electrons, each of which has charge e and travels independently of others. In the FQHE, the current is carried by quasi-particles with charge $e/3$, $e/5$, depending on experimental conditions. In previous studies (the presentation by D C Glattli at the First Mesoscopic Conference in Chernogolovka in 1997, see Ref. [94]), the existence of quasi-particles with one-third of an electron's charge was announced, which is the same fraction as that of the respective QHE fractional state. An outstanding ambiguity is, therefore, whether these studies measured the charge or conductance. Reznikov et al. reported [95] the observation of quasi-particles with a charge $e/5$ in the $2/5$ fractional state, from measurements of shot noise in a 2DEG. They performed measurements of two-terminal conductance (in units of the quantum conductance) versus voltage applied to the gates of the quantum point contacts. The results gained imply that charge can be measured independently of conductance in the FQHE regime, generalizing the previous observation of fractionally charged quasiparticles. In terms of composite fermions, the fractional charge appears as what is called the 'local charge' of an composite fermion, defined as the sum of its intrinsic charge $(-e)$ and the charge of the screening cloud around it. Its value at $\nu = \nu^*/(2p\nu^* \pm 1)$ can be shown by simple arguments to be $-e/(2p\nu^* \pm 1)$. So, this fractional charge is a manifestation of quantized screening by the quantum fluid of composite fermions.

In the talk by N B Zhitenev et al. (Bell Labs, Lucent Technologies) a new scanning probe technique on a submicrometer scale has been developed and applied for imaging localized electronic states in the quantum Hall regime [96]. The concept of electron localization has long been accepted to be essential in the physics of the QHE. The exact quantization of the Hall resistance and the zero of the diagonal resistance over a range of filling factors close to integer are attributed to the localization of electronic states at the Fermi energy in the interior of the 2D-electron gas. As the electron density is changed, charging of the individual localized states may occur by single-electron jumps, causing associated oscillations in the local electrostatic potential. The authors searched for such a manifestation of localized states in the QHE regime with the use of a scanning electrometer probe and observed localized potential signals, at numerous locations, that oscillated with changing electron density. In the general case, the corresponding spatial patterns are rather complex, but well-defined objects are often seen, which evidently arise from individual localized states. These objects interact and from time to time form a lattice-like arrangement (analog of 2D-electron Wigner lattice).

9. Quantum wells, dots and wires

During this session the collective behavior of the interwell excitons in coupled quantum wells, the scattering processes connected with 2D-excitonic polaritons in GaAs-based microcavities, the tunneling processes between strongly disordered electron systems in GaAs heterojunctions, spectroscopy of electron – electron scattering in 2DEG, the theory of the electron g factor in ultra-small metallic grains, quantum dots, and the peculiar behavior of mesoscopic charge density waves were discussed.

We will start by considering 2D systems, coupled double quantum wells, and superlattices, which are very attractive, in particular, because of the fundamental possibility of spatial separation of photoexcited electrons and holes between adjacent quantum wells. In double QWs with an applied electric field which tilts the band, spatially separated electrons and holes are bound to interwell excitons due to the Coulomb interaction. Interwell excitons are long-lived compared to intrawell excitons, so they can easily be accumulated to high enough density, and a gas consisting of these excitons can be cooled down to fairly low temperatures. As a result of broken inversion symmetry, interwell excitons have a dipole moment even in the ground state. The theory predicts various possible scenarios for collective behavior in a dense system of spatially separated electrons and holes. Particularly, it was shown that the condensed dielectric excitonic phase (the analog of a Bose condensate) can only occur in the presence of lateral confinement (random or artificially prepared) in the QW plane. Under this confinement and the associated external compression, it is easier for interwell excitons to build up to the high critical densities sufficient for the appearance of a collective excitonic phase. In the talk presented by V B Timofeev (Institute of Solid State Physics, Chernogolovka), luminescence spectra of interwell excitons in electrically biased double GaAs/AlGaAs QWs were studied. In these n-i-n structures, the electron and hole in the interwell excitons are separated between neighboring QWs by a thin AlAs tunneling barrier. It was found [97] that under resonant excitation by circularly polarized light the luminescence of the interwell exciton line exhibits dramatic narrowing as the exciton concentration grows and simultaneously the degree of circular polarization of the exciton photoluminescence (PL) increases substantially. On resonant excitation by linearly polarized light, the alignment of the interwell excitons increased as a threshold process with increasing optical pumping. By analyzing time-resolved spectra, it was established that under these conditions the radiative recombination rate increases by around an order of magnitude. The observed effect occurs at below-critical temperature and is interpreted in terms of the collective behavior of interwell excitons (Bose condensation). Studies of PL spectra in a magnetic field showed that the collective exciton phase is dielectric and in this phase the interwell excitons still retain their individual properties.

The results of an above experiments on the evolution of PL spectra of spatially interwell excitons with temperature and concentration were interpreted in the theoretical talk by A S Ioselevich (L D Landau Institute). The author claims that the broad, low-energy PL line, presented at all T and rather low densities originates from 'incoherent' excitons bound to individual impurities. However, due to screening effects, the local levels can accommodate only the excitons with a concentration below the threshold, $n \leqslant n_c$; above the threshold (for $n > n_c$) the excess excitons remain delocalized (or localized only at very large scale). Kosterlitz – Thouless type spatial coherence may develop within this group of delocalized interwell excitons at sufficiently low T, which results in a narrow, high-energy peak in the PL spectra, observed only at high concentration and low temperatures.

There has been a great interest in the study of 2D exciton – polariton phenomena in semiconductor microcavities during the last few years, and the report by V D Kulakovskiĭ et al. (Institute of Solid State Physics, Chernogolovka) concerned this subject (see Ref. [2], p. 54). Semiconductor microcavities are planar Fabry – Perot resonators in which the mirrors are formed from distributed Bragg reflectors, alternating quarter-wavelength layers of materials of low and high dielectric constant. In the case considered, the authors used a typical GaAs-based microcavity with six InGaAs quantum wells placed in the center of cavity. For experiments in the strong exciton – photon coupling regime at least 20 AlGaAs/AlAs mirror repeats are commonly employed, to achieve the required high cavity finesse. In a microcavity, the photon and exciton states are both quantized along the growth direction (z), and due to that a one-to-one correspondence in k-space between exciton and photon states is realized. However, along the growth direction the photon has a finite lifetime due to leakage through the mirrors. So, 2D excitons are thus directly coupled to external photons, and as a result polariton states and polariton-related phenomena in the strong coupling limit can be studied directly in optical experiments. This situation contrasts strongly with that in either bulk material or quantum wells, and is the principal reason why new polariton physics can be studied in microcavities. One intriguing property of 2D-excitonic polaritons is connected to their bosonic nature. Recall that the mixed exciton – photon state (or lower polariton branch) in microcavities is characterized by an extremely small effective mass ($< 10^{-4} m_0$, m_0 — the free electron mass). Due to that, the high filling at the excitonic polariton band bottom ($k = 0$) can be achieved at relatively low densities ($na^2 \ll 1$, where n is the excitonic polariton density and a is the Bohr radius). In this case, the bosonic nature of polaritons can favor stimulated interband (intraband) scattering and allow, in principle, Bose condensation. Polariton – polariton scattering has been measured by Kulakovskiĭ et al. in a GaAs-based microcavity by polarized photoluminescence technique using circularly and elliptically polarized resonant photoexcitation into the lower 2D-polariton branch. The scattering has been found to be strongly enhanced when the singlet biexciton state is allowed as an intermediate one, i.e. under elliptically polarized excitation. An exponential growth of $k = 0$ low polariton emission was found at high excitation densities for the main spin population leading to an emission polarization degree higher than that of excitation. This effect indicates that stimulated scattering appears due to the bosonic nature of excitonic polaritons when the high filling of excitonic polaritons at $k = 0$ is reached.

The tunneling processes between strongly disordered 2D electrons in GaAs heterostructures were studied and discussed in the talk presented by Yu V Dubrovskiĭ et al. (Institute of Microelectronics Technology, Chernogolovka, Grenoble High Magnetic Field Laboratory, the Universities of Nottingham and Sheffield), see Ref. [98]. 2D-electron accumulation layers are formed by extremely narrow layers of Si donors (Si delta doping) in GaAs on either side of the AlGaAs tunnel barrier. It was found that at liquid helium temperature the high magnetic field creates a Coulomb gap in the tunneling density of states, due to electron correlations in the layers, that depends linearly on the magnetic field. The temperature dependence of the magnetic field variation of the equilibrium tunneling conductance reveals features which could be tentatively interpreted as a manifestation of the insulator – quantum-Hall-insulator transition. Besides, strong interaction between Landau levels of different 2D subbands has been also observed as anticrossing of the related peak positions with the magnetic field.

Electron – electron scattering processes in a 2DEG were the focus of discussions in the talk presented by H Buhmann et al. (Würzburg University and Institute of Low Temperature Physics, Khar'kov) (see Ref. [2], p. 57]). The main concern of this work was a decisive difference in momentum transfer of 2D electrons compared with 3D case. These differences are due to the conservation of energy and momentum controlled by a parameter $\max(T, \epsilon)/\epsilon_F$, which characterizes the degree of degeneracy (here ϵ is the nonequilibrium electron energy counted from the Fermi energy ϵ_F). In contrast to the 3D case, electron – electron collisions in 2DEG lead to scattering at very small angles, $\phi \sim \epsilon/\epsilon_F$. However, the probability of collisions of electrons with almost opposite momenta, $\mathbf{p} = -\mathbf{p}_1$, is drastically increased compared to the 3D case. In these experiments, the electron-beam injection and detection via quantum point contact (QPC) were performed to study the scattering of a nonequilibrium electron distribution in a 2DEG in a GaAs-based heterojunction. The QPC are defined electrostatically by applying a negative voltage to lithographically processed Schottky gates (such a technique was used for reasonably good control of the electron-beam opening angle). Thus, the characteristic scattering distribution function was obtained directly from magnetic field-dependent beam deflection experiments. It was found that the scattering distribution function broadens with increasing electron kinetic energy in contrast to 3D systems where width is energy-independent. Furthermore, a pronounced dip occurs at a small angle. These observations exhibit conclusive evidence for the manifestation of 2D density-of-states effects and the importance of the small-angle e – e scattering process in 2D systems.

The problem of the g factor of individual electron levels in metallic nanoparticles and in quantum dots was considered in the talk given by A I Larkin (University of Minnesota and L D Landau Institute). First of all, the effect of the spin-orbit scattering was shown to be much stronger than in the usual Fermi-liquid approach. The reason is that spin-orbit scattering rate τ_{so}^{-1} should now be compared to the level spacing δ, instead of the Fermi energy E_F. A strong effect of spin-orbit scattering occurs at $\tau_{so}\delta \ll 1$. For a nanoparticle or quantum dot of general shape, both the spin and orbital motion contribute to the total magnetic moment. For 3D diffusive nanoparticles, both contributions are shown to be small, resulting in $g \sim \max[(\delta\tau_{so})^{1/2}, (l/L)^{1/2}]$. If electron scattering occurs at the boundaries (i.e. $l \geqslant L$), the g factor is of order 1. In a 2D quantum dot the orbital contribution to the g factor is the main one; it reaches values of the order of $\sqrt{k_F l}$. For GaAs quantum dots the orbital contribution is further enhanced by the small effective mass of electrons: $g \sim (m/m^*)\sqrt{k_F l}$. These results are published in Ref. [99].

Although mesoscopic effects in superconductors have been very intensively studied for several years, almost nothing was known about the behavior of another kind of correlated-electron state — charge density wave — in micron-size samples. The fabrication technique and results of the first experimental study of mesoscopic effects in quasi-1D charge-density-wave compounds $NbSe_3$ and TaS_3 were presented by H van der Zant from Delft University of Technology (see Ref. [2], p. 61). In the compound $NbSe_3$, a considerable decrease of the depinning electric field E_T was found along with a decrease of the length between voltage contacts down to about 1 μm. Even more striking are the results obtained on TaS_3 samples: the current – voltage characteristics of micron-size segments were strongly segment-dependent. Some of the segments show a negative differential resistance; occasionally, even the full resistance of a segment was measured to be negative. However, on longer scales — with the distance between contacts larger than 10 μm — all the $I(V)$ characteristics were quite regular. No theory is available to describe these phenomena.

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