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Gravitational-wave astronomy: new methods of measurements

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Abstract. The current status of gravitational-wave astronomy is reviewed. Advances in ground-based antennae are discussed and new methods of measurement that promise a considerable improvement in sensitivity are described. The promise held out by the improved antenna technologies is discussed.

1. Historical introduction. Searching for source models and main characteristics of antennae

The purpose of this paper is to describe to the reader the state of the art in the part of astrophysics usually called gravitational-wave astronomy. This part includes the programs of studies and developments, as well as the discussion of unsolved problems, which are related to ground-based antennae using free masses. During the next 4 – 8 years, these antennae are expected to bring qualitatively new astrophysical information: the registration of bursts of gravitational radiation from astrophysical catastrophes that occurred hundreds of millions of years in the past. The main part of the paper is devoted to the key problem, the free-mass antenna sensitivity, and to the development of qualitatively new methods of measurements. The first introductory section is devoted to the reader not acquainted with this part of physics, and also includes a short historical description of the development of gravitational-wave astronomy. Sections 2 and 3 give the state of the art in the construction of ground-based antennae as of the moment of writing of this paper and point to possible ways of solving some non-trivial problems of measurements, which determine the antenna's sensitivity. In Sections 4 and 5 we give a short description of possible consequences resulting from successive operation of antennae, for gravitational-wave astronomy and for other fields of physics.

The emission of gravitational waves by masses moving with alternating accelerations, predicted by A Einstein in 1918 (see [1] and also [2, 3]), was first met without interest by experimentalists. The reason for this was the smallness of the effect for any reasonable laboratory experiment. The power of gravitational radiation for small derivatives of the acceleration is

$$\dot{\mathcal{E}}_{\rm grav} = \frac{G}{45c^5}\left(\frac{\partial^3 D_{\alpha\beta}}{\partial t^3}\right)^2, \tag{1}$$

where G is the gravitational constant, c is the speed of electromagnetic and gravitational wave propagation, $D_{\alpha\beta}$ is the quadrupole moment of masses

$$D_{\alpha\beta} = \int_V \rho(3x^\alpha x^\beta - \delta_{\alpha\beta} r^2)\,{\rm d}v\,. \tag{2}$$

The factor $Gc^{-5} \approx 10^{-60}$ (in CGS units) is due to the quadrupole character of radiation from gravitational 'charges' (gravitational masses). The quadrupole character, in turn, is the consequence of the experimental fact, usually called the equivalence principle, which is one of the postulates of general relativity (GR). For two equal point masses M separated by a distance l and rotating around the barycenter with a frequency ω, equation (1) takes the form

$$\dot{\mathcal{E}}_{\rm grav} = \frac{128}{5}\,\frac{G}{c^5}\,M^2 l^4 \omega^6\,, \tag{3}$$

which shows that for $M = 10^6$ g, $l = 10^2$ cm, $\omega = 3 \times 10^2\ {\rm s}^{-1}$, the value of $\dot{\mathcal{E}}_{\rm grav}$ is as small as 10^{-23} erg s^{-1}.

It is such estimates that underlay the pessimism of experimentalists in detecting this radiation in the laboratory.

Now it seems evident that the smallness of the factor Gc^{-5} can be 'compensated' by a high value of the factor M^2, if $M \simeq 10^{33}$ g (i.e. of order of the solar mass). In other words, one can expect a significant values of $\dot{\mathcal{E}}_{\rm grav}$ from astrophysical

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sources. However, 30 years had passed after Einstein's prediction when Fock [4] pointed out a good prospect for astrophysical sources: In 1948 he noted that for the planet Jupiter orbiting around the Sun $\dot{\mathcal{E}}_{\rm grav} \simeq 4\times \times 10^9\,{\rm erg\,s^{-1}} \simeq 400$ W. A simple examination of the catalogs of known binary stars [6] made at the beginning of the 1960s, immediately revealed that there are about ten binary systems with orbital periods $\omega \simeq 10^{-3}\,{\rm s^{-1}}$ (one turn per several hours) and $l \simeq 10^{11}$ cm, which emit $\dot{\mathcal{E}}_{\rm grav} \simeq 10^{30}-10^{31}\,{\rm erg\ s^{-1}}$ (i.e. about 1% of the electromagnetic power of the Sun).

The next logical step was to answer the question: Are there any explosive sources (i.e. astrophysical catastrophes) with a mass of the order of the solar mass but with a frequency $\omega_{\rm grav}$ much higher than in close binary stars? At the beginning of the 1960s, some papers evaluated $\dot{\mathcal{E}}_{\rm grav}$ and $\omega_{\rm grav}$ for simple models of such catastrophes. At this stage, a great contribution was made by Ya B Zeldovich, N S Kardashev, I D Novikov, I S Shklovskiĭ [6, 7]. The discovery of pulsars (rotating neutron stars with a large magnetic moment whose axis does not coincide with the rotational one) by J Bell and A Hewish in 1967 [8] was an important argument favoring the existence of such sources. A few years later, J Taylor and R Hulse discovered a pulsar in a binary system with a neutron star companion [9]. Careful measurements of the orbital period decrease in this binary system due to the energy losses by gravitational radiation allowed the validity of Eqn (1) to be checked for the first time with an accuracy of a few percent. Clearly, at the moment of coalescence of the two components the power $\dot{\mathcal{E}}_{\rm grav}$ should be maximal. This type of cataclysm has become 'candidate number one' for experimentalists, who by that time already had some experience in constructing such antennae (see below). A detailed analysis of the coalescence process of binary neutron stars has been conducted for many years in several groups. Although this analysis has not yet been completed (see the review by K S Thorne [10] and references therein), one can say that a certain consensus between theoreticians has been achieved, which states that the total energy emitted should be about $\mathcal{E}_{\rm grav} \simeq 10^{52}$ ergs and the wave packet of the radiation should last for several seconds with an increasing mean frequency from tens to several hundreds Hz. The key question for experimentalists, namely, how often such events occur, has not been definitely solved as yet, and there exist both pessimistic and optimistic prognoses.

In the 'semi-optimistic' scenario (prognosis) by H Bethe and G Brown [11] the coalescence of two neutron stars should happen on average once per 10^4 years in one galaxy. This implies that in a space volume with radius $R = 10^{26}$ cm (i.e. about 30 Mpc), which comprises near 10^5 galaxies, about 10 coalescences (and, hence, bursts of gravitational radiation) should occur during one year. Therefore, the terrestrial observer could register about one gravitational-wave burst with intensity $\tilde{I} \simeq 10-0.1$ erg $\rm s^{-1}\,cm^{-2}$ with changing frequency at hundreds Hz during an integration time of one month. It is such intensities and event rates that are expected to be detected by two large ground-based free-mass antennae LIGO[1] and VIRGO[2] (see review [2] and references therein). These and other projects will be discussed below.

[1] LIGO from Laser Interferometric Gravitational-wave Observatory. The project was initiated by California and Massachusetts Institutes of Technology.

[2] VIRGO is the name of the Franco-Italian project.

A gravitational wave is the wave of acceleration gradients (components of the Riemann curvature tensor) perpendicular to the direction of the wave vector alternating in time and space, which propagates with the speed of light. For example, if a sine-like gravitational wave propagates along the z-axis, in one half period the acceleration gradient is positive along the x-axis and is negative along the y-axis. Over the next half a period, the direction of the gradients reverses. According to a convenient expression by K S Thorne, the gravitational wave is a ripple over the static curvature. Thus, it is impossible to discover a gravitational wave at one point. However, this becomes possible using two point masses separated by a finite distance L or an extended body. In the first case one needs to measure the change in the distance $\Delta L_{\rm grav}$ between the masses, which for some optimal orientation of the wave and the two masses is

$$\Delta L_{\rm grav} = \frac{1}{2}\,hL\,, \tag{4}$$

where h is the dimensionless amplitude of the wave (metric perturbation amplitude). For further description it is important to stress that this shift in the position is due to a changing force with an amplitude

$$F_{\rm grav} = \frac{1}{2}\,hLm\omega_{\rm grav}^2\,, \tag{5}$$

where $\omega_{\rm grav}$ is the gravitational wave frequency. The value $h\omega_{\rm grav}^2$ is just the amplitude of the wave component of the Riemann tensor. Therefore, both gravitational-wave emitters and detectors are of quadrupole type. The term 'free-mass antennae' means that the frequency of real mass suspensions is much smaller, and the frequency of their internal mechanical modes is much higher, than the gravitational-wave frequency $\omega_{\rm grav}$. It is relevant to notice that a plain gravitational wave, i.e. far away from the source, has two independent polarizations turned with respect to each other by 45° and usually denoted by h_+ and $h_\times$. So the total amplitude of the wave h in equations (4) and (5) is

$$h = \sqrt{h_\times^2 + h_+^2}\,. \tag{6}$$

It is not difficult to express h through the intensity $\tilde{I}$:

$$h = \frac{1}{\omega_{\rm grav}}\sqrt{\frac{16\pi G}{c^3}\,\tilde{I}}\,. \tag{7}$$

Assuming $\tilde{I} = 10$ or 0.1 erg $\rm s^{-1}$ $\rm cm^{-2}$ and $\omega_{\rm grav} = 10^3$ rad $\rm s^{-1}$, we obtain from Eqn (7) $h \simeq 10^{-21}$ or $h \simeq 10^{-22}$, respectively. If $L = 4\times 10^5$ cm and $m = 10^4$ g (these are the distance between the test bodies and their mass, respectively, in the LIGO project), Eqns (4) and (5) imply that the amplitudes $\Delta L_{\rm grav} \simeq 2\times 10^{-16}$ cm or 2×10^{-17} cm should be measured, which are due to forces $F_{\rm grav} \simeq 2\times 10^{-6}$ dyne or 2×10^{-7} dyne, respectively. These estimates will be used below to evaluate the required parameters of antenna and measurement equipment. It is important to emphasize that the force $F_{\rm grav}$ reveals itself in the relative motion of one test mass with respect to another, and hence the measuring devise should be set up between the masses.

As was already noted, the antenna should not necessarily use two separated masses: one can take an extended massive body and measure variable tensions inside it caused by $F_{\rm grav}$. The first to suggest such an antenna was J Weber.

In 1968, he announced the discovery of gravitational radiation in the coincidence scheme with two such antennae [13]. The sensitivity reached in his experiments corresponded to $h \simeq 10^{-16}$ (i.e. five orders of magnitude (!) worse than what is being planned at the initial stage of the LIGO and VIRGO projects). At the beginning of the 1970s, Weber's experiments were repeated by several independent groups (and in particular, by the MSU – IKI group [14]) using similar detectors. The tests gave negative results: no bursts were discovered. Despite this fact, Weber, in my opinion, deserves recognition by experimentalists as the pioneer of a new astrophysical field. One of the reasons for the failure of his experiments was the absence of a reliable astrophysical prognosis of the event rates, their amplitude and frequency.

The invention of lasers in 1961 by T Meiman had an important effect on choosing the scheme for the antenna. Immediately after this discovery, in 1961, M E Gertsenshtein and V I Pustovoit [15] suggested using two widely separated freely-suspended massive mirrors (as the test masses), which form an optical Fabry – Perot resonator. The pumping of this resonator with powerful coherent laser radiation should provide a high sensitivity to slight oscillations of such mirrors caused by a gravitational wave. This concept underlies the LIGO and VIRGO projects. In the middle of the 1970s, R Drever and R Weiss had already gained significant experience with small laboratory models of such an antenna. Together with K S Thorne, they organized a triumvirate, which justified the necessity of developing a large-scale antenna based on this principle (it is this antenna that has been called LIGO). K S Thorne played a special role in this matter. Being an outstanding theoretician and specialist in general relativity, he simultaneously has a deep understanding of experimental physics. Apparently, his intuition suggested that the accumulation of astrophysical information should allow soon meanwhile provide experimentalists with a reliable prediction of the values of expected h, $\omega_{\rm grav}$, and event rates, and that the period could be spent in getting experience using a large prototype of the full-scale antenna. In 1981, the National Science Foundation (NSF) in the USA started to finance the design and construction of a prototype of the large antenna (which has $L = 4 \times 10^3$ cm). After 15 years of work, researchers from Caltech and MIT have reached a sensitivity for this prototype of $h \simeq 10^{-19}$ (with an integration time $\tau \simeq 10^{-2}$ s and a signal-to-noise ratio of order one). Thus, about two decades after Weber's experiments, the sensitivity has been increased by three orders of magnitude (see [16] and references therein). In 1996, NSF began financing the construction of the full-scale antennae ($L = 4 \times 10^5$ cm). The construction has been completed in the last year, and now the gradual adjustment of two such antennae is under way with the aim of reaching a sensitivity of $h \simeq 10^{-21}$ by the fall of 2001 (the LIGO-I project), after which a long-term recording of the distance variations between the mirrors in the coincidence scheme must begin. In about 2005, significant modifications are planned, among which are a change of the mirrors' suspension and their insulation from noise. As a result, the sensitivity is expected to increase up to $h \simeq 10^{-22}$ (the LIGO-II project). Originally, the LIGO project was purely national. However, the NSF encouraged cooperation not only between American universities and institutes, but also the participation of foreign groups (in particular, two Russian groups: one from M V Lomonosov Moscow State University (MSU), another from the Institute of Applied Physics of Russian Academy of Sciences). Presently, more than 200 researchers participate directly in the LIGO project, and there is a large exchange with participants form the VIRGO project (which uses one antenna with arms $L = 3 \times 10^5$ cm), as well as with the more modest (in the size of L) projects GEO-600 and TAMA. For a detailed description of wave solutions in general relativity and many other interesting events related to gravitational-wave astronomy, two large reviews [17, 18] are recommended to the reader. The author has restricted himself to a very short historical introduction above, which, however, is sufficient to get acquainted with the experimental achievements over the last decade and an understanding of unsolved problems.

2. Thermal and non-thermal noise in free-mass antennae

Through a cross-section with area

$$\lambda_{\rm grav}^2 = \frac{4\pi^2 c^2}{\omega_{\rm grav}^2} \tag{8}$$

during half a period of a gravitational wave a very large number of gravitons pass:

$$N_{\rm grav} = \frac{\widetilde{I} c^2}{4\pi\hbar\omega_{\rm grav}^4} \simeq 10^{45} \frac{\widetilde{I}}{0.1\ {\rm erg\ s^{-1}\ cm^{-2}}} \times \left(\frac{\omega_{\rm grav}}{10^3\ {\rm rad\,s^{-1}}}\right)^{-4}. \tag{9}$$

It is this ensemble of gravitons that produces the gravitational force $F_{\rm grav}$ acting on the test masses of the antenna. The large value of $N_{\rm grav}$ allows us to consider $F_{\rm grav}$ as being a classical force. It is seen from estimate (9) that this can be done even for values of $\widetilde{I}$ much smaller than quoted in the previous section. The classicality of $F_{\rm grav}$ means that there is no need to quantize the gravitational field.

According to the quantum theory of measurements, there is no limit on detecting a small classical force acting on a mass. Consequently, the gravitational antenna sensitivity is expected to increase with improving methods of measuring small $\Delta L_{\rm grav}$ (or other observables related to $F_{\rm grav}$). This important feature has already been used in planning the two 'steps' LIGO-I and LIGO-II, which we discussed in Section 1. Apparently, after having reached the sensitivity $h \simeq 10^{-22}$, the LIGO-III stage with a smaller level of h will be constructed. However, this does not imply that there are no constraints at all to the antenna sensitivity, which is dependent on the Planck constant $\hbar$. In reality they do exist and are determined by the specific method of measurement of the response of the system consisting of two masses on $F_{\rm grav}$. These quantum limitations will be considered in Section 3, and in this section we shall describe the most important sensitivity constraints determined by random actions of thermal and non-thermal origin.

The mirror in the LIGO-I (a fused-quartz cylinder 25 cm in diameter with a thickness of 10 cm) has a mass of about 10^4 g. It is suspended on a thin steel fiber loop with a free length of about 20 cm (the fiber girds about the middle of the cylindric surface). If one considers such a suspended mirror as a Galilean pendulum with a point-like mass and takes into account that the pendulum oscillation frequency is much smaller than $\omega_{\rm grav} \simeq 10^3$ s^{-1}, then the obvious condition for detection of $F_{\rm grav}$ (provided that the only source of random

force action is the thermostat) will be

$$F_{\text{grav}} > \sqrt{\frac{4kTH}{\tau}} = \sqrt{\frac{4kTm}{\tau_{\text{m}}^{*}\tau}}, \tag{10}$$

where T is the thermostat temperature, H is the friction coefficient in the pendulum oscillation mode, and $\tau_{\text{m}}^{*} = m/H$ is the mode relaxation time. Formula (10) is a direct consequence of the fluctuation-dissipative theorem (FDT) for this simple model. Assuming in (10) $F_{\text{grav}} = 2 \times 10^{-6}$ dyne (i.e. for the sensitivity level of LIGO-I) and $\tau = 10^{-2}$ s, we obtain that it is necessary to have $\tau_{\text{m}}^{*} > 4 \times 10^{4}$ s for a signal-to-noise ratio of the order of one. In LIGO-I, such a suspension on steel fibers gives $\tau_{\text{m}}^{*} \simeq 10^{5}$ s. Clearly, this is not sufficient for LIGO-II, in which F_{grav} must be smaller by an order of magnitude.

Adding the condition that the signal-to-noise ratio is 10, formula (10) implies that τ_{m}^{*} must be 4×10^{8} s, i.e. about 12 years at $T = 300$ K, $m = 10^{4}$ g, $\tau = 10^{-2}$ s. Note that condition (10) holds for the so-called viscous friction model, in which $H = \text{const}$. If we consider the losses in the suspension as in the structure friction model, the requirement on τ^{*} will be an order of magnitude smaller. My colleagues from the MSU V P Mitrofanov, O A Okhramenko, and K V Tokmakov, starting from 1991, developed the mirror suspension for the LIGO using fibers made of a super-pure fused quartz (whose internal losses correspond to a mechanical quality of $Q_{\text{m}} \simeq 3 \times 10^{7}$ [19]). Several new technological tricks suggested and realized by them have allowed the achievement of $\tau_{\text{m}}^{*} \simeq 1.7 \times 10^{8}$, i.e. about 5.4 years [20], which is close to what is required in LIGO-II. Such a long relaxation time of mechanical oscillations corresponds to a relative amplitude decrease by 0.3% in five days. It is this value that was measured in the experiment. The volume and goal of the present paper does not permit us to consider the technological details of these methods in more detail. However, the following note is pertinent. Although FDT suggests no recommendations as to what should be done to increase τ_{m}^{*}, nevertheless, by using not too strict semi-empirical relationships confirmed experimentally, one can affirm that in such a suspension under real laboratory conditions $\tau_{\text{m}}^{*} \simeq 10^{9}$ s and even higher values can be reached. As will be clear from the next section, the value of the dimensionless ratio $\tau/\tau_{\text{m}}^{*} \simeq 5 \times 10^{-11}$ already achieved today plays an important role in attaining the so-called standard quantum limit of the sensitivity.

The simple model considered above (a point-like mass and the unique pendulum mode) is not full: it is also necessary to take into account the contribution to the fluctuation action on the mirror's center of mass from thermal fluctuations over the entire length of the fiber. In terms of the Langevin language for Brownian motion, it is necessary to take into account spectral components of numerous violin modes of proper oscillations of the fiber near ω_{grav}, as well as the transfer function of the fiber oscillations to the mirror's barycenter. The calculations, which we omit here, give the following condition: for $h \simeq 10^{-22}$ the quality of the lowest frequency modes of the fiber must be at the level 10^{7}. The measurements by Mitrofanov and Tokmakov have shown that the quality of the low-frequency eigen modes of the same fibers made of the super-pure quartz falls within the range $5 \times 10^{7} - 1 \times 10^{8}$ [21].

There is one more feature of the fluctuation force action of the fiber on the mirror. This feature is that the sources of fluctuations are within the fiber itself and cause its motion. This motion (the change of the fiber coordinates relative to, say, a platform to which the fiber is attached) can be measured by a separate sensor. The calculations by S P Vyatchanin and Yu M Levin [22] have shown that such a measurement allows one to subtract about 99% of random shifts, caused by thermal fluctuations inside the fiber, from the mirror's motion.

To conclude this part of the section, one can say that the problem of 'suppression' of random fluctuations of the mirror's barycenter position, caused by the thermostat in the suspension fibers, can also be solved in the case where the planned sensitivity must be much better than $h \simeq 10^{-22}$.

When considering fluctuations in the mirror suspension fibers, we have so far restricted ourselves to the case of thermal equilibrium, to which FDT can be applied. However, in addition to such fluctuations, there are others which do not obey FDT. It is easy to see that to attain larger τ_{m}^{*}, it is advantageous to decrease the diameter of the fiber (of steel or fused quartz), i.e. it is profitable to approach the stress close to the fiber disruption threshold. The free energy density in polycrystal solids and amorphous bodies is usually of the order of 10^{6} erg g^{-1}. As established long ago, at the tension close to solid body disruption, an acoustic emission is observed, which is the consequence of a partial redistribution of the free energy (vacancy group jumps, dislocation creation, etc.) At present, no theoretical models of such processes have been proposed.

It is important to note that the kinetic energy $m\omega_{\text{grav}}^{2}\,\Delta L_{\text{grav}}^{2}/2 \simeq 2 \times 10^{-22}$ erg for LIGO-I is many orders of magnitude smaller than the total free energy storage in the fiber. In other words, even a tiny fraction of the free energy in the fiber transformed by tension into acoustic emission (i.e. into additional to thermal oscillations of the fiber), can mimic F_{grav}. Recently, A Yu Ageev and I A Bilenko (also from the MSU group), when making detailed measurements of the temporal structure of the Brownian oscillations of the steel string used in LIGO-I have found random short-term changes in the oscillation amplitude exceeding the ordinary values, which correspond to the Langevin model of the Brownian motion:

$$\sqrt{\overline{\delta x^{2}}} = \sqrt{\frac{kT}{m\omega_{\text{m}}^{2}}}\sqrt{\frac{2\tau}{\tau_{\text{m}}^{**}}}, \tag{11}$$

where m is the effective mass of a mode with frequency ω_{m} and relaxation time τ_{m}^{**} [23]. These rare bursts exactly represent excessive (non-thermal) noise. As established from the measurements, such bursts, which can mimic metric variations at the level $h \simeq 10^{-21}$, are observed in the fibers, on average, once every several hours under tensions of about 50% of the breaking stress. Decreasing the tension strongly decreases the false event rate. A coincidence scheme between two antennae, of course, makes it easier to reject such 'signals' imitating the action of F_{grav}, however it is natural that the experimentalists consider it as the last 'line of defense'.

So far, no measurements of the excess fluctuations (of non-thermal origin) in the fibers of super-pure quartz have been performed. This is a much more complicated problem, since for such fibers the value of τ_{m}^{**} is about four orders of magnitude higher than for steel fibers, and, correspondingly, the sensitivity to small fiber oscillations must be at least two orders of magnitude better. A new method of measurements recently suggested by I A Bilenko and M L Gorodetskiĭ [24] will, in my opinion, allow this problem to be solved.

Not only thermal fluctuations in the suspension fibers prevent accurate measurements of small $F_{\rm grav}$, which causes oscillations $\Delta L_{\rm grav}$ of the mirror's center of mass. The optical Fabry–Perot resonator is formed by two almost flat (with a curvature radius of about 10 km) surfaces of two quartz cylinders. The mirrors are constructed from a multi-layer coating of the quartz cylinders' surface, which has a high reflectivity. It is clear that the laser interferometer measures not the displacement of the cylinder's barycenter, but the sum of this displacement and the proper internal oscillations of the cylinder. If they are due to internal forces (for instance, due to the inner friction inside the material which, according to FDT, is the source of the volume forces), they do not shift the barycenter, but can give an appreciable noise contribution to the quantity being measured.

To calculate this contribution, A Gillespi and F Raab [25], using the same Langevin model for modes, estimated the fraction of the total variance of the mean coordinate of the mirror (i.e. the cylinder's surface) near $\omega_{\rm grav}$. Note that $\omega_{\rm grav}$ is significantly lower than the proper frequencies of the cylinder's modes, so the contributions from low-frequency wings of several tens of modes have been taken into account. This calculations showed the possibility of attaining a level close to $h \simeq 10^{-22}$ only at low frequencies $\omega_{\rm grav} \leqslant 10^3$ rad s^{-1} at a signal-to-noise ratio close to one. The same calculations for a sapphire cylinder (whose internal losses are an order of magnitude smaller than for quartz, and the density and velocity of sound is two times as large) show that it is possible to gain three times in sensitivity near $\omega_{\rm grav} \simeq 10^3$ rad s^{-1}, if the losses inside the material are of so-called structural character.

Unfortunately, fluctuations of the cylinder's surface (the mean coordinate of the surface of the mirror) in the frequency domain of interest include not only pure Brownian fluctuations, which can be calculated in the linear Langevin model. There is at least one more effect of a non-linear origin that contributes almost as much as Brownian fluctuations. This effect is the consequence of anharmonism of the lattice, which gives rise to thermal expansion and temperature fluctuations δT. If for some reason the temperature inside the layer adjacent to the mirror's surface l changes by some value δT, the displacement of the outer surface of the mirror will be significant even for tiny temperature changes:

$$\delta l = \alpha_{\rm T} l \delta T = 10^{-17}\,{\rm cm}\ \frac{\alpha_{\rm T}}{5\times 10^{-7}\,{\rm K}^{-1}}\ \frac{l}{10^{-2}\,{\rm cm}}\ \frac{\delta T}{2\times 10^{-9}\,{\rm K}}\,, \tag{12}$$

where $\alpha_{\rm T}$ is the coefficient of linear expansion. As seen from estimate (12), under these conditions δl is of order of $\Delta L_{\rm grav} \simeq 2\times 10^{-17}$ cm for δT of about 1 nanokelvin. Such temperature fluctuations can result from ordinary equilibrium fluctuations, whose total variance is [26]

$$\langle \delta T^2 \rangle = \frac{kT^2}{C\rho V}\,, \tag{13}$$

where C is the specific heat capacity, ρ is the density, and V is the volume. In order to find the fluctuations, corresponding to (13), of the mean coordinate of the mirror's surface near the frequency $\omega_{\rm grav}$ of interest here, it is necessary to know the frequency dependence of the dissipation inside the mirror's material. Assuming the losses inside the mirror to correspond only to the thermoelastic model, M L Gorodetskiĭ and S P Vyatchanin obtained analytical expressions (see [27, 28]) for the spectral density of fluctuations of the mean coordinate in the mirror's surface inside a circle of radius r_0. The introduction of the parameter r_0 is significant, because only from such a spot does the laser beam get information about the mirror's motion. Calculations show that for $r_0 \simeq 1.6$ cm, which was initially planned for LIGO-II, and the mirror made of fused quartz, the sensitivity limitations due to this effect are three times smaller than due to 'purely' Brownian limitations. At present, monocrystal sapphire is being widely discussed as a possible 'candidate' for the mirror material. For this material, the above effect is twice as large as the Brownian one for the same r_0. There are at least two possibilities to reduce the contribution of this effect to the total fluctuations, which do not relate to the mirror's barycenter motion. The first possibility is to appreciably increase r_0 (see [27]), the second suggests subtracting fluctuations of the mirror's surface coordinate (recall that they are due to internal forces, which do not shift the position of the mirror's barycenter).

It is quite possible that the analysis [27, 28] neglects other fluctuation processes that could lead to additional random variations of the mirror's surface and, correspondingly, to increase the threshold value of h (in particular, excessive noise). Clearly, only direct measurements can give a definite answer.

Concluding this section, we should note that here we have described the state of the art in solving two of the most difficult experimental problems. It is also relevant to stress that our description had a semi-qualitative character with simple numerical estimates. In the cited references, the reader can find a lot of details which were omitted, in particular, the spectral representation of the limiting sensitivity in the frequency range 10 rad s^{-1} $< \omega_{\rm grav} <$ 10^4 rad s^{-1}. Due to limit of space and bearing in mind the purpose of the present paper, we have not considered the solutions of simpler problems, such as antiseismic insulation of the mirrors and a strategy of monitoring other possible sources of force action on the mirrors.

Finally, we can emphasize that the problem of choice of the mirror and its suspension proved to be, in essence, a rather difficult experimental task, which has required a lot of time. This can be illustrated as follows. It took in total 15 years to reach a sensitivity of $h \simeq 10^{-19}$ in the LIGO-I prototype, and about 8 years to obtain a relaxation time of $\tau_{\rm m}^* \simeq 5.4$ years for LIGO-II [20].

3. Antenna measurement system, the sensitivity limits of quantum origin and the problem of energy in the system

The scheme of the measurement system in LIGO (with some simplifications) is shown in Fig. 1.

Its principal elements include two optical Fabry–Perot resonators formed by two pairs of mirrors AA′ and BB′. The distance between the mirrors in each pair is $L = 4\times 10^5$ cm. Optical oscillations inside them are excited by laser *1* (with the wavelength $\lambda = 1064$ nm) through beam-splitter *2*, which together with additional mirrors *3* and *4* connects the resonators. Mirrors A and B are 'deaf', in other words their reflection coefficient R is close to one. Mirrors A and B are more transparent with a lower R. In LIGO-II, mirrors with fineness $\mathcal{F} = \pi/(1-R) \simeq 3\times 10^3$ are planned. The concept of this scheme was first suggested by Drever [29]. Later this scheme was analyzed in detail and modified by several groups (see references cited in [12]).

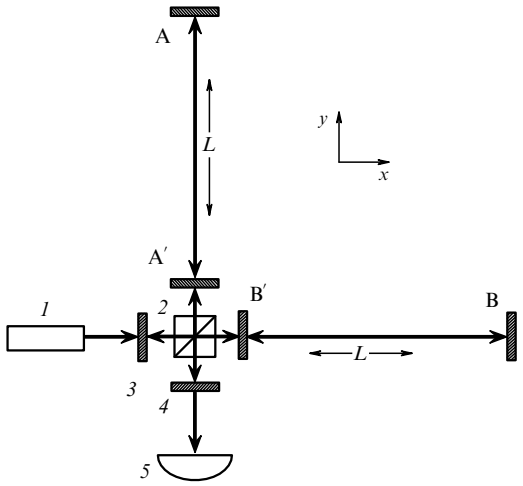

Figure 1.

If a gravitational wave propagates along the axis x, the force $F_{\rm grav}$ acts between the mirrors AA′ and their separation changes with amplitude $\Delta L_{\rm grav} \simeq hL/2$, while the distance between B and B′ does not change. If the wave propagates along the y-axis, the roles of the mirrors reverse. When the wave propagates along the normal to the plane xy and the wave polarizations are optimal with respect to the axes x and y, both pairs of mirrors will oscillate in antiphase so that the total response will be twice as large.

A special control scheme (with a feed-back) allows adjustment of the mutual positions of the mirrors and the tuning of resonance mode frequencies in these schemes accordingly. Note that in the feed-back the upper boundary of monitoring frequencies is much lower than $\omega_{\rm grav}$, so the mirrors react on $F_{\rm grav}$ as essentially free masses. The accuracy of mirror position control up to this upper boundary is small fractions of $\lambda/\mathcal{F}$.

For a certain position adjustment of all six mirrors, the amplitude of oscillations of the phase difference of the optical field in two arms $\Delta\varphi$ is proportional to h.

The depth of modulation of the power flux coming into detector 5 is, in turn, proportional to $\Delta\varphi$. Note that the tuning of the optical system is such that the detector is working almost in the 'dark field' regime.

If a burst of gravitational radiation has a wave-packet form with duration τ and frequency $\omega_{\rm grav}$, one can obtain simple formulas for the two cases: when $\pi/\omega_{\rm grav}$ is smaller than the relaxation time of optical oscillations $\tau^*_{\rm opt}$, and $\pi/\omega_{\rm grav} > \tau^*_{\rm opt}$ in the opposite case. The time $\tau^*_{\rm opt}$ can be expressed through R and $\mathcal{F}$ and in one of the LIGO-II variants under discussion is of order

$$\tau^*_{\rm opt} = \frac{L}{c(1-R)} = \frac{\mathcal{F}L}{\pi c} \simeq 4\times 10^{-3}\,{\rm s}\,\frac{\mathcal{F}}{10^3}\,\frac{L}{4\times 10^5\,{\rm cm}}\,. \quad (14)$$

If $\pi/\omega_{\rm grav} \geqslant \tau^*_{\rm opt}$, then

$$\Delta\varphi \simeq \frac{2\pi L}{\lambda(1-R)}\,\frac{\Delta L_{\rm grav}}{L} \simeq \frac{\mathcal{F}L}{\lambda}h \simeq 4\times 10^{-10}\,{\rm rad}\,\frac{\mathcal{F}}{10^3} \times \frac{L}{4\times 10^5\,{\rm cm}}\,\frac{h}{10^{-22}}\left(\frac{\lambda}{10^{-4}\,{\rm cm}}\right)^{-1}. \quad (15)$$

If $\pi/\omega_{\rm grav} < \tau^*_{\rm opt}$, then

$$\Delta\varphi \simeq \frac{1}{2}h\omega_{\rm opt}\,\frac{\pi}{\omega_{\rm grav}} \simeq 4\times 10^{-10}\,{\rm rad}\,\frac{h}{10^{-22}}\,\frac{\omega_{\rm opt}}{2\times 10^{15}\,{\rm s}^{-1}} \times \left(\frac{\pi/\omega_{\rm grav}}{4\times 10^{-3}\,{\rm s}}\right)^{-1}. \quad (16)$$

These numerical estimates show that the values of $\Delta\varphi$ from (16) and (15) are nearly equal at $\pi/\omega_{\rm grav} \simeq \tau \simeq \tau^*_{\rm opt}$.

It is important to note that measurements of small quantities $\Delta\varphi$ over a short time interval τ in such an interferometer are essentially the matching of the optical field oscillations in resonators AA′ and BB′. The ratio $\Delta\varphi/\tau$ is the deviation of one frequency with respect to other, and the ratio $\Delta L_{\rm grav}/L = h/2$ is the relative value of this deviation equal to $\Delta\omega_{\rm opt}/\omega_{\rm opt}$. From this point of view, the sensitivity $h \simeq 10^{-19}$ already reached in the LIGO prototype [16] (in which $L = 4\times 10^3$ cm) is, undoubtedly, an outstanding result. For comparison, it is sufficient to mention that the smallest relative deviations of two stable generators (Allan's variance) measured so far are only about 10^{-17}.

At the end of this section, we shall use the above estimates for $\Delta\varphi \simeq 10^{-10}$ rad (for LIGO-II).

As was noted in the Introduction, in gravitational-wave antennae at a certain sensitivity level there appear some kind of 'barriers', sensitivity limitations of quantum origin. They depend on the choice of the observable and measurement procedure. Such restrictions were predicted in experiments with macroscopic masses in the coordinate measurements with a finite averaging time in 1967 [30]. A few years later [31] it became clear that the same type of limitations can be directly derived from Heisenberg's uncertainty relations for coordinate measurements, taking into account the finiteness of the measurement time τ. Clearly, there exist a whole family of such limits if measurements of an observable involve the direct measurement of a coordinate (geometrical distance, field strength, etc.) They are called 'standard quantum limits'. For example, in the case of a free mass we are interested in, the standard quantum uncertainty of its coordinate is

$$\Delta x_{\rm SQL} = \sqrt{\frac{\hbar\tau}{2m}} \simeq 2\times 10^{-17}\,{\rm cm}\left(\frac{\tau}{10^{-2}\,{\rm s}}\right)^{1/2}\left(\frac{m}{10^4\,{\rm g}}\right)^{-1/2}, \quad (17)$$

if two point observations of the coordinate x are made at the beginning and the end of a time interval τ.

By equating the free mass coordinate uncertainty to $\Delta x_{\rm SQL}$, one can find an analytical expression for the standard limit of the metric perturbation amplitude $h_{\rm SQL}$ caused by a wave packet with duration τ and mean frequency $\omega_{\rm grav}$ in the scheme of the LIGO-II interferometer (with four test masses)

$$h_{\rm SQL} = \sqrt{\frac{8\hbar}{m\omega^2_{\rm grav}L^2\tau}} \simeq 2\times 10^{-23}\left(\frac{m}{10^4\,{\rm g}}\right)^{-1/2} \times \left(\frac{\omega_{\rm grav}}{10^3\,{\rm s}^{-1}}\right)^{-1}\left(\frac{L}{4\times 10^5\,{\rm cm}}\right)^{-1}\left(\frac{\tau}{10^{-2}\,{\rm s}}\right)^{-1/2}. \quad (18)$$

Equation (18) is correct for $\tau \geqslant 2\pi/\omega_{\rm grav}$. As seen from estimates (17) and (18), the planned LIGO-II sensitivity $h \simeq 10^{-22}$ is very close to $h_{\rm SQL}$. In other words, the contribution of purely quantum fluctuations to the total measurement error will be significant (see [27] for more detail).

A 'recipe' to get around this obstacle has been known for quite a long time. As noted in Section 1, for antennae

there are practically no sensitivity limits of quantum origin. It is only necessary to choose other observables to detect $F_{\rm grav}$. The 'recipe' usually referred to as quantum non-demolition (QND) measurements was initially suggested for gravitational antennae (see review [32]). However, it also proved attractive for opticians studying quantum phenomena, and QND-measurements have been successfully realized in optical experiments, although with a rather large number of photons (see review [33]). Recently, the same measurements were performed with microwave quanta as well: the presence of a single microwave quantum in a resonator was measured without absorption of the quantum [34]. Up to the present time, this has not yet been done with mechanical test masses (a free mass or an oscillator). However, without solving this, a much more difficult, problem, it is impossible to increase the gravitational antenna sensitivity keeping the values of L and M.

Possible schemes for QND-measurements in antennae have been analyzed by the MSU group over last few years. The first suggestion was to use larger fineness values $\mathcal{F}$ in mirrors thereby drastically increasing $\tau^*_{\rm opt}$ [35]. At the already reached $\mathcal{F} = 2 \times 10^6$ [36] the value $\tau^*_{\rm opt} \simeq 10$ s for $L = 4 \times 10^5$ cm. Such a long relaxation time opens the possibility of using the small parameter $\tau/\tau^*_{\rm opt} \simeq 10^{-3}$, which determines the attainable degree of 'squeezing' of the quantum state of the optical field inside resonator. Naturally, the meter itself should be installed inside the resonator and should not absorb optical photons. The variant of such a meter, suggested in [35], was rather the description of a thought experiment than a realistic engineering scheme.

The second attempt to develop such a meter [37] led to the scheme shown in Fig. 2.

Three free mirrors with a maximal $\mathcal{F}$ (and, correspondingly, $\tau^*_{\rm opt}$) form an optical Fabry–Perot resonator. An external laser (not shown in the figure) excites oscillations in the resonator. Inside it, near the tilted mirror B, an additional fourth mirror D is inserted with reduced fineness and, correspondingly, with relatively high transmittance, which should be specified. In such a scheme, all modes in the resonator ABC are split into doublets due to mirror D. At some transmittance of mirror D, when the upper component of the doublet is excited, the ponderomotive force produced by the optical field shifts mirror D if mirror C is displaced. If such a resonator receives some amount of energy of about

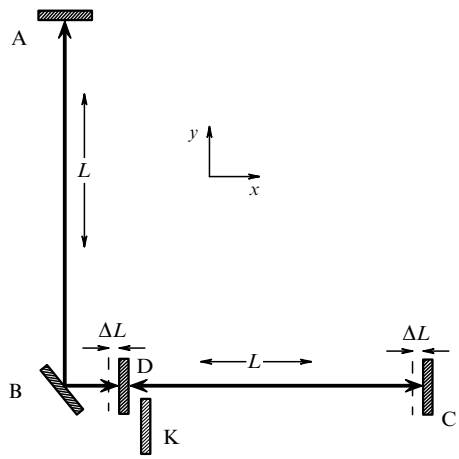

Figure 2.

what is needed to reach $h_{\rm SQL}$, the displacement is $D \simeq hL/2 = \Delta L_{\rm grav}$.

The meter itself must be placed between mirror K, located off the optical field (the ponderomotive force does not act on it), and mirror D. For an ideal QND device (for instance, for speed measurements), the finite value of $\mathcal{F}$, as was calculated by M L Gorodetskii and F Ya Khalili, bounds the sensitivity of such an antenna with the limit

$$h = \frac{h_{\rm SQL}}{\sqrt{\omega_{\rm grav}\tau^*_{\rm opt}}} = 10^{-2} h_{\rm SQL} \left(\frac{\omega_{\rm grav}}{10^3\ {\rm s}^{-1}}\right)^{-1/2} \left(\frac{\tau^*_{\rm opt}}{10\ {\rm s}}\right)^{-1/2}. \tag{19}$$

The analysis of the speed meter recently made by the MSU group in collaboration with K S Thorne using the parameters attainable in the modern cryogenic microwave electronics, showed that one can reach a sensitivity not better than $h \simeq 0.3 h_{\rm SQL}$ [38]. Possibly, to find a much simpler technological solution, it would be convenient to turn the free masses D and K into a mechanical oscillator with eigen frequency near $\omega_{\rm grav}$, by 'adding' to them a stiffness of optical origin with a very low noise level [39], or to use symphoton quantum states in a different antenna topology [40].

The last problem to discuss in this section is the amount of admissible energy inside resonators (or circulating power W). For an ordinary (coordinate) measurement scheme (see Fig. 1) in LIGO-II the projected circulating power $W \simeq 10^{13}$ erg s^{-1} $\simeq 1$ MW. Provided that the mirrors have multi-layer reflection coatings of highest quality, the power dissipated in the coating will be of order 10^7 erg s$^{-1} \simeq 1$ W. The absorption of each optical photon with energy $\hbar\omega_{\rm opt} \simeq 2 \times 10^{-12}$ erg will give rise to a local burst consisting of around 50 additional thermal photons, which in turn, because of a small free-path length (both in quartz and sapphire), will lead to a local shot-like heating of the mirror surface. Due to a non-zero thermal expansion coefficient $\alpha_{\rm T}$, the mirror surface will fluctuate. Such a specific thermo-photon noise was analyzed in detail in paper [27]. The calculations showed that $W \simeq 10^{13}$ erg s^{-1} is indeed very close to the limiting admissible value, if one wish to approach $h_{\rm SQL}$. Clearly, a possible solution is the use of squeezed quantum states in the main resonators, which can be prepared at $\tau/\tau^*_{\rm opt} \ll 1$.

4. Other sources of gravitational waves. Other antennae

The main purpose of the present article is to describe achievements in development of ground-based free-mass antennae and the discussion of novel methods of measurements that can substantially increase the sensitivity. In this section, we restrict ourselves to only a brief enumeration of other directions of studies in gravitational-wave astronomy.

In previous sections we used a simple example to illustrate the conditions required to reach a given sensitivity: a gravitational-wave burst has a duration of $\tau \simeq 10^{-2}$ s and a mean frequency of $\omega_{\rm grav} \simeq 10^3$ rad s^{-1}. The free-mass antennae operate over a very broad frequency range 10 rad s$^{-1} \leqslant \omega_{\rm grav} \leqslant 10^4$ rad s^{-1}. Bursts of radiation from coalescing neutron stars (the preferred source in the LIGO and VIRGO prognoses) must have τ of the order of several seconds and a changing frequency from tens to a few hundred Hz. The *a priori* knowledge of waveforms of such bursts would significantly facilitate the detection and increase the

signal-to-noise ratio. This is one of the problems that is being studied by some theoretical astrophysics groups. Ignorance of the exact equation of state of neutron star material makes this problem even more difficult. In addition, two coalescing components may have strongly different values and orientations of angular momentum. Experimentalists still hope that theoreticians hold their promise to calculate around 10^5 burst waveforms from such sources when the antennae start working in a stationary observational regime.

Coalescing binary neutron stars are not the only sources to be sought by gravitational wave antennae. Stronger bursts can be expected from coalescing neutron stars and black holes and coalescing binary black holes. One more unsolved theoretical problem are the waveforms from such sources and their expected event rate.

Non spherically-symmetric supernova explosions can also generate bursts of gravitational radiation. The event rate of such events in a galaxy is about two orders of magnitude higher than that of coalescing neutron stars in the model by H Bethe and G Brown. Attempts to construct a reliable model of the initial stage of the supernova explosion have failed so far.

With increasing sensitivity, ground-based antennae will be capable of registering gravitational wave noise backgrounds. Some of them should have the same origin as the cosmic microwave background discovered in the mid-60s. The first models of the relic gravitational radiation suggested by L P Grishchuk [41] and A A Starobinskiĭ [42] were further developed by other scientists. Of these new models, in my opinion, the most interesting is the model by R Brustein and G Veneziano [43] based on the superstring theory. This is essentially the first direct prediction of the superstring theory. A notable feature of this model is that it has no divergence between 'before' and 'after' the Big Bang.

At present, two other projects are also developing free-mass antennae. Both these programs invoke satellites. The first of them exploits the Doppler effect as a measure of the 'response' on metric perturbations, as was suggested many years ago [44]. These antennae have a threshold sensitivity of $h_{\min} \simeq 10^{-14} - 10^{-15}$ for gravitational wave bursts with a mean frequency 10^{-2} rad s$^{-1} \leqslant \omega_{\text{grav}} \leqslant 10^{-4}$ rad s^{-1}. The sensitivity is determined, first of all, by the non-stability of microwave autogenerators, which are used in the Earth-satellite channel. In such a program, it would be natural to have two pairs of free masses (the first satellite — Earth and the second satellite — Earth) and to use a coincidence scheme. Unfortunately, there are no such dedicated programs and the experimentalists have to use, as a 'by-product', free hours of communication with single satellites, whose principal goal is to study remote planets of the Solar system (see references in [45, 46] for more detail).

Another program of free-mass antennae (the LISA project) also uses satellites and plans to search the same frequency range of ω_{grav} as the previous one. In many respects this program is similar to LIGO and VIRGO: it intends to use three (drag-free) satellites at the terrestrial distance from the Sun and separated by $L \simeq 5 \times 10^{11}$ cm (i.e. by six orders of magnitude larger than the mirrors in LIGO and VIRGO). Distance variations between the satellites should be measured by a laser interferometer. At the present time, a significant number of ground-based laboratory tests of model units of such antennae and many calculations have already been done, however the final decision about the practical realization of this program has not yet been taken (see, e.g. [47] and a special issue of journal [48]).

5. New physical information which can be obtained with gravitational-wave antennae

In conclusion of this paper, it is relevant to enumerate what new can be expected by physics from using ground-based gravitational-wave antennae.

1. The discovery of bursts of gravitational radiation and the study of their rate of occurrence can give information about the space density of binary neutron stars in galaxies and the contribution they give to the so-called dark matter. Burst waveforms can possibly be used to study the preferential equation of state of the neutron star matter. Short burst waveforms will, perhaps, allow the construction of a model of the initial stage of supernova explosions.

2. More than twenty five years ago General Relativity (GR) turned into an engineering discipline for high-precision space navigation. This essentially means that GR has been checked to a high degree of accuracy, however high-precision navigation inside our Solar system implies at the same time that GR is valid only in the case where gravitational potential is much smaller than c^2. The validity of GR in the ultrarelativistic case (when gravitational potential is of order of c^2) has never been tested. The possibility of such a test will emerge if theoretical astrophysicists can predict the waveforms of the signal from coalescing black holes and if such bursts are really detected.

3. The detection of the relic gravitational wave background will, undoubtedly, provide an invaluable contribution to cosmology. It is however possible that detected chaotic fluctuations of the mirror's barycenter, which are not due to thermal (or non-thermal) fluctuations in the suspension or inside the mirror itself and the fluctuating action of the quantum detector, will not be correlated with the motion of the two pairs of mirrors in a way that the wave components of the Riemann tensor produce. In such a case the prediction of S Hawking [49] (see also [50] and [51]) about the interaction of space-time fluctuations on the Planckian scale with ordinary matter may be realized. These fluctuations are sometimes called the space-time foam. Hawking's prediction is that such an interaction appears in the decoherence of the wave function of an ordinary body, i.e. in small random displacements of the body's barycenter.

2. It is natural to expect that methods of measurements developed for LIGO and VIRGO can be used in other fields of physics. It is worthwhile noting that when QND-detectors of the relative velocity of motion of masses, which allow the determination of the linear momentum with an accuracy better than the standard quantum limit

$$\Delta P_{\text{SQL}} = \sqrt{\frac{\hbar m}{2\tau}},$$

are used in antenna measurement systems, the energy, determined through the momentum, will be measured with an error $\Delta\mathcal{E}$ less than $\hbar/\tau$ [52].

To conclude, it should be said that all the expenses on the LIGO project over 20 years, most of which were spent to construct buildings and vacuum equipment, are less than a fourth part of the cost of a nuclear submarine. Yet mankind continues manufacturing several such submarines each year, which, in contrast to LIGO, are incapable of making it any wiser.

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