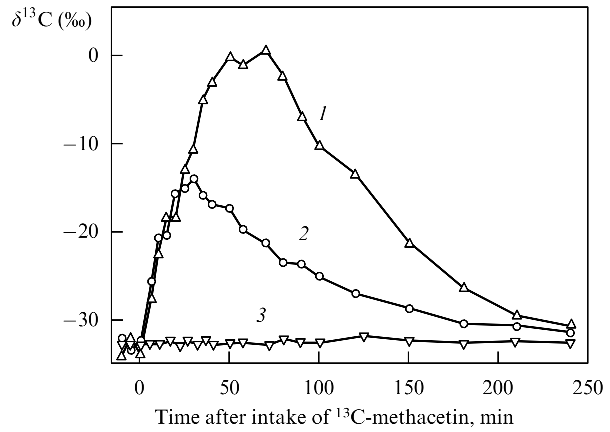

Figure 4. Dynamic of the carbon isotopic ratio in exhaled CO_2 during the carrying out of a ^{13}C-methacetin breath test: in the absence of liver pathology (*1*); with liver deficiencies of average seriousness (*2*); with developed cirrhosis of the liver (*3*).

C_2H_5OH, etc.), construction of multi-component analytical systems, and the broadening of the range of clinical problems that can be resolved using these methods.

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Dissipative optical solitons

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Spatial and temporal optical solitons, i.e. light beams or pulses for which the diffraction or dispersion-induced linear extension is compensated by nonlinear focusing, provide a striking example of self-organization of coherent radiation and formation of particle-like field structures. Their potential is equally important for optical data processing because solitons are natural units of information. Conservative solitons realized in transparent (with negligibly small energy dissipation) nonlinear optical media are better known than other varieties [1, 2]. It is significant that conservative solitons exhibit a continuous spectrum of basic characteristics (e.g., maximum intensity), which accounts for the drift of these characteristics under the influence of fluctuations. Another type of soliton, autosolitons or dissipative optical solitons (DOS's), was initially predicted for wide-aperture nonlinear interferometers excited by external radiation [3, 4] and for laser systems with saturable absorbers [5, 6]. DOS's are substantially different from conservative solitons, first of all in the discrete spectrum of major characteristics (the DOS energy balance is satisfied only for discrete values of maximal intensity). For this reason, the drift of soliton parameters under the effect of fluctuations is suppressed, and the unusual stability achieved may be of value for a number of applications. DOS's are formed under hard (threshold) conditions, and the loss of stability of uniform field distribution is not a necessary prerequisite for their existence. Unlike non-optical systems in which the mechanism of spatial coupling depends on diffusion [7], optical schemes are normally dominated by the diffraction mechanism of transverse linkage with characteristic diffractional field oscillations. The latter mechanism is conducive to a considerable broadening of the range of DOS types. Theoretical and experimental studies that followed the publication of Refs [3–6] further increased the number of systems in which DOS's can be realized and their new varieties and properties revealed. The objective of the present paper is to summarize advances in this area of research (see also review [8] and monograph [9]).

Schemes with inertialess nonlinearity are primarily described by the equation for the slowly changing field E envelope (dimensionless variables are used):

$$\frac{\partial E}{\partial \varsigma} = (\delta + \mathrm{i})\Delta_{\mathrm{d}} E + f\big(|E|^2\big)E + E_i \,. \qquad (1)$$

The evolutionary variable ς has the sense of time t for resonator schemes and the longitudinal coordinate z in resonatorless schemes, δ is the effective diffusion coefficient (rapid medium relaxation approximation), the Laplace operator Δ describes diffraction and dispersion and has the dimension $d = 1, 2, 3$ in the space of 'transverse coordinates' $\mathbf{r}$ (the latter variant is realized for a continuous nonlinear medium with frequency dissipation), f is a complex function of intensity $I = |E|^2$ which characterizes the nonlinearity of the medium including amplification and absorption, and E_i is the external radiation amplitude.

Nonlinear interferometers ($d = 1,2$). An example is the Fabry–Perot wide-aperture interferometer filled with a nonlinear medium and excited by external radiation ($E_i \neq 0$). The shape of the DOS field is found analytically for nonlinearity of the threshold type and numerically for other types of nonlinearity. DOS's are characterized by local intensity overshooting (towards higher or lower values for 'bright' and 'dark' DOS's respectively), which contrasts with the constant intensity corresponding to the transversely-uniform distribution. External radiation determines both the frequency and the phase of radiation in the form of DOS's. DOS's can also exist in the absence of bistability of transversely-uniform states [10]. However, the greatest variety of DOS's occurs under bistability conditions. Such DOS's can be represented as the coexistence of two regimes, one corresponding to a branch of uniform states and undergoing excitation on a small part of the aperture of an interferometer and the other corresponding to one more state which is realized in the remaining aperture region (Fig. 1). Interpretation of DOS's as a bound state of switching waves is equally constructive [10]. It implies the discreteness of individual DOS spectra, with the DOS's differing in the width of intensity overshoot and the number of intensity

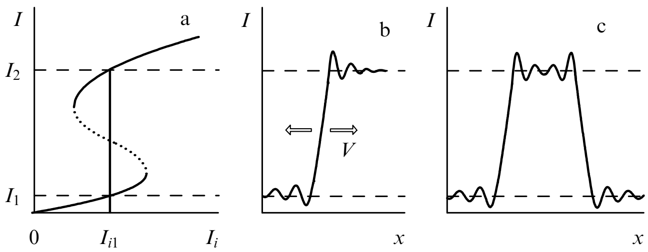

Figure 1. Variant of interpretation of dissipative solitons: (a) hysteresis dependence of output radiation intensity I on input radiation intensity I_i in the plane wave approximation; (b) intensity profile for a switching wave, x is the transverse coordinate, front velocity v is determined by intensity I_i; (c) intensity profile for a dissipative optical soliton.

oscillations (a quantum number analogue) in the central region (Fig. 1b). Similarly, there are combined 'two' and 'multiparticle' DOS's with a discrete set of equilibrium distances between single constituent DOS's. Symmetrical combined DOS's are stationary while asymmetric ones propagate transversely. Regions in which isolated and combined DOS's exist do not coincide. The latter may occur even in a certain intensity range of incident radiation where individual DOS's are non-existent. With a rise in the number of bound DOS's, their parameter spectrum first becomes fuller and then continuous (zonal) for an infinite chain of DOS's. This is analogous to the spectrum modification upon the transition from an atom to a two- or many-atomic molecule and to a chain of atoms (model of a solid). When the interferometer is excited with a wide beam, DOS's arise as the power gradually diminishes.

Polarized DOS's are described by coupled equations for $E_{1,2}$, i.e. two polarization components of the field (for a transversely one-dimensional magneto-optical interferometer with the electronic Kerr nonlinearity [11]; a similar description is valid for two- and three-frequency DOS's in interferometers with quadratic nonlinearity):

$$\mathrm{i}\,\frac{\partial E_1}{\partial t}+\frac{\partial^2 E_1}{\partial x^2}+2\big(|E_1|^2+2|E_2|^2\big)E_1-E_1=\mathrm{i}E_{i1}\,,$$
$$\mathrm{i}\,\frac{\partial E_2}{\partial t}+\frac{\partial^2 E_2}{\partial x^2}+2\big(|E_2|^2+2|E_1|^2\big)E_2+E_2=\mathrm{i}E_{i2}\,. \qquad (2)$$

Figure 2 shows cross profiles of these amplitudes for two types of DOS $\big(|E_{i1}|^2=|E_{i2}|^2\big)$. Here, DOS's exist even at intensities of external radiation which fail to ensure bistability of transversely-uniform states.

The real scheme contains transverse inhomogeneities of the intensity I_i and phase Φ_i of external radiation as well as inhomogeneities of the losses and optical length of the interferometer. Large-scale (compared with the DOS width) inhomogeneities lead to the transverse motion of the DOS. For small and smooth inhomogeneities, such a motion is described by the 'Aristotelean' (first order) equation of motion for the coordinates $\mathbf{r}_0$ of the DOS 'center of gravity' (for simplicity, only external radiation inhomogeneities are taken into account and the medium is inertialess, $\delta = 0$ [12]):

$$\dot{\mathbf{r}}_0=\alpha\,\mathrm{grad}\,\Phi_i+\beta\,\mathrm{grad}\,I_i\,. \qquad (3)$$

The first term in the right-hand part of Eqn (3) corresponds to the geometric drift of the obliquely incident external radiation (the so-called Galilean invariance). In other words, this term describes the motion of a DOS with a transverse velocity proportional to the local slope of the external radiation wave front. The motion driven by the intensity gradient (second term) is naturally interpreted in a DOS model as the bound state of switching waves because these waves have different velocities due to local changes in the intensity of incident radiation. It follows from (3) that the DOS becomes localized in the maximum intensity region at $\beta > 0$. Effects of the two forms of inhomogeneity can be mutually compensated. Specifically, in the simplest (one-dimensional) geometry, it follows from (3) that, in the case of oblique incidence of an external radiation beam, the DOS is stationary if the incidence angle is smaller than a certain critical value. In two-dimensional geometry, the excitation of an interferometer by external radiation with a wave front dislocation leads to the DOS rotating about the dislocation with a constant angular velocity. It may be concluded that artificial introduction of transverse inhomogeneities makes it possible to control DOS localization and manipulate it. Such a possibility was used to create a DOS-based scheme of multichannel memory (experimentally realized in Ref. [13]), shift registry, and full optical summator [12].

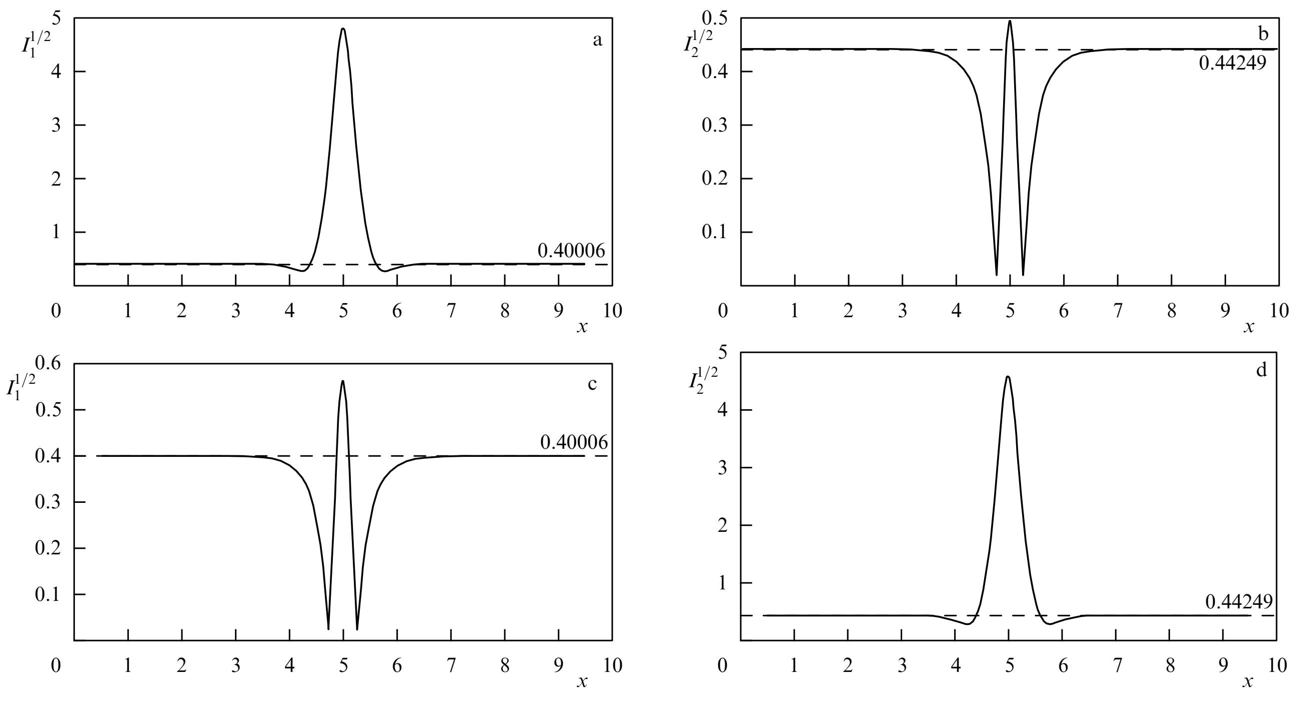

Figure 2. Profiles of amplitudes $|E_1| = I_1^{1/2}$ (a, c) and $|E_2| = I_2^{1/2}$ (b, d) for the first (a, b) and second (c, d) types of dissipative magneto-optical solitons.

A layer of nonlinear medium with an additional mirror. In this system, an external signal enters the simplified governing equation in a somewhat different way than in Eqn (1):

$$\frac{\partial E}{\partial \varsigma} = (\delta + \mathrm{i})\Delta_{\mathrm{d}} E - E + f\big(|E|^2\big) E_i \,. \qquad (4)$$

Nevertheless, in a scheme free from inhomogeneities, this equation retains the same symmetry properties that are inherent in Eqn (1), that is translational invariance, symmetry relative to a change in the sign of transverse coordinates, and Galilean symmetry (at $\delta = 0$). Accordingly, major DOS properties are similar to those described in the previous section.

Laser systems. Amplification may make up for the absence of an external signal [in Eqn (1) $E_i = 0$]. Systems of this kind include lasers with saturable absorption ($d = 1, 2$), single-mode optical fibers ($d = 1$), planar waveguides ($d = 1, 2$), and continuous media ($d = 1, 2, 3$) with nonlinear amplification and absorption and frequency dispersion. The field of the simplest (stationary and symmetrical) DOS has the form $E = A(r)\exp(-\mathrm{i}\alpha\varsigma)$, where $r = |\mathbf{r}|$ and α are the shifts of radiation frequency and phase velocity respectively ($\alpha = 0$ for schemes with an external signal). Hence, an ordinary differential equation follows from (1) for $A(r)$ in which the eigenvalue of α needs to be determined. The order of this equation can be reduced by taking advantage of the arbitrariness of the field phase. Mathematically, DOS's are homoclinal trajectories, i.e. solutions of this equation leading to a uniform generationless state $A = 0$ at the periphery (far from the center) of the DOS. The dependence of the spectral parameter α on the linear amplification coefficient for fundamental DOS's of different dimensions d is illustrated by Fig. 3. Also stable are 'excited' DOS's, such as two-dimensional ones with wave front dislocations of the second and highest orders or asymmetrical DOS's which rotate with a constant angular velocity [9].

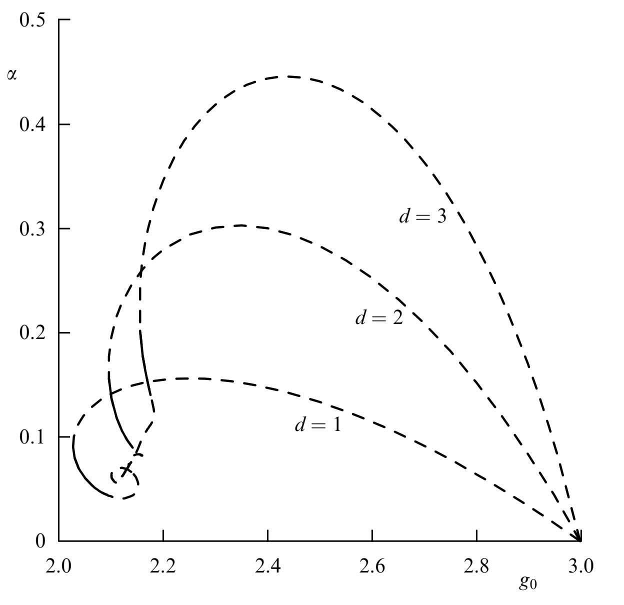

Figure 3. Dependence of spectral parameter α on the amplification coefficient g_0 for laser solitons with geometric dimension d. Hatched portions of the lines correspond to unstable structures.

For a laser with transverse inhomogeneities, Eqn (1) assumes the form ($\delta = 0$)

$$\frac{\partial E}{\partial \varsigma} = \mathrm{i}\Delta_{\mathrm{d}} E + \big[f\big(|E|^2\big) + \tilde{f}\big(|E|^2, \mathbf{r}\big)\big]E \,, \qquad d = 1, 2 \,. \qquad (5)$$

In this case, $f' = \operatorname{Re}\tilde{f}$ describes inhomogeneities of both absorption and amplification while $f'' = \operatorname{Im}\tilde{f}$ describes inhomogeneities of the resonator's optical pathways. Such DOS parameters as the intensity profile q_I and wave front q_{ph} curvatures follow the characteristics of smooth inhomogeneities adiabatically. Then, the equation for the DOS center of gravity assumes the form [cf. Eqn (3)]

$$\ddot{\mathbf{r}}_0 = \mathbf{B}(\mathbf{r}_0) + (\dot{\mathbf{r}}_0, \nabla_{\mathrm{d}})\,\mathbf{D}(\mathbf{r}_0)\,,$$

$$\mathbf{B}(\mathbf{r}_0) = 2\nabla_{\mathrm{d}} f'' - 2\,\frac{q_{\mathrm{ph}}}{q_I}\,\nabla_{\mathrm{d}} f'\,, \qquad \mathbf{D}(\mathbf{r}_0) = \frac{1}{2q_I}\,\nabla_{\mathrm{d}} f' \,. \quad (6)$$

Given the possibility of DOS propagation with an arbitrary speed in a homogeneous system containing a inertialess medium, this is a second-order equation ('Newton's mechanics') which includes 'friction' in the presence of inhomogeneities of losses or amplification. Friction provides for the uptake of a DOS by a localized inhomogeneity. Marked inhomogeneities in a laser system are exemplified by the resonator's mirror edges. A DOS approaching the edge with a low transverse velocity v ($v < c\lambda/w$, where c is the velocity of light, λ — wave length, and w — DOS width) is reflected from it and moves to the center of the resonator at a somewhat lower speed. The reflection may lead to a modification of the DOS type.

There are different regimes of DOS interaction, both weak and strong [9]. For inertialess media with the Galilean invariance of Eqn (1), the specific features of this interaction primarily depend on its duration which, in turn, is a function of the relative transverse velocities of colliding DOS's. At high velocities v ($v \gg w|\delta\varepsilon|/\lambda$, where $\delta\varepsilon$ is the characteristic value of nonlinear dielectric permeability), the interaction is weak and the DOS's undergo no serious distortion even if they pass through one another. As the speed becomes lower, the following strong interaction regimes are consecutively realized. The first one corresponds to the birth of a new, additional soliton from the collision of the two DOS's. The second regime leads to two DOS's merging into a single one. If two DOS's propagate at a low speed, they first come into close proximity and then repulse each other. Also stable are bound laser DOS's (antiphasic and synphasic structures; for the latter, the condition $\delta \neq 0$ is essential).

In addition, the Bloch equations for a density matrix are used to take relaxation of the medium into account. In this case, transverse velocity is no longer arbitrary, and the symmetry of the driving equations with respect to the Galilean transformation is broken. This results in new types of DOS. They are stationary, 'slow' and 'fast' DOS's, depending on the parameters of the scheme, as well as DOS's with oscillating shape and speed of transverse movement. A change in the parameters leads to hysteresis jumps between these regimes (Fig. 4). 'Slow' and 'fast' DOS's differ not only in velocity (the module is fixed and the direction is arbitrary at fixed parameters), but also in width and peak intensity. A 'fast' DOS can be interpreted as a local pulse of modulated quality propagating over the aperture.

Many of the various DOS types described above have been obtained in experiment, viz. in a spatio-temporal

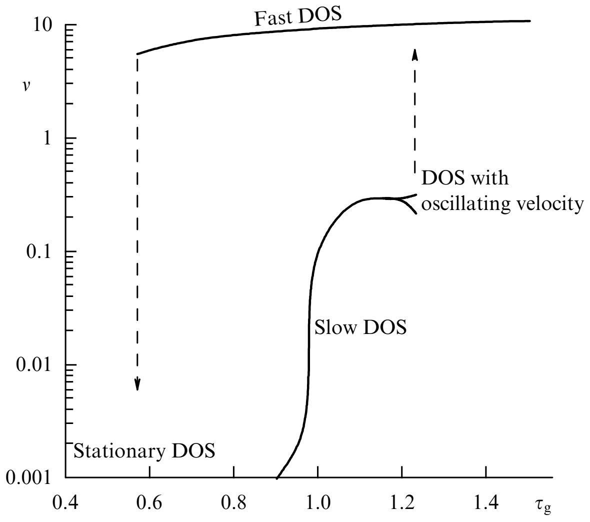

Figure 4. Hysteresis-related consecutive replacement of different types of dissipative optical solitons with the transverse velocity v during gradual alteration of the amplification relaxation time τ_g. As τ_g grows from small values, a stationary soliton sets in. At $\tau_g \approx 0.9$, it loses stability and undergoes transformation to a slowly moving soliton. The latter also loses stability at $\tau_g \approx 1.2$; a soliton with oscillating velocity and shape is realized at τ_g values of up to 1.25. Thereafter, this regime collapses, and a fast soliton is formed. With decreasing τ_g, the fast soliton regime gives place to a stationary soliton.

modulator scheme with optical feedback and spatial filtration [13], in a sodium vapor cell in a magnetic field with a feedback mirror [14], in a laser with saturable absorber [15], in an equivalent scheme with photorefractive crystals [16], and in passive quantum-well semiconductor interferometers [17]. Minimization of DOS size is an essential precondition for their practical application to parallel information processing, given the current technical level of electronic circuits. Recent studies have demonstrated the possibility of creating 'optic needles', i.e. extremely narrow conservative spatial solitons with width smaller than the radiation wavelength in a linear medium) [18]. There is every reason to believe that the analogous supernarrow DOS can also be realized.

To summarize, the dissipative optical solitons predicted 10 years ago have since been well-described theoretically and observed in experiment. There is a surprising variety in their types, which include: steady-state and pulsating solitons; stationary, rotating, and moving freely at a constant speed or intermittently; solitons having a regular front and dislocations of higher orders; scalar and vector solitons; one- and multi-frequency ones; single and bound, one-, two-, and three-dimensional solitons. Semiconductor interferometers and quantum-well lasers appear to be best suited for the practical purpose of optical data processing.

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