

**Figure 5.** Numerical simulation of the magnetization process in a dipole lattice (1:2) consisting of 36 chains of 6 dipoles each; the external magnetic field is applied parallel to the chains. Horizontal line shows one of the feasible transitions between hysteresis branches; A and B are the states with similar magnetization distributions.

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### References

- 1. Simonds J L Phys. Today 48 (4) 26 (1995)
- Rozenbaum V M, Ogenko V M, Chuĭko A A Usp. Fiz. Nauk. 161 (10) 79 (1991) [Phys. Usp. 34 883 (1991)]
- 3. Morup S Europhys. Lett. 28 (9) 671 (1994)
- 4. Sugavara A, Sheinfein M R Phys. Rev. B 56 (4) R8499 (1997)
- 5. Gider S et al. Appl. Phys. Lett. 69 3269 (1996)
- 6. Lok J G S et al. Phys. Rev. B 58 12201 (1998)
- 7. Bukharaev A A et al. Fiz. Tverd. Tela 40 1277 (1998) [Phys. Solid
- State 40 1163 (1998)]
  8. Gusev S A et al. Pis'ma Zh. Eksp. Teor. Fiz. 68 475 (1998) [JETP Lett. 68 509 (1998)]
- 9. Usov N A Personal communication
- 10. Ishii Y, Nakazava Y J. Appl. Phys. 81 1847 (1996)
- 11. Gross M, Kiskamp S Phys. Rev. Lett. 79 2566 (1997)
- 12. Bak P, Bruinsma R Phys. Rev. Lett. 49 249 (1982)

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# Theory of coherent oscillations in a resonant tunneling diode

V F Elesin, I Yu Kateev, A V Krasheninnikov, A I Podlivaev

#### 1. Introduction

Resonant tunneling and negative differential conductance in a resonant tunneling diode (i.e., in a double-barrier quantum well or quantum dot) are known to be related to the appearance of a resonance level energy due to the spatial quantization phenomenon [1, 2]. Negative differential conductance accounts for the possibility of generating electromagnetic waves. Oscillations in a resonant tunneling diode at a frequency of up to 712 GHz were demonstrated in Refs [3, 4].

However, the extensive application of oscillations based on resonant tunneling diodes is hampered by their relatively low powers and frequencies. What is responsible for such a situation and how a further rise in the power can be achieved remains unclear, probably because it is intrinsically difficult to give a theoretical description of the resonant tunneling diode. Although there are a relatively large number of theoretical works [4-6] on the subject based primarily on the use of numerical methods, many aspects of the problem need to be elucidated in greater detail, e.g.,

(1) is there a fundamental limitation on the frequency of oscillations in the resonant tunneling diode? According to a widespread viewpoint [5, 6], the frequency of the resonant tunneling diode is restricted by the resonance level width  $\Gamma$  (or by the inverse characteristic lifetime of an electron in a quantum well,  $\tau_{\gamma}^{-1} = \Gamma$ ,  $\hbar = 1$ ). This is actually true of 'classical' generators, e.g., the Gunn diode.

(2) Is the resonant tunneling diode a 'quantum' or a 'classical' oscillator, with the properties determined by the parameters of negative differential conduction?

(3) How does the resonant tunneling diode generation power depend on the bias voltage and structure parameters?

The present paper reports selected results recently obtained in an attempt to answer these and other questions.

#### 2. Linear theory of the resonant tunneling diode

The studies described in Refs [7, 8] were conducive to the development of the analytical theory of coherent oscillations in the resonant tunneling diode in the approximation linear in the electromagnetic field *E*. The coherent regime suggests that the electron's inverse lifetime in a quantum well  $\tau_{\gamma} = \Gamma^{-1}$  is shorter than the time of loss of coherence. For the simplest model of a double-barrier structure, an exact solution of the Schrödinger equation with open boundary conditions was found along with active (*J*<sub>c</sub>) and reactive (*J*<sub>s</sub>) polarization currents which determine the rate of the field increase (*J*<sub>c</sub>) and its frequency  $\omega$ .

Using small parameters  $\omega/\varepsilon_{\rm R}$  and  $\Gamma/\varepsilon_{\rm R}$  inherent in the resonant tunneling diode ( $\varepsilon_{\rm R}$  is the resonance level energy), expressions for the polarization currents  $J_{\rm c}$  and  $J_{\rm s}$  can be reduced to a simple and clear form:

$$\widetilde{J}_{c}(\omega,\delta) = \frac{4J_{c}}{e^{2}EaQ} = -\frac{\Gamma^{2}\delta}{\left[\left(\delta+\omega\right)^{2}+\Gamma^{2}\right]\left[\left(\delta-\omega\right)^{2}+\Gamma^{2}\right]},$$
(1)

$$\widetilde{J}_{s}(\omega,\delta) = \frac{\Gamma\omega[\delta^{2} - \omega^{2} - 3\Gamma^{2}]}{\left[\delta^{2} + \Gamma^{2}\right]\left[\left(\delta + \omega\right)^{2} + \Gamma^{2}\right]\left[\left(\delta - \omega\right)^{2} + \Gamma^{2}\right]} \cdot (2)$$

Here,  $\delta = \varepsilon - \varepsilon_R$ ;  $\varepsilon$  is the resonance energy of electrons entering the quantum well from the emitter with a velocity Q(in our model, the energy  $\varepsilon$  is equivalent to a constant voltage applied to the quantum well); and a is the quantum well size.

The character of the frequency dependence of the current  $\widetilde{J}_{c}(\omega, \delta)$  is critically related to the ratio  $\delta/\Gamma$ . If  $\delta < \Gamma$ , then the current  $\widetilde{J}_{c}$  has a maximum at  $\omega = 0$  (Fig. 1) and drops with frequency as  $1/\omega^4$ . Near the maximum at  $\omega \ll \Gamma$ , the current  $\widetilde{J}_{c}$  is expressed through the differential conductance of the constant current  $J_{0}(\delta)$ :

$$\widetilde{J}_{c} = \frac{\partial J_{0}(\delta)}{\partial \varepsilon}, \qquad J_{0} = \frac{\Gamma^{2}}{2(\delta^{2} + \Gamma^{2})}.$$
 (3)

The largest increase is achieved at  $\delta = \Gamma/\sqrt{3}$ , where the negative differential conductance has a maximum absolute value. It is  $\delta = \Gamma/\sqrt{3}$  that is normally chosen in both experimental and theoretical studies. In this case, the



Figure 1. Frequency dependence of the polarization current  $J_c$ : solid line, numerical solution; dashed line, analytical solution.

increment rapidly decreases with frequency at  $\omega > \Gamma$ :

$$\widetilde{J}_{\rm c}(\omega,\delta) \approx -\frac{\delta\Gamma^2}{\omega^4};$$
(4)

in other words, there is a limiting frequency  $\omega \approx \Gamma$ . The regime of oscillation in a resonant tunneling diode at  $\delta < \Gamma$  with  $\tilde{J}_c$  maximum at point  $\omega = 0$  is naturally referred to as the 'classical' regime.

In the opposite case  $(\delta > \Gamma)$ , the current  $\widetilde{J}_c$  attains a maximum at a frequency

$$\omega_{\rm m}^2 = \delta^2 - \Gamma^2 \tag{5}$$

and equals

$$\widetilde{J}_{\rm c}(\omega_{\rm m}) = -\frac{1}{4\omega} \,. \tag{6}$$

The new maximum can be accounted for by quasi-resonant transitions between states with energies  $\varepsilon$  and  $\varepsilon_R$ , since Eqn (5) gives a 'quasi-resonance' condition  $\omega \gg \Gamma$  at  $\omega = \varepsilon - \varepsilon_R$ . This regime of oscillation in the resonant tunneling diode can be called the 'quantum' regime.

When choosing the value of  $\delta = \sqrt{\omega^2 + \Gamma^2}$  outside the maximum of the negative differential conductance, it is possible to obtain an oscillation frequency  $\omega$  which is significantly higher than the resonance level width  $\Gamma$ . This means that there is no limitation on the oscillation frequency in the 'quantum' regime and that a resonant tunneling diode can work either in the 'quantum' or the 'classical' regime, depending on the  $\delta/\Gamma$  ratio.

#### 3. Analytical non-linear theory

In the low-frequency limit ( $\omega \ll \Gamma$ ), it is possible to derive an expression for the polarization current  $\widetilde{J}_c$  over a wide range of fields [9]:

$$\widetilde{J}_{c} = -\frac{\Gamma}{\sqrt{2}\widetilde{E}^{2}} \frac{\sqrt{\sqrt{x^{2} + y^{2}} - x}}{\sqrt{x^{2} + y^{2}}},$$
(7)

where

$$x = 1 + \frac{\widetilde{E}^2(\Gamma^2 - \delta^2)}{(\Gamma^2 + \delta^2)^2}, \quad y = \frac{2\delta\Gamma\widetilde{E}^2}{(\Gamma^2 + \delta^2)^2}, \quad \widetilde{E} = eEa.$$

This makes it possible to find the dependence of the generation power on the parameters of the resonant tunneling diode and the pumping current Q (Fig. 2). The figure shows that in the 'quantum regime' ( $\delta > \Gamma$ ) the generator works in a rigid regime and exhibits hysteresis.



Figure 2. Dependence of the generation power on the dimensionless pumping current Q for different parameters of the resonant tunneling diode: solid line, numerical solution; dashed line, analytical solution.

# 4. Nonlinear theory of the resonant tunneling diode with a numerical solution of the Schrödinger equation

Two methods were employed to obtain a numerical solution to the Schrödinger equation. One [10] was designed to solve a system of equations for partial wave functions described in Ref. [11]. The other sought a direct solution of the timedependent Schrödinger equation [12]. Figure 3 presents the computed dependences of the oscillation amplitude on the dimensionless pumping current Q in the 'quantum' and 'classical' regimes of resonant tunneling diode generation. It is clear that in the former case the generation is more efficient and, as soon as fields as large as  $eEa/\Gamma > 4$  are achieved, the field amplitude becomes higher than that in the 'classical' regime.

To conclude, the exact analytical and numerical expressions obtained in the present study for the polarization currents that influence both the magnitude and the frequency of oscillation of a resonant tunneling diode indicate that the generation occurs in both 'classical' and 'quantum' regimes.



Figure 3. Computed dependences of the generation amplitude on the dimensionless pumping current Q in the 'classical' and 'quantum' regimes of the resonant tunneling diode generation.

Moreover, high-power generation in a resonant tunneling diode is feasible in a new 'quantum' regime at superhigh frequencies, which may vary in a broad range.

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#### References

- 1. Esaki L, Tsu R Appl. Phys. Lett. 22 562 (1973)
- 2. Keldysh L V Fiz. Tverd. Tela 4 2265 (1962)
- 3. Brown E R et al. Appl. Phys. Lett. 55 1777(1989); 58 2291 (1991)
- 4. Frensley W R Rev. Mod. Phys. 62 745 (1990)
- 5. Liu H C, Sollner T C L G Semicond. Semimet. 41 359 (1994)
- 6. Liu H C Phys. Rev. B 43 12538 (1991); erratum 48 4977 (1993)
- 7. Elesin V F Phys. Low-Dim. Struct. (5/6) 79 (1998)
- 8. Elesin V F Zh. Eksp. Teor. Fiz. 116 704 (1999) [JETP 89 377 (1999)]
- 9. Elesin V F Phys. Low-Dim. Struct. (2000) (in press)
- Elesin V F, Krasheninnikov A V Phys. Low-Dim. Struct. (7/8) 65 (1999)
- 11. Elesin V F Zh. Eksp. Teor. Fiz. 112 483 (1997) [JETP 85 264 (1997)]
- 12. Elesin V F, Podlivaev A I, Kateev I Yu *Fiz. Tekh. Poluprovodn.* (2000) (in press)

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# Effects of exciton – electron interaction in quantum well structures containing a two-dimensional electron gas

## V P Kochereshko, R A Suris, D R Yakovlev

It has been thought until recently that the effect of an electron (hole) gas on excitonic states is exclusively confined to their destruction as a result of screening or filling phase space and also to exciton scattering on electrons (holes). However, it was found in 1993 [1] that interactions between excitons and an electron gas in quantum-well structures can be much more diverse. Specifically, such interactions can give rise to trions, i.e., bound states of excitons and electrons (holes), and to combined exciton-electron processes also referred to as combined exciton-cyclotron resonance [2]. The present paper reports experimental manifestations of exciton interaction with a two-dimensional electron gas (2D EG) in structures with quantum wells at relatively low electron densities, when the Fermi energy of the two-dimensional electron gas does not exceed trion binding energy.

We used  $ZnSe/Zn_{0.89}Mg_{0.11}S_{0.18}Se_{0.82}$ - and CdTe/ Cd<sub>0.7</sub>Mg<sub>0.3</sub>Te-based structures each having a single 8 A wide quantum well delta-doped with donors, 10 A from the well. The electron surface density in the quantum well ranged from  $10^9$  to  $5 \times 10^{11}$  cm<sup>-2</sup>. We investigated reflectivity, photoluminescence, and photoluminescence excitation spectra in magnetic fields of up to 8 T in the Faraday geometry.

Figure 1a presents typical reflectivity (R) and photoluminescence (PL) spectra taken from a "specially undoped" reference ZnSe/ZnMgSSe structure with a single quantum well. At energies on the order of 2.82 eV, the reflectivity spectrum of this sample contained a distinct reflection line of a quasi-two-dimensional exciton (X) in the quantum well.



**Figure 1.** Reflectivity (*R*) and photoluminescence (*PL*) spectra in an exciton (*X*) and trion (*X*<sup>-</sup>) resonance region taken from ZnSe/Zn<sub>0.89</sub>Mg<sub>0.11</sub>So<sub>0.18</sub>Se<sub>0.82</sub>-based structures with a single 8 A wide quantum well: (a) undoped structure; (b) modulated doped structure with an electron density  $n_e = 6 \times 10^{10}$  cm<sup>-2</sup>.

The reflectivity spectrum of a doped ZnSe/ZnMgSSebased structure having a quantum well (Fig. 1b) contains two lines. One is the reflection line of the quasi-twodimensional exciton observed in the undoped structure, the other is a new reflection line  $X^-$  shifted 5 meV toward the long-wavelength region relative to line X. We believe that the reflection line  $X^-$  may be associated with the formation of a charged exciton – electron complex (trion), which is in fact a bound state of an exciton and an electron. Trionic states in ZnSe-based quantum-well structures were examined in greater detail in Refs [3, 4].

Photoluminescence (PL) spectra of the two structures contain both the exciton PL line and the trion PL line. This is due, first, to the low level of background impurities in the specially undoped structure and, second, to the short exciton – electron binding time comparable to the duration of exciton radiative recombination. The shortness of the trion