

CONFERENCES AND SYMPOSIA

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A scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences (RAS) was held at the P L Kapitza Institute for Physical Problems, RAS on 24 November 1999.

The following reports were presented at this session:

(1) **Magarill L I, Romanov D A, Chaplik A V** (Institute of Semiconductor Physics, Siberian Branch of RAS, Novosibirsk, Russia) "Low-dimensional electrons in curvilinear nanostructures";

(2) **Devyatov E V, Shashkin A A, Dolgopolov V T** (Institute of Solid-State Physics RAS, Chernogolovka, Russia), **Hansen V** (Institut für Angewandte Physik, Universität Hamburg, Hamburg, Germany), **Halland M** (Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow, United Kingdom) "Tunneling measurements of the Coulomb pseudogap in a two-dimensional electron system in a quantizing magnetic field";

(3) **Gusev S A, Nozdrin Yu N, Sapozhnikov M V, Fraerman A A** (Institute of Microstructure Physics RAS, Nizhni Novgorod, Russia) "Collective effects in artificial two-dimensional lattices of ferromagnetic nanoparticles";

(4) **Elesin V F, Kateev I Yu, Krasheninnikov A V, Podlivaev A I** (Moscow Engineering Physics Institute, Moscow, Russia) "A theory of coherent oscillations in a resonant tunneling diode";

(5) **Kochereshko V P, Suris R A, Yakovlev D R** (Ioffe Physicotechnical Institute, St. Petersburg, Russia) "Effects of exciton – electron interaction in quantum-well structures containing a two-dimensional electron gas";

(6) **Krasil'nik Z F, Novikov A V** (Institute of Microstructure Physics RAS, Nizhni Novgorod, Russia) "Optical properties of strained $Si_{1-x}Ge_x$ and $Si_{1-x-y}Ge_xC_y$ heterostructures".

Abridged versions of these reports are given below.

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Low-dimensional electrons in curvilinear nanostructures

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It is known that the transition to a lower-dimensional curvilinear space in classical mechanics is reduced to the mere introduction of constraints diminishing the effective number of particle's degrees of freedom. It is therefore possible to use, from the 'very beginning', curvilinear coordinates. In the quantum context, when it comes to waveguides, in which one or two characteristic dimensions become much smaller than all the remaining lengths of the problem, the wave equation must be transformed in the spirit of the adiabatic approximation (degrees of freedom are divided into fast and slow). This leads to an adiabatic potential of purely geometrical nature.

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Using an elliptical ring as an example, the present paper demonstrates that the result depends on the manner of reduction to a one-dimensional model. In other words, the system remembers the potential which restricts the particle's motion in an n-dimensional problem giving rise, in the limit, to an $(n-1)$-dimensional wave equation. The situation is considerably simplified for surfaces of constant curvature (e.g., sphere, circular cylinder) because in such cases the adiabatic potential is constant. Selected examples of curvilinear low-dimensional systems are considered below.

1. Spirally folded quantum well (a roll)

For cylindrical surfaces, the problem is apparently reduced to a one-dimensional model (e.g., an electron in a curved quantum wire). By introducing coordinates s (length of the arc of the curve) and h (distance to the curve along its radius of curvature at point s) and averaging over the ground state of the fast motion along the coordinate h, one arrives at the one-dimensional Schrödinger equation with an adiabatic potential

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial s^2}-\frac{\hbar^2}{8mR^2(s)}\psi=E_{\parallel}\psi\,. \tag{1}$$

Here, $R(s)$ is the radius of curvature as a function of the position of the point at the curve and $E_{\parallel}$ is the energy of motion along the curve. Hence, the particle is attracted to regions of maximum curvature.

Numerical calculations for the Archimedes' spiral (in polar coordinates, equation $\rho=L\varphi$) indicate that the number of bound states (i.e., levels with energy $E_{\parallel}<0$) increases with increasing number of turns. For 1, 2 and 3 turns, the number of negative levels is 1, 2 and 3, respectively (for an infinite spiral, the number of negative levels is also infinite because the asymptotic potential in (1) behaves like a Coulomb one; i.e., it is proportional to $-\hbar^2/16mLs$). A similar numerical calculation of electromagnetic wave absorption caused by transitions between the bound states shows that, owing to the anisotropy of the spiral in its plane, the intensities of all absorption lines strongly depend on the direction of linear polarization of the wave.

2. Excitons and trions in quantum rings

The two-particle Schrödinger equation for an electron and a hole traveling in a quantum ring can easily be split into an equation for relative motion (variable $\vartheta=\varphi_e-\varphi_h$) and an equation for the center of masses $\varphi_c=(m_e\varphi_e+m_h\varphi_h)/M$,

where $M = m_e + m_h$, and φ_e and φ_h are the azimuthal coordinates of the electron and hole on the ring. The total wave function must be *independently* periodic in φ_e and φ_h with a period of 2π. At the same time, the wave function of relative motion $\psi(\vartheta)$ must satisfy the Bloch theorem, since the electron – hole interaction potential is proportional to $|\sin(\vartheta/2)|^{-1}$.

Combination of these two conditions gives the expression for the energy of exciton bound states:

$$W_n = E_n - \Delta_n \cos\left(\Phi + \frac{Jm_e}{M}\right), \quad n = 0, 1, 2, \ldots \tag{2}$$

Here, E_n are the energy levels of a one-dimensional Coulomb system, Δ_n are the amplitudes corresponding to electron tunneling toward a hole along the ring, J is the total azimuthal moment of the exciton as a whole, and Φ is the magnetic flux through the ring measured in flux quanta hc/e. Therefore, the *internal* exciton energy turns out to be a periodic function of the magnetic field.

Also, it follows from Eqn (2) that in the absence of a magnetic flux ($\Phi = 0$) the binding energy of the system depends on its total mechanical moment J. In the corresponding one-dimensional rectilinear problem, the internal energy is certainly independent of the center-of-mass momentum (the relativity principle). However, the uniform circumferential motion is in principle detectable by 'an internal observer', obviously not due to centrifugal effects. The reason is purely quantum in nature and relates to the possibility of tunneling through the Coulomb barrier around the region encircled by the ring.

A similar line of reasoning applied to the case of trions (charged h–e–e complexes) gives evidence that the binding energy also oscillates with magnetic flux. In this case, however, the oscillation period depends on the effective mass ratio

$$\Delta\Phi = \frac{m_h + 2m_e}{2M}\,. \tag{3}$$

3. Ballistic magnetoconductance of the cylindrical sector

Let us consider the sector $-\varphi_0 \leqslant \varphi \leqslant \varphi_0$ of a circular cylinder of radius R placed in a homogeneous magnetic field **B** perpendicular to the cylinder axis (axis z). Angle φ is counted from the direction **B**. Two-dimensional electrons of the sector can 'see' only the normal component of the magnetic field, which therefore becomes effectively inhomogeneous. The degeneracy of the Landau levels is thus removed, and the electron energy depends on the location of the suspension point of the magnetic oscillator.

For the internal states in a sufficiently strong magnetic field (magnetic length l_B is much smaller than R), it is easy to obtain

$$E_n(p) = \frac{\hbar eB}{mc}\sqrt{1 - \frac{p^2 l_B^4}{\hbar^2 R^2}}\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots \tag{4}$$

Here, p is the momentum along the z axis; the momentum region $p > \hbar R/l_B^2$ corresponds to boundary states; the spectrum is found by means of numerical calculation; and the energy grows asymptotically as p^2.

The Hall conductance G_H is computed analytically in the limiting case of $l_B \ll R$ (i.e., when the contribution of the boundary states is negligible) if the Fermi level lies in a group of nonoverlapping bands $E_n(p)$. For example, in the ultraquantum limit, when only band $n = 0$ is populated, the Hall conductance is written as

$$G_H = \frac{2e^2}{h}\left[1 - \frac{1}{\varphi_0}\arcsin\left(\sin\varphi_0 - \varphi_0\frac{\nu}{2}\right)\right], \tag{5}$$

where $\nu = 2\pi N_s l_B^2$ is the filling factor and N_s is the surface density of carriers.

In the general case, the results of numerical calculations show that the magnetic-field dependence of the Hall conductance $G_H(B)$ at zero temperature has inflection points corresponding to the coincidence of the Fermi level with the tops of consecutive Landau bands (Fig. 1). The longitudinal ballistic conductance of a cylindrical sector (unlike that of a flat strip) exhibits a nonmonotonic dependence on the Fermi energy: there are sites of decreasing $J(V_g)$, where J is the current along the cylinder axis and V_g is the gate voltage, which alters energy of the Fermi system.

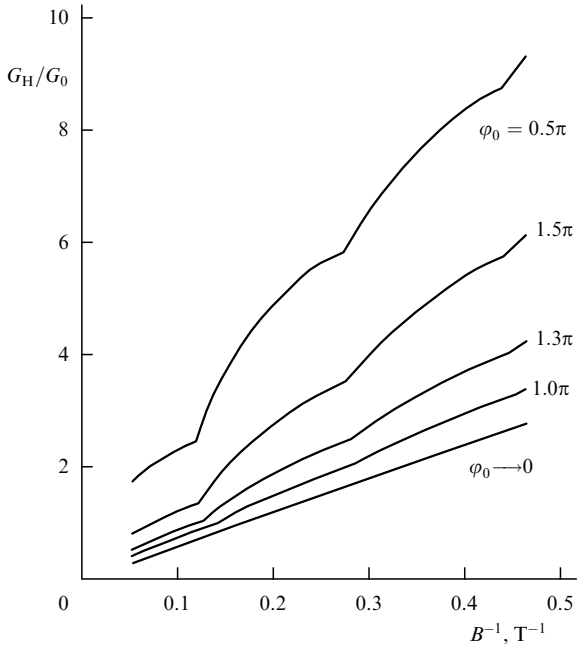

Figure 1. Magnetic-field dependence of the Hall conductance for a cylindrical sector at different φ_0 values. The straight line corresponds to a flat strip.

4. Spin – orbit interaction at the cylindrical surface

The spin – orbit coupling of two-dimensional electrons is described by two contributions to the Hamiltonian. One of them known as the Rashba model is written in the invariant form as

$$\hat{V}_1 = \gamma_1[\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}]\,\mathbf{n}\,. \tag{6}$$

Here, $\hat{\sigma}_i$ and $\mathbf{p}$ are the Pauli matrices and the operator of the two-dimensional momentum, respectively; $\mathbf{n}$ is the normal to the surface; γ_1 is the constant of spin – orbit interaction, which differs from zero only for structures asymmetrical in the direction of the normal (a typical example is a GaAs/GaAlAs heterojunction).

The second contribution arises from terms that are cubic in the momentum in the volume Hamiltonian. It can be

written in the explicit form only for a certain choice of coordinate axes. Given that the normal to the surface is parallel to axis z, we have

$$\widehat{V_2} = \gamma_2(\hat{\sigma}_x\hat{p}_x - \hat{\sigma}_y\hat{p}_y)\,, \tag{7}$$

with $\gamma_2 \neq 0$ being valid also for a symmetric quantum well. For two-dimensional planar systems, Hamiltonians V_1 and V_2 are unitarily equivalent, their spectra are identical and depend only on the moduli $|\gamma_1|$ and $|\gamma_2|$. These statements are invalid in the case of curved surfaces.

The Schrödinger equation including the spin–orbit interaction admits an exact analytical solution for the case of a hollow circular cylinder (nanotube). The energy eigenvalues turn out to be noninvariant with respect to a change in the sign of the spin–orbit interaction constant: they contain the products $\gamma_1 R$ and $\gamma_2 R$. This suggests a difference between the energy spectra of concave and convex systems (it should be recalled that in the model under consideration we are dealing with an oriented surface, i.e., directions $\mathbf{n}$ and $-\mathbf{n}$ are physically nonequivalent).

In the experiment, the sign of γ_1 can manifest itself in the absorption of an electromagnetic wave, linearly polarized along the cylinder axis, by nanotube electrons. Depending on the senses of γ_1 and the curvature, the absorption maximum at the spin-flip transition is shifted to the right or to the left from the position corresponding to the planar structure of the same material. The contribution V_2 to the thermodynamic and optical characteristics of the system after summation over states are independent of the sign of γ_2.

Taken together, the above considerations suggest the possibility, in principle, to distinguish between the contributions V_1 and V_2 when measuring one and the same response of the systems differing only in the sense of the curvature (e.g., GaAs/GaAlAs heterojunction) bent in such a way that electrons occur either on the internal or external surface of the cylinder.

5. Effect of spin–orbit interaction between two-dimensional electrons on the magnetization of nanotubes

The Schrödinger equation including spin–orbit coupling can be solved also in the case when a homogeneous magnetic field aligned parallel to the nanotube axis is applied to the system. Given a zero longitudinal momentum and a magnetic flux through the nanotube equal to a half-integer number of flux quanta, there is crossing of terms related to different spin projections. This peculiarity of the energy spectrum is responsible for the anomalies in the system's magnetization behavior.

In the absence of spin–orbit interaction, linear susceptibility corresponds to diamagnetism. If the spin–orbit coupling for a certain range of parameters of the problem is included, the sign of susceptibility may be changed (diaparatransition). For the same reason (term crossing), the magnetic susceptibility of the nanotube is characterized by marked dispersion in the low-frequency region (several orders of magnitude lower than the frequency of the electron rotational quantum $\hbar^2/2mR^2$).

We have also demonstrated that the incidence of an electromagnetic wave linearly polarized along the nanotube axis (with the external magnetic field oriented in the same direction) gives rise to a constant magnetic moment proportional to the wave intensity. This photoinduced magnetization is proportional to γ_1^2 (in the Rashba model), shows resonant dependence on the wave frequency, and attains maximum at the spin-flip transition frequency.

The effect discussed in the present paper is akin to the photogalvanic one in that the preferred direction of the circular current that induces magnetic moment is given by the vector product $[\mathbf{B} \times \mathbf{n}]$, where the normal $\mathbf{n}$ is directed along the cylinder radius. The second-order response to the electric field of the wave contains second and zero harmonics, the latter being responsible for constant magnetization.

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Tunneling measurements of the Coulomb pseudogap in a two-dimensional electron system in a quantizing magnetic field

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It is well known that the injection of a charge in the case of tunneling into a two-dimensional electron system placed in a quantizing magnetic field is highly sensitive to multiparticle effects. Tunneling may occur into the edge of a two-dimensional system (lateral tunneling) [1–3] and into its entire plane (vertical tunneling) [4–10]. It has been demonstrated in Refs [4, 5] that vertical tunneling is sensitive to both real spectral gaps at integer filling factors and the Coulomb pseudogap undergoing a shift concurrent with the Fermi level shift.

A survey of tunneling between identical highly mobile two-dimensional systems in the ultraquantum limit has revealed a pseudogap with an exponentially small tunneling density of states [6]. Since the experiments in [4–6] have been performed on samples of different quality and in different magnetic field limits, it long remained unclear whether the gaps observed were of similar or different nature. A recent study [7] has demonstrated that all previously obtained results may be reproduced using one sample. Therefore, it is inferred that Refs [4–6] describe the same pseudogap. Ref. [8], in which the method of an earlier study [6] was employed, reports surprising evidence that the width of the pseudogap at a filling factor $\nu = 1/2$ can be proportional to the magnetic field.

An enhancement of the pseudogap in the vicinity of $\nu = 1$ accompanied by the appearance of a double-humped structure in the tunneling resistance has been documented in experiments on tunneling from a three-dimensional electron system into a highly mobile two-dimensional electron gas [9].