LETTERS TO THE EDITORS

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Comment on the paper by S É Shnoll et al.

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The article entitled "Realization of discrete states during fluctuations in macroscopic processes" by S É Shnoll and colleagues was published in *Phys. Usp.* **41** 1025 (1998) together with comments by D S Chernavskiĭ as a reviewer.

We totally agree that this article was likely to attract keen interest of the audience, and appreciate the editorial decision to publish it together with the reviewer's afterword. And precisely on account of the appeal of this publication, we would like to communicate our remarks to the editors and to share our comments with the authors and with the reviewer.

Knowing Simon Él'evich Shnoll and Dmitriĭ Sergeevich Chernavskiĭ as leading biophysicists, noted in this country and abroad, we would never question the scientific quality of the studies whereupon this publication is based. We fear, however, that they are lacking those meticulous assessments of statistical significance that help us separate firmly established *scientific facts* from sometimes very interesting *plausible suppositions* or *scientific hypotheses*. Of course, we do not mean that there is no place for such plausible suppositions on the contrary, we hold that they are very important, but there must be a clear distinction between *facts* and *hypotheses*. This is especially important for the younger generation of scientists.

It seems that the team of researchers did not include that 'killjoy statistician' who (like *Yasha the Statistician* from the popular novel by I Grekova *Beyond the Entrance Lodge*¹) would curb the creative and bold statements of the authors by tediously insisting on a statistical evaluation of their reliability. So, since we did not see the presence of this 'killjoy statistician', we shall try if only partly to fill his shoes.

(1) We totally agree with the authors that the common statistical goodness-of-fit tests (like the χ^2 criterion) are "insensitive to the fine structure of the distributions" (p. 1026). This does not imply, however, that it is not possible to construct such a measure of similarity of the histograms

¹ *I Grekova* — pen-name of Elena Sergeevna Ventzel, professor, doctor of technology, author of widely known textbooks on probability theory, learned papers and popular science books. *Yasha the Statistician* was probably modeled after Yakov Borisovich Shor, professor, doctor of technology, a noted scientist concerned with the application of statistical methods to analysis and control of reliability and quality of industrial products. E S Ventzel and Ya B Shor were acquainted and held each other in high esteem.

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Received 25 January 1999 Uspekhi Fizicheskikh Nauk **170** (2) 213–214 (2000) Translated by A S Dobroslavskiï; edited by A Radzig that would take detailed account of their fine structure, and use this measure for estimating the statistical significance of various suppositions in *complete accordance with the general principles of validation of statistical hypotheses* (as described, for example, in D Hudson's *Statistics. Lectures on Elementary Statistics and Probability* (Geneva, 1964) [Translated into Russian (Moscow: Mir, 1967)], or in the fundamental monograph of H Cramer entitled *Mathematical Methods of Statistics* (Stockholm: Almquist and Wiskell, 1946) [Translated into Russian (Moscow: Mir, 1975)].

(2) Let us give an illustration of what we mean here by constructing and applying the measure of similarity.

(By no means do we intend to lecture the esteemed authors and insist on using this particular measure. Moreover, we are almost confident that some measure of similarity of histograms — perhaps even more efficient than ours — has been used by the authors for the selection of similar histograms. We only want to indicate what we feel is missing from this interesting article, and what we would expect to see in the future publications of its authors.)

So, assume that we want to assess the statistical significance of our conclusion that "histograms of the distributions belonging to the Special Group (A) exhibit much greater similarity to one another than the histograms from the Ordinary Group (B)".

For short, we shall refer to the histograms from group *A* as *Special Histograms*, and to those from group *B* as *Ordinary Histograms*. For example, *Special Histograms* may be the histograms describing the measurement results on radio-activity of specimens, which are differed in the observation periods of 24 hours, 27 days, and 365 days — in the opinion of the authors, these histograms "point to the existence of a cosmophysical factor that determines the shape of the histograms" (p. 1030).

In our opinion, the similarity of a pair of histograms X and Y can be quantitatively estimated, for example, with the following *measure*:

$$Z = \frac{1}{n} \sum_{k=1}^{n} Z_k \,, \tag{1}$$

where $Z_k = 1$ with the proviso that

$$\left|\frac{x_k/Sx - y_k/Sy}{x_k/Sx + y_k/Sy}\right| \leqslant \varepsilon,$$

and otherwise $Z_k = 0$; $Sx = \sum_{k=1}^n x_k$, $Sy = \sum_{k=1}^n y_k$; x_k , y_k are the numbers of observations within the *k*th interval of histograms *X* and *Y*, respectively; *n* is the number of intervals used in the construction of histograms, and ε is the parameter that defines the 'interval-by-interval' similarity of histograms under comparison using the 'single-interval similarity estimates', i.e. values of Z_1, \ldots, Z_n which are averaged over all

intervals to give the similarity measure Z (1). The total number of intervals n can be selected large enough, in order to take into account all the important structural features of the distributions, and the value of the parameter ε can be adjusted so as to make the measured similarity estimate agree with our intuitive assumptions.

(3) For statistical analysis of the similarity of *Ordinary and Special Histograms*, one has to find the mean values and the mean square values of similarity measures (1) for all pairs of histograms from groups A and B, and then estimate the statistical significance of deviation of the mean similarity measures based on the known rules of validation of statistical hypotheses (see, for example, Cramer's book mentioned earlier).

We believe that the similarity measure (1) can be used for estimating the statistical significance of deviations exhibited by *Special* and *Ordinary Histograms*. The fine structure of the distributions will then be taken into account, while the application of this measure complies with all basic principles of validation of statistical hypotheses.

The number of intervals n and the value of the parameter ε should be selected so as to achieve the highest possible statistical significance of the deviations between estimated similarities of *Ordinary* and *Special Histograms* (which may require several iterations). The histograms themselves can be transformed in various ways (as done by the authors), provided that the procedure is strictly the same for both *Ordinary* and *Special Histograms*.

(4) If the authors agree with us, we would be very much eager to know the results of statistical validation of their very interesting and important assertions.