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FROM THE HISTORY OF PHYSICS

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The discovery of the Planck constant: 'roentgenoscopy' of the scientific situation (1900). Missed opportunities in the choice of the Second Step

(on the centenary of the First Step of quantum theory)

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Contents

From the Editorial Board. **The birth of quantum theory is inherently associated with the report Max Planck (1858 – 1947) presented at the December 14, 1900 Physical Colloquium in Berlin. It is the purpose of the present Physics – Uspekhi article to mark the centenary of this event.**

Each experiment should answer, as far as it is possible, just one question.
M A Leontovich

1. Introduction

It is quite natural that the centenary of the discovery of the Planck constant $h \equiv 2\pi\hbar$ or the quantum of action sharpens the interest in retrospective analysis of the First Step of quantum mechanics. Of course, all historians of physics are unanimous in their recognition of the grandeur of the step (really the only one which took place). However, as soon as the analysis widens to the broad lands of '*virtual history*' [1], coming to mutual juxtaposition of the entire series of 'virtual' First Steps [2] in reference to their heuristic power in achieving *the principal goals* of quantum theory — the judgements of these historians often become sceptical. Thus, the known historian of science M Jammer [3] considers the very energy quantization of a radiation oscillator (further EQRO) to be 'conceptually too complicated' and on the basis of this attitude he refuses to recognize the 'heuristic optimality' of this First Step, using such intense expressions as 'it arouses regret', 'there cannot be any doubt', 'it would require far less intellectual effort' and so on. And the already mentioned F Hund, based on the considerable temporal distance between the discovery of h (1900) and what was certainly the key step in the 'main stream' of quantum theory — the Bohr theory of the atom (1913), proceeds even further, claiming: "It can be said that the discovery of the quantum of action h turned out to be a premature birth".

I would like to raise an objection against Jammer's and Hund's estimates which, in my opinion, put too low an 'heuristic potential' of Planck's EQRO. And my objection appears not for the sake of a 'full-scale' assertion of the superiority of this First Step over the alternatives — for example, the fours in Hund's list of alternatives given in footnote 2. And even not for the sake of widely taken heuristicity inseparable from such attributes of the actual truly stirring drama of ideas in the history of quantum physics as 'wanderings', 'blunders', postulates '*ad hoc*' [3] and mere 'blindness' with which the monographs [2, 3] [4] are so rich. But first of all — for the sake of demonstration of '*self-sufficiency*' of EQRO. This self-sufficiency consists in that EQRO, as it

[1] The term from Ref. [1] is concerned with the history of Russia and Russian language. The author hopes it will acquire a general 'scientific citizenship' as well.

[2] Eminent theoretician F Hund in a special section (headed 'Could history have developed another way') of his interesting book [2] indicates four mental alternatives of this kind (in a uniform statement beginning with 'Could quantum theory have started from ...?'): (1) the light quantum?; (2) the low-temperature physics?; (3) the combination principle of spectra? (4) the experimental discovery of interference in cathode rays?

[3] This is the example of the critical *self-appraisal* of A Sommerfeld concerning the key point of one of his early papers on the bremsstrahlung theory (see Ref. [4], Vol. 2).

[4] As a concrete illustration of the general statement in work [5]: "Quantum concepts, especially those of photons, found their way into physics with great difficulty", I would like to mention Hund's answer [2] to the question "Why was particle – wave dualism of matter accepted so late?" It comes out to the assertion that Einstein "was overengrossed in the gravitation theory" and was not inferred to generalize (literally in one line!) his own relation $(E, \mathbf{p}) = \hbar(\omega, \mathbf{k})$ for photons to the case of material waves, starting from the relation $E^2/c^2 - p^2 = m^2c^2$ for particles.

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turns out, allows direct and heuristic continuation — the genuine Second Step thus revealing the surprisingly great 'immediate' (1900!) prognostic power of Planck's discovery by itself, first of all in the aspect of correct description of *the interaction between radiation and matter* [5].

Already from the general orientation of Jammer's and Hund's grudges against EQRO it is clear that the recipe for the realization of the Second Step consists (or, to be exact, *consisted* immediately after 1900!) of eliciting $\hbar$ 'from the blocks of *thermodynamics*' and in transferring it ($\hbar$) in the state of 'merging' with some simple and for the time being classical *electrodynamical* system, in other words — in 'intrusion of $\hbar$' into some classical radiative process. Such process turned out to be *bremsstrahlung* (BS) of electrons in a Coulomb attractive field, viz. the basic mechanism of *X-ray emission* (discovered in 1895, i.e. before the advent of $\hbar$). The theoretical 'grand piano in the bushes' (ready for necessary *adaptation for 1900*) proved to be the conception of *Kramers' electrodynamics* (KrED), proposed and qualitatively developed by the author of this paper in 1986–1987 (reports at the seminars of V L Ginzburg and S T Belyaev in Moscow, further works [6–9] and others).

Anticipating the exposition of the essence of 'adaptation of KrED for 1900', permit me to say some words on that very conception. KrED is the foundation of a new, essentially classical, method for description and evaluation of quantum transitions (first radiative, then collisional as well) of *arbitrary* extent of inelasticity $\hbar\omega/E$, brought about by electrons of not high (*quasi-classical*) energies, moving in a central generally non-Coulomb attractive potential (further CAP). In the general framework of quantum electrodynamics [10], KrED could take an intermediate position between its 'fully developed' (in the sense of an arbitrary degree of electron *motion nonclassicity*) domain and the limit $\hbar = 0$, i.e. classical electrodynamics [11]. The proposal and development of the KrED conception were preceded by the extended (and initially motivated by the problems of studying radiative processes in plasma containing multiply charged ions) work of the author jointly with V I Gervids and A B Kukushkin on the theory of electron BS both in Coulomb and non-Coulomb potentials — quantum, classical and semiclassical theories (e.g., Refs [12–14]). In its turn, in the course of the conception KrED, the corresponding ('Kramersian', i.e. including the domains of the deeply inelastic transitions) divisions in the theories of nonstatic (so-called polarization) BS [15, 16], radiation cascades [15, 17], *multiphoton* static + polarization BS [18] and so on were also developed.

2. Microscopic mechanism of a bremsstrahlung event

As a basis of our qualitative analysis we shall take a model of the 'intrusion of $\hbar$' into the logical structure of classical electrodynamics using the example of bremsstrahlung (BS) of nonrelativistic electrons in the field of a nucleus or positive ion. The choice of the model is appropriate not only because BS was already known by 1900 [6], but also due to its relative simplicity and, as we shall convince ourselves, a certain *resonance* effect.

The starting point for our analysis is the 'electrodynamic largeness (EDL) of $\hbar$', namely $\hbar \gg e^2/c$ (where e is the elementary charge, and c is the speed of light). It is obvious that this brings about the inequality $\hbar\omega \gg \Delta E[\rho_{\rm eff}(\omega)]$ [7], where $\Delta E(\rho)$ is the classical value of the total bremsstrahlung energy emitted by an electron with the impact parameter $\rho_{\rm eff}(\omega)$ which is 'responsible' for emitting radiation of the frequency ω (see below for details). This inequality means, as one can easily see, that in the framework of classical electrodynamics it is impossible to form a portion of energy of the scale $\hbar\omega$ in the wave zone of an emitting electron through classical (continuous) outflow of the energy of the electromagnetic field, which unavoidably requires strong feedback (energy exchange) between an emitted field and emitter (and suppression of emitting channels in comparison with corresponding radiationless channels) and means *a strong fluctuativity* of the BS event. As one can see, the role of 'strong coupling constant' is played by $(\hbar c/e^2) \gg 1$.

Such considerations near 1900 could anticipate the content of the first (more radical) Bohr postulate 'forthcoming in 1913'. To tell the truth, realization of the classical 'smooth' mechanism of radiation (let it be bremsstrahlung or atomic) obviously requires the inequality $E_{\rm stat} \gg \hbar\omega$ to be satisfied, where $E_{\rm stat}$ is the energy 'in front' of the wave zone ($r \lesssim \lambdabar$) of an emitting dipole d in a layer of the same order $\lambdabar$ in thickness, equal in order of magnitude to

$$\frac{\mathcal{E}^2_{\rm near}(\lambdabar)}{8\pi}\, 4\pi\lambdabar^2\, \lambdabar \sim \frac{d^2}{\lambdabar^3} \qquad (r \lesssim \lambdabar)\,.$$

Introducing the dipole characteristics, namely, the size a, charge q, and particle velocity v, we get $d \sim qa$, $a\omega \sim v$, so that under the condition

$$\frac{q^2}{\hbar c}\left(\frac{v}{c}\right)^2 \gg 1\,,$$

which can be satisfied, say, for macroscopic emitter, the mechanism of radiation proves to be classical. On the other hand, for a hydrogen atom ($q = e$) we have from 'virial' considerations $mv^2 \sim e^2/a$, so that the mechanism of radiation could be classical under the condition $\hbar a \ll e^4/mc^3$ which obviously cannot be satisfied. (To convince oneself of this, the system of atomic units can be used to advantage: $e = m = \hbar = 1$, $c = 137$.) So, an atom emits *nonclassically* not due to the stationarity of the (excited) state but just due to the fact that it is too large and friable a system unable to give such an acceleration e^2/ma^2 to an electron, which would be sufficient to 'produce' an amount of electromagnetic energy of order of its typical value $\hbar\omega \sim \hbar v/a$ during each revolution around the nucleus.

Similarly, the inequality $\hbar \gg e^2/c$ could be a strong suggestive reason in favor of the fluctuative character of *spontaneous radiation*, and therefore of the initial (1916) Einsteinian analogy between this phenomenon and radioactivity.

It is also interesting to note that quite a number of years after 1913, already almost at the dawn of quantum mechanics, Wentzel [19] in his semiclassical theory of BS (which later

[5] By the way, according to Ref. [2] still in 1909 this very interaction was considered both by Planck himself and Einstein as a 'core of difficulties'.

[6] In an equivalent manner, assuming a *Coulomb character* of the attraction center, we also do not sin against the truth (of that time) because the Thomson 'non-Coulomb' atom model *was preceded* by other Coulomb models by Perrin, Nagaoka and Lenard (see, for example, Sommerfeld [4], Vol. 1), not to mention ions.

[7] Just due to relativistic smallness of the right-hand side in comparison with the left one which is of an 'atomic' order of magnitude.

became one of forerunners for the theory of Coulomb excitation of nuclei), without any regard for the EDL of $\hbar$, just returned to the Bohr postulates, this time for the mechanism of BS. Of course, after the advent of quantum mechanics (1925–1926) these postulates retained only historical interest. As for the quantity $(e^2/\hbar c) \ll 1$, it remained in its 'legitimate' place as a small parameter of perturbation theory in quantum electrodynamics. We considered it appropriate to elicit the important role of the *inverse* quantity $(\hbar c/e^2) \gg 1$ for the recognition of fluctuative character of the *field* aspect of the radiation in microworld.

We would like to emphasize that the above (anticlassical!) fluctuativity of the microscopic mechanism of radiation already at those remote times could not promise particular *observable* effects from considerations connected with *statistical averaging* of excitations of radiation oscillators over their Poisson distribution. Later on quantum mechanics strictly confirmed this intuitive feeling: *in the limiting case of the purely classical motion* of the electron it gives the result of *classical theory* for the averaged intensity of radiation, from which $\hbar$ drops out (see, for example, Ref. [29]) [8].

Nevertheless, it is instructive to trace the mechanism of an *elementary* BS event, in which 'strong coupling' between its field and trajectorial aspects manifests itself. To this end let us turn to Fig. 1, where two limiting electron trajectories in a Coulomb attractive field are depicted — rectilinear and parabolic. The electron accelerates considerably only moving along the latter trajectory and due to this fact it can effectively excite the 'high-frequency' radiation oscillator ω, having performed thus the BS event. Using the exact *purely classical* (and at the same time *prequantum*!) relationship for 'comet perihelion' (the turning point of a parabola) r_0, one arrives at

$$\frac{\omega_{\text{rot}}(r_0)}{E^{\text{kin}}_{\max}(r_0)} = \frac{2}{M}\,, \tag{1}$$

where M is the angular momentum of the electron, $\omega_{\text{rot}}(r_0)$ is the angular velocity of its orbital revolution around the center of field at the turning point r_0 (the other notation is obvious):

$$\omega_{\text{rot}}(r_0) = \frac{M}{mr_0^2} = \frac{v_{\max}}{r_0} = \sqrt{\frac{2(E+|U(r_0)|)}{mr_0^2}}\,, \tag{2}$$

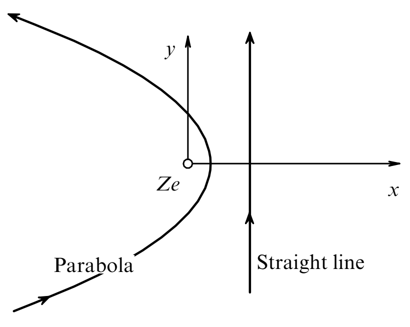


Figure 1. Limiting cases of electron trajectories in a Coulomb attractive field. The horizontal segment in the BS spectrum corresponds to a quasi-parabolic part of the flights $[\rho \ll a = (Ze^2/mv^2)]$. The logarithm in the BS spectrum corresponds to a quasi-rectilinear part of the flights $[\rho \gg a = (Ze^2/mv^2)]$.

[8] However, fluctuative character of the photon emission is of significant importance in synchrotron radiation (see, for example, Ref. [21]).

and taking into account the condition of 'rough [9] resonance' between the radiation frequency ω and the angular velocity of revolution $\omega_{\text{rot}}(t_0)$:

$$\omega_{\text{rot}} \sim \omega\,, \tag{3}$$

we find, multiplying both sides of Eqn (1) by $\hbar$:

$$\frac{\hbar\omega}{E^{\text{kin}}_{\max}(r_0)} \sim \frac{\hbar}{M} \ll 1\,. \tag{4}$$

This relation has a clear meaning — the electron 'shared' only a small part of its kinetic energy with the field oscillator; the recoil in BS is small, i.e. the electron trajectory is approximately conserved.

3. Connection of rotational component of the electron motion with the BS spectrum. Criterion for quasi-classical character of motion. Two kinds of classical BS spectra

The missing of $\hbar$ from the observed (statistically averaged) frequency distribution of BS from an individual electron trajectory enables us to consider this case in purely classical manner and, therefore, to bring to light the limits of applicability of the existence domain for the type (3) resonance which, obviously, does not contain the energy integral E of the electron trajectory. (The SOEI effect — 'switching off' the energy integral.) To this end let us take advantage of another exact formula, in this case for the Coulomb field of the charge Ze [6, 8, 9] [10]:

$$\omega_{\text{rot}}(r_0) = \frac{Z^2me^4}{M^3}\left[1+\sqrt{1+\left(\frac{Mv}{Ze^2}\right)^2}\,\right]^2, \tag{5}$$

where $v = \sqrt{2E/m}$ is the electron velocity at infinity, $M = mv\rho$ (ρ is the electron impact parameter).

The limit of the parabola (SOEI) is realized in the case of smallness of the second term under the square root, i.e. for close flights $\rho \ll (Ze^2/mv^2)$. With due regard for Eqn (3) this corresponds to the frequency range

$$\omega \gg \frac{mv^3}{Ze^2} \equiv \tilde{\omega}\,. \tag{6}$$

Along with this classical low-frequency restriction, in the SOEI domain there exists a quantum high-frequency restriction on the classical trajectory of electron motion itself. It is clear that this restriction expresses the relative smallness of the *recoil energy* of the electron in the BS event [the left-hand side of Eqn (4)] and taking into account Eqns (1) and (5) one arrives at the desired relationship [it is easy to verify that the role of correction under the root is even less important than in deriving formula (6)]:

$$\omega \ll \frac{Z^2me^4}{\hbar^3} \sim \omega_{\text{Bohr}}\,. \tag{7}$$

[9] It turns out [8] that the 'roughness' of this resonance is characterized by a factor ≈ 3 in the right-hand side of Eqn (3). This inference has a simple qualitative meaning — the parabola within its 'emitting' section occupies approximately 1/3 of the arc of a circle.

[10] Unfortunately, in Refs [6, 9] this formula contains by one (but different!) misprint.

The 'Bohr' frequency scale arising here restricts from above the *combined* spectrum of bremsstrahlung and photo-recombination radiation of a hydrogen-like atom — 'bremsstrahlung in a wide sense' [22]. Together with the 'SOEI frequency' $\tilde{\omega}$ defined by Eqn (6) and the frequency

$$\omega_{\max} = \frac{E}{\hbar} = \frac{mv^2}{2\hbar} \,,$$

i.e. the high-frequency boundary of 'bremsstrahlung proper' spectrum, frequency (7) form a geometric progression with positive powers of the denominator $Ze^2/\hbar v$. With the classical trajectory (limit $\hbar \to 0$) underlying the whole our analysis, only the inequality $Ze^2/\hbar v \gg 1$ can be compatible. Therefore, we have derived incidentally the criterion of *quasi-classicity* of electron motion in a Coulomb field with means at the 'level of 1900', which coincides with the 'future' (after 1925) quantum-mechanical criterion

$$\frac{Ze^2}{\hbar v} \gg 1 \,. \tag{8}$$

Let us now bring to the light the shape of the BS spectrum produced by a homogeneous electron beam, moving classically in the Coulomb field of the nucleus Ze. This problem was solved by Kramers [23] only in 1923, and it would be hardly appropriate to run so far ahead 'from 1900'. (However, it would be relevant to mention that Kramers himself in his exact and fairly complicated Fourier analysis used a 'Handbuch' of Bessel functions of sufficiently venerable age — 1904!) We shall satisfy ourselves with a *qualitative* estimate on the basis of order-of-magnitude relation (3). Really, in this case the frequencies of the motion, responsible for the BS of 'high' (in fact, as it can be seen further, 'not too low'!) frequencies ω, can, obviously, be only angular velocities of revolution $\omega_{\text{rot}}(r_0)$ of electrons on turning sections of highly curved, *quasi-parabolic* trajectories.

The classical 'effective emission' of an electron beam is defined as $d\varkappa(\rho) = \Delta E(\rho)\, 2\pi\rho\, d\rho$, where $\Delta E(\rho)$ is the total energy emitted during the flight with the impact parameter $\rho \ll Ze^2/mv^2$. The quantity $\omega_{\text{rot}}(r_0)$ *in this case* is uniquely connected with $\rho = M/mv$, so that $d\varkappa(\rho)$ transforms itself into $d\varkappa(\omega_{\text{rot}})$ and we identify this distribution due to Eqn (3) with the sought BS spectrum $d\varkappa(\omega)$. (The consequences resulting from the nonstrictness of this identification do not advance beyond the uncertainty of order 1 in the numerical coefficient.)

Further, introducing $v_{\max}$, i.e. the electron velocity at the turning point r_0, we find from the laws of conservation of energy and angular momentum:

$$\frac{mv^2}{2} = \frac{mv_{\max}^2}{2} - \frac{Ze^2}{r_0} \,, \quad mv\rho = mv_{\max} r_0 \,. \tag{9}$$

For the domain under consideration $\rho \ll a \equiv Ze^2/mv^2$, $r_0 \cong \rho^2/2a \ll \rho$, $v_{\max} \cong 2av/\rho \gg v$, and

$$\omega_{\text{rot}} = \frac{v_{\max}}{r_0} \sim 4\left(\frac{a}{\rho}\right)^3 \tilde{\omega} \gg \tilde{\omega} \,,$$

where $\tilde{\omega} = mv^3/Ze^2$ [see Eqn (6)]. Further, according to the relationship for the intensity of dipole radiation

$$\Delta E(\rho) = \int_{-\infty}^{\infty} \frac{2e^2}{3c^3} w^2(t)\, dt \sim \frac{e^2}{c^3} (w^2)_{\max} (\Delta t)_{\text{eff}} \,, \tag{10}$$

where $w_{\max} = Ze^2/mr_0^2$ is the maximum electron acceleration, $(\Delta t)_{\text{eff}} \sim r_0/v_{\max}$ is the effective duration of a BS event. Using these relations, we can easily find that

$$d\varkappa(\rho) \propto \frac{d\rho}{\rho^4} \propto d\omega_{\text{rot}} \,, \tag{11}$$

i.e. the distribution of effective radiation over ω_{rot} and, hence, its distribution over ω (the observed BS spectrum) are homogeneous. Gathering all the literal factors, we get the 'Kramers plateau' to an order of magnitude (Fig. 2):

$$d\varkappa(\omega) \sim \frac{Z^2 e^6}{c^3 m^2 v^2}\, d\omega \quad \text{for} \quad \omega \gg \tilde{\omega} = \frac{mv^3}{Ze^2} \,. \tag{12}$$

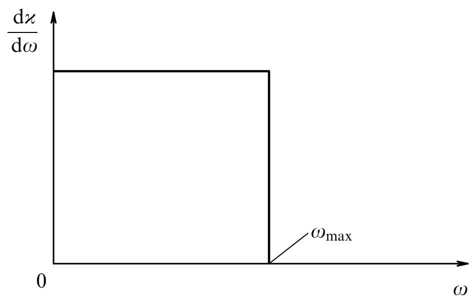


Figure 2. Kramers plateau — classical BS spectrum corresponding to the parabolic limit ($v = 0$).

This plateau describes the *overwhelming* part of the classical BS spectrum. Really, this spectrum extends up to $\omega = \omega_{\max} = mv^2/2\hbar$, so that $\omega_{\max}/\tilde{\omega} \sim (Ze^2/\hbar v) \gg 1$ [see Eqn (8)].

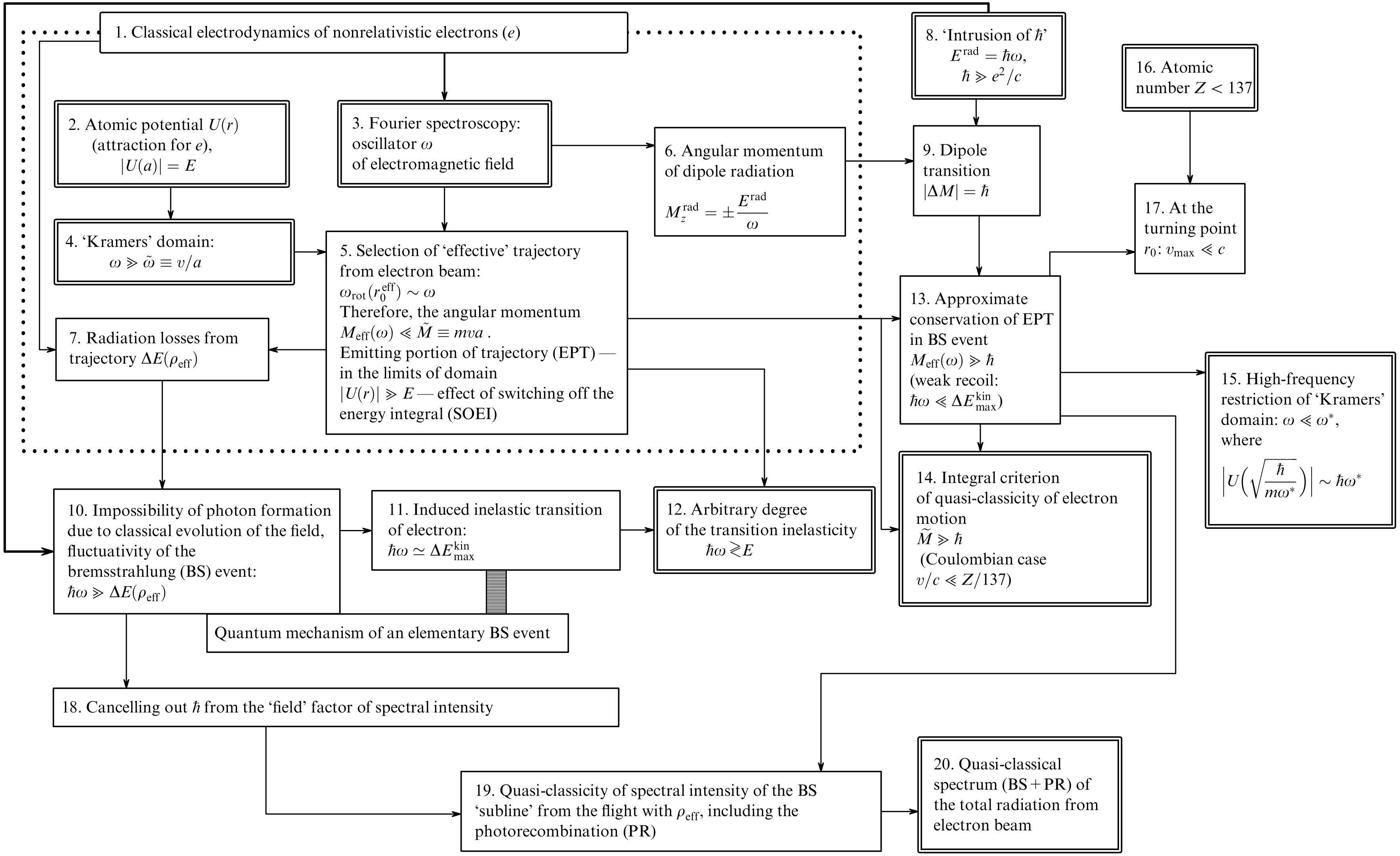
The range of 'soft' frequencies ($\omega \ll \tilde{\omega}$) in the BS spectrum, even when the condition (8) is satisfied, cannot be described with the 'plateau' (12), and with diminishing $Ze^2/\hbar v$ the shape of the spectrum $d\varkappa(\omega)/d\omega$ characteristic for this spectral region (as we shall see further, logarithmic) gradually spreads to an increased portion of its full width. Let us estimate $d\varkappa(\omega)$ for small ω. It is obvious that almost rectilinear flights with $\rho \gg a$ are responsible for the emission of these frequencies. So that the corresponding electron accelerations resemble 'bursts' with a duration of order ρ/v. The Fourier expansions of these bursts involve the frequencies from $\omega = 0$ to $\omega \sim v/\rho$ (the greater frequencies ω are exponentially suppressed). Therefore, the contribution of a flight with a given ρ to the 'effective radiation' is, roughly speaking, a 'spectral line' with a rectilinear profile and an energy of order $\Delta E(\rho)[d\omega/(v/\rho)]$, so that the total $d\varkappa(\omega)$ is given by

$$d\varkappa(\omega) \sim \frac{d\omega}{v} \int_{\rho_{\min}}^{\rho_{\max}(\omega)} \rho \Delta E(\rho) 2\pi\rho\, d\rho \,, \tag{13}$$

where, obviously, $\rho_{\min} \sim a = Ze^2/mv^2$, and $\rho_{\max} \sim v/\omega$ (this is the maximum value of ρ contributed to BS of the frequency ω); the contribution of the greater ρ corresponds to *incomplete* flights, for which relation (10) is not valid; though $\Delta E(\rho)$ has the former general form (10), for the quasi-rectilinear flights under consideration $r_0 \approx \rho$, and $(\Delta t)_{\text{eff}} \sim \rho/v$.

Substituting all this into Eqn (13) and dividing the result by formula (12), we find the sought relative excess of the logarithmic intensity in the spectrum over the 'horizontal' one

Table 1. Scheme of the 'origin' of Kramers' electrodynamics (KrED) using the example of maintenance of a classical bremsstrahlung spectrum of electrons in an atomic potential inspite of quantum nature of the elementary mechanism of radiation.



[the so-called Gaunt factor $g(\omega)$]:

$$g(\omega) \sim \ln \frac{\tilde{\omega}}{\omega}, \quad \text{for} \quad \omega \ll \tilde{\omega} = \frac{mv^3}{Ze^2}, \quad \frac{Ze^2}{\hbar v} \gg 1. \qquad (14)$$

From the above it is clear that with increasing v, as the parameter $Ze^2/\hbar v$ diminishes from large values (8) to values of order unity, the logarithmic spectral region more and more excludes the 'residual' plateau. Consequently, according to the change in the shape of the observed BS spectrum with increasing v, one can directly follow the evolution of such an interesting characteristic of the electron beam as the degree of classicity/nonclassicity of the motion of its particles in a Coulomb field (this is an illustration to the epigraph by M A Leontovich).

4. The basic results

(1) Using simple analytical means of classical physics of the pattern 'no later than 1900' (the characteristics of 'comet' perihelion plus the theory of dipole radiation by Hertz, 1889) in combination with Planck's energy quantization of a radiation oscillator (EQRO) in this work, the initial (1900) prognostic power of the discovery of the universal constant $\hbar$ was analyzed. The reality of the Second Step was shown, i.e. the effective continuation of EQRO by including it in the 'interpenetration' (see Table 1 — scheme of the KrED 'origin') with the simplest radiation process — bremsstrahlung (BS) of an electron beam moving classically in a Coulomb attractive field. The emphasis is put on the 'initiating' role of the inequality $\hbar \gg e^2/c$ as a manifestation of 'strong coupling' between the field and trajectorial aspects of the BS process, leading to the *fluctuativity* in the elementary emission event, and the role of this coupling is traced in connection with the 'selection' from the beam (for realization of this event) of *quasi-parabolic* trajectories, along which the electrons experience maximum accelerations and due to this fact enter with their turning angular velocities $\omega_{\text{rot}}(r_0)$ into *rough resonance* with the frequency ω of the field oscillator, resulting in *maximal BS 'emitting'* of the electron beam (Fig. 2).

(2) Because of the decisive role of quasi-parabolic trajectories, the important effect of *switching off the energy integral* (SOEI), practically overlooked in the scientific literature but playing the key role in bringing to light the range of applicability of classical theory of the BS process, was analyzed. The matter is that for overwhelming BS frequency range the emitting section of the quasi-parabolic trajectory does not practically depend upon the (asymptotic) value of E and, consequently, passes through that domain of r, where the electron is already strongly accelerated in the attractive potential, i.e. at small distances from the center, $Ze^2/r \gg E$.

In particular, the SOEI effect was overlooked in the otherwise exact Fourier analysis [11, p. 243] where the fairly complicated argument of MacDonald functions $K_{1/3}$ and $K_{2/3}$, just corresponding to the parabolic limit, simply was not brought to the stage (as it has been shown above — obligatory!) where E drops out.

For the same reason, the criterion $\hbar\omega \ll E$, practically accepted in the scientific literature as the 'condition of classicity of the bremsstrahlung spectrum', is also wrong. The correct criterion is $\hbar\omega \ll E_{\text{max}}^{\text{kin}}$ [see Eqns (4) and (7)], and this can be seen directly from Fig. 1. Opportune (and very long standing, in addition) recognition of the simple relations exposed above could prevent the permanent confusion accompanying the discussion of this, really simple, issue.

(3) Two classes of the BS spectra were brought to light above — 'flat' and 'logarithmic' ones, corresponding to the limits of parabolic and quasi-rectilinear trajectories (Fig. 1 is taken from Ref. [24]). The first of these spectral classes (Fig. 3) is utterly missed in the book by Jackson [25], very valuable in all other aspects.

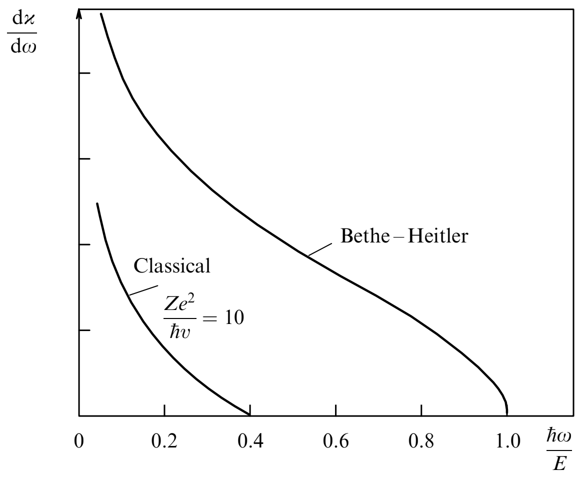


Figure 3. Figure 15.3 from all three editions of the book [25] by Jackson, including the edition of 1999. The right-hand Born (i.e. anticlassical) BS spectrum — the logarithmic one, is *correct*. The left-hand spectrum, supposedly 'classical', is *wrong* because really it involves only quasi-rectilinear flights, whereas the quasi-parabolic ones (namely those which prevail in classics!) are 'lost'.

(4) It was shown that in the analysis of the 'pattern of 1900' the criteria of quasi-classicity of the motion in a Coulomb field could be foreseen (in complete agreement with the forthcoming quantum mechanics), and not only the integral one, $Ze^2/\hbar v \gg 1$, but the local one as well, $r \gg \hbar^2/Zme^2$, where r is the radial coordinate (this criterion can be derived from the high-frequency restriction on classicity through the obvious 'parabolic' correspondence $r_0 \to r$, $\omega_{\text{rot}}(r_0) \to \omega_{\text{Bohr}}$).

(5) The microscopic mechanism of an elementary BS event with participation of the field and trajectory, considered in Section 2, is only one example of the '*dephenomenologization*' of the quantum description, which is favored by taking into account the EDL-of-$\hbar$ effect and the fluctuativity of the radiation processes, caused by it. The examples of 'dephenomenologization' of the first Bohr postulate mentioned above as well as Einstein's 'radioactive analogy' for spontaneous emission also pertain here. Another example, considered in Ref. [8], analytically proves the approximate mutual equivalence of the 'phenomenological' equation $\hbar\omega = E_1 - E_2$ (the Bohr second postulate) and the 'microscopic' relationship $\hbar\omega \approx (E_{\text{max}}^{\text{kin}})_1 - (E_{\text{max}}^{\text{kin}})_2$, which visually expresses the emission of a light quantum $\hbar\omega$ as a result of a (weak) discontinuity of the electron trajectory.

5. Conclusions

Our general conclusion consists in the assertion that already the initial (1900) prognostic power of the discovery of the Planck constant gave the foundation for a correct description of the radiation interaction with *classical* particles, which enabled, firstly, the avoidance of the erroneous interpretation concerning the basic *quantum* criteria of the theory division mentioned (this interpretation holds up till now in the scientific literature due to the insufficient transparency of the often cumbersome exact analytical results [11]) and, secondly, using the EDL of $\hbar$ as well (also for *classical* particles) to achieve a certain 'dephenomenologization' of the quantum description, i.e. a visual interpretation of the quantum radiation event.

In other words, the Planck EQRO was not just the First Step of quantum theory, but from the very beginning it had a considerable 'analytical resource' to foresee the essential (quasi-classical) share of the theory.

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[11] Suffice is to say that the limiting transition to classics ($\hbar \to 0$) in exact quantum-mechanical BS theory by Sommerfeld [4] for the first time was carried out [26] only 20 years after the creation of the theory itself, and in its final form even later [13].