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The Sagnac effect: correct and incorrect explanations

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Contents

Abstract. **Different explanations for the Sagnac effect are discussed. It is shown that this effect is a consequence of the relativistic law of velocity composition and that it can also be explained adequately within the framework of general relativity. When certain restrictions on the rotational velocity are imposed, the Sagnac effect can be attributed to the difference in the time dilation (or phase change) of material particle wave functions in the scalar (or correspondingly vector) gravitational potential of the inertial forces in a rotating reference system for counterpropagating waves. It is also shown that all the nonrelativistic interpretations of the Sagnac effect, which are unfortunately sometimes found in scientific papers, monographs and textbooks, are wrong in principle, even though the results they yield are accurate up to relativistic corrections in some special cases.**

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1. Introduction

The Sagnac effect [1 – 3] (see also reviews [4 – 8] and the review part in Ref. [9]) is understood as a phase shift of one counterpropagating wave with respect to another wave of this mode in a rotating ring interferometer, the shift being directly proportional to the angular velocity of rotation, the area enclosed by the interferometer, and the wave frequency. The Sagnac effect applies to a kinematic effect of the special theory of relativity (STR) [10] and, as shown in Ref. [11], ensues from the relativistic law of velocity composition. Along with the Michelson – Morley experiment [12, 13], a Sagnac experiment furnishes one of the basic experiments of relativity theory. The Sagnac effect outside the optical range has been described for radiowaves [14], X-rays [15], and nonelectromagnetic waves, i.e. de Broglie waves of material particles (electrons [9, 16], neutrons [17, 18], calcium [19], sodium [20], and caesium [21] atoms). Also, this effect was theoretically predicted for γ-rays [22], acoustic waves, surface acoustic and surface magnetostatic waves (the so-called 'slow' waves) [11, 23] and the de Broglie waves of π-mesons [24].

In principle, the Sagnac effect can also be recorded as the difference of times spent by a macroscopic body to pass a closed circular path on a rotating disk when moving in the direction of rotation and in the opposite sense, the body's speed with respect to the disk being the same in each case.

It has been shown [4–9] that the Sagnac effect for both optical and nonelectromagnetic waves is explained in several totally different ways including those that reject the theory of relativity. Nevertheless, most of these interpretations of the Sagnac effect yield, in some special cases, correct results despite their obvious incorrectness. This accounts in part for the lack of a complete understanding of the Sagnac effect and has given some authors reason to regard it as 'puzzling' [25].

The objective of the present work is to provide a strict derivation of the expression for the magnitude of the Sagnac effect in the framework of STR for the most general case, i.e. for waves of arbitrary nature including light waves in a ring interferometer filled with an optical medium characterized by a refractive index n and arbitrary dispersion. It will be shown that the magnitude of the Sagnac effect is independent of these medium parameters not only in the nonrelativistic limit but also for an arbitrary rotational velocity.

To the best of my knowledge, this problem has not thus far been given a rigorous and comprehensive consideration in the framework of STR for the most general case. A strict derivation of the expression for the Sagnac effect in terms of STR in optics was undertaken in Ref. [10] but only for the case of a ring interferometer containing no optical medium. Our previous work [11] describes the derivation of the expression for the size of the Sagnac effect for arbitrary waves using STR; however, it neglects small relativistic corrections. Some other works consider the Sagnac effect in the framework of STR only for light waves [5, 26–33] and with a number of approximations.

The Sagnac effect is known also to have been adequately explained in the framework of the general theory of relativity (GTR) (see, for instance, Refs [34, 35]). In this case, the authors used a metric tensor in the co-moving frame of reference, attached to a rotating interferometer.

Moreover, the Sagnac effect can be attributed to the different time dilation in rotating reference systems associated with the phase front motions of counterpropagating waves. Such a possibility is due to the difference in phase velocities of counterpropagating waves with respect to the inertial reference system when the ring interferometer rotates. The same result can be obtained proceeding from the difference between the Newtonian (nonrelativistic) scalar gravitational potentials of centrifugal forces for counterpropagating waves in the above frames of reference (by virtue of the equivalence principle).

Also, it will be shown that for certain restrictions upon the rotational velocity of a ring interferometer the Sagnac effect may be considered to result from different time dilation for counterpropagating waves in a co-moving frame of reference attached to the rotating ring interferometer. This time difference is due to the different signs of the Newtonian scalar gravitational potentials of Coriolis forces [35], in consequence of the equivalence principle as well. The problem appears to have been tackled for the first time in this light. The Sagnac effect for quantum mechanical objects, i.e. material particles, can be interpreted as a result of the action of the vector gravitational potential of Coriolis forces on the change of the wave function phase [9].

It is worth noting that neither the centrifugal nor Coriolis forces give rise directly to the Sagnac effect because it is a kinematic effect unrelated to any force.

Another objective of this work is to consider various nonrelativistic interpretations of the Sagnac effect and clearly demonstrate that they are incorrect.

2. Correct explanations of the Sagnac effect

In what is forthcoming, an explanation of the Sagnac effect will be considered correct if it leads to an exact expression for the phase difference between counterpropagating waves in a rotating ring interferometer without imposing particular limitations on the parameters of the system. Such parameters are the linear rotational velocity of the ring (the turntable upon which the interferometer is mounted), the velocities of waves (including de Broglie waves) or a material object with respect to a co-moving reference system rotating together with the ring (certainly, they must not exceed the speed of light), the material particle mass, etc. All correct explanations of the Sagnac effect are based on the application of relativity theory.

2.1 Sagnac effect in the special theory of relativity

Let light or an arbitrary wave move in a circular orbit (Fig. 1), as in a fibre ring interferometer (FRI) [36] or a conventional ring interferometer where the number of circularly arranged mirrors and prisms ensuring complete internal reflection tends to infinity. Hereinafter we shall take into consideration neither changes in the geometric parameters of the interferometer caused by centrifugal forces nor the transverse shift of counterpropagating waves induced by centrifugal forces associated with the bending of their path in the interferometer. This shift accounts for an equally insignificant enlargement of the enclosed area of the ring for the two counterpropagating waves [37, 38]. Hence, there is no phase difference between them.

In the majority of cases (see, for instance, Ref. [10]), the expression for the Sagnac effect in the framework of STR is derived taking advantage of the invariance of the interval $x^2 + y^2 + z^2 - c^2t^2$ (where x, y, z are the wave front coordinates, and t is the time). Following our earlier work [11], we propose to address the problem in question in a simpler and physically more illustrative way based on the relativistic law of velocity composition. This approach allows the expression for the Sagnac effect to be obtained (without loss of generality) in the case of arbitrary waves, in particular, electromag-

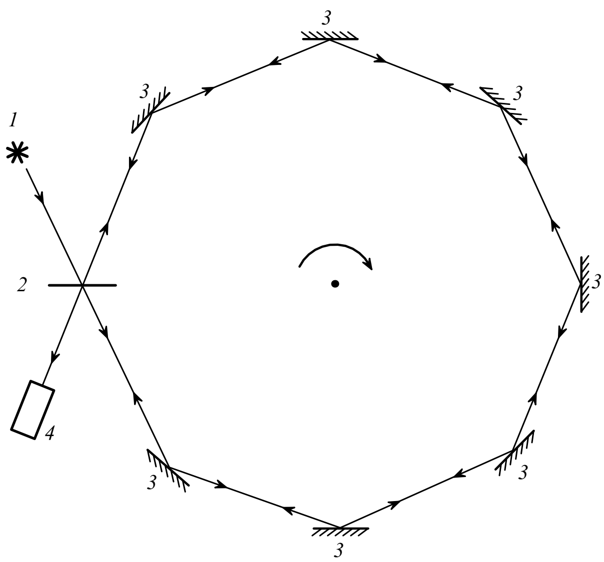

Figure 1. Ring interferometer: *1* — radiation source, *2* — beam-splitting plate (semitransparent mirror), *3* — mirrors, *4* — photodetector. The arrow shows the direction of interferometer rotation.

netic waves, in a ring interferometer containing an optical medium with an arbitrary refractive index and dispersion.

Let us consider the Sagnac effect in the most general form, that is for arbitrary waves spreading in an arbitrary medium with a phase velocity $v_{\rm ph}^{\pm}$. Illustrated below are the expressions for the path length $l^{\pm}$ in a laboratory (stationary) system of coordinates K, where the special theory of relativity holds *a priori* (sign plus corresponds to a wave travelling in the direction of rotation, and sign minus to a wave propagating in the opposite direction):

$$l^{\pm} = 2\pi R \pm R\Omega t^{\pm}\,, \tag{1}$$

$$v_{\rm ph}^{\pm} = \frac{v_{\rm ph} \pm R\Omega}{1 \pm v_{\rm ph} R\Omega/c^2}\,. \tag{2}$$

Here, R is the ring radius, Ω is the angular velocity of rotation, c is the speed of light in vacuum, and $t^{\pm} = l^{\pm}/v_{\rm ph}^{\pm}$ are the times spent by counterpropagating waves to complete one trip around the enclosed area of the ring.

Let us now consider the physical meaning of the phase velocities $v_{\rm ph}^{\pm}$ of counterpropagating waves in a rotating ring interferometer. The problem is not so simple as it appears at first sight. The fact is the device recording the interference fringes produced by counterpropagating waves (a photodetector or photographic plate, for light waves) (see Fig. 1) fails to show any changes in the pattern at a constant angular velocity Ω and monochromatic harmonic oscillations being studied. It records a constant phase difference between the counterpropagating waves, and it is difficult to draw a definitive conclusion about their velocities.

The Sagnac effect can be addressed in a simpler way if two counterpropagating pulses of a certain nature are assumed to travel in a ring interferometer. In this case, their velocities are group velocities which can also be obtained by the relativistic summation (with an appropriate sign) of the pulse group velocity and linear velocity $R\Omega$. Such pulses, if sufficiently short, reach the interferometer beam-splitter (which is displaced some distance while they propagate round the circle) at different times and therefore produce no interference pattern. The propagation time difference measured by one of the available methods will characterize the magnitude of the Sagnac effect. Recruiting the interference of counterpropagating waves increases the sensitivity of measuring the angular velocity of rotation by many orders of magnitude compared with its estimation from the difference between the times necessary for each pulse to reach the detector. For the event of electromagnetic waves in the absence of an optical medium, the phase and group velocities of light coincide, and the travelling time difference between the counterpropagating waves can be computed for group velocities (see, for instance, Ref. [10]). The calculated results thus obtained may be used to measure interference between counterpropagating waves. However, in the most general case (especially for waves of arbitrary nature), the computation of the parameters describing counterpropagating wave interference requires that all intermediate calculations be made for phase velocities of the waves; hence, the necessity of an adequate definition.

We shall now demonstrate why it is possible to use the phase velocity of the wave for the computation despite the lack of its correspondence to any real displacement of a physical object in space or energy transfer. It is known (see, for instance, Ref. [34]) that the parameters ict, $\mathbf{r}$ give rise to the 4-vector in a four-dimensional Minkowski space-time. The expression for the wave phase has the form

$$\mathbf{kr} \pm \omega t = \phi = inv$$

(where $\mathbf{k} = \mathbf{x}^0 k_x + \mathbf{y}^0 k_y + \mathbf{z}^0 k_z$ is the vector formed by wave numbers k_x, k_y, k_z; $\mathbf{r} = \mathbf{x}^0 x + \mathbf{y}^0 y + \mathbf{z}^0 z$; $k_i = 2\pi n_i/\lambda$; ω is the circular wave frequency; n_i is the index of refraction in the ith direction, $i = x, y, z$; $\mathbf{x}^0$, $\mathbf{y}^0$, $\mathbf{z}^0$ are the orthogonal unit vectors, and λ is the wavelength) because the phase as a scalar quantity is invariant with respect to Lorentz transformations. For this reason, the quantities ω, $\mathbf{k}$ also give rise to the 4-vector in a generalized four-dimensional space of wave numbers, all components of which show similar translational properties during the transition from the frame of reference K to the frame of reference K'. Then $\Delta\mathbf{r}/\Delta t = v_{\rm g}$ is the group velocity of the wave, and $\omega/k = v_{\rm ph}$ is its phase velocity, where $k = (k_x^2 + k_y^2 + k_z^2)^{1/2}$; therefore, both $v_{\rm g}$ and $v_{\rm ph}$ have identical translational properties during the transition from the frame of reference K to the frame of reference K'.

Let us define the phase velocity of the counterpropagating waves each as a linear velocity of its fixed phase point travel around the ring.

According to Eqns (1), (2), the times $t^{\pm}$ are given by

$$t^{\pm} = \frac{2\pi R(1 \pm v_{\rm ph} R\Omega/c^2)}{v_{\rm ph}(1 - R^2\Omega^2/c^2)}\,. \tag{3}$$

The propagation time difference between counterrunning waves is found as

$$\Delta t = t^{+} - t^{-} = \frac{4\pi R^2 \Omega}{c^2(1 - R^2\Omega^2/c^2)}\,. \tag{4}$$

Thus, the difference between the circulation times of counterpropagating waves is independent of their phase velocity. Consequently, the propagation time difference due to the Sagnac effect is unrelated to whether the ring interferometer is filled with an optical medium or not. Moreover, it follows that for arbitrary waves (e.g. acoustic waves, the velocity of which is much lower than the velocity of light) such a time difference is equal to that of electromagnetic waves provided the wave frequency, enclosed area of the rotating ring interferometer, and angular velocity of rotation are identical.

It is also apparent from expression (4) that waves producing an interference pattern on the beam-splitting mirror at the ring input (i.e. those reaching it simultaneously after a full circulation) leave it with a time lag Δt. This in no way affects the visibility of the fringe pattern since we are dealing here with monochromatic harmonic oscillations. However, if an oscillation spectrum of finite width is utilized in a ring interferometer and Δt exceeds the correlation time of these oscillations, the fringe pattern visibility may fall considerably.

The majority of authors concerned with the assessment of the Sagnac effect size confine themselves to the calculation of the quantity Δt. It has been mentioned above, however, that in experiment the phase difference of two waves is measured rather than the times during which the counterpropagating waves travel their respective paths round the ring; hence, the necessity of a relevant expression.

In order to calculate a phase difference (attributable to the Sagnac effect) between counterpropagating waves at the exit from a ring, it is more expeditious to pass to the co-moving frame of reference K' rotating together with the interferometer. This is necessary for several reasons. Firstly, the interference pattern is recorded by a photodetector or imprinted on a photographic plate fixed in the frame of reference attending rotation (in fact, interference fringes are already apparent at the beam-splitter of the ring interferometer). Secondly, in the laboratory (stationary) system of coordinates K, the beam-splitter serves as both the source and actually the detector of radiation and, besides, is in motion. This may lead to a completely inadequate conclusion that the classical Doppler effect takes place in such a situation [39] (this mistake will be considered at a greater length in Section 5.3). The passage to the co-moving frame of reference K' attending the interferometer rotation (where the beam-splitter of the ring interferometer is immobile) obviates the necessity for considering this question here.

In accordance with the Lorentz transformations [40], the propagation time difference between counterrunning waves in the frame of reference K' is equal to

$$\Delta t' = \Delta t\sqrt{\left(1 - \frac{R^2\Omega^2}{c^2}\right)} = \frac{4\pi R^2\Omega}{c^2(1 - R^2\Omega^2/c^2)^{1/2}} \,. \tag{5}$$

The phase difference (attributable to the Sagnac effect) between the counterpropagating waves at the ring output is

$$\Phi_{\mathrm{S}} = \omega\Delta t' = \frac{4S\Omega\omega}{c^2(1 - R^2\Omega^2/c^2)^{1/2}} = \frac{8\pi S\Omega\nu}{c^2(1 - R^2\Omega^2/c^2)^{1/2}} \,, \tag{6}$$

where ν is the frequency of the radiation source in the frame of reference K' if the source is located at distance R from the center of rotation, i.e. exactly on the ring; $\omega = 2\pi\nu$ is the circular frequency of the radiation source, and $S = \pi R^2$ is the ring area.

If the radiation source is full at the center of rotation, then its radiation frequency is shifted by $R^2\Omega^2/(2c^2)$ to a higher frequency range with respect to the frequency of the same source lying at distance R from the center of rotation [41]. This effect was observed in experiment [42].

When a radiation source is placed in a fixed frame of reference K, the shift of its frequency on passage to the frame of reference K' attending the ring rotation depends on how radiation enters the ring interferometer. If the radiation from a stationary source falls directly on the moving beam-splitter of the ring interferometer, its frequency changes by $R\Omega/c$ due to the classical Doppler effect. The sign of this change depends on whether the beam-splitter approaches the source or moves away. This mode of radiation coupling into a ring interferometer is highly inconvenient because the radiation is transmitted over a small part of the ring revolution. For FRI connected with a fixed radiation source by a sufficiently long section of a single-mode optical fiber (SMOF), such a method ensures radiation transfer throughout many revolutions of the ring [43]. However, it should be borne in mind that linear and torsion strains in the connecting SMOF section lead to a change of its optical length, while its motion produces Fresnel – Fizeau light drag effect [39, 44] which is rather difficult to take into account in the analysis of experimental findings [45]. It is much more convenient to feed radiation into the ring interferometer from above, through a system of mirrors directing a light beam at the center of ring rotation. In this case, the radiation does not undergo a frequency shift, and the ring rotation leads to no optical disalignment. Thereupon the radiation runs from the center of rotation along the radius into the ring interferometer and undergoes a frequency change (rise), as mentioned above, only to second order in $R\Omega/c$. Such a way of radiation coupling into the ring interferometer was first proposed by F Harress [46] (see Refs [4 – 6, 8, 26, 47 – 49] for the description of experiments [46]) followed by B Pogany [50 – 52] and A Dufour and F Prunier [53]. A Einstein [54] analyzed the experiments of Harress [46] and showed that, using this method of radiation feeding into a ring interferometer, the radiation frequency does not change to first order in $R\Omega/c$.

Two important conclusions can be drawn from expression (6).

I. The phase difference between counterpropagating waves due to the Sagnac effect depends on the wave frequency ν rather than the wave phase velocity. It follows in particular that the phase difference between counterpropagating waves, attributable to the Sagnac effect in the optical range with $v_{\mathrm{ph}} = c/n$, depends neither on the refractive index n of the optical medium filling the interferometer nor on its dispersion $\mathrm{d}n/\mathrm{d}\lambda$, regardless of the ratio $R\Omega/c$. The authors of Ref. [10] supposed that such a situation holds only in the nonrelativistic limit. The dependence of the counterpropagating wave phase difference on the refractive index n in a single-mode optical fiber making the FRI was discussed in Ref. [55]. Later on, it was shown by means of rather complicated computations using the expression for the Fresnel – Fizeau drag coefficient in an optical medium [33, 56] that the phase difference between counterpropagating waves, attributable to the Sagnac effect, does not depend on the refractive index n. However, it did not put an end to the discussion about the effect of dispersion of the refractive index $\mathrm{d}n/\mathrm{d}\lambda$ [57]. (See also the works of Einstein [54] and Post [5] as well as our papers [8, 11] concerning this issue.) The phase velocity of an arbitrary wave being independent of medium dispersion, the dispersion can in no way influence the magnitude of the Sagnac effect.

II. In expression (2), we used the relativistic law of the composition of phase velocity v_{ph} and the ring rotational velocity $R\Omega$, in correspondence with the opinion that the Sagnac effect constitutes an STR effect [10, 11]. It will be shown in Section 5.2 that application of the Galilean law of the velocity composition to the consideration of the Sagnac effect for waves propagating in material media leads to the erroneous conclusion that this effect does not show itself.

To illustrate the use of expression (6), let us consider the Sagnac effect for electromagnetic waves in an interferometer filled with an optical medium. The light wavelength is then $\lambda = c/\nu$. In this case, expression (6) assumes the form

$$\Phi_{\mathrm{S}} = \frac{8\pi S\Omega}{\lambda c(1 - R^2\Omega^2/c^2)^{1/2}} \,. \tag{7}$$

It was mentioned in a previous paragraph that the influence of the refractive index of a medium and its dispersion on the magnitude of the Sagnac effect remains a matter of controversy. Below, there are expressions for the shift of interference fringes produced by counterpropagating waves in a ring interferometer, Δz ($\Phi_{\mathrm{S}} = 2\pi\Delta z$), and for the frequency difference of counterpropagating waves in a ring laser, $\Delta\nu$ (the frequency difference between counterpropagating waves, attributable to the Sagnac effect in a ring laser, is inversely proportional to the refractive index [5]). Both the interferometer and the laser cavity of perimeter L contain an optical medium with an index of refraction n and dispersion $\mathrm{d}n/\mathrm{d}\lambda$. The expressions are given for different situations of medium and cavity rotation with the proviso that $\Omega = \text{const}$ [5, 8].

(1) Both the interferometer and the medium rotate together as a whole:

$$\Delta z = \frac{4S\Omega n^2(1-\alpha)}{\lambda c} = \frac{4S\Omega}{\lambda c}\,, \qquad \Delta\nu = \frac{4S\Omega}{\lambda L n}\,, \tag{8}$$

where $\alpha = 1 - 1/n^2$ is the Fresnel drag coefficient, S is the enclosed area of the ring interferometer or the cavity of the ring laser, and λ is the light wavelength in a vacuum.

(2) The interferometer rotates, while the medium is stationary:

$$\Delta z = \frac{4S\Omega n^2}{\lambda c}\,, \qquad \Delta\nu = \frac{4S\Omega n}{\lambda L}\,. \tag{9}$$

(3) The interferometer is nonrotating and the medium rotates within fixed boundaries (this case may correspond to the rotation of a glass cylinder or pumping a fluid through a cuvette placed inside a stationary ring interferometer):

$$\Delta z = \frac{4S\Omega n^2\alpha}{\lambda c} = \frac{4S\Omega n^2}{\lambda c}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n}\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right),$$

$$\Delta\nu = \frac{4S\Omega n}{\lambda L}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n}\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right), \tag{10}$$

where $\alpha = 1 - 1/n^2 - (\lambda/n)(\mathrm{d}n/\mathrm{d}\lambda)$ is the Fresnel drag coefficient with the Lorentz correction [54].

(4) The interferometer is at rest, while the medium executes a translational motion and has movable boundaries:

$$\Delta z = \frac{4S\Omega n^2\alpha}{\lambda c} = \frac{4S\Omega n^2}{\lambda c}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n^2}\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right),$$

$$\Delta\nu = \frac{4S\Omega n}{\lambda L}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n^2}\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right), \tag{11}$$

where $\alpha = 1 - 1/n^2 - (\lambda/n^2)(\mathrm{d}n/\mathrm{d}\lambda)$ is the Laub drag coefficient.

The Sagnac effect in its pure form is realized only in Situation 1 when all parts of the interferometer rotate as a whole, and a shift of interference fringes is indicative of the rotation with respect to the inertial reference system. In all other situations, there is always a static part of the interferometer with respect to which one may measure rotation by a mechanical or any other technique. Situation 3 is in perfect correspondence with H Fizeau's experiments [59, 60] in which water was pumped through a cuvette placed in a stationary ring interferometer. Situation 4 fully reproduces the experiments of P Zeeman [61, 62] (see also reviews [6, 49]), in which among other things a bar of quartz or flint having optical surfaces perpendicular to a beam of light moved progressively in a stationary ring interferometer along the beam direction. These experiments were designed to verify the form of the Laub drag coefficient and may be regarded as a demonstration of the modified Fresnel – Fizeau drag effect. The physical meaning of the appearance of a phase difference between counterpropagating waves in Situation 2 corresponding to the experiments of Dufour and Prunier [53] is more difficult to define. If the situation is considered in a frame of reference attached to the rotating interferometer and then the filling medium rotates with respect to this reference system, it may be stated that the Fresnel – Fizeau light drag takes place. The Sagnac effect also occurs in this case. Therefore, Situation 2 corresponds to the presence of both the Sagnac effect and the Fresnel – Fizeau drag effect in the frame of reference attached to the rotating interferometer.

Expressions for the form of the drag coefficient in Situations 1, 3, and 4 were obtained by Einstein [54] back in 1914 in the realm of STR.

Experiments on ring interferometers corresponding to Situations 1, 2, and 3 have been described in detail in our review [8] (see also Refs [5, 43, 49]). Situation 4, besides having been reproduced in Zeeman's experiments [61, 62], also prevailed in the notorious experiments of W Kantor [63], in which two thin optical plates attached to the opposing arms of a fixed ring interferometer moved in opposite directions, and their motion at the instant of measurement was close to translational. The results obtained in experiments [63] led the author to the wrong conclusion that the moving plates, regarded as the source of radiation, emitted it at a velocity exceeding that of light. Despite the higher accuracy of similar experiments repeated by other authors [64], they failed to confirm this conclusion. We present here for the first time an adequate expression for the shift of interference fringes in Situation 4.

The form of the drag coefficient was verified experimentally in the 1970s [65, 66].

The interference of counterpropagating waves of different nature is not the sole tool for detecting the Sagnac effect. If a macroscopic body moves round a circle with velocity v in relation to a rotating reference system, the difference between the circulation times in opposite directions is also described by expression (4).

If the body is a material particle to which a de Broglie wave corresponds, there appears a possibility, as mentioned above, of using the interference of counterpropagating waves for recording the Sagnac effect.

A Einstein was the first to consider wave propagation in a rotating ring interferometer with the aid of STR. His work [54] written in 1914 contained a discussion of Harress' experiments [46]. In this work, Einstein was not interested in the derivation of an expression for the phase difference between counterpropagating waves due to the Sagnac effect. He concentrated instead on the narrower problem of finding an expression for the form of the light drag coefficient in the event of a co-moving ring interferometer and optical medium. His intention was to show that this expression did not depend on the optical medium's dispersion. H Witte considered the Sagnac effect in the framework of STR in the same year [67, 68].

In 1920, M Laue [26] undertook a theoretical analysis of the experiments conducted in Ref. [46], proceeding from the relativistic law of velocity composition. From a didactic point of view, it was a lame attempt since M Laue neglected the terms of second and higher orders in v/c in the expression for the travel times of counterpropagating waves. As a result, he eventually restricted the consideration to the classical kinematic analysis of the Sagnac effect (see Section 5.2) based on the Galilean law of the composition of counterpropagating wave velocities and the platform rotational velocity as he had done in an earlier work [69]. Moreover, Laue actually tried in Ref. [69] to reduce the Sagnac effect to the Fizeau drag effect [59, 90] (see Section 5.4).

L Silberstein [27] arrived at an expression for the phase difference of counterpropagating waves due to the Sagnac effect by taking advantage of the invariance property of the interval

$$ds^2 = c^2\,dt^2 - dr^2 - r^2\,d\varphi^2 - dz^2$$

(where t is the time, and r, φ, and z are cylindrical coordinates). However, the author of the cited work considered such an explanation unsatisfactory and further addressed the Sagnac effect from the standpoint of STR, ether theory (see Section 5.1), and the action of Coriolis forces in the path of counterpropagating light beams (see Section 5.5).

In the works of A Lunn [70], C Runge [71], C Corps [28], P Langevin [72], A Metz [29 – 32], C Møller [73], E Post [5], S Ezekiel [33], H Arditty and H Lefevre [37, 38, 74, 75], J Wilkinson [76], D Allan, M Weiss, and N Ashby [77], A Logunov and Yu Chugreev [10, 78], D Dieks and G Nienhuis [79], D Dieks [80], F Hehl and W Ni [81], G Vugal'ter and G Malykin [11], the Sagnac effect is also considered from the STR standpoint.

The analysis of the Sagnac effect in a laboratory (inertial) coordinate system within STR is based on the fact that this theory allows relativistic kinematic transformations to be considered not only for a point making a uniform motion but also for a point undergoing acceleration [82].

There was a long-standing opinion after STR was created [40] that all events in noninertial (e.g. rotating) reference systems should be considered only in the domain of GTR (see, for instance, Refs [82 – 84]). However, the use of GTR is unnecessary when purely kinematic effects are considered. Here is a quotation from Einstein's work [85]: "Kinematic equivalence of two coordinate systems is actually not restricted to the case when systems K and K' make rectilinear uniform motions. From the kinematic standpoint, this equivalence is fairly well satisfied, for instance, if one system uniformly rotates with respect to the other." W Wien [86] appears to have been the first to notice that STR can be used for the description of phenomena in noninertial systems after he came to know the results obtained by H Minkowski in work [87]. In the absence of gravitational fields, when there is no space curvature, noninertial frames of reference can be described in terms of STR in the most general way, for arbitrary accelerations and not only for kinematic events [88, 89]. In Ref. [10], such an approach was applied to the computation of the Sagnac effect in a reference system attached to a rotating ring interferometer. In that case, the authors used a metric tensor in the four-dimensional Minkowski space-time [87] for which the curvature tensor was zero.

There is a third method for the computation of the Sagnac effect in the framework of STR. It consists in the calculation of the Lorentzian time dilation difference in the moving reference systems K^+ and K^- which attend the fixed phase points of counterpropagating waves. This difference can be conveniently illustrated as the difference between the readings of two clocks sent along a circular path with radius R. The clocks are transported with equal but opposite linear velocities (v) relative to the center of a disk rotating at an angular velocity Ω. The center of the ring and the center of rotation coincide, as before (the reference systems K^+ and K^- may be substituted by a set of instantaneously attending inertial frames of reference). At the instant of time the clocks' positions in space coincide, they undergo synchronization. The difference between the readings of the clocks is measured after the one travelling in the direction of disk rotation (the corresponding parameters are designated $+$) and the other moving in the opposite direction ($-$) have completed approximately one circular trip each and met again. Without any loss of generality, it may be assumed that $v = v_{ph}$, i.e. the clock velocity with respect to the disk is identical with the phase velocity of counterpropagating waves. Certainly, this is true if $v_{ph} \leqslant c$.

If clock synchronization occurs at the point on the ring occupied by the beam-splitting mirror, the clocks (similar to the pulses of light above) will meet again at another point of the ring after having completed one revolution each. A simple computation indicates that this point falls $R\Omega v_{ph}/c^2$ radians behind the new angular position of the beam-splitter (synchronization point) which it will occupy when the clocks reencounter each other in space. Therefore, the propagation time difference between counterrunning waves, attributable to the Sagnac effect and computed using expressions (1) – (6) (i.e. from their travel times between beam-splitters in a rotating ring interferometer), and the time lag in synchronized clocks (fixed phase points of the wave front) travelling in their own reference systems from the beam-splitter to the meeting point should differ but insignificantly. The relative difference measures $R\Omega v_{ph}/c^2$. Thus, an accurate comparison between the calculated time differences relevant to counterpropagating waves, obtained by these methods, encounters difficulty. Further calculations will be done in neglect of small relativistic corrections which are of no practical value in the present case because the exact expressions for the magnitude of the Sagnac effect have already been found [see Eqns (4) – (7)]. Here, it is more important to illustrate the physical meaning of this effect in reference systems K^+ and K^- attached to the fixed phase points of counterpropagating wave fronts.

In accordance with the Lorentz transformations, the times spent by the clocks to trace around the ring in the clock-attending rotating reference systems K^+ and K^- are

$$t^{K^\pm} = t\sqrt{1 - \frac{(v_{ph}^\pm)^2}{c^2}}\,, \tag{12}$$

where $t = 4\pi R/(v_{ph}^+ + v_{ph}^-)$ is the time needed for counterpropagating waves to pass from the beam-splitter of a ring interferometer to their meeting point in the laboratory (fixed) reference system, and $v_{ph}^\pm$ are the phase velocities of counterpropagating waves in the laboratory frame of reference [see expression (2)].

The difference between times t^{K^+} and t^{K^-} after checking the clocks is

$$\Delta t^{K^+,K^-} = t^{K^-} - t^{K^+} \simeq \frac{4\pi R^2\Omega}{c^2\sqrt{1 - v_{ph}^2/c^2}} = \frac{4S\Omega}{c^2\sqrt{1 - v_{ph}^2/c^2}}\,, \tag{13}$$

where S is the area enclosed by the ring.

Expression (13) gives the difference between the readings of clocks which are in motion in different frames of reference K^+ and K^-. The velocities of these frames of reference with respect to a stationary reference system K are very close in absolute figures. Nevertheless, they are somewhat different [see expression (2)] and opposite in sign. The calculation of the time difference Δt in the fixed frame of reference K gives

$$\Delta t \sim \frac{\Delta t^{K^+,K^-}}{\sqrt{1 - v_{\mathrm{ph}}^2/c^2}} \sim \frac{4S\Omega}{c^2}\,. \tag{14}$$

Expression (14) coincides with expression (4) to within small relativistic corrections. In other words, the propagation time difference between counterpropagating waves, calculated by the latter method, does not depend on their phase velocity either. It is also worth noting that in expression (4) $t^+ > t^-$ while in expression (13) $t^{K^+} < t^{K^-}$. This is due to the dissimilarity between the time difference of counterpropagating waves passing around the rotating ring from one beam-splitter to another in the laboratory frame of reference K and the time difference of counterpropagating waves which travel from the beam-splitter to the meeting point in their intrinsic frames of reference K^+ and K^-.

2.2 Sagnac effect in the general theory of relativity

It has been shown in the foregoing that the Sagnac effect is a corollary to the relativistic law of velocity composition, i.e. it presents a kinematic effect of STR. Despite this, many authors consider the Sagnac effect in terms of GTR.

Let us try to understand why these authors choose to tackle the problem in the domain of GTR. This is what M-A Tonnelat writes in his book [35] after having described the results of Sagnac's experiment: "Thus, the following ensues from these experiments concerning systems with acceleration. In the case of accelerated motions, it appears possible to determine absolute motion. In the absence of reference systems based on other solids, it needs to be assumed that such absolute motions (e.g. rotation of the Earth in experiments with a Foucault pendulum) occur with respect to a vacuum, i.e. absolute space. In the light of criticism met by the special theory of relativity, this assumption should be regarded as unsatisfactory, and the question may be posed: is not such absolute motion related by necessity to the presence of other masses, i.e. the existence of remote stars? Mach held this view." This quotation prompts two propositions.

(1) It is supposed that the Sagnac effect may be due to the action of inertial forces which arise in a rotating frame of reference.

(2) It is conjectured whether inertial forces themselves may result from the effect of large remote masses, i.e. the Mach principle [90] is verified.

In a sense the first proposition appears to be true. As will be shown below (see Sections 3.1 and 3.2), the Sagnac effect is not an immediate consequence of the action of inertial forces on a macroscopic body, material particle or wave in a rotating frame of reference. Nevertheless, it may be interpreted (with certain limitations and assumptions) as a corollary to different time dilations in rotating frames of reference, attached to the motion of counterpropagating wave phase fronts in the potential of the equivalent gravitational field of centrifugal accelerations (due to different counterrunning wave velocities, hence different centrifugal accelerations), with respect to the inertial coordinate system as a ring interferometer rotates. Alternatively, the Sagnac effect may arise from the different signs of the potential in the equivalent gravitational fields of Coriolis accelerations for counterpropagating waves; in other words, it may be due to inertial forces. It is worthwhile to note that the scientific literature contains many allusions to the similarity between the Sagnac effect and the Foucault pendulum (see, for instance, Ref. [91]). It should be borne in mind, however, that neither the centrifugal force nor Coriolis force may be the direct cause of the Sagnac effect occurrence since both are perpendicular to the motion of a wave front (macroscopic body or material particle) during its travel along a circular path and produce no work. The Sagnac effect relates to the category of kinematic effects unrelated to any force.

The validity of the second proposition remains to be established. Classical mechanics does not take the nature of inertial forces as its main subject matter [82, 92–94]; nevertheless, there has been a long-standing debate on whether these forces are real or fictitious [95]. In the early GTR history, Einstein was convinced of the validity of the Mach principle [85, 96, 97]. However, he never again applied it to explain the nature of inertial forces after he had acquainted himself with a paper by W de Sitter [98]. (See Ref. [99] for the evolution of Einstein's views of the problem.) The works of J Lense and H Thirring [100–103] gave rise to the opinion that the existence of centrifugal and Coriolis forces in a rotating frame of reference was due to the Lense–Thirring effect caused by the rotation of all masses in the Universe relative to an observer located in the rotating frame of reference. This opinion was shared by H Weyl [104]. However, it became clear by the 1960s that it is impossible to strictly prove a Mach principle in GTR in this form, too [99, 105, 106]. The main difficulty is that in considering the relative motion of distant masses with respect to a rotating frame of reference one has to deal with supraluminal relative speeds; this renders the calculation of a Lense–Thirring effect not feasible. Indeed, to an observer on the Earth, rotating at an angular velocity of only 1 rps, the Moon (to say nothing about other masses in the Universe) will travel with a velocity exceeding the velocity of light. Difficulties encountered in applying the Mach principle to GTR stimulated the development of alternative gravitation theories, in particular, Brans–Dicke theory [107, 108] and Logunov's relativistic gravitation theory [109]. At present, there are many gravitation theories different from GTR; they are described in Ref. [110]. A characteristic feature of the majority of such theories consists in that they yield practically the same predictions as GTR, given weak gravitational fields (what occurs, for example, within the bounds of the solar system).

The classical analysis of the Sagnac effect in terms of GTR was undertaken by L D Landau and E M Lifshitz in their *Course of Theoretical Physics* [34]. They used a metric tensor to calculate the propagation time difference between counter-running waves in a frame of reference attending rotation. In Ref. [34], elements of the metric tensor in the rotating frame of reference that lacked gravitational fields (i.e. in the absence of space curvature) were calculated by the same method as in Ref. [10], that is taking advantage of an interval invariance. Therefore, the discussion of whether purely kinematic problems in rotating frames of reference in the absence of gravitating masses should be considered in terms of GTR or STR is a mere scholasticism; indeed, it is a question of definition (from the mathematical standpoint, there is no difference between calculations in GTR [34] and STR [10]).

In the presence of real gravitational fields, the expression for the gravitational potential U contains additional terms corresponding to these fields, and the Sagnac effect can be computed using only GTR [111–116]. It appears more rational to use GTR if a ring interferometer rotates with a significant angular acceleration and the angular velocity considerably changes during the by-pass time around a ring for counterpropagating waves [57, 117].

The Sagnac effect was first considered in the framework of GTR by P Langevin [118] and L Silberstein [27] in 1921. Later on, the same approach was used in the works of L D Landau and E M Lifshitz [34], M-A Tonnelat [35], C Heer [119], E Post [120, 121], A M Khromykh [122], É M Belenov and E P Markin [117], A I Bakalyar and D P Luk'yanov [123], A M Volkov, A A Izmest'ev, V A Kiselev, and G V Skrotskiĭ [111, 124, 125], B F Fedorov, A G Sheremet'ev, and V N Umnikov [126], A Ashtekar and A Magnon [127], S I Bychkov, D P Luk'yanov and A I Bakalyar [128], L Stodolsky [129], I V Shpak and A V Solomin [116], J Anandan [130,131], V F Fateev [57, 115], W Chow, D Gea-Banacloche, L Pedrotti, V Sanders, W Schleich, and M Scalli [132], and A G Sheremet'ev [36]. A Sommerfeld [91] also believed that the Sagnac effect can be strictly considered only in the framework of GTR.

The Sagnac effect is easy to calculate in GTR when a ring interferometer or the cavity of a ring laser is devoid of an optical medium. In the presence of an optical medium, the appropriate calculation in GTR is much more difficult than in STR and often leads to the advent of mistakes.

Here are a few illustrative examples of such mistakes. So, it follows from Refs [124, 125] that in a rotating ring laser the polarization planes of counterpropagating waves will be turned between themselves through an angle which is numerically equal to their phase difference attributable to the Sagnac effect. It will be shown below (see Section 5.6) that this is an erroneous result which was previously obtained in Ref. [133] by a different (incorrect) method, that is by taking into account photon orbital momenta in a rotating frame of reference.

Difficulties also arise when GTR is used to evaluate the influence exerted on the Sagnac effect by the refractive index of the medium filling a ring interferometer or the cavity of a ring laser and by the dispersion of this index. It has been shown in the foregoing that the phase difference between counterpropagating waves in a ring interferometer due to the Sagnac effect depends neither on the refractive index nor on its dispersion if the interferometer (or cavity) uniformly rotates together with the filling medium as a single entity. Conversely, it follows from Refs [116, 134] that in this case the dispersion of the SMOF refractive index appears in the expression for the phase difference between counterpropagating waves, while the light drag coefficient has the form of the Laub coefficient [54, 58]. This discrepancy obscures the problem under consideration.

For all that, the use of GTR for the calculation of the Sagnac effect showing itself in an interferometer filled with an optical medium does not necessarily leads to a mistake. By way of example, it has been shown in Refs [35, 57, 115] that in a rotating interferometer with a co-moving medium the fringe shift depends neither on the refractive index nor its dispersion. However, the problems of interest are easier to consider in the framework of STR.

The main result of this section is that the use of GTR for the calculation of the Sagnac effect is warranted but not rational. It complicates the computation and sometimes leads to mistakes. The use of GTR is relevant when a ring interferometer rotates with a large angular acceleration. In addition, if the gravitational field effect is to be taken into account, GTR should be necessarily used.

2.3 Methods for computing the Sagnac effect for electromagnetic waves in anisotropic media

The Sagnac effect for electromagnetic waves is not infrequently calculated using Maxwell equations. Certain authors (see Refs [5, 38, 111 – 116, 119, 122, 124, 125, 132, 135 – 147]) calculated the Sagnac effect by solving Maxwell equations in a rotating frame of reference. This is a merely computational method rather than an original approach to the consideration of the Sagnac effect. In each particular case, one of two available options is used for the calculation, either STR [5, 137, 142, 145, 146] or GTR [111 – 116, 119, 122, 124, 125, 132, 135, 138 – 141, 144, 147]. This method comes to mind when it is necessary to compute the Sagnac effect in the presence of an optical medium, in particular, when the medium is an anisotropic one. In Ref. [136], the Maxwell equations are used in combination with an incorrect approach to the evaluation of the Sagnac effect (the reader is referred to Section 5.7 for a detailed discussion). It is worthwhile to note that the Maxwell equations are invariant with respect to the Lorentz transformations and therefore correspond to STR.

Meanwhile, it is well known that the medium's anisotropy in a ring interferometer is much easier to take into consideration using Johns matrices [148] since they are derived from the Maxwell equations.

The Johns matrices were utilized in Refs [149 – 163] to calculate the unrelated-to-rotation phase shift of an outgoing interference signal related to the polarizational nonreciprocity of the FRI contour [149 – 153, 164 – 166] made of anisotropic SMOF. This phase shift adds to the phase shift attributable to the Sagnac effect and leads to an error in the measurement of the angular velocity of rotation. If the SMOF of which the FRI contour is made exhibits random irregularities of birefringence, a drift of the FRI zero point occurs, characterized by a mean value and dispersion [151 – 163].

When Silberstein [27] contemplated in 1921 the then forthcoming experiments of A Michelson and H Gale [167, 168], which he inspired and partly supported, he pointed to the possibility of recording the Lense – Thirring effect caused by rotation of the Earth [100 – 103] using an optical ring interferometer with a large reflecting surface. In 1981, M Scully and co-authors [169] reexamined this problem and discussed the feasibility of applying FRI to the measurement of certain GTR effects including the Lense – Thirring effect. According to our estimates, such experiments could be realized with a FRI having one SMOF loop about 7 km in diameter [161, 163] because mode coupling on SMOF irregularities leads to a drift of the interference pattern zero at the exit from FRI. Another difficulty results from the rotation of the Earth, which accounts for rather a large shift of the fringe pattern origin due to the Sagnac effect in such a big FRI. The use of a source of polychromatic radiation allows the FRI zero drift to be significantly diminished [152 – 155]. In this case, the visibility of the interference pattern falls off with a rise in Φ_S because this quantity is proportional to the optical frequency of light, in agreement with expression (6). This finding has been confirmed in experiment [170]. We employed the method of Johns matrices to show that the presence of random irregularities in SMOF is responsible for a significant improvement of the interference pattern visibility at very large Φ_S values [171]. Thus, calculations using Johns matrices are conducive to the solution of rather complicated problems pertaining to the assessment of polarizational nonreciprocity and visibility of the interference pattern in FRI with a randomly anisotropic medium.

The rotation-related anisotropy of a medium was considered in Ref. [172] using Maxwell equations. It should be emphasized, however, that the effect in question is very small at the real angular velocities of rotation (according to Ref. [172], rotation-induced circular birefringence is $\Delta n_c \sim 10^{-18}$) and can hardly be observed in experiment.

When an anisotropic optical medium is in motion with respect to the interferometer (see, for instance, the experiments described in Refs [43, 45, 53], the Maxwell equations need to be solved in order to take into account the bending of the ray trajectory related to the Fizeau light drag [173].

2.4 Main results of the analysis of the Sagnac effect in the framework of relativity theory

Thus, the Sagnac effect in the realm of relativity theory may be addressed in the following contexts.

In the framework of STR:

— in a laboratory (stationary) system of coordinates taking advantage of the relativistic law of velocity composition;

— in a co-moving system of coordinates attached to a rotating ring interferometer using a metric tensor in plane four-dimensional Minkowski space-time;

— in a co-moving system of coordinates attached to the wave fronts (fixed phase points) of counterpropagating waves using the relativistic law of velocity composition and Lorentz transformations.

In the framework of GTR:

— in a co-moving coordinate system attached to a rotating ring interferometer using a metric tensor in the absence of space curvature, i.e. in the absence of gravitational fields.

The magnitude of the Sagnac effect for electromagnetic waves and, in particular, light in anisotropic media can be computed by solving the corresponding Maxwell equations or using Johns matrices.

3. Conditionally correct explanations of the Sagnac effect

By conditionally correct are meant such explanations of the Sagnac effect which give approximate expressions for the phase difference between counterpropagating waves in a rotating ring interferometer, when certain constraints are imposed on the parameters of the system, viz. linear velocity of a ring rotation, wave (material particle or body) velocity with respect to the co-moving frame of reference attending rotation, mass of a material particle, etc. Moreover, these explanations imply a number of assumptions and certain *a priori* suppositions which, generally speaking, ensue from nowhere. Nevertheless, conditionally correct explanations allow the physical meaning of the Sagnac effect to be clearly demonstrated. The whole variety of conditionally correct explanations of the Sagnac effect can be reduced to the examination of effects of the Newtonian nonrelativistic scalar potential or vector potential of the equivalent gravitational field of inertial forces (centrifugal or Coriolis forces) on the time dilation in a rotating frame of reference or on the phase change of the material particle wave function. In other words, this approach to the evaluation of the Sagnac effect is based on the equivalence principle.

However, this does not mean that the Sagnac effect is considered in the framework of GTR. The potential of the equivalent gravitational field of inertial forces was used by Einstein [174, 175] for the calculation of time dilation in the accelerated frames of reference before he created GTR; not infrequently, it failed to yield a correct result. Suffice it to say that the deflection of a light beam in the solar gravitational field was underestimated as being two times lower than its real value [175]. Eventually, Einstein obtained the correct solution in the framework of GTR [96] taking into consideration the space curvature ascribed to the solar mass action.

3.1 Sagnac effect as a consequence of the distinction between nonrelativistic scalar gravitational potentials of centrifugal forces in frames of reference attached to counterpropagating waves

This section is focused on a simple and physically illustrative derivation of the expression assessing the size of the Sagnac effect, based on the relativistic law of velocity composition, the equivalence principle, and the time dilation phenomenon in a gravitational field.

Let us consider two rotating frames of reference, K^+ and K^-, co-moving the travel of fixed phase points (phase fronts) of counterpropagating waves or of a certain clock the velocity of which equals the phase front velocity (as before, the plus sign refers to a wave propagating in the direction of rotation of a ring interferometer, and the minus sign to a wave travelling in the opposite sense). These rotating frames of reference give rise to centrifugal accelerations

$$a_c^{\pm} = (\Omega^{\pm})^2 R\,,$$

where $\Omega^{\pm} = v_{ph}^{\pm}/R$ are the angular velocities of the fixed phase points of counterpropagating waves, measured in the frame of reference K. Absolute values of accelerations $a_c^{\pm}$ differ because of different $(v_{ph}^{\pm})^2$ [see expression (2)]. In agreement with the equivalence principle [34, 35], the rotating reference systems K^+ and K^- may be substituted by the inertial systems K_{in}^+ and K_{in}^-, respectively, in which the gravitational fields are involved and give rise to gravitational forces coincident in terms of size and direction with the centrifugal forces in the noninertial frames of reference K^+ and K^-. The presence of a gravitational field has been shown to lead to the corresponding time dilation [35, 41, 83]. The time dilation in the frames of reference K_{in}^+ and K_{in}^- with respect to time t in a laboratory (stationary) frame of reference K is [34, 35]

$$t^{K_{in}^{\pm}} = t\sqrt{1 + \frac{2U_{in}^{\pm}}{c^2}}\,, \tag{15}$$

where $t = 4\pi R/(v_{ph}^+ + v_{ph}^-)$ is the time for which counterpropagating waves move round a ring from the beam-splitter to the point of their intersection in the laboratory (stationary) reference system [see Section 2.1]; $U_{in}^{\pm}$ denote the nonrelativistic gravitational potentials of the equivalent fields of centrifugal forces in the frames of reference K_{in}^+ and K_{in}^-. It should be noted that the computing method under consideration is valid if $U/c^2 \ll 1$ [34].

The magnitude of the nonrelativistic gravitational potential is related to the acceleration in the following way [34]:

$$a = -\operatorname{grad} U\,.$$

Hence, one finds

$$U = -\int a\, \mathrm{d}l\,,$$

where $\mathrm{d}l$ is the element of length along a certain direction. The potential U is determined here up to the constant because the lower limit of integration may be chosen arbitrarily. For gravitational fields corresponding to real masses, the limit of integration is usually chosen in such a way that the condition $U = 0$ holds at infinity [34, 41]. Here, we shall choose it so that the condition $U = 0$ is satisfied at the center of rotation ($R = 0$), where there is no centrifugal acceleration. Then [34]

$$U_{\mathrm{in}}^{\pm} = -\int_0^R (\Omega^{\pm})^2 R\, \mathrm{d}R = -\frac{(\Omega^{\pm})^2 R^2}{2} = -\frac{(v_{\mathrm{ph}}^{\pm})^2}{2}\,.$$

The travel time of counterpropagating wave phase fronts (clock circulation around an enclosed area) in system K in the case of disk rotation is $t \sim 2\pi R/v_{\mathrm{ph}}$, whence

$$t^{K_{\mathrm{in}}^{\pm}} = t\sqrt{1 - \frac{(v_{\mathrm{ph}}^{\pm})^2}{c^2}}\,, \qquad (16)$$

where $t \sim 2\pi R/v_{\mathrm{ph}}$. Expression (16) completely coincides with Eqn (12) obtained in the framework of STR. By this means the Sagnac effect is attributable here to the fact that one clock shows a smaller lapse of time than the other due to the difference in gravitational potentials equivalent to centrifugal accelerations in the co-moving reference systems K_{in}^{+} and K_{in}^{-}. In contrast, the Sagnac effect in STR results from the fact that one clock is slower than the other throughout their journey in counter directions on a rotating disk, in accordance with different Lorentzian time contractions in the co-moving frames of reference K^{+} and K^{-}. Thus, in the absence of real gravitational fields corresponding to gravitating masses, the explanations of the Sagnac effect in the framework of STR based on different Lorentzian time contractions [see expressions (12)–(14)] and that based on Eqn (16) are absolutely identical, in agreement with the equivalence principle.

3.2 Sagnac effect as a consequence of sign distinction between nonrelativistic scalar gravitational potentials of Coriolis forces for counterpropagating waves in a frame of reference attaching rotation

Using the nonrelativistic gravitational potential, it is possible to compute the Sagnac effect in an inertial frame of reference K'_{in} equivalent to the frame of reference K' attached to a rotating ring interferometer. In this frame of reference, the absolute phase velocities of counterpropagating waves are the same and equal to v_{ph}. Bodies in motion (and, in particular, clocks) accompanying fixed phase points of counterpropagating waves undergo Coriolis acceleration, besides centrifugal acceleration. Its absolute value is $2\Omega v_{\mathrm{ph}}$ and the direction depends on whether a body travels in the direction of rotation (in such a case, it coincides with the direction of centrifugal acceleration) or in the opposite direction (then it is opposite to the direction of centrifugal acceleration).

There is, however, a peculiarity pertaining to the definition of the gravitational potential of the Coriolis force. The thing is that the Coriolis force, similar to the Lorentz force (see Refs [176–178] for the analogy between the two), is not found to belong to a potential one. Both are referred to as gyroscopic forces [179] (I E Tamm calls the Lorentz force a 'solenoidal force' in his textbook [180]). A characteristic feature shared by the two forces is that they do not perform any work because they are always oriented perpendicular to the body's velocity. Nevertheless, the notion of a scalar potential may be introduced for such forces, with certain constraints and reservations. For example, the role of the potential function for a magnetic field is played by a function the decrease of which is equivalent to the work done by the ponderomotive forces of a magnetic field; however, this function is not identical to the magnetic field potential energy [180]. An adequate description of the potential function of a magnetic field can be obtained provided the path of integration does not enclose the source of the magnetic field, i.e. a current-carrying conductor. Specifically, the potential function may be defined inside a long current-carrying solenoid. The Coriolis force being formally (for the purpose of a mathematical description) analogous to the Lorentz force [176–178] inside a solenoid (the doubled angular velocity of rotation 2Ω in the respective expressions is substituted by magnetic induction B), it is possible to define the scalar gravitational potential of Coriolis forces.

Let us choose the limits of integration in the expression for gravitational potential following the aforedescribed procedure such that the condition $U = 0$ be fulfilled in the center of rotation where centrifugal acceleration is absent. This ensures in particular the equality of potentials for counterpropagating waves at a point corresponding to the center of rotation. The potentials for counterpropagating waves in the coordinate system K'_{in} (which is inertial and equivalent to the noninertial system K' accompanying rotation) are given by

$$U'^{\,\pm}_{\mathrm{in}} = -\int_0^R \left[\Omega^2 R \mp 2\Omega v_{\mathrm{ph}}\right] \mathrm{d}R = -\frac{\Omega^2 R^2}{2} \mp 2\Omega v_{\mathrm{ph}} R\,.$$

The travel time of the fixed phase points attendant to counterpropagating waves (of circular clock motion) in the system K'_{in} on a rotating disk is

$$(t^{\pm})^{K'_{\mathrm{in}}} = t\sqrt{1 + \frac{(U'_{\mathrm{in}})^{\pm}}{c^2}} = t\sqrt{1 - \frac{\Omega^2 R^2}{2c^2} \mp \frac{2\Omega v_{\mathrm{ph}} R}{c^2}}\,, \qquad (17)$$

where $t = 4\pi R/(v_{\mathrm{ph}}^{+} + v_{\mathrm{ph}}^{-})$ is the time required by counterpropagating waves to pass from the beam-splitter to their meeting point in a laboratory (stationary) frame of reference (see Section 2.1).

Expanding $(t^{\pm})^{K'_{\mathrm{in}}}$ in powers of the small parameter

$$\frac{2\Omega v_{\mathrm{ph}} R}{2c^2} \ll 1\,,$$

and neglecting the effect of the gravitational potential corresponding to the centrifugal acceleration:

$$\frac{\Omega^2 R^2}{2c^2} \ll \frac{2\Omega v_{\mathrm{ph}} R}{c^2}\,,$$

leads to an approximate expression for the travel time difference between the fixed phase points of counterpropagating waves, or clocks the velocities of which coincide with the velocities of these points:

$$\Delta t^{K'_{\mathrm{in}}} \simeq \frac{t\, 2\, v_{\mathrm{ph}} \Omega R}{c^2} = \frac{4S\Omega}{c^2}\,. \qquad (18)$$

Expression (18) coincides, to within small relativistic corrections, with expression (5) for the travel time lag between counterpropagating waves in the frame of reference K' attached to a rotating ring interferometer. Hence, it may be concluded that the Sagnac effect in a frame of reference attending the rotation should be considered as a consequence of the time dilation distinction for counterpropagating waves due to the effect of the nonrelativistic gravitational potential corresponding to Coriolis forces and having different signs for the two counterpropagating waves.

3.3 Sagnac effect in quantum mechanics as a consequence of effects of the vector potential of Coriolis forces on the wave function phases of counterpropagating waves in a frame of reference attaching rotation

The vector potential of the Coriolis force can be introduced correctly, unlike its scalar potential which is introduced with many assumptions and reservations. In agreement with quantum-mechanical laws, the vector potential acts on the phase of a wave function. Similar to the scalar potential of the Coriolis force, its vector potential has no influence on the particle's coordinate and velocity. The calculation of the phase difference between de Broglie counterpropagating waves in a rotating ring interferometer was undertaken in Refs [9, 17, 130, 131, 181–183]. The authors utilized the solutions of the corresponding Schrödinger, Dirac, and Klein–Gordon equations for the purpose [182]. The calculations were usually done with regard for the Wentzel-Kramers–Brillouin (WKB) approximation [9, 17, 181]. The phase difference between counterpropagating waves was determined from the following expression [9]

$$\Phi_{\mathrm{S}} = \frac{8\pi E S\Omega}{hc^2}\,, \tag{19}$$

where E is the total energy of a material particle, and h is the Planck constant.

The substitution of $E = h\nu$ (where ν is the de Broglie wave frequency of a material particle or light quantum frequency) into Eqn (19) yields expression (6) accurate to within small relativistic corrections. Conversely, the substitution of $\nu = E/h$ into (6) leads to expression (19). Therefore, quantum-mechanical calculations of the imaginary part of a wave function are not at all necessary to compute the phase shift of de Broglie counterpropagating waves in a rotating ring interferometer, attributable to the Sagnac effect. The result can be just as well obtained by simple kinematic calculations in the framework of STR because the method is suitable for waves of arbitrary nature (see above considerations). The substitution of the expression for the total energy of a material particle with the nonzero rest mass $E = mc^2$, where

$$m = m_0\sqrt{1 - \frac{v_m^2}{c^2}}$$

is the particle's relativistic mass, m_0 is its rest mass, and v_m its velocity, into expression (19) gives the well-known relationship for the Sagnac effect, as applied to de Broglie waves [24]:

$$\Phi_{\mathrm{S}} = \frac{8\pi S\Omega m}{h}\,. \tag{20}$$

It follows from Eqn (20), in particular, that the phase difference due to the Sagnac effect for the de Broglie waves is independent, in the nonrelativistic limit, of the particle's velocity v_m, i.e. of the de Broglie wavelength $\lambda_m = h/(mv_m)$ [24]. Thus, the utilization of a polychromatic beam of material particles does not lead in this case (contrary to the case of electromagnetic waves) to the impaired visibility of the interference pattern.

It is worthwhile to note that for several reasons there are no ring interferometers measuring de Broglie waves. Most popular are Mach–Zehnder equal-arm interferometers in which particles meet after each has completed half of its closed path length. Due to this, the effective area of Mach–Zehnder interferometers goes half that of ring interferometers of a similar configuration (therefore, expressions (19), (20) contain 4 instead of 8).

4. Attempts to explain the Sagnac effect by analogy with other effects

Drawing an analogy between different effects has nothing to do with an attempt to reduce one effect to another. It has the purpose of clearly explaining the physical meaning of an effect by comparison with a similar one which is simpler, better known and easier to understand. In this context, such an approach should not be regarded as incorrect. Indeed, drawing an analogy may facilitate understanding in selected cases when the effects of interest have close physical interpretations. Whenever there occurs a merely formal similarity between two effects, the analogy is of little help for understanding their physical meaning even though it can facilitate the application of the mathematical apparatus, well-developed for one effect, to the explanation of the other.

4.1 Analogy between the Sagnac and Aharonov–Bohm effects

Certain authors (see Refs [79, 127, 130, 181–186]) make an analogy between the Sagnac and Aharonov–Bohm effects [187–189]. The latter consists in the action of the vector potential of an electromagnetic field (i.e. the Lorentz force) on the wave function of a charged elementary particle and thus leads to a shift of interference fringes resulting from the superposition of the de Broglie waves of two particle beams. It occurs even in the absence of a magnetic field in the area of particle motion, when neither trajectories nor velocities of the particles change. To simplify the arguments of the authors of Refs [79, 127, 130, 181–186], the vector potential of a gravitational field equivalent to Coriolis forces in a rotating frame of reference leads to a shift of the interference pattern produced by counterpropagating waves for both uncharged (photons, neutrons, neutral atoms) and charged (electrons, mesons, etc.) quantum particles; it is analogous to the vector potential of the Lorentz forces. However, such an analogy is formal and superficial, the similarity of these effects being confined to the fact that in both cases the vector potential of nonpotential gyroscopic forces acts on the wave function phase. In fact, the two effects are significantly different because the Aharonov–Bohm effect [187–189] exists only for quantum objects and vanishes in the case of macroscopic bodies, whereas the Sagnac effect occurs for both quantum and macroscopic objects.

One important feature of rotation-related effects is worth special noting. Unlike translational motion, rotation shows evidence of being an absolute (not relative) phenomenon. Rotation of a real body functioning as a source of an electric, magnetic or gravitational field about a motionless field sensor and rotation of a field sensor about a given body result in different readings of the sensor. The following relevant examples are borrowed from Ref. [5].

(1) Rotation of a ferromagnetic rod induces a magnetic field. In other words, a sensor placed in a fixed coordinate system detects a magnetic field, whereas rotation of a coordinate system (sensor) about the rod fails to furnish a magnetic field in an instrument (Barnett's experiments [190, 191]).

(2) It has been shown by Schiff [192] that the rotation of a charged spherical capacitor induces an external magnetic field, whereas the rotation of a reference system (sensor) about a fixed charged spherical capacitor fails to reveal a presence of the magnetic field.

(3) Experiments of Dufour and Prunier [53] have demonstrated that in a stationary ring interferometer filled with a rotating optical medium and in a rotating ring interferometer filled with a stationary medium, the phase differences between counterpropagating waves are distinct despite similar angular velocities [see expressions (9) and (10)].

The rotation of a massive body results in the appearance of vector potential of a gravimagnetic field (by analogy with the induction of a magnetic field by a rotating charge). In this case, the gravitational Aharonov – Bohm effect should be expected to occur and lead (for rotating macroscopic bodies) to a minor phase shift of counterpropagating waves in an optical ring interferometer or to a frequency difference between counterpropagating waves in a ring laser [193] (see also Refs [194 – 197]). It is worthy of note that this effect has not been observed thus far. It is the author's opinion that the gravitational Aharonov – Bohm effect is closely related to the Sagnac effect [193]. However, this is a wrong inference. To begin with, the Sagnac effect is associated with a rotating frame of reference, whereas the gravitational Aharonov – Bohm effect relates to the rotation of a real massive body. Secondly, the Sagnac effect leads to a change in velocities of counterpropagating waves and holds not only for quantum-mechanical particles (photons, electrons, etc.) but also for ordinary acoustic and magnetic waves. In contrast, the Aharonov – Bohm effect (the gravitational one, in particular) does not influence particle velocities in counterrunning beams and alters only their wave function phases.

It is worthwhile to note that the rotation of a massive body gives also rise to the Lense – Thirring effect [100 – 103], an analogue of electromagnetic induction, and causes a change in the velocity of bodies, including photons; in other words, it changes their wave function moduli. As yet, this GTR effect has not been recorded. The very first experiments to observe the light drag by a rotating body were performed on a ring interferometer by O Lodge at the end of the 19th century [198]. The author hypothesized that the light, together with the luminiferous ether, should be entrained by a rotating mass. Also, the rotation of a massive body should lead to the splitting of spectral frequencies of electromagnetic oscillations of an emitting atom [199], a change of the frequency difference between counterpropagating waves in a ring laser [111], and the rotation of the plane of polarization of light [200 – 202]. None of these effects has been observed.

The gravitational Aharonov – Bohm effect as well as the Lense – Thirring effect occur independently of the Sagnac effect and differ from it in that they are associated with the presence of rotating masses instead of rotating frames of reference. When the phase differences between counterpropagating waves at the output of a ring interferometer are measured, these effects may also be apparent in addition to the Sagnac effect. To conclude, there is a certain degree of analogy between the Sagnac and Aharonov – Bohm effects, but it is formal and does not reflect their true physical meaning.

4.2 Sagnac effect as a Berry phase manifestation

Works [7, 9, 80] consider the Sagnac effect as a manifestation of the Berry phase [204] (see also reviews [203, 205 – 209]).

The Berry phase (geometric, topological phase) manifests itself as a change of a system's parameter in the course of its spatial evolution. It is exemplified by a change of the quantum-mechanical state of a particle flux in the course of its spatial evolution [204] (Berry phase); a change in the polarization of radiation travelling in a curvilinear trajectory [209 – 212] (Rytov effect), which gives rise to an additional phase incursion [203, 205 – 207, 209, 213]; a cumulative phase incursion of the radiation in the event when a change in the polarization state occurs during its propagation through the medium [205 – 207, 209, 214, 215] (Pancharatnam phase); a change of the quantum-mechanical state of a neutron spin in the course of its spatial evolution [208], or a change in the angular position of a solid if the associated axis draws a solid angle during its motion [94, 216, 217] (Ishlinskiĭ effect). It needs to be emphasized that our studies [218, 219] have demonstrated that the Ishlinskiĭ effect has a relativistic analogue in the form of Thomas precession [220].

A classical example of the Berry phase manifestation in quantum mechanics is an additional phase incursion of electrons due to the Aharonov – Bohm effect [187 – 189].

In fiber ring interferometers, the Berry phase is manifested itself as additive to the Sagnac effect and produces a phase shift of the interference pattern at the output of FRI. Such a shift is unrelated to rotation and is therefore responsible for an error in the measured angular velocity of rotation. The authors of Ref. [221] reported on an experiment in which the Rytov effect in FRI was due to the nonplanar bending of a weakly anisotropic SMOF of which the FRI contour was made. A phase shift of the interference pattern at the exit from an FRI was due to the polarizational nonreciprocity of its contour [166] and may be interpreted as a nonreciprocal geometric phase of counterpropagating waves [164, 165], i.e. a Berry phase manifestation.

In ring interferometers transmitting de Broglie waves of charged material particles, e.g. electrons, the Berry phase due to the Aharonov – Bohm effect produces a shift of the origin of the interference pattern unrelated to rotation. When material particles carry no charge but have a nonzero spin and magnetic moment (e.g. neutrons), the Berry phase (induced by the Aharonov – Casher effect [222] resulting from the evolution of the spin quantum-mechanical state) also accounts for a phase shift of the interference pattern unrelated to rotation [181]. Even in the case of a zero vector potential of the magnetic field near the material particle trajectories (i.e. when the Aharonov – Bohm and Aharonov – Casher effects are nonexistent), the spins of the particles travelling in reverse ways are found to be oppositely directed because of Thomas precession (an STR effect) [218 – 220]). This leads to a rotation-unrelated shift of the origin of the interference pattern [131, 223].

It should be noted that the Berry phase may be a source of erroneous readings of not only FRI but also of other types of gyroscopic devices. For example, the Ishlinskiĭ effect [94, 216, 217] accounts for incorrect readings of paired mechanical gyroscopes and strongly monitored gyroscopes, which are actually used as spatial gyrocompasses (directional pickups). In NMR gyroscopes [133], the Aharonov – Casher phase [222] related, in this instance, to the evolution of the quantum-mechanical state of a nuclear spin is responsible for erroneous angular velocity measurements [224, 225].

The Berry phase is a manifestation of nonholonomicity (nonintegrability) because it is impossible to calculate it starting from the initial and final states of the system [205 – 207]. The authors of Refs [226, 227] considered the Sagnac effect as a manifestation of nonholonomicity in a rotating frame of reference. These papers were published before Ref. [204] — that is, prior to the introduction of the Berry phase notion.

A major conclusion from this section can be formulated as follows: both the analogy between Sagnac and Aharonov – Bohm effects and the interpretation of the Sagnac effect as a Berry phase manifestation are purely formal and fail to clarify the physical meaning of the effect in question.

5. Incorrect explanations of the Sagnac effect

The overwhelming majority of incorrect explanations of the Sagnac effect proceed either from the downright negation of relativity theory or from the neglect of it and an attempt to reduce this kinematic effect of STR to some other effect well-known in classical physics. Several incorrect explanations arise from a poor understanding of relativity theory or erroneous calculations.

5.1 Sagnac effect in the theory of stationary (not entrained) luminiferous ether

This explanation of the Sagnac effect is based on the concept of a 'luminiferous ether' which is not entrained by a rotating interferometer. Surprisingly, it is still in use although it was the very first explanation proposed to address the Sagnac effect [8].

Let the light travel around an enclosed circular area (see Fig. 1). If the rotating interferometer contains no optical medium, the expression for the phase velocity of light $v_{\rm ph}^{\pm}$ in the system of coordinates K' attached to the interferometer has the form

$$(v'_{\rm ph})^{\pm} = c \mp R\Omega \,. \tag{21}$$

The travel times of counterpropagating waves around a ring are

$$(t')^{\pm} = \frac{2\pi R}{(v'_{\rm ph})^{\pm}} = \frac{2\pi R}{c \mp R\Omega} \,, \tag{22}$$

respectively. The propagation time difference is given by

$$\Delta t' = (t')^{+} - (t')^{-} = \frac{4\pi R^2 \Omega}{c^2 (1 - R^2\Omega^2/c^2)} \,. \tag{23}$$

This result is accurate to within small relativistic corrections. Expression (23) is distinct from the corresponding expression (5) obtained in the framework of STR by the coefficient $(1 - R^2\Omega^2/c^2)^{1/2}$. This distinction can be explained only in terms of STR; namely, the number π in the rotating coordinate system K' is $(1 - R^2\Omega^2/c^2)^{1/2}$ times that in the inertial coordinate system [34, 35]. This fact was disregarded in expressions (22), (23). The multiplication of quantity $\Delta t'$ in expression (23) by $(1 - R^2\Omega^2/c^2)^{1/2}$ yields the quantity $\Delta t'$ that appears in expression (5).

This approach to the evaluation of the Sagnac effect was applied by O Lodge [198, 228], A Michelson [167, 168, 229], and M G Sagnac [1 – 3], staunch advocates of the 'luminiferous ether' theory. L Silberstein [27] also considered the concept of 'luminiferous ether' as an alternative explanation of the Sagnac effect. It should be emphasized that the contribution of Silberstein to the understanding of the Sagnac effect remains open to different interpretations. On the one hand, he was one of the first to scrutinize the Sagnac effect in the framework of both STR and GTR [27]. On the other hand, the same paper considered this effect based on the 'luminiferous ether' concept and in terms of the direct action of Coriolis forces in the path of counterpropagating waves in a ring interferometer (see Section 5.5). This further confused matters. L Silberstein was the author of one of the best monographs on STR that appeared in the early 20th century [230]. At the same time, he published an enthusiastic review of the experimental results obtained by D Miller [231, 232] who reproduced the Michelson – Morley experiments [12, 13] and argued to have confirmed the existence of a 'luminiferous ether'. As a matter of fact, this review was in keeping with a note by A Timiryazev [234], an ardent opponent of relativity theory. The experiments of D Miller shattered the belief of certain investigators in the validity of STR. In response, S I Vavilov published his famous work [4] in which he demonstrated the groundlessness of the 'luminiferous ether' theory. It was shown in a later work [235] that the experiments of D Miller were not free from a systematic error attributable to a temperature drift of the arm lengths in the Michelson interferometer.

The interpretation of the Sagnac effect in the context of the 'luminiferous ether' theory is not simply a fact picked up from the past history of physics. It continues to be treated from the same standpoint even now. By way of example, Winterberg [236] considers both the Sagnac and Aharonov – Bohm effects as proceeding from the 'ether' concept. Also, it is proposed that a ring interferometer be used for the measurement of the Earth's translational velocity relative to a certain absolute space filled with a stationary ether medium [237], in obvious conflict with the theory of relativity.

In other words, there are still attempts to approach the effects of interest beyond the scope of STR (these authors usually maintain that the Sagnac effect is a first-order effect in $R\Omega/c$, thereby baselessly concluding that it is possible to manage without STR). A characteristic example of the implicit application of the 'ether' concept is provided by the calculation, in obvious conflict with STR, of the magnitude of the Sagnac effect using the supraluminal speed of one of the counterpropagating waves. This method has been used in a well-known experimental work by Bershteĭn [14], reviews [238, 239], and some textbooks for the faculties of physics at higher education institutions. See, for instance, the corresponding courses of electrodynamics [240] and optics [241].

The same method for the calculation of the Sagnac effect was applied by S I Vavilov [4], L I Mandel'shtam in his lectures on the theory of relativity [84], G Joos in a course of theoretical physics [242], R Ditchburn in a course of physical optics [243], and V A Ugarov in a supplement to his course of STR. True, these authors used the said method as a didactic device to prove the nonexistence of 'luminiferous ether' and to show that the experiments under consideration suggest the impossibility of the 'ether' being dragged by rotation, at variance with the Michelson – Morley results [12, 13].

For an interferometer filled with an optical medium having an index of refraction n, the Sagnac effect was computed in terms of the 'luminiferous ether' concept based on the Fresnel hypothesis that the ether is partially entrained by a moving optical medium [44]. Let us write down the expression for phase velocities of counterpropagating waves in the laboratory (nonrotating) system of coordinates K, as proposed by Bershteĭn [14]:

$$v_{\mathrm{ph}}^{\pm} = \frac{c}{n} \pm (1-\alpha) R\Omega \,, \qquad (24)$$

where $\alpha = 1 - 1/n^2$ is the Fresnel drag coefficient [6, 14, 44]. In the coordinate system K' attending rotation, the expression for $(v'_{\mathrm{ph}})^{\pm}$ will have the form

$$(v'_{\mathrm{ph}})^{\pm} = \frac{c}{n} \mp \frac{R\Omega}{n^2} \,. \qquad (25)$$

The times necessary for counterpropagating waves to complete one trip around the ring are

$$(t')^{\pm} = \frac{2\pi R}{c/n \mp R\Omega/n^2} \,. \qquad (26)$$

Then the propagation time difference is given by

$$\Delta t' = (t')^{+} - (t')^{-} = \frac{4\pi R^2 \Omega}{c^2\big(1 - R^2\Omega^2/(c^2 n^2)\big)} \,. \qquad (27)$$

Thus, we have again obtained the result accurate to within small relativistic corrections. It has been previously shown that the magnitude of the Sagnac effect does not depend on the refractive index of the filling optical medium when it is co-moving with the rotating ring interferometer.

The question is why does the notoriously wrong premise of a 'luminiferous ether' allow for a result accurate to within small relativistic corrections? The answer may be as follows.

(1) It is concluded from the assumption of an ether resistant to the drag by a rotating interferometer that such a medium must be stationary in a fixed (laboratory) system of coordinates. Hence, the speed of light in this system should be constant regardless of its direction, in agreement with the special theory of relativity (a fixed coordinate system is an inertial one).

(2) If an interferometer is filled with an optical medium, the Fresnel drag coefficient is used as described in Ref. [14]. This coefficient can be derived from the relativistic law of velocity composition as the first approximation [40, 54].

It appears from the above that the 'luminiferous ether' theory yields virtually correct results and can be used for the purpose in question. This is a wrong conclusion, however, because the explanation based on this theory contains internal contradictions. In order to explain the Sagnac effect, we had to assume that the ether is dragged neither by a rotating interferometer nor even by the rotation of the Earth [167, 168]. At the same time, the negative results of Michelson – Morley's experiments [12, 13] and their subsequent versions (see reviews [4, 6]) can be accounted for in the framework of the 'luminiferous ether' concept only if the ether is supposed to be completely dragged by the Earth's translational motion.

To summarize, the 'luminiferous ether' theory has been shown above to lay down contradictory conditions. Specifically, it states that the ether must be totally dragged along by the Earth's translational motion but fails to be dragged by its rotation [4, 84, 242, 243]. However, this discrepancy does not seem to confuse the advocates of the 'ether' theory. S I Vavilov [4] wrote: "It is however conceivable that ether can be stationary and yet in a mechanical motion... The notion of an ether dragged by a moving body and at the same time remaining 'irrotational' (or vortex-free) was being developed by Stokes in the last century... It is difficult... to definitively disprove the idea that the Earth is embedded in a shell of ether, but there is no foothold either on which to base further development of this idea." It is worthwhile to note that G Stokes' model of an ether alluded to in this extract contains irremovable internal contradictions (see Ref. [6] for a comprehensive discussion of this problem).

It should be emphasized that the explanation of the Sagnac effect in terms of the concept of a 'luminiferous ether' not dragged by rotation contains one more important contradiction which, to my knowledge, has never been considered before. In order to obtain a correct result in the framework of the 'ether' theory, one has to accept the assumption that the ether is motionless with respect to the inertial laboratory reference system attached to the center of rotation of a ring interferometer.

If the ether is in uniform translational motion with respect to the center of rotation of the interferometer, then the expression for the phase shift of counterpropagating waves, attributable to the Sagnac effect, will be different from Eqn (23). It is exactly this premise that is used in Ref. [237] to measure the Earth's translational velocity relative to the ether. The author hypothesizes that two identical ring interferometers to the centers of which two different inertial reference systems are attached should record different phase shifts of counterpropagating waves. However, this hypothesis is in conflict with a key postulate of STR according to which all inertial frames of reference are equivalent.

5.2 Sagnac effect from the standpoint of classical kinematics

The concept of a 'luminiferous ether' which fails to be dragged by rotation is applicable only to the computation of the Sagnac effect for electromagnetic waves. The magnitude of this effect for arbitrary waves in the most general case [7 – 9, 11] is more appropriate for assessment by means of classical kinematic calculations based on the estimation of the rotation-induced displacement of the beam-splitting mirror at the entrance to a ring interferometer (see Fig. 1) during the time needed for counterpropagating waves to complete their trip around the ring. Also, the Galilean law of velocity composition of each counterpropagating wave, v_{ph}, with the linear rotational velocity $R\Omega$ may be used. At first sight, this approach allows the Sagnac effect to be computed not only for electromagnetic waves but also for waves propagating in a material medium with the drag coefficient equalling 1 (e.g. for acoustic waves, surface acoustic and surface magnetostatic waves — the so-called 'slow' waves [11, 23] as well as for the de Broglie waves of material particles: electrons [9, 16], neutrons [17, 18], mesons [24], calcium [19], sodium [20], and caesium [21] atoms).

Let us first consider the application of the classical kinematic method to the computation of the Sagnac effect in the optical range for the most general case, when the interferometer is filled with a co-moving optical medium having refractive index n. In the absence of interferometer rotation, the time needed for each counterpropagating wave to traverse its path length in order to complete one full circulation is $t = 2\pi Rn/c$. In the case of rotation, the displacement of the beam-splitter for a time t is

$$\Delta l = \frac{2\pi R^2 \Omega n}{c} \,,$$

and the optical paths for counterpropagating waves are given by

$$l^{\pm} = 2\pi R \pm \frac{2\pi R^2 \Omega n}{c} \,,$$

while the velocities of counterpropagating waves in a laboratory frame of reference are defined by expression (24). In this case, the propagation time difference between counter-running waves in the laboratory system of coordinates K takes the form

$$\begin{aligned}\Delta t = t^+ - t^- &= \frac{l^+}{v_{\text{ph}}^+} - \frac{l^-}{v_{\text{ph}}^-} \\ &= \frac{2\pi R + 2\pi R^2 \Omega n/c}{c/n + \alpha R\Omega} - \frac{2\pi R - 2\pi R^2 \Omega n/c}{c/n - \alpha R\Omega} \\ &= 4\pi R^2 \frac{\Omega}{c^2}\left[1 - n^2\left(1 - \frac{1}{n^2}\right)R^2 \frac{\Omega^2}{c^2}\right]^{-1} \approx \frac{4\pi R^2 \Omega}{c^2} \,, \qquad (28)\end{aligned}$$

where $\alpha = 1 - 1/n^2$. The expression thus obtained coincides with expression (4) to within small relativistic corrections. In other words, the classical kinematic approach yields a practically correct result for the Sagnac effect in the optical range.

M Laue was the first to employ the classical kinematic approach to the computation of the Sagnac effect in the optical range in 1911 [69]. In the analysis of experimental situation, he dealt with a four-mirror interferometer and calculated the difference between the optical paths of counter-propagating waves taking into account the rotation of each mirror during the time needed for a counterpropagating wave to reach it. The calculation described in the previous paragraph for counterpropagating waves travelling around the ring is much simpler and leads to the same result as in Ref. [69]. This calculating method for the evaluation of the Sagnac effect magnitude has also been used in a number of studies [5, 6, 36, 38, 74, 75, 91, 126, 128, 245–247]. A similar explanation of the Sagnac effect is offered in the two latest editions of *Physical Encyclopedia* [248, 249]. The classical kinematic approach was also used to calculate the Sagnac effect in Refs [238–242].

However, the classical kinematic method leads to serious mistakes if applied to the computation of the Sagnac effect for nonelectromagnetic waves propagating in a material medium whose drag coefficient equals unity (e.g. for surface acoustic and magnetostatic waves, also called 'slow' waves [11, 23]). Let us consider, with reference to work [23], a ring interferometer transmitting 'slow' waves, using the kinematic approach. Let the phase velocity of a 'slow' wave be $v_{\text{sl}_{\text{ph}}}$; then, the counterpropagating wave velocities in a laboratory frame of reference are

$$v_{\text{sl}_{\text{ph}}}^{\pm} = v_{\text{sl}_{\text{ph}}} \pm R\Omega \,,$$

respectively. In the absence of interferometer rotation, the time needed for each counterpropagating wave to trace around a ring path is

$$t = \frac{2\pi R}{v_{\text{sl}_{\text{ph}}}} \,.$$

If the interferometer rotates with an angular velocity Ω, the device generating counterpropagating waves in the ring and also used to transform 'slow' waves back to an electric signal (and thus playing the same role as the beam-splitter in an optical ring interferometer) will be shifted (for time t) to a distance

$$\Delta l = R\Omega t = \frac{2\pi R^2 \Omega}{v_{\text{sl}_{\text{ph}}}} \,.$$

The optical path for counterpropagating waves will equal

$$l^{\pm} = 2\pi R \pm \frac{2\pi R^2 \Omega}{v_{\text{sl}_{\text{ph}}}} \,,$$

and the propagation time difference between counterpropagating waves will be

$$\begin{aligned}\Delta t = t^+ - t^- &= \frac{l^+}{v_{\text{sl}_{\text{ph}}}^+} - \frac{l^-}{v_{\text{sl}_{\text{ph}}}^-} \\ &= \frac{2\pi R + 2\pi R^2 \Omega / v_{\text{sl}_{\text{ph}}}}{v_{\text{sl}_{\text{ph}}} + R\Omega} - \frac{2\pi R - 2\pi R^2 \Omega / v_{\text{sl}_{\text{ph}}}}{v_{\text{sl}_{\text{ph}}} - R\Omega} = 0 \,. \qquad (29)\end{aligned}$$

Thus, based on the kinematic computing method, we arrived at the conclusion (just as the authors of Ref. [23] did) that there is no Sagnac effect for 'slow' waves. A similar result could be obtained for some other types of waves, e.g. ordinary acoustic waves. But this is a completely wrong result because calculations in the framework of STR [11] indicate that the Sagnac effect for slow waves does exist, with the propagation time difference between counterpropagating waves being $\simeq 4\pi R^2 \Omega \omega / c^2$.

Errors inherent in the classical kinematic method applied to the computation of the Sagnac effect are due to the use of the Galilean instead of the relativistic law of velocity composition, viz. the velocity of wave propagation in a medium and the velocity of rotation. When the Sagnac effect in the optical range is considered, the use of the Fresnel drag coefficient corresponds in the first approximation to the relativistic law of velocity composition (see above). In this case, the result of computation is accurate to within small relativistic corrections. In the remaining cases, calculations using the classical kinematic technique yield erroneous results.

The computation of the Sagnac effect for the de Broglie waves of material particles using the classical kinematic method also leads to a zero result [79]. The author of Refs [130, 131] distinguishes between the classical and relativistic Sagnac effect in the optical range but emphasizes that the same effect in quantum mechanics (i.e. for de Broglie waves) can be only relativistic. A similar conclusion has been drawn in Refs [79, 80]. The author of Ref, [80] writes that "the nonrelativistic Sagnac effect is a paradox". However, such a view does not arise from certain specific features of the Sagnac effect in quantum mechanics (it is always a relativistic effect [10, 11]). Rather, it originates from the impossibility of introducing a certain coefficient for the de Broglie waves of material particles or acoustic waves, by analogy with the Fresnel drag coefficient in optics. On the one hand, such a coefficient plays a part of the first approximation to the relativistic law of velocity composition. On the other hand, it must give the illusion of using the Galilean law of velocity composition with a certain correction coefficient (that is, the illusion that the consideration remains confined to the classical theory).

5.3 Sagnac effect as a manifestation of the classical Doppler effect off a moving beam-splitting mirror

This explanation of the Sagnac effect is based on the consideration of a rotating semitransparent mirror (in the general case, a beam-splitting device) of a ring interferometer in a stationary frame of reference as a movable radiation source. The source emits radiation of a shorter wavelength in the direction of rotation and a longer one in the opposite sense than it would do in its absence. The advocates of this approach maintain that similar ring lengths for counterpropagating waves allow a different wavelength number, which accounts for the phase shift of the interference pattern at the exit from the interferometer. However, such an approach is altogether wrong because the radiation source and detector must be in motion relative to each other if the Doppler effect is to be manifest. In the case being considered, the beam-splitter of the ring interferometer serves as both the source and the detector of radiation; naturally, it cannot be in motion with respect to itself. It is worthwhile to mention here a work by A Einstein [54] showing that the frequency (hence, the wavelength) of light in a rotating frame of reference remains unaltered in the first approximation in $R\Omega/c$ if the ring interferometer rotates with the filling medium as a single whole.

Generally speaking, it is quite permissible to use the classical Doppler effect in calculations related to light beam interference in a noninertial frame of reference undergoing a certain acceleration along a straight line [250]. Such a possibility is due to the fact that at the moment of emission the velocity of the radiation source differs from that of the absorbing detector. This may also be explained proceeding from the equivalence principle — that is, assuming that the nonrelativistic gravitational potential is different at the moments of emission and detection [250, 251], the same as the frequencies of the emitted and detected signals. However, in the absence of angular accelerations in a rotating frame of reference, the beam-splitter which simultaneously serves as the radiation source and detector always has the same absolute linear velocity.

The inadequacy of the method in question for the computation of the Sagnac effect is best illustrated by its application in optics where the size of the Doppler effect does not depend on whether the space-filling medium between the source and the detector is in motion or not [6, 39]. In the laboratory coordinate system K where the beam-splitter moves with a linear velocity $R\Omega$, the light wavelength attributable to the Doppler effect for counterpropagating waves in a ring interferometer must be

$$\lambda^{\pm} = \lambda\left(1 \pm \frac{R\Omega n}{c}\right).$$

Here, λ is the light wavelength in the absence of rotation [39]. The wave numbers and optical paths for the counterpropagating waves are

$$k^{\pm} = \frac{2\pi n}{\lambda^{\pm}}\,, \qquad l^{\pm} = (2\pi R \pm R\Omega t)n\,,$$

respectively, where $t = 2\pi R n/c$ is the time of wave passage around the ring, and n is the refractive index. The phase difference between counterpropagating waves due to the Sagnac effect is given by

$$\Phi_{\mathrm{S}} = k^{+}l^{+} - k^{-}l^{-} = \frac{4\pi^2 R n}{\lambda}\left(\frac{1 + R\Omega n/c}{1 - R\Omega n/c} - \frac{1 - R\Omega n/c}{1 + R\Omega n/c}\right)$$
$$= \frac{16\pi S\Omega n^2}{\lambda c\left(1 - R^2\Omega^2/(c^2 n^2)\right)} \approx 16\,\frac{\pi\Omega S n^2}{\lambda c} = 16\,\frac{\pi\Omega S n^2 \nu}{c^2}\,, \quad (30)$$

where $\nu = c/\lambda$ is the light frequency.

Even in the absence of an optical medium in the interferometer, this result is twice that ensuing from expression (7); moreover, the quantity Φ_{S} in expression (30) is proportional to n^2. In order to obtain the 'correct' expression, the authors employing the present method to calculate the Sagnac effect assume that the optical paths for counterpropagating waves are equal, viz.

$$l^{+} = l^{-} = 2\pi R\,.$$

In other words, the velocities of counterpropagating waves are considered in the reference system K, while the optical paths in the reference system K'; in addition, the case of $n = 1$ is adopted.

Therefore, an error for the case of an interferometer containing an optical medium must be defined by the coefficient $2n^2$.

In reality, however, the magnitude of the Sagnac effect is independent of the medium's refractive index. This inference is drawn from both strict calculations using the special theory of relativity [5, 11, 54] and experimental works reported by B Pogany [50–52] and I L Bershteĭn [14] (see also reviews [4, 8]). For all that, the dependence of the FRI sensitivity on the refractive index was a matter of heated discussion as early as the very first FRIs were constructed [43, 55–57, 134]. Experimental investigations of the Sagnac effect in FRI [43, 56] also demonstrated that the phase shift of counterpropagating waves due to this effect was independent of the SMOF refractive index.

It is interesting to trace where the origin of the view that the Sagnac effect may be considered as a consequence of the Doppler effect in a rotating interferometer is rooted. Dufour and Prunier [252] considered the Sagnac effect to be a manifestation of the Doppler effect off a rotating beam-splitting mirror. A Sommerfeld who used the classical kinematic method to compute the size of the Sagnac effect wrote in his *Optik* [91]: "We could simplify the above calculations if we began from the Doppler effect which takes place here because the semitransparent plate H acts as a movable source that emits radiation of different wavelengths in the forward and backward directions." In this case, however, the corollaries of Sommerfeld's authority were altogether negative. The publication of his work [91] has given rise to numerous allegations that the Sagnac effect is due to the Doppler effect off a moving beam-splitter (see, for instance, Refs [36, 126]). Likewise, this phenomenon is regarded as a potential cause of the Sagnac effect in certain papers [18, 80], reviews [36, 74, 126, 239, 253], a review section of paper [9], a monograph on fiber gyroscopy [75], a course of wave optics [254], two latest editions of *Physical Encyclopedia* [248, 249], and a number of other works (see, for instance, Refs [117, 126]). The encyclopedic *Quantum Electronics* [255] and the textbook *Fundamentals of Quantum Electronics* [256] treat the Doppler effect as the sole possible cause of the splitting of counterpropagating wave frequencies in a rotating ring laser. In other words, they totally ignore the Sagnac effect. The author of Ref. [135] argues that the Sagnac effect is a variant of the Doppler effect, i.e. a change of counterpropagating wave frequencies in a rotating frame of reference. H Lefevre, a known expert on fiber gyroscopy, concludes the examination of the causes underlying the Sagnac effect in the domain of STR and the involvement of the Doppler effect in the following way [74, 75]: "These two explanations are equivalent but take care not to use them simultaneously!"

Using the method under consideration, let us compute the size of the Sagnac effect for waves travelling in a material medium, e.g. ordinary acoustic or 'slow' waves. In the case of a Doppler shift in a reference system K, the length of acoustic waves propagating in opposite directions in a ring interferometer must be [39]

$$\lambda_{\mathrm{s}}^{\pm} = \lambda_{\mathrm{s}} \left(1 \mp \frac{R\Omega}{v_{\mathrm{s}}} \right),$$

where λ_{s} is the wavelength in the absence of rotation, and v_{s} is the velocity of sound in the medium. The wave numbers and sound-path lengths for counterpropagating waves are

$$k_{\mathrm{s}}^{\pm} = \frac{2\pi}{\lambda_{\mathrm{s}}^{\pm}}, \qquad l^{\pm} = 2\pi R \pm R\Omega t,$$

respectively, where $t = 2\pi R/v_{\mathrm{s}}$ is the time of wave passage around the ring, and the phase difference between counterpropagating waves due to the Sagnac effect is given by

$$\begin{aligned} \Phi_{\mathrm{S}} &= k_{\mathrm{s}}^{+} l^{+} - k_{\mathrm{s}}^{-} l^{-} = \frac{4\pi^2 R}{\lambda_{\mathrm{s}}} \left(\frac{1 + R\Omega/v_{\mathrm{s}}}{1 - R\Omega/v_{\mathrm{s}}} - \frac{1 - R\Omega/v_{\mathrm{s}}}{1 + R\Omega/v_{\mathrm{s}}} \right) \\ &= \frac{16\pi^2 R^2 \Omega}{\lambda_{\mathrm{s}} v_{\mathrm{s}} (1 - R^2\Omega^2/v_{\mathrm{s}}^2)} \approx \frac{16\pi S\Omega}{\lambda_{\mathrm{s}} v_{\mathrm{s}}} = \frac{16\pi S\Omega \nu_{\mathrm{s}}}{v_{\mathrm{s}}^2}, \end{aligned} \tag{31}$$

where $\nu_{\mathrm{s}} = v_{\mathrm{s}}/\lambda_{\mathrm{s}}$ is the sound frequency.

This resultant expression is incorrect. Specifically, it contains the speed of a material wave instead of the speed of light, which accounts for a $2(c/v_{\mathrm{s}})^2$ times higher estimated magnitude of the Sagnac effect than the real one [11]. Also, it is worth noting here that the Doppler effect in acoustics is possible on condition that the source or the detector is in motion at different speeds with respect to the medium [39]. However, the beam-splitter of a ring interferometer, which is both the source and the detector, is static relative to the medium.

The authors of Refs [18, 80] resort to the Doppler effect in an attempt to explain the phase shift of counterpropagating de Broglie waves of material particles. The length of the counterpropagating de Broglie waves in the reference system K must be

$$\lambda_m^{\pm} = \frac{h}{m v_m^{\pm}} = \frac{h}{m(v_m \pm R\Omega)},$$

where h is the Planck constant, m is the particle mass, and v_m is the velocity of a material particle (group velocity of the spreading wave packet). The wave numbers and path lengths for counterpropagating waves in the frame of reference K are

$$k_m^{\pm} = \frac{2\pi}{\lambda_m^{\pm}}, \qquad l^{\pm} = 2\pi R \pm R\Omega t,$$

respectively, where $t = 2\pi R/v_m$ is the time necessary for a wave to complete one trip around the ring. The phase difference between counterpropagating waves due to the Sagnac effect is determined through

$$\Phi_{\mathrm{S}} = k_m^{+} l^{+} - k_m^{-} l^{-} = \frac{16\pi^2 R^2 \Omega m}{h(1 - R^2\Omega^2/v_m^2)} \approx \frac{16\pi S\Omega m}{h}. \tag{32}$$

Thus, the result is again incorrect being twice as large as that given by expression (20). In order to obtain the correct expression, the authors of Refs [18, 80] assumed that $l^{+} = l^{-}$.

The main conclusion from this section is that the Sagnac effect is in no way related to the Doppler effect. The erroneous interpretation of the former as the corollary of the latter is due to the fact that both are first order effects in v/c.

5.4 Sagnac effect as a manifestation of the Fresnel – Fizeau drag effect

F Harress [46] was the first to suggest that the phase difference between counterpropagating waves in a rotating ring interferometer is due to the Fresnel – Fizeau light drag [44, 59, 60]. This author is known to have pioneered experimental investigations of the effect of interest (see reviews [4 – 9, 49]. He believed that in the absence of any filling medium in a ring interferometer, its rotation should cause no phase difference between counterpropagating waves. In such a situation, the expression for the counterpropagating wave velocity in a rotating frame of reference must have the form

$$v_{\mathrm{ph}}^{\pm} = \frac{c}{n} \pm \alpha R\Omega,$$

where $\alpha = 1 - 1/n^2$ is the Fresnel drag coefficient. This line of reasoning had led to erroneous conclusions from the analysis of his experimental data which were later corrected by P Harzer [47] and A Einstein [54].

M Laue [26] undertook the explanation of the Sagnac effect in terms of STR concurrently with its reduction to the Fresnel – Fizeau drag effect and interpretation of Harress' experiments [46] as an analogue of the experimental studies reported by P Zeeman [61, 62]. K Kovacs [257] described an experiment designed to prove the validity of this approach. One of the arms of a ring laser (a glass bar) was in reciprocating motion while two others each rotated about one of its ends and thus ensured the continuity of optical paths for counterpropagating waves. In the mean time, the arm bearing a gas discharge tube remained motionless. This experiment was actually a laser-based variant of the previous Kantor's experiments [63].

However, Einstein [54] had long before demonstrated that the Sagnac effect should by no means be considered as a consequence of Fresnel – Fizeau drag. Indeed, it was shown to occur even in the absence of any optical medium in the ring interferometer. Moreover, the magnitude of the Sagnac effect was unrelated to the presence of such a medium. An increase of the light beam paths in a ring interferometer due to the presence of an optical medium with the index of refraction n was completely compensated for by lowering the Fresnel drag coefficient $\alpha = 1 - 1/n^2$.

The problem was revised with the advent of the first FRIs [55 – 57]. Certain recent reports again tend to relate the Sagnac effect in ring interferometers [258] or ring lasers [135] filled with an optical medium to the Fresnel – Fizeau light drag.

5.5 Sagnac effect and Coriolis forces

As shown in Sections 3.2 and 3.3, the Sagnac effect may be regarded as a consequence of different time dilations or phase change of the de Broglie wave of a material particle for counterpropagating waves under the influence of the non-relativistic gravitational scalar or respectively vector potential of Coriolis forces in a rotating frame of reference. It was proposed in a paper by Silberstein [27] (see also Ref. [259]) that the Sagnac effect should be explained in terms of the direct action of Coriolis forces on counterpropagating waves, in addition to other feasible explanations (in the framework of STR, GTR or the 'luminiferous ether' concept). The author of Ref. [27] thought that the effect of Coriolis forces on counterpropagating waves in a three-mirror ring interferometer accounted for the optical path of a wave travelling in the direction of rotation in the form of a triangle with somewhat convex sides; a wave spreading in the opposite direction had an optical path in the form of a triangle with somewhat concave sides. For this reason, the triangles had different areas. Hence, the relative time delay between the counterpropagating waves, the additional travel time of each wave dependent on the Sagnac effect being proportional to the closed contour area [35].

After a little while, however, A Lunn [70] showed that the triangles are actually equal in area even though their contours for counterpropagating waves are not quite coincident during rotation (the contribution of the deflection of each counterpropagating light beam caused by the Coriolis forces to a change of the contour area is totally compensated for by the contribution from the altered angle of incidence on the next mirror). It is easiest to demonstrate the equality of contour areas for counterpropagating waves in a fixed frame of reference where Coriolis forces are lacking. In such a case, only rotations of reflecting mirrors at given moments need to be taken into consideration as was done by M Laue [69].

5.6 Sagnac effect as a consequence of the difference between photon orbital momenta in counterpropagating waves

In Ref. [133] (see also Ref. [260]), the Sagnac effect was evaluated by the method taking into account changes of the orbital momenta of macroscopic photon orbits and the energy of generated photons in a rotating ring laser. This approach implies that the orbital momenta of macroscopic photon orbits for counterpropagating waves vary during rotation. Accordingly, photon energies and counterpropagating wave frequencies are also different. In reality, the distinction in the frequencies of the counterpropagating waves in a rotating ring laser can be accounted for by their different optical paths due to the Sagnac effect.

The explanation of the Sagnac effect in Ref. [133] is similar to the above interpretation of this effect as a consequence of the Doppler shift — either is based on the assumption of different frequencies of counterpropagating waves. It is worthwhile to emphasize the incorrect result of computation of the Sagnac effect in a ring resonator taking into account a change in the orbital momenta of macroscopic photon orbits in a rotating frame of reference. A Einstein has long showed [54] that in a rotating frame of reference the light frequency to first order in v/c remains unaltered. In order to change photon frequency in a fiber ring resonator, it needs to be in accelerated motion perpendicular to the light guide wall [261].

Nevertheless, consideration of the altered orbital momenta of macroscopic photon orbits in a rotating frame of reference in Ref. [133] yielded an adequate expression for the frequency difference between counterpropagating waves in a rotating ring laser. This is, however, a mere coincidence (just as in the above case of treating the Sagnac effect as a result of the Doppler shift), all the three effects being those of first order in v/c.

At the same time, it follows from Ref. [133] that for the counterpropagating waves being trapped, their planes of polarization make an angle numerically equal to the phase difference between counterpropagating waves attributable to the Sagnac effect. This means that in a rotating frame of reference the propagation velocities of right-handed and left-handed circularly polarized waves are different, the sign of the difference being dependent on whether a wave travels in the direction of rotation or in the opposite direction. To the best of my knowledge, the latter effect has never been observed in experiment despite the fact that it is a first order effect in v/c and as such is a sufficiently large one. It should be recalled that a similar erroneous result was obtained in Refs [111, 112, 114, 124, 125, 140] in the framework of GTR (see Section 2.2).

The error contained in Ref. [133] is closely akin to that in Refs [127, 130, 181 – 186] where the Sagnac effect is regarded as completely identical to the Aharonov – Bohm effect. Indeed, the rotation of a massive body results in the rotation of the plane of polarization of a light beam passing nearby [111, 199 – 202, 262] in a trajectory of nonplanar curvature. In this case, the rotation of the plane of polarization is actually a manifestation of the Rytov effect [210 – 212] for optically inhomogeneous media, the inhomogeneity resulting from the space curvature caused by a rotating mass, i.e. being due to the Lense – Thirring effect [100 – 103]. This, in turn, affects the propagation velocity of photons acquiring right- and left-hand polarization. This means that even a vacuum exhibits circular birefringence in the vicinity of a rotating mass. However, the rotations of a real body and a frame of reference lead to different physical phenomena (see above). The authors of Refs [111, 112, 114, 124, 125, 133, 140, 260] arrived at the wrong conclusion that optical inhomogeneity is also inherent in a rotating reference system and accounts for different propagation velocities of photons subject to right- and left-hand polarization.

It is worthy of note that a light beam travelling in the direction of rotation genuinely undergoes rotation of the plane of polarization, i.e. Fermi's polarization drag [263, 264] which is very small in size [265] and remains of little practical value for the measurement of angular velocities of rotation. It will be recalled that the Sagnac effect is absent when the axis of rotation coincides with the direction of optical wave propagation, that is when Fermi polarization effect takes place. This accounts for the possibility of measuring the Fermi polarization effect using a ring laser which rotates about one of its arms filled with an optical medium [266].

5.7 Sagnac effect as a manifestation of electromagnetic field inertial properties

Paper [136] offers the derivation of an expression for the frequency difference between counterpropagating waves due to the Sagnac effect in a ring laser, using the inertial properties of an electromagnetic field in a ring resonator. This work considers the rotation-induced deformation of the standing wave structure in a ring resonator represented by an ideal metal cylinder with the coefficient of reflection equalling unity. The author of Ref. [136] describes his approach as "a variant of the ether theory adapted to the special theory of relativity at low linear rotational velocities''. We consider such an approach to be in some measure analogous to the foregoing interpretation of the Doppler effect as a cause of the Sagnac effect. Also, we believe that the expression for the frequency difference between counterpropagating waves in a ring laser accurate to within small relativistic corrections was obtained in Ref. [136] by virtue of proportionality between this difference and the ratio v/c, where $v = R\Omega$, and also because the author dealt with a ring resonator containing no optical medium.

It should be noted that the inertial properties of waves (or wave packets, for that matter) are made use of in such gyroscopic instruments as solid-state wave gyroscopes [267] and also gyroscopes whose principle of action is grounded on the macroscopic quantum properties of superfluid helium [268–270]. These instruments along with the Foucault pendulum and mechanical gyroscopes [216, 217, 271] are applied to determine the angular position in space. In contrast, devices in which the Sagnac effect provides the working principle (optical ring interferometers, ring lasers, ring interferometers transmitting de Broglie waves, acoustic and 'slow' waves) serve as angular velocity pickups. This makes the fundamental distinction between instruments based on the Sagnac effect and those in which the property of physical bodies or wave packets to maintain orientation in space is employed.

5.8 Sagnac effect in incorrect gravitation theories

Numerous theories of gravitation proposed thus far also include the so-called incorrect gravitation theories, the corollaries of which are in conflict with the results of classical experiments designed to verify GTR within the boundaries of the solar system. Works [110, 272, 273] describe a multistage procedure which having been applied allows us to consider a gravitation theory as correct or incorrect based on the evaluation of post-Newtonian parameters.

It appears from the above-cited works that the scalar theory of H Yilmaz [25, 274–276] is an incorrect gravitation theory which came into being (along with a number of the aforementioned correct theories of gravitation [107–110, 272, 273]) as a consequence of difficulties incident to the application of the Mach principle in GTR. According to Refs [110, 272, 273], Yilmaz's theory states, in particular, that the angular velocity of the planets' rotation and precession of their perihelia depend on the relative velocity between the frame of reference attached to the central body about which a planet moves in its orbit and a privileged reference system imagined to be at rest with respect to the center of mass in the Universe. It should be mentioned that the discovery of the relict radiation anisotropy [277] provoked the discussion of potential effects of the reference system velocity relative to the privileged reference system on certain physical phenomena, e.g. isotropy of the speed of light [169, 278, 279]. However, experiments on a laser with an absorption cell have demonstrated that there is no anisotropy of the speed of light to an accuracy of 2×10^{-13} [280]. The accuracy of measuring the anisotropy of the speed of light could be further improved by three orders of magnitude using a Michelson fibre-optic interferometer with arm lengths from 1 to 10 km [281].

H Yilmaz has investigated the Sagnac effect (which he terms 'mysterious') in considerable detail in the framework of the gravitation theory [25, 274–276]. It follows from the results of Ref. [25] that the refractive indices of vacuum for counterpropagating waves in a reference system attending rotation are different:

$$\frac{1}{n^{\pm}} = 1 \pm \frac{\Omega R}{c} \, .$$

The author of the work cited concludes that this gives rise to the Sagnac effect. It should be recalled, however, that the magnitude of the Sagnac effect is totally independent of the refractive index of a medium.

5.9 Other incorrect explanations of the Sagnac effect

There are more incorrect explanations of the Sagnac effect, which are however less known than the foregoing ones. They are just listed below without offering much comment upon each.

In Refs [282–284] (see also Ref. [9]), the Sagnac effect is considered as a result of broken time reversion for counterpropagating waves in a ring interferometer (resonator) in a rotating frame of reference. It is known that the time is irreversible in the constant gravitational field created by a rotating body [34]. However, it has been shown earlier in this paper that the rotation of a massive body is not equivalent to that of a coordinate system about a motionless massive body. In other words, time is reversible in a rotating frame of reference. Moreover, the Sagnac effect occurs regardless of the presence of a massive body inside a ring interferometer.

In Ref. [223], the Sagnac effect is considered in terms of the locality hypothesis broadly understood. In doing so, the magnitude of the Sagnac effect is computed using the following relationship between the photon energies E and E' in the stationary reference system and co-moving reference system attached to rotating object, respectively:

$$E' = \gamma(E - \Omega L) \, ,$$

where $\gamma = (1 - R^2\Omega^2/c^2)^{-1/2}$, L is the photon orbital momentum, Ω is the angular velocity of rotation, and R is the radius of rotation. Evidently, the calculation method used in Ref. [223] is analogous to that discussed previously [133], which takes into account changes of the orbital momenta of macroscopic photon orbits and the energy of generated photons in a rotating ring laser. This method is incorrect and leads to some mistakes (see above).

In Ref. [285], the Sagnac effect is considered to be a corollary to the validity of the Fermat principle in a rotating frame of reference. The motion of a free material particle in a gravitational field is governed by the Fermat principle (the principle of least time) which states that a travelling particle seeks a path such that its world point follows a geodesic in the four-dimensional space-time [34]. However, a photon in a ring interferometer is not a free particle [261]; it undergoes multiple successive reflections by the mirrors and progresses through a light-guiding core of the SMOF toward the FRI.

In Ref. [286], the Sagnac effect is considered to be a manifestation of the adiabatic invariant in a rotating frame of reference.

According to Rosenbloom [287], the Sagnac effect relates to the Schwarzschild solution [288] for a rotating reference system. The author of Ref. [287] puts forward the hypothesis that, through the force of the equivalence principle, gravitational fields are applied to photons in a rotating frame of reference, and the velocity of one counterpropagating wave in a ring interferometer increases while that of the other decreases. It should be emphasized that such a situation occurs only with the availability of angular acceleration [117]; therefore, the explanation of the Sagnac effect in Ref. [287] is incorrect. Moreover, the author considers the Sagnac effect to be an analogue of light beam deflection in the gravitational field, proceeding from the erroneous results of an earlier work by Einstein [175]. The paper [289] treats the Sagnac effect almost in the same manner as Ref. [287]. However, the author addresses the subject with greater caution stating that the standard computation of the Sagnac effect is not necessarily incorrect. Moreover, he emphasizes the possibility of obtaining the correct result in a stationary reference system in Minkowski space, i.e. in the framework of STR.

The author of Ref. [227] makes the wrong assumption that an accelerated motion (including rotation) is relative rather than absolute in character and as a consequence this results in the Sagnac effect.

Finally, Dieks [80] considers the Sagnac effect as a consequence of the twin paradox in a rotating frame of reference.

6. Conclusions

This review familiarizes the reader with selected works concerning the Sagnac effect, which the author considers to be especially important for its understanding. It illustrates a wide variance of opinions as regards the physical causes of this phenomenon. Evidently, no other effect has given rise to such different and sometimes mutually exclusive interpretations.

The Sagnac effect was at different times the focus of interest of O Lodge, A Michelson, M Laue, M G Sagnac, A Einstein, P Langevin, L Silberstein, C Runge, B Pogany, A Sommerfeld, F Zernike, C Møller, M-A Tonnelat, J Sakurai, and Y Aharonov. In this country, it attracted the attention of S I Vavilov, L I Mandel'shtam, L D Landau and E M Lifshitz, I L Bershteĭn, G V Skrotskiĭ, V A Ugarov, and A A Logunov.

It has been shown before that the Sagnac effect is a corollary to the relativistic law of velocity composition (the propagation velocity of an arbitrary wave and rotational velocity of an interferometer). This means that the Sagnac effect constitutes a kinematic effect of STR [10, 11]. Also, it has been demonstrated that calculations of the Sagnac effect in terms of GTR and STR are virtually equivalent in the absence of gravitating masses, i.e. in the absence of space curvature. Nevertheless, certain authors question the relevance of computation in accelerated frames of reference (in particular, in rotating frames of reference) using STR. On February 2, 1952, R Shankland addressed A Einstein for clarification (in connection with the discussion about the influence of Earth's rotation on the results of the Michelson – Morley experiments [12, 13] and those of D Miller [231, 232]). The answer was [290]: ''O yes, that's right, so long as there is no gravity; in all other cases the special theory of relativity is applicable. Although the approach of the general theory of relativity is probably better, it is not called for.''

Summarizing this review, it may be concluded that all explanations of the physical nature of the Sagnac effect are incorrect except the relativistic one, even though in some specific cases they are likely to yield a result accurate to within relativistic corrections. This is due to the fact that the Sagnac effect, similar to certain other effects sometimes regarded as its substitutes, is a first-order effect in v/c. This accounts for the large number of incorrect explanations, from reducing the Sagnac effect to a corollary of the Galilean law of velocity composition or some other known first-order effects (Doppler, Fresnel – Fizeau, Aharonov – Bohm, Lense – Thirring effects, etc.) to a vague and confused interpretation in the framework of the incorrect gravitation theory [25] or a reanimated luminiferous ether concept [136, 236, 237]. It has been shown above that the Sagnac effect often occurs concurrently with some other effects, which makes it difficult to distinguish between them and accounts for inadequate conclusions.

The numerical coincidence of the results obtained by the correct method and incorrect computation does not necessarily suggest the possibility to explain one and the same phenomenon in two or more different ways. The validity of this inference is exemplified by the expression for the speed of light in a moving medium derived by O Fresnel in 1818 based on the partial luminiferous ether drag theory [44]. This expression is still in use in contemporary optics, giving a highly accurate result, but it disregards a very small correction for medium dispersion, which can be obtained only in the framework of STR [54]. It is not to be forgotten, however, that this work by O Fresnel played a somewhat negative role (besides a positive one) in the progress of physics. Specifically, it provided a foundation on which the theory of luminiferous ether had rested for almost 90 years until it was superseded by STR. Had not O Fresnel postulated the entrainment of the luminiferous ether by a moving medium, the inconsistency between the experimental results of Michelson and Morley [12, 13] and the luminiferous ether theory would have led scientists to the discovery of the relativistic law of velocity composition (and, possibly, even STR) earlier than it actually came. Another example is the explanation of the gravitational red shift as a consequence of a change of its potential energy when moving in a gravitational field. This explanation completely ignores conclusions drawn from GTR but yields a correct result in absolute value. The authors of Ref. [41] give evidence that this explanation is incorrect.

Thus, the possibility of obtaining a correct result by an 'incorrect' method may hinder the understanding of the physical nature of the effect in question and retard the development of an adequate theory.

I dedicate this work to the memory of my father B F Malykin (4 May 1912 – 26 June 1999) who inspired me to concentrate on this most intriguing physical problem.

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