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Holey fibers

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Contents

Abstract. A brief review of the conceptual and technological progress achieved in various areas of physics due to the extensive use of holey fibers is provided. The cladding in these new fibers consists of an array of hollow silica fibers fused and processed at a high temperature. The basic properties of such fibers are considered and physical problems that can be solved with the use of such structures are discussed.

1. Introduction

The creation of holey fibers (HFs), i.e. fibers with a cladding having the form of a two-dimensional (often periodic) array of closely packed glass capillaries drawn at a high temperature, is one of the most significant achievements in optical technology within the last five years. Since the very first papers reporting the fabrication of holey fibers back in 1996 [1], much attention has been focused on the investigation of the remarkable properties of such fibers. The field of application of these fibers is now ever expanding, resulting in a fast growth in the number of research groups using holey fibers in their studies.

One of the main advantages of holey fibers is that they support single-mode waveguiding over a very broad spectral range. In the case of conventional fibers (Fig. 1a), the spectral region of single-mode waveguiding is comparatively not too large [2, 3]. As the frequency of optical radiation increases, a single-mode conventional fiber becomes multimode, while lowering the radiation frequency increases optical losses.

The situation radically changes in the case of holey fibers (Fig. 1c) which allow the spectral region of single-mode waveguiding to be considerably expanded [4] due to the fact that the effective refractive index of the composite cladding in this circumstances changes as a function of radiation frequency not only because of material dispersion, but also because of variations in the distribution of the light field throughout the fiber cladding (see Section 3). Fibers of this type are of considerable interest in the context of many problems of fiber optics [5 – 16], nonlinear optics [17 – 22], atomic optics [17, 23, 24], the physics of photonic crystals and

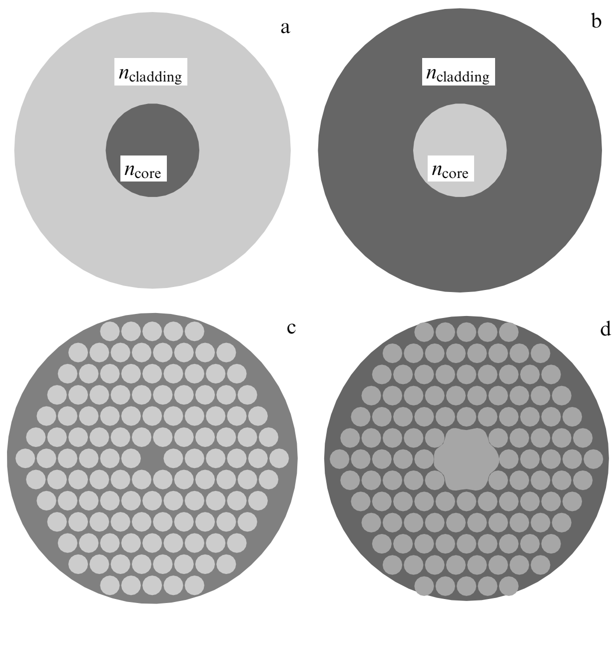

Figure 1. Types of optical fibers: (a) conventional fiber with a solid cladding ($n_{\text{core}} > n_{\text{cladding}}$), (b) hollow fiber with a solid cladding ($n_{\text{core}} < n_{\text{cladding}}$), (c) holey fiber, and (d) hollow-core holey fiber. Darker areas correspond to materials with higher refractive indices.

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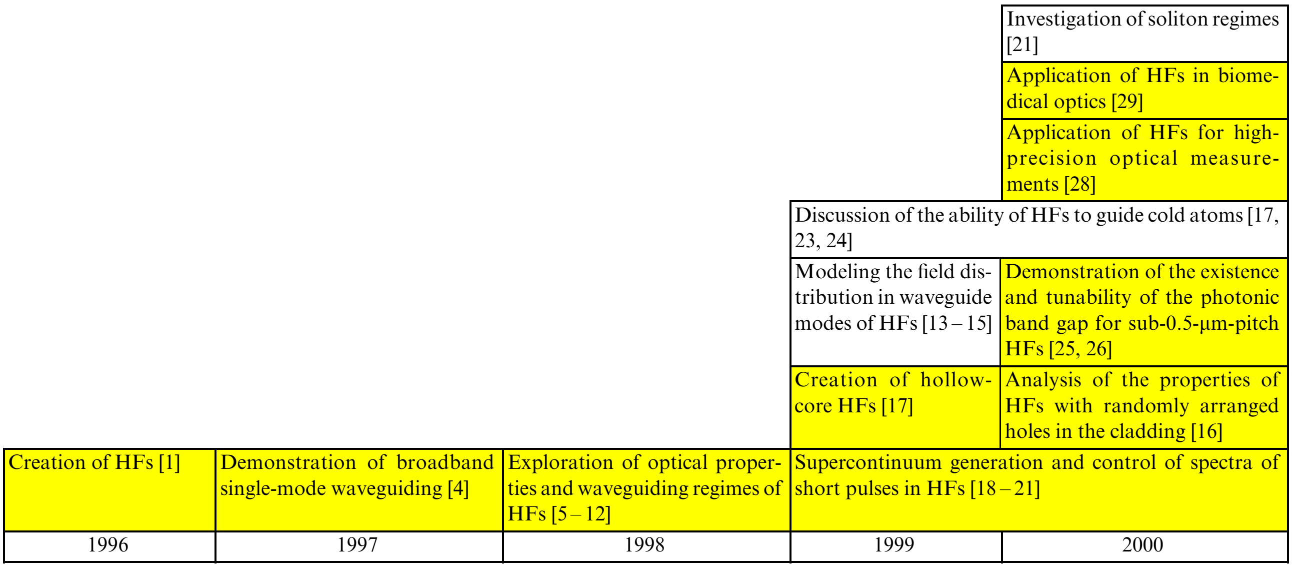

Figure 2. Diagram illustrating the development of the concept and the growth of holey fiber applications from the moment of creation of these fibers (1996) until the present time (2000). The shaded cells indicate experimental results, while unshaded cells correspond to theoretical proposals.

quantum electrodynamics [23 – 27], high-precision optical frequency measurements [28], biomedical optics [29], and optical data transmission [21] (see Fig. 2).

In the case of a periodic arrangement of holes, the cladding of a holey fiber produced by a close packing of hollow-core fibers has the structure of a two-dimensional photonic crystal (Fig. 1c, d). Due to the periodicity of air holes in glass, the transmission spectrum of such a structure displays photonic band gaps (PBGs) for certain directions of the wave vector. Within these frequency ranges, radiation cannot penetrate into the fiber cladding. Whereas a fiber without a hole at the center is used for a core in such a waveguide, the fiber core can be considered as a lattice defect with respect to the otherwise perfect two-dimensional photonic crystal. Such fibers seem to offer much promise for extending many of the exciting proposals discussed recently by the photonic-crystal community [30 – 38] to the optical range.

Although only a few research groups can now fabricate holey fibers (see Fig. 2), the well-developed and very efficient international scientific collaboration promotes rapidly growing applications of holey fibers in different areas of modern physics. A prominent example of such a cooperation was provided recently by researchers from the University of Bath (Bath, UK), who fabricated holey fibers for high-precision optical measurements performed by the group of Professor T Hänsch at the Max Planck Quantum Optics Institute (Garching, Germany) [28]. Remarkably, the time gap between the demonstration of the ability of holey fibers to generate supercontinuum and the application of this property of holey fibers to optical metrology was as short as several months.

The progress achieved in the last few years in the technology of fabrication and the methods of characterization of holey fibers and the extensive expansion of holey fibers into various areas of science have resulted in a rapid growth in the number of scientific publications devoted to these optical devices [1 – 28] and they have received publicity from periodicals (e.g., see Refs [39 – 44]).

All these circumstances along with the above-mentioned rapidly progressing collaboration aimed at utilizing holey fibers for the solution of different urgent problems of modern physics necessitate some preliminary systematization of the results obtained in this field with an overview of the main properties of this type fibers and the problems that can be solved with the aid of them. These are the objectives to be pursued by this brief review.

The layout of the paper is the following. The history of the creation of holey fibers and the development of the technology for their fabrication is briefly discussed in Section 2. Section 3 analyzes the physical factors underlying broadband single-mode waveguiding in holey fibers. Section 4 is devoted to the role of the photonic band gap in the propagation of radiation through holey fibers. Issues related to the dispersion of holey fibers are considered in Section 5. Section 6 gives a short overview of the main applications of holey fibers. Finally, some general results of our analysis are summarized in the Conclusions.

2. The short history of holey fibers

2.1 The pioneering experiments

The history of holey fibers is not very long. However, it seems very instructive to examine its main stages, since holey fibers provide a glowing example of how new approaches in optical technology open the way for the solution of many urgent problems stimulating a conceptual progress in different areas of physics.

The fabrication of fibers with a composite cladding where air holes were arranged in a two-dimensional periodic structure in glass (Fig. 1c) was first reported by Knight et al. [1] in 1996. Subsequent studies have shown that fibers of this type support single-mode waveguiding over a remarkably broad spectral range, allowing radiation energy losses to be considerably reduced in the single-mode regime [4].

The authors of Ref. [1] called the fibers they created photonic-crystal fibers. However, since the periodicity of the structure in the fiber cladding is not necessary for broadband single-mode waveguiding, the term 'holey fibers' [18] was introduced on an equal footing with the term 'photonic-crystal fibers' to define a broader class of fibers with a composite cladding. The role of the structure periodicity in the fiber cladding and the photonic band gap in the transmission spectrum of this structure will be discussed in greater detail in Section 4.

The first experiments by Knight et al. [1] devoted to the creation and investigation of holey fibers have stimulated intense investigations into the waveguide properties of holey fibers, which allowed the mode structure of radiation in such fibers to be revealed [12], the boundaries of the single-mode regime to be found [4], the frequency dependence of laser-radiation divergence to be determined [45], and the dispersion of holey fibers to be measured [46]. Holey fibers also hold much promise for nonlinear-optical applications. In particular, the high degree of localization of the light field within a small area around the core of a holey fiber suggests a way of considerably improving the efficiency of nonlinear-optical interactions [18]. Recent experiments [19–21] have shown that these properties of holey fibers can be employed to control the spectra of ultrashort laser pulses and to generate a supercontinuum.

2.2 Hollow-core holey fibers

The creation of hollow-core holey fibers [17] has opened a new phase in the development of the concept of holey fibers. A hollow core in guides of this type (Fig. 1d) is produced by removing several fibers around the center of the stack of fibers employed to fabricate a holey fiber. An important distinguishing feature of such fibers is that, physically, waveguiding in these fibers is due to the existence of the photonic band gap in the transmission spectrum of the fiber cladding rather than to total internal reflection (as is usually the case in conventional fibers and holey fibers considered in the previous section). Fibers of this type offer much promise for the creation of gas sensors and gas-phase analysis as well as the development of new spectral elements, and laser guiding of cold atoms.

Due to the fact that the optical breakdown threshold for gases is much higher than the optical damage threshold for solids, hollow-core holey fibers (similar to hollow-core fibers with a solid dielectric cladding, see Fig. 1b) can be employed in experiments with high-power short light pulses [47–56]. This would open new horizons in the physics of high-intensity light fields by providing an opportunity to implement waveguiding regimes for high-intensity laser pulses, thus improving the efficiency of nonlinear-optical interactions. Such an enhancement of effects of nonlinear-optical interactions could be employed to generate ultrashort light pulses and to produce high-order optical harmonics.

The idea of employing hollow-core holey fibers for the laser guiding of cold atoms is also currently being discussed. The configuration of a photonic-crystal lattice and a defect in such a lattice constructed in Refs [23, 24] produce the spatially distributed electric potential allowing the guiding of cold atoms along the lattice defect of the photonic crystal due to the dipole force acting on atoms in the field of blue-detuned laser radiation. The estimates presented in Refs [23, 24] demonstrated that the transverse temperatures and transverse localization degrees of atoms attainable with photonic-crystal fiber atom guides are much higher than those characteristic of laser guiding of atoms in hollow-core fibers with a solid cladding.

2.3 Sub-500-nm-pitch holey fibers

Until recently, investigations into the optical properties of holey fibers have been restricted to propagation regimes when the radiation wavelength is much less than the pitch of the photonic-crystal cladding and the core diameter. In these regimes, the existence of a photonic band gap has, in fact, no influence on the propagation of light in a fiber. The physical principles of photonic-crystal fibers with a photonic band gap of the cladding, tunable in the visible and near-IR spectral ranges, have been recently demonstrated in Refs [25, 26].

Optical properties of such holey fibers were studied in Ref. [26] by means of direct numerical integration of the Maxwell equations with the use of the finite-difference time-domain (FDTD) technique. This analysis demonstrated that the transmission spectrum of a photonic-crystal cladding in a holey fiber may display a photonic band gap within the frequency range characteristic of the available efficient femtosecond lasers if a two-dimensional periodic structure with a period less than 500 nm is employed as a cladding in such a fiber. Photonic-crystal fibers meeting this requirement were fabricated and experimentally studied in Refs [25, 26].

The analysis of transmission spectra of these holey fibers measured in the direction perpendicular to the direction of waveguiding has revealed the existence of a photonic band gap within the wavelength range from 930 to 1030 nm. The possibility of tuning the photonic band gap of the photonic-crystal cladding in holey fibers by the filling air holes in the cladding with alcohol was also experimentally demonstrated in Refs [25, 26].

The experimentally examined possibility of tuning the photonic band gap of a photonic-crystal cladding in holey fibers by using various materials to fill the air holes in the fiber cladding offers broad opportunities for tuning the dispersion of holey fibers and controlling the luminescence of molecules over a broad spectral range.

3. Broadband single-mode waveguiding

One of the most important properties of holey fibers, allowing the solution of an extensive class of urgent problems and making holey fibers especially useful for many practical applications, is the ability of these fibers to considerably expand the spectral range of single-mode waveguiding in comparison with conventional fibers.

Physically, this property of holey fibers depends upon the fact that the degree of light-filling of air holes in the cladding of holey fibers varies as the radiation wavelength changes. As a result, the difference in the refractive indices of the fiber core and the fiber cladding becomes wavelength-dependent, considerably expanding the spectral range of single-mode waveguiding.

Mathematically, the criterion of single-mode waveguiding for a conventional fiber can be written as follows [2, 4]

$$V = \frac{2\pi\rho}{\lambda} \left(n_{\mathrm{core}}^2 - n_{\mathrm{cladding}}^2\right)^{1/2} < 2.405\,, \tag{1}$$

where ρ is the radius of the fiber core, λ is the radiation wavelength in a vacuum, n_{core} is the refractive index of the fiber core, and n_{cladding} is the refractive index of the fiber cladding.

In the limiting case when the radiation wavelength λ is much greater than the pitch Λ of the structure, the effective refractive index of the cladding, n_{cladding}, is equal with a high accuracy to the weighted-mean of the refractive indices of glass and air in the structure. As the wavelength λ decreases, light becomes mainly concentrated in the regions with a higher refractive index.

This qualitative analysis explains why light with a shorter wavelength 'sees' more glass and less air. Thus, as the radiation wavelength becomes shorter, the difference between the refractive index of the fiber core and the effective refractive index of the fiber cladding decreases, which allows the criterion of Eqn (1) to be met over a sufficiently broad wavelength range.

As shown in Refs [4, 34], the parameter V in the case of a holey fiber can be replaced by the effective parameter

$$V_{\text{eff}}^2 = \left(\frac{2\pi\Lambda}{\lambda}\right)^2 \left\{ f(n_1^2 - 1) + \frac{\int |\nabla\psi|^2 \,\mathrm{d}x\,\mathrm{d}y}{k \int |\psi|^2 \,\mathrm{d}x\,\mathrm{d}y} \right\}, \qquad (2)$$

where Λ is the pitch of the structure in the fiber cladding, n_1 is the refractive index of the glass employed to fabricate the fiber, the function ψ describes the profile of the field amplitude, ∇ is the transverse part of the Laplacian, f is the air-filling ratio of the structure, and x and y are the coordinates in the cross section of the fiber.

Figure 3 presents the parameter V_{eff} calculated by Birks et al. [4] as a function of the ratio Λ/λ in the case of glass with a refractive index equal to 1.45 for different ratios of the hole diameter d to the pitch Λ. As can be seen from these dependences, a holey fiber remains single-mode over a broad spectral range for sufficiently small values of d/Λ.

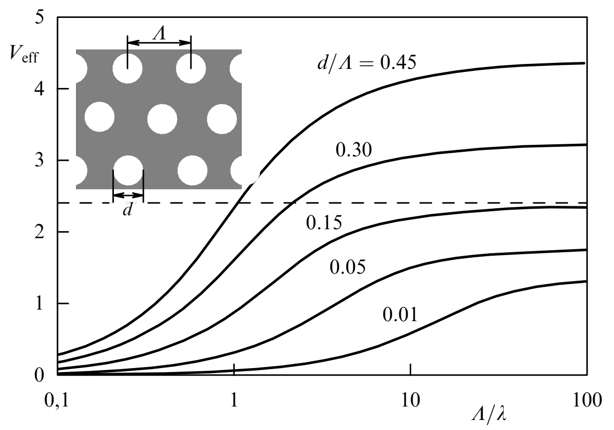

Figure 3. Parameter V_{eff} as a function of the ratio Λ/λ calculated by Birks et al. [4] for glass with a refractive index equal to 1.45 and different ratios of the hole diameter d to the pitch Λ. The dashed line shows the cutoff value of the parameter V_{eff}. The inset displays a triangular lattice of air holes in dielectric.

4. The role of the photonic band gap

When air holes are arranged periodically in the cladding of a holey fiber, a photonic band gap may be observed in the transmission spectrum of the fiber, measured in the direction perpendicular to the direction of waveguiding. Radiation with wavelengths falling within this spectral range cannot penetrate into the cladding. The core of a holey fiber then plays the role of a defect in a two-dimensional photonic-crystal lattice. Such structures, in fact, suggest a new way of solving the problem of fabricating photonic crystals (e.g., see Refs [22, 57 – 59] and references therein). Fibers of this type can be employed to control the emission of atoms and molecules and to localize light within the PBG frequency range. These issues are actively discussed now in connection with new ideas in fundamental physics and numerous applications [30 – 38, 60, 61].

Physically, photonic crystals (or photonic band-gap structures) [30 – 38] are a new type of artificial material characterized by a spatial periodicity of optical characteristics with the sizes of crystal unit cells on the order of the wavelength of visible light. Due to the periodic modulation of their optical properties, photonic crystals ensure specific regimes of light propagation within certain ranges of wavelengths and wave vectors.

In particular, the coupling of electromagnetic waves in such structures gives rise to photonic band gaps for certain directions of the wave vector. If a closed photonic band gap exists for some periodic structure, then electromagnetic radiation cannot propagate in such a structure within some frequency range regardless of the wave vector directions and polarization of this radiation. Such photonic band gaps are similar to electron forbidden gaps observed in semiconductors.

Photonic crystals open new horizons in various areas of physics. There are several reasons why researchers are interested in photonic band gaps, and continue trying to create photonic crystals. The initial motivation behind creating PBG structures was that photonic crystals make it possible to control elementary optical phenomena, including spontaneous emission, molecular fluorescence, emission of a dipole, etc. (see Ref. [31]). With a more general attitude to this issue, photonic crystals suggest a way of controlling photons and creating various photonic devices in pretty much the same fashion as electrons can be controlled in the atomic lattice, permitting the development of numerous electronic devices.

Although the creation of three-dimensional photonic crystals is still a serious technological problem, the possibilities of fabricating photonic crystals with different types of photonic-crystal lattice and tunable photonic band gaps are currently being discussed [32, 33, 57, 59, 62 – 64]. Thus, potentially, the possibilities for controlling photons in photonic crystals are much broader than those for controlling electrons in an atomic lattice.

Yet another important factor that stimulates research into photonic crystals is the elegant new physics of PBG structures, which differs in many respects from, for example, the dynamic theory of X-ray diffraction owing to the high contrast of refractive indices attainable in optics.

Finally, photonic crystals provide a new way for solving classical problems of nonlinear optics, including frequency conversion [65 – 67], pulse compression [35], optical switching [68 – 73], and the creation of bistable elements [74].

As is clear from the analysis performed in Section 3, the periodicity of holes in the fiber cladding is of no importance for broadband single-mode waveguiding. This conclusion is confirmed by the results of experiments [16] performed with the use of holey fibers where the air holes in the cladding were arranged randomly rather than periodically.

Thus, the existence of a photonic band gap is not necessary for broadband single-mode waveguiding in holey fibers. This is why the term 'holey fibers' was introduced for fibers where the arrangement of air holes in the cladding was not necessarily periodic. This term is now widely used parallel with the term 'photonic-crystal fibers,' which implies that a photonic band gap exists in the transmission spectrum of the fiber cladding.

However, we should emphasize that, as can be seen from Eqns (1) and (2), the frequency characterizing the arrangement of holes in the cladding (the PBG frequency in the case of periodically arranged holes) determines the characteristic frequency scale for the structure considered. Consequently, the tunability of a PBG in the frequency range [25, 26] would imply the tunability of optical properties of holey fibers, including the range of single-mode waveguiding and waveguide dispersion. Obviously, the periodicity of air holes in the cladding of a holey fiber is important whenever the photonic band gap is required to implement some physical idea.

As shown in Ref. [26], the forbidden band structure of photonic energies for a photonic-crystal cladding of holey fibers can be calculated by means of direct numerical integration of the Maxwell equations with the use of the FDTD technique [75]. A detailed description of this numerical procedure can be found in Ref. [76].

Simulations performed in studies [26, 76] employed periodic boundary conditions, while the initial conditions were taken in the form of an incident plane wave with wave vectors specifying the relevant boundary conditions. Calculations were carried out for an array of cylindrical air holes with a radius r and an infinite length, arranged in a two-dimensional periodic triangular-lattice structure in a dielectric (Fig. 1c). The first Brillouin zone for such a photonic-crystal lattice has the form of a regular hexagon illustrated in Fig. 4a.

Applying the FDTD method to numerically integrate the Maxwell equations and performing the Fourier transform of the field evolution obtained with the use of this procedure for arbitrary points of the simulation area, one can determine the eigenfrequencies corresponding to the chosen values of the wave vectors. Then, the band structure of photonic energies can be calculated and the form of the photonic band gap can be found by repeating this procedure for all the wave vectors from the first Brillouin zone.

The above-described approach was taken to calculate the band structure of photonic energies for a two-dimensional photonic-crystal lattice with $r/a = 0.4$ and $n = 1.5973$. The band structure of photonic energies for E- and H-polarized radiation fields is presented in Fig. 4b, c, respectively. The results of these simulations show that the creation of holey fibers with a photonic band gap lying within the wavelength range characteristic of the accessible efficient femtosecond titanium sapphire and forsterite lasers requires the fabrication of two-dimensional PBG fiber structures with a pitch less than 500 nm [26].

The technology employed to fabricate holey fibers with such parameters was similar to that described in Ref. [1] and was based on the following procedure. Identical glass capillaries were stacked into a periodic structure which was then fused at a high temperature, in order to eliminate air gaps between the capillaries, and drawn. The resulting structure was cut into segments. These segments were also stacked into a periodic array, which underwent the drawing process again. These technological stages were repeated until a pitch (the distance between the centers of the air holes) of 32 μm was achieved. At the final stage, a holey fiber with a desirable pitch was obtained by drawing the 32-μm-pitch structure inside a glass shell. The initial inner diameter of glass capillaries used in this process was about 1 mm.

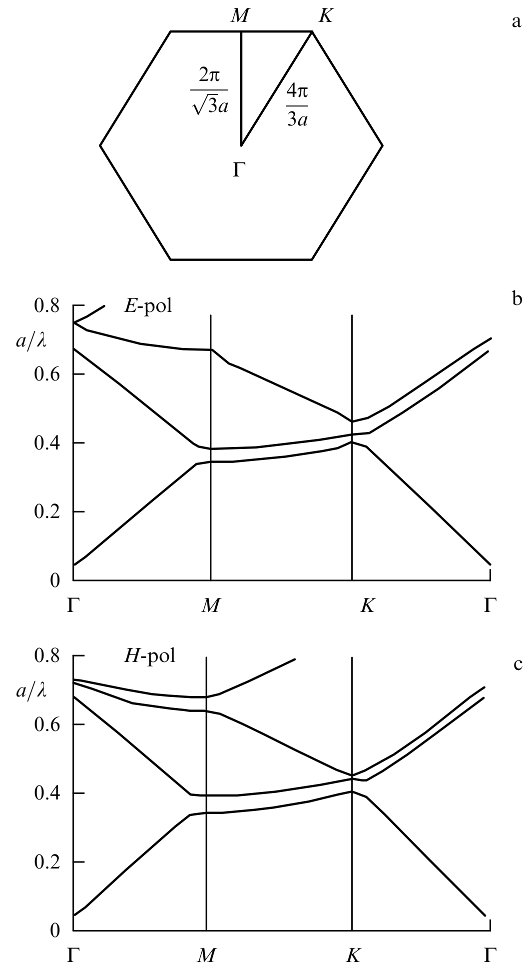

Figure 4. (a) The first Brillouin zone corresponding to a triangular lattice of air holes in a dielectric (Fig. 1c), and (b, c) the band structure of photon energies calculated for (b) E- and (c) H-polarized radiation fields in a photonic-crystal cladding of a holey fiber with the ratio r/a of the hole radius to the pitch equal to 0.4 and $n = 1.5973$ [26].

The above-described technique allowed the fabrication of optical fibers where the cladding had the structure of a two-dimensional photonic crystal containing several hundreds of periodically arranged air holes with a pitch ranging from 460 nm up to 32 μm (Fig. 5). Special precautions were taken to keep the circular shape of air holes and to eliminate the air gaps between neighboring capillaries. Two-dimensional photonic crystals with the lattice period of the PBG structure less than 0.5 μm and the ratio of the diameter of air holes to the pitch of the structure equal to 0.4 were fabricated by utilizing this technology. An image of such a PBG structure obtained with the use of a scanning electron microscope is shown in Fig. 6. A defect was introduced into this two-dimensional photonic-crystal structure by replacing the central capillary with a normal fiber made of another type of glass. This central fiber served as a core in the photonic-crystal waveguide (Fig. 5).

The periodic structure in the cladding of a holey fiber gives rise to photonic band gaps in the transmission spectrum of the structure measured in the direction perpendicular to the

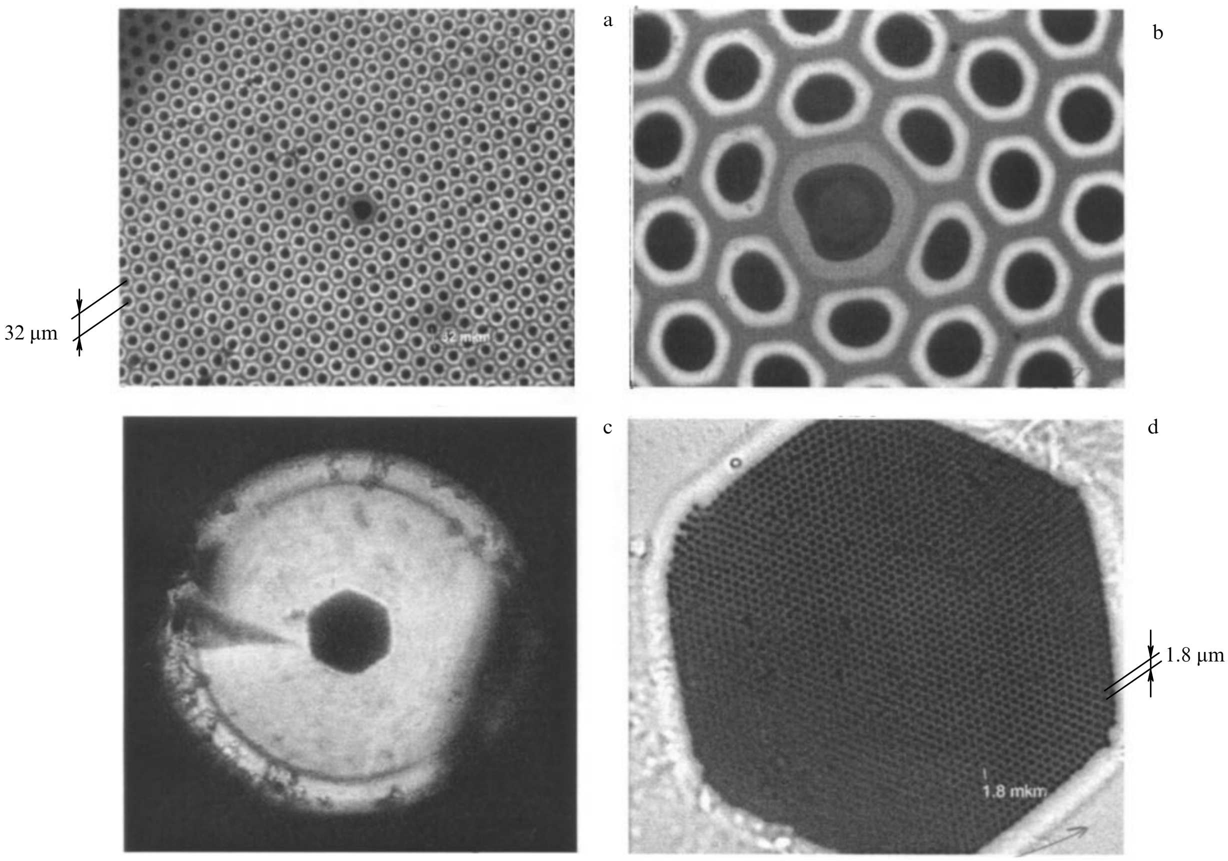

Figure 5. Microscope cross-sectional images of holey fibers with pitches of the cladding structures equal to (a, b) 32 μm and (c, d) 1.8 μm [19].

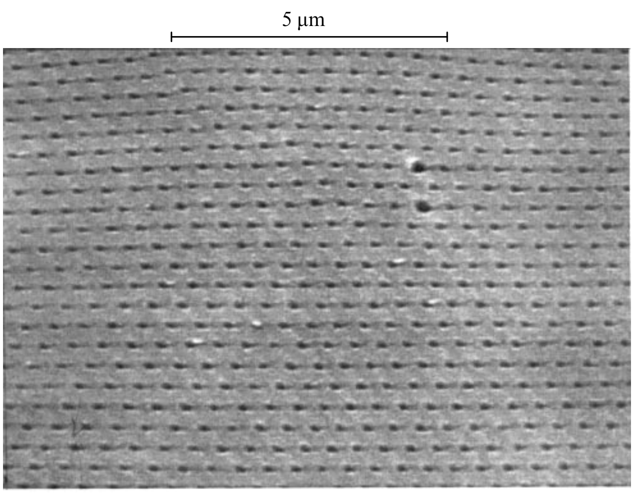

Figure 6. Scanning-electron-microscope images of the photonic-crystal cladding in a holey fiber [26]. The horizontal bar at the top of the figure corresponds to a scaled distance of 5 μm.

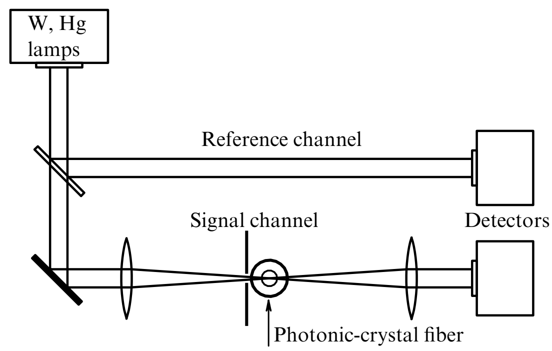

Figure 7. Experimental setup for measuring the transmission spectra of photonic-crystal fibers [19].

direction of waveguiding. Two-dimensional periodic structures with a period less than 500 nm allowed us to observe photonic band gaps falling within the wavelength range characteristic of widespread modern lasers. The experimental setup for measuring the spectra of holey fibers was built around a Hitachi-333 spectrophotometer and included the signal and reference channels (Fig. 7).

A 5-cm-focal-length quartz lens was used in the signal channel to ensure the predominant illumination of the central part of the sample having a photonic-crystal structure. This lens focused the light beam on a slit diaphragm with an aperture $d = 250$ μm. A holey-fiber sample was placed immediately behind the diaphragm. Radiation transmitted through the sample was collimated with a quartz lens, which was identical to the focusing lens. Transmission spectra were measured over the range of wavelengths from 400 to 1400 nm. The mercury lamp of the spectrometer was replaced by a tungsten lamp in the neighborhood of 870 nm. To be able to measure transmission spectra of a photonic-crystal sample for different directions of the wave vector in the first Brillouin zone of the photonic-crystal lattice, we rotated the sample around its axis corresponding to the direction of waveguiding.

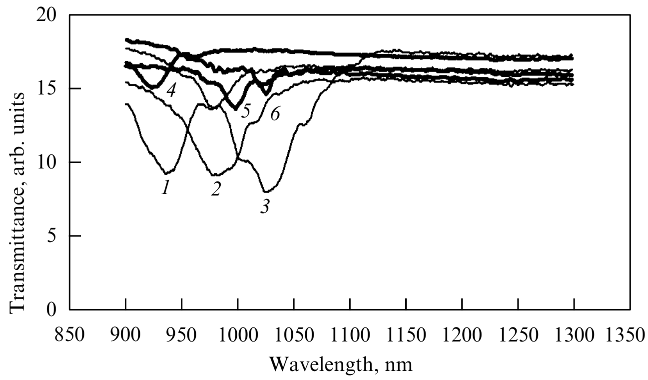

Figure 8. Photonic-crystal fiber transmission spectra measured for different directions of the wave vector in the first Brillouin zone of the photonic-crystal lattice of the PBG cladding (see Fig. 4a) with (*1–3*) air-filled and (*4–6*) alcohol-filled holes.

Transmission spectra measured for a holey fiber with a PBG structure period less than 500 nm are presented in Fig. 8. The photonic band gap in the transmission spectra of such samples was observed within the range of wavelengths from 930 to 1030 nm. Since air holes periodically arranged in the fiber cladding form a hexagonal lattice, the position of the photonic band gap in the transmission spectrum changes, depending on the angle of the structure rotation about the direction of an incident radiation. Comparison between the simulated results and experimental data (cf. Fig. 4b, c and Fig. 8) shows that the position of the photonic band gap can be satisfactorily described within the framework of the numerical approach based on the direct integration of the Maxwell equations using the FDTD technique.

The structure of the photonic band gap in the transmission spectrum of a photonic-crystal structure depends on the relation between the refractive indices of materials forming the periodic structure. In the case of holey fibers, this circumstance opens up the opportunity of tuning the photonic band gap by filling the air holes in the PBG structure with various materials whose refractive indices differ from unity.

Changes in the photonic band gap of holey-fiber samples, arising when the air holes of the PBG structure are filled with alcohol, were explored in Refs [25, 26]. As can be seen from the results of the measurements presented in Fig. 8 (curves *4–6*), the position and the width of the photonic band gap noticeably change in this event.

Thus, direct numerical integration of the Maxwell equations with the use of the finite-difference time-domain technique reveals the possibility of creating holey fibers with a photonic-crystal cladding whose photonic band gap lies within the frequency range characteristic of the accessible efficient femtosecond titanium sapphire and forsterite lasers if a two-dimensional periodic structure with a period less than 500 nm is employed as a cladding in such fibers. The fabrication of holey fibers meeting this requirement allowed the existence of a photonic band gap within the wavelength range of 930–1030 nm to be experimentally confirmed for such structures by measuring the transmission spectra of holey fibers in the direction perpendicular to that of waveguiding. This photonic band gap is satisfactorily described within the framework of the numerical approach making use of the direct integration of the Maxwell equations by the FDTD technique. The experimentally demonstrated possibility of tuning the photonic band gap of a photonic-crystal cladding in holey fibers by utilizing various materials to fill the air holes in the fiber cladding offers broad opportunities for controlling the dispersion of holey fibers.

5. Dispersion of holey fibers

Dispersion is a key parameter of a waveguide, which eventually determines whether a waveguide can be employed for various applications, including soliton propagation regimes, generation of short pulses, supercontinuum and harmonic generation, etc. Analysis of the dispersion of holey fibers encounters serious difficulties because of the complex field distribution in the waveguide modes of a holey fiber. Currently, there are several approximate methods intended for describing the dispersion of holey fibers. These methods will be briefly considered below.

Ranka et al. [21] analyzed the dispersion of holey fibers by replacing the refractive index of a holey cladding by an effective refractive index. Using this approach, the authors of Ref. [21] demonstrated that holey fibers may display anomalous dispersion and points of zero group-velocity dispersion in the visible range. The experimental results presented in this paper are in qualitative agreement with these theoretical predictions.

Holey fibers featuring an anomalous dispersion in the visible region, created by Ranka et al. [21], and the possibility of generating a supercontinuum, which was experimentally demonstrated by these authors, allow holey fibers to be employed for various spectroscopic applications, the generation of ultrashort light pulses, and the establishment of soliton propagation regimes for laser pulses.

However, the method of dispersion calculations employed in Ref. [21] creates only some general impression concerning the dispersion behavior in holey fibers. A more accurate approach to dispersion calculation should include the peculiarities of the field distribution in a holey fiber or provide a general recipe for calculating the effective refractive index of the fiber cladding.

The method developed in Refs [13, 14] implies the calculation of the dispersion of holey fibers by using the plane-wave expansion. In particular, the approach proposed by Broeng et al. [14] involves the solution of the eigenvalue problem corresponding to the vector equation for the magnetic field $\mathbf{H}(\mathbf{r})$:

$$\nabla \times \left[\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \frac{\omega^2}{c^2} \mathbf{H}(\mathbf{r}) , \tag{3}$$

where ε is the dielectric constant.

Then, using the Bloch theorem and expanding the periodic component of the solution to Eqn (3) in a Fourier series, the authors of Ref. [14] represent the result as

$$\mathbf{H}(\mathbf{r}) = \sum_{\mathbf{G}} \sum_{\gamma=1,2} \mathbf{h}_{\mathbf{k}+\mathbf{G},\gamma} \exp\left[\mathrm{i}(\mathbf{k}+\mathbf{G})\mathbf{r} \right] , \tag{4}$$

where $\mathbf{k}$ is the wave vector, $\mathbf{G}$ is the reciprocal lattice vector, and γ indicates two field directions perpendicular to the vector $\mathbf{k}+\mathbf{G}$. For a given wave vector $\mathbf{k}$, variational methods can be employed to find the solution to this problem.

This approach provides sufficiently detailed information on the dispersion of a holey fiber. However, powerful computers are usually necessary to implement this method. The method for computing the dispersion of holey fibers developed by Monro et al. [15, 37] seems to be less timeconsuming. The authors perform the expansion of the field in a holey fiber in functions that account for the radiation field localization in a fiber core.

The expansion thus obtained is then substituted into the relevant wave equation for the electric field, and the resulting eigenfunction and eigenvalue problem is solved using numerical methods. This procedure yields the field distribution throughout the fiber and the propagation constants. The approach proposed in Refs [15, 37] seems to be very convenient in use and allows the field distribution in a holey fiber to be described with an adequately high accuracy. Importantly, this method can be efficiently employed to analyze the influence of the cladding structure (not necessarily a periodic one) on the field distribution and the dispersion of waveguide modes in holey fibers [16].

6. Applications of holey fibers: achievements and outlook

6.1 Nonlinear optics

Recent experiments devoted to the investigation of spectral broadening of short pulses and supercontinuum generation in holey fibers [19–21] provide a prominent example of how the properties of holey fibers can be employed to solve urgent problems of modern physics. In fact, the possibility of the holey fibers generating a supercontinuum starting with subnanojoule femtosecond pulses, which was demonstrated experimentally by Ranka et al. [21], opens up new horizons in different fields of physics, including numerous spectroscopic applications, the generation of short light pulses, controlling the spectra of short pulses, etc. The ability of holey fibers to generate broadband radiation even with low-power input pulses was recently employed for the purposes of optical frequency metrology [28].

Holey fibers generating broadband radiation also seem to hold much promise for applications in linear and nonlinear spectroscopy, where broadband sources of coherent radiation allow spectra of different systems to be recorded with a single laser pulse, which is extremely important for the analysis of nonstationary media and the investigation of ultrafast phenomena.

Along with broadband dye lasers [77–79], supercontinuum-generating sources are currently employed more and more extensively for various applications in short-pulse spectroscopy [80]. Since supercontinuum generation involves nonlinear-optical processes [80], high intensities of incident light are usually required to ensure a spectral brightness of supercontinuum emission sufficient for spectroscopic applications.

The idea of enhancing the efficiency of nonlinear-optical interactions and, consequently, the efficiency of spectral broadening and supercontinuum generation in holey fibers as compared to conventional fibers is based on robust low-loss single-mode waveguiding, which is supported, as demonstrated in Refs [1, 4], by holey fibers within a broad frequency range, and local field enhancement which is characteristic of waveguide modes of holey fibers [18] (the localization of the light field in defect modes of two-dimensional photonic crystals has also been analyzed in Refs [76, 81, 82]).

As mentioned above, the analysis of radiation propagation through holey fibers (especially in the case of nonlinear-optical interactions in such fibers) encounters many difficulties because of the complex distribution of the radiation field in waveguide modes of holey fibers. Therefore, we will restrict our consideration here to simple but physically instructive arguments illustrating the possibility of enhancing nonlinear-optical interactions in holey fibers. In the case of self-phase modulation, for example, the nonlinear phase incursion of a pulse propagating in a hollow-core fiber is given by (see Ref. [50])

$$\Phi_{SPM} = \frac{8\pi}{3n_0}\tilde{\gamma}_1^n \frac{P}{S_{\text{eff}}} L\,, \tag{5}$$

where P is the power of radiation, L is the interaction length, n_0 is the refractive index of the medium filling the fiber core, S_{eff} is the effective area of the waveguide mode, and

$$\tilde{\gamma}_1^n = \frac{3\pi\omega^2}{2K_p^n c^2}\,\mathbf{e}_p^{n^*}\hat{\chi}^{(3)}(\omega;\omega,-\omega,\omega)\,\mathbf{e}_p^n\,\mathbf{e}_p^{n^*}\,\mathbf{e}_p^n\,\frac{\int\int\left[f_p^n(\boldsymbol{\rho})\right]^4\rho\,\mathrm{d}\rho\,\mathrm{d}\theta}{\int\int\left[f_p^n(\boldsymbol{\rho})\right]^2\rho\,\mathrm{d}\rho\,\mathrm{d}\theta} \tag{6}$$

is the nonlinear interaction coefficient that accounts for the field distribution $f_p^n(\boldsymbol{\rho})$ in the eigenmode of the fiber, ρ and θ are the polar coordinates in the cross section of the fiber, K_p^n is the propagation constant of the light pulse, $\hat{\chi}^{(3)}(\omega;\omega,-\omega,\omega)$ is the nonlinear-optical susceptibility responsible for self-phase modulation, and $\mathbf{e}_p^n$ is the unit vector of field polarization in the light pulse.

As can be seen from Eqn (5), the efficiency of nonlinear-optical interactions can be improved in holey fibers due to the reduction of the effective area S_{eff} of the waveguide mode and the increase in the interaction length L. The results of this qualitative analysis were confirmed in the experiments performed by Broderick et al. [18].

In the case of frequency-nondegenerate nonlinear-optical processes, such as optical harmonic generation and parametric interactions of light pulses, we can also expect the enhancement of nonlinear-optical frequency conversion due to the decrease in the effective area S_{eff} of the waveguide mode and a sufficiently large interaction length L. An additional enhancement can be achieved in this case through the improvement of phase and group-velocity matching when the material dispersion is compensated by the dispersion of waveguide modes (see Refs [49, 50]).

An important advantage of holey fibers is that they provide an opportunity to implement nonlinear-optical interactions of short pulses in the waveguiding regime around zero group-velocity dispersion, i.e. under conditions when the group-velocity dispersion

$$D^n = \frac{\partial^2 K_p^n}{\partial\omega^2} = -\frac{1}{(u_p^n)^2}\left(\frac{\partial u_p^n}{\partial\omega}\right), \tag{7}$$

where u_p^n is the group velocity of a light pulse in a waveguide mode, is rather low.

Due to the fact that the material dispersion can be compensated by the waveguide dispersion in holey fibers, the latter, as demonstrated by Ranka et al. [21], allow the zero point of group-velocity dispersion to be shifted to the visible range. The authors of Ref. [21] were able to generate a supercontinuum with a bandwidth exceeding 550 THz in a 75-cm-long holey fiber around zero group-velocity dispersion for 100-fs pulses with an energy less than 1 nJ.

The wavelength of laser radiation lay relatively far from the photonic band gap of the fiber cladding in these experiments (as well as in many other studies devoted to spectral broadening of laser pulses in holey fibers). However, the results of these experiments demonstrate quite clearly that the light-field confinement in the fiber core leads to a substantial increase in the efficiency of spectral broadening of short laser pulses as compared with conventional fibers.

Thus, holey fibers offer much promise for improving the efficiency of nonlinear-optical processes. The effective enhancement of nonlinearity due to light localization in the core of a holey fiber was systematically studied by Broderick et al. [18].

6.2 Controlling the spectrum of short pulses

In this section, we shall discuss in greater detail the results of experimental studies devoted to the propagation of femtosecond laser pulses in holey fibers. These experiments [19, 20] demonstrate that the use of holey fibers allows the efficiency of spectral broadening of femtosecond pulses to be considerably increased as compared with conventional fibers. This finding may have important implications for light pulse compression and the creation of new efficient broadband radiation sources. Furthermore, these results confirm the possibility of enhancing the efficiency of nonlinear-optical interactions due to the confinement of the light field in the core of a holey fiber.

The ability of holey fibers to control the spectra of short light pulses can also be employed in biomedical optics [29], for instance, to deliver radiation and to tune the chirp of short light pulses in optical coherence tomography making use of femtosecond laser pulses [83, 84].

The technology of fabricating holey fibers employed in experiments [19, 20] was similar to the procedure described in Refs [1, 4] and discussed earlier in Section 4. Holey fibers with a cladding in the form of a two-dimensional photonic crystal with periodically arranged air holes were fabricated for these experiments. The pitch of the PBG structure in the cladding of fibers employed in experiments [19, 20] ranged from 1.4 to 32 μm, and the ratio of the diameter of air holes to the pitch of the PBG structure was equal to 0.4. The fiber core was produced by replacing the central capillary with an ordinary fiber made of glass of another type.

Experiments [19, 20] devoted to the spectral broadening of short laser pulses in holey fibers were carried out with the use of a laser system consisting of a Ti:sapphire laser, an eight-pass preamplifier, and a four-pass final amplifier. This laser system generated 150-fs pulses with a repetition rate of 10 Hz. The maximum energy of these pulses was as high as 100 mJ. In holey-fiber experiments, the energy of laser pulses was varied over the range from 1 nJ to 15 μJ. The contrast ratio of femtosecond pulses measured at 1-ps delay from the pulse maximum was estimated as 10^{-4}.

The laser beam was focused onto the entrance of a holey fiber. Experiments were performed with lenses having different focal lengths. In particular, for the comparison of spectral broadening achieved in holey and conventional supercontinuum-generating fibers, measurements with a 10-cm-focal-length lens were carried out. This lens focused a laser beam into a 100-μm spot.

For holey fibers with a core diameter ranging from 0.56 to 1.28 μm, this regime of radiation focusing corresponded to the excitation of the waveguide mode with a nearly plane wave (this regime of excitation of defect modes in photonic crystals was theoretically investigated in Refs [63, 68, 69]). Only a small fraction of light energy was coupled into the waveguide mode under these conditions, while a considerable part of the light beam was scattered due to the roughness of the fiber end. The spectra of light pulses at the output of the holey fiber were analyzed with the help of a monochromator and a CCD camera, which was used to image the output slit of the monochromator.

Analysis of the spectra of light coming out of the holey fiber revealed a considerable spectral broadening of the femtosecond pulses. With a transform-limited 150-fs laser pulse at the input of the fiber (Fig. 9a), a spectral width of about 40 nm at the level of 0.3 of the maximum intensity was achieved at the output of the fiber for pulses with a moderate intensity. A considerable fraction of the radiation energy was contained in the wings of the spectrum under these conditions (Fig. 9b).

The insets to Fig. 9a, b show the CCD images of a laser beam at the output of the monochromator, with the horizontal direction corresponding to the spatial beam profile and the vertical direction corresponding to the spectral profile of the laser pulse. The spectral broadening of the femtosecond pulses coming out of a holey fiber is manifested as an increase in the vertical sizes of CCD images. The image shown in the inset to Fig. 9b also indicates the presence of irregular modulation in the spectrum of a femtosecond pulse at the output of a holey fiber.

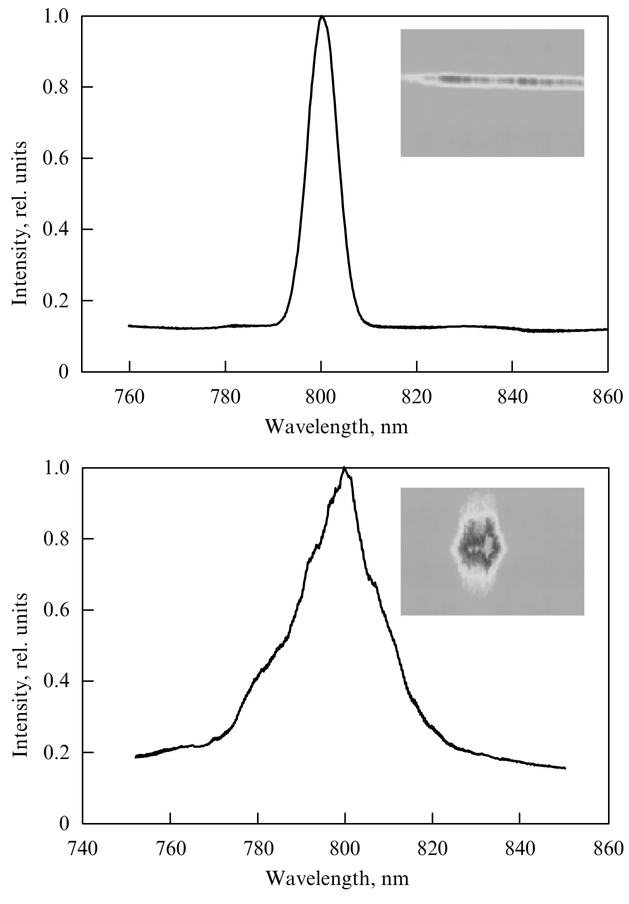

Figure 9. Spectral broadening of a femtosecond light pulse in a holey fiber [19, 20]: (a) the spectrum of a 150-fs Ti:sapphire laser output pulse, and (b) the broadened spectrum of the pulse at the output of the holey fiber with the pitch of the photonic-crystal cladding equal to 1.7 μm. The insets show the images of the beam patterns recorded by a CCD camera, with the horizontal direction corresponding to the spatial beam profile and the vertical direction corresponding to the spectral profile of the laser pulse.

The main features of spectral broadening of femtosecond pulses observed in these experiments were similar to the features of spectral broadening of short pulses in conventional fibers. The properties of such phenomena are well understood now and are described in detail in the extensive literature (e.g., see Ref. [80]). Importantly, even with considerable losses due to scattering when coupling radiation into a holey fiber, the spectral broadening of femtosecond pulses in such a fiber was much more efficient than in a conventional silica fiber with an entrance of much higher quality.

Thus, the results of experiments [19, 20] demonstrated efficient spectral broadening of 150-fs pulses of a Ti:sapphire laser in holey fibers. This effect can be employed both in pulse compression and the creation of new sources of broadband radiation. In the experiments described above, holey fibers ensured large propagation lengths due to robust waveguiding over a broad frequency range, which permits the requirements on the intensity of incident radiation in the spectral control of ultrashort laser pulses and supercontinuum generation to be considerably weakened.

Holey fibers can also be employed to improve the efficiency of other nonlinear-optical interactions. The further enhancement of nonlinear-optical processes in holey fibers can be achieved by using radiation wavelengths lying closer to the photonic band gap of the photonic-crystal cladding, utilizing materials with higher nonlinearities for the fiber fabrication, and improving the phase matching by separately controlling different components of holey-fiber dispersion.

The creation of hollow-core holey fibers [17] (see Fig. 1d) suggests new ways for improving the efficiency of nonlinear-optical processes. The energy of light pulses that can be coupled into a fiber of this type without damaging the fiber itself is much higher than the energy that can be coupled into a fiber with a solid core. Nonlinear processes in hollow-core fibers and dielectric solid cladding (Fig. 1b) are currently widely used to produce ultrashort light pulses and generate high-order harmonics [52 – 54].

Hollow-core holey fibers also hold much promise for improving the sensitivity of nonlinear-optical spectroscopy [85]. The use of such fibers would reduce optical losses, increase the degree of localization of the light field in the fiber core, improve phase matching for nonlinear-optical wave-mixing processes, and enhance the sensitivity of modern methods for coherent nonlinear spectroscopy [86].

7. Conclusions

Thanks to their remarkable properties, holey fibers allow many important fundamental and technological problems to be solved, opening new possibilities for research in various fields of modern physics. Figure 10 briefly summarizes the basic properties of holey fibers and shows some of the problems and those areas of physics where progress could be achieved by the application of holey fibers.

Broadband single-mode waveguiding is one of the most important characteristics of holey fibers. This property, as can be seen from Fig. 10, seems to be extremely useful for various applications of holey waveguides in fiber optics and other fields of physics.

The photonic band gap is not necessarily involved in the waveguiding of laser radiation in holey fibers. However, holey fibers with a photonic-crystal cladding permit the solution of several fundamental problems in the physics of photonic crystals, including local field strengthening, controlling the emission of atoms and molecules, enhancing the efficiency of nonlinear-optical processes, and studying quan-

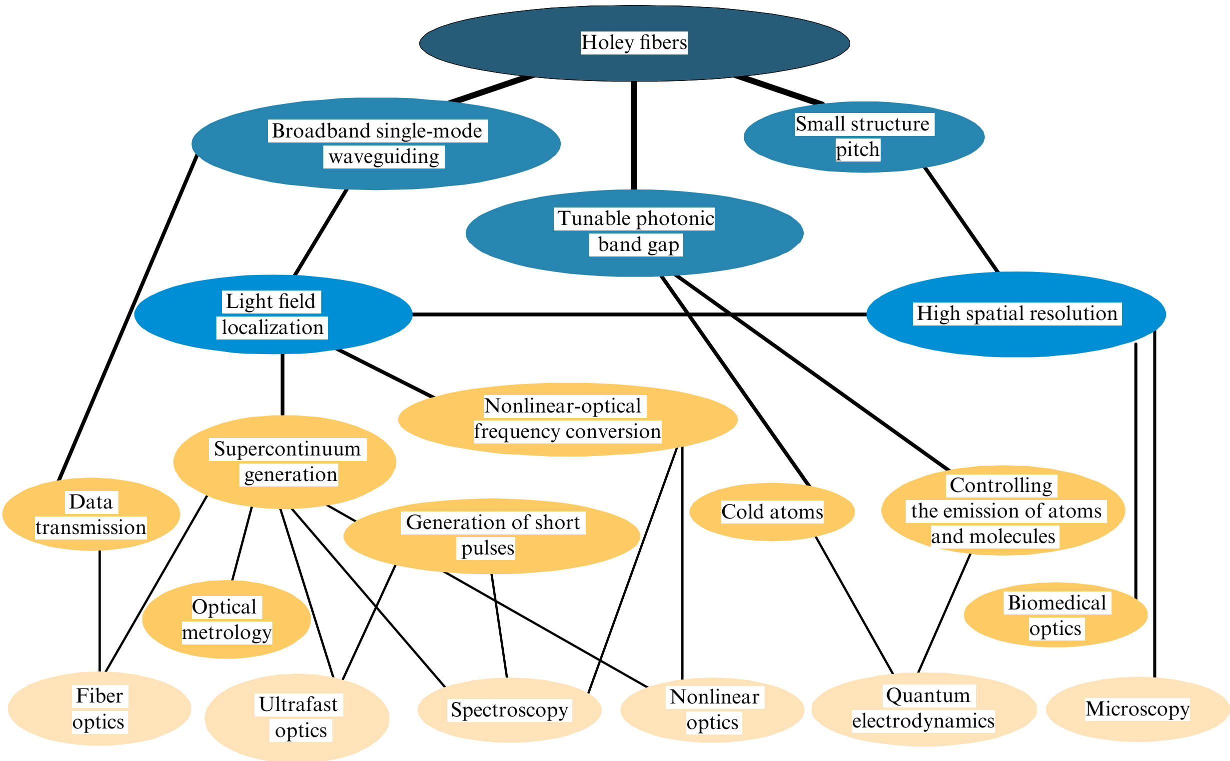

Figure 10. Basic properties and main areas of applications of holey fibers.

tum-electrodynamic aspects of the atomic coupling with radiation modes of microcavities and photonic crystals. The experimentally demonstrated tunability of the photonic band gap for a photonic-crystal cladding of a holey fiber implies that the dispersion of holey fibers can be controlled.

The results of experimental and theoretical studies show that all these remarkable properties of holey fibers can be employed to solve urgent problems in nonlinear optics, produce broadband radiation, generate ultrashort light pulses, improve the efficiency of nonlinear-optical frequency conversion, and implement optical switching. The ability of holey fibers to efficiently generate broadband radiation opens new possibilities in spectroscopy, the generation of short pulses, and high-precision optical frequency measurements. Hollow-core holey fibers can be employed for high-order harmonic generation in the field of high-intensity laser radiation and for the laser guiding of cold atoms. The possibility of using holey fibers for controlling the spectra of short light pulses and image transmission suggests new approaches in many fields of applied optics, including biomedical optics. Finally, the tunability of the dispersion in the case of holey fibers provides additional flexibility in data transmission.

The properties of holey fibers are currently being intensely studied by several research groups, and a massive effort is being made to improve the structure and the characteristics of these fibers. At the same time, extensive scientific collaboration aimed at applying holey fibers in various fields of physics is developing very rapidly. This activity gives grounds to believe that new prominent results in this rapidly progressing area of optics may be achieved in the near future.

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