PACS number: 42.65.Tg

From femtosecond to attosecond pulses

A V Kim, M Yu Ryabikin, A M Sergeev

1. Introduction

The modern level of optical technologies permits the generation of laser pulses of less than 5 fs duration [1], which is equal only to several optical cycles. The generation of such short pulses has become possible, on the one hand, due to the discovery of a new class of laser crystals with an extremely broad amplification band, which allows one potentially to generate pulses of duration of 2 – 3 periods of the optical field (one of the important representatives of this class is a Ti:sapphire crystal), and on the other hand, due to the use of the fast Kerr nonlinearity for laser self-mode-locking owing to the induced lens in the refractive index and (or) spectral broadening caused by self-phase modulation. The characteristic response time of the Kerr nonlinearity in transparent condensed media is typically shorter than 1 fs [2]. It is natural that after such impressive achievements the question arises of the possibility of generation of electromagnetic pulses of even shorter duration. It is obvious that the problem of going from femtosecond to attosecond pulses requires the advancement from the visible spectral region to shorter wavelengths and studies of physical mechanisms capable of producing the ultrafast nonlinear response of a medium needed for overcoming the attosecond barrier. Hopes of a successful solution of this problem are mainly related to advances in the physics of the interaction of strong optical fields with a matter during ultrashort time intervals.

One of the most important and extensively studied nonlinear phenomena taking place upon the interaction of strong laser fields with matter is the generation of high-order harmonics of optical radiation in gases. In fields with intensities above 10^{14} W cm^{-2}, a specific feature of the spectrum of emission transmitted through a gas is the presence of a broad plateau which extends up to frequencies exceeding the incident radiation frequency several hundreds times. From the point of view of nonlinear optics, the spectral transformation of optical radiation to such high frequencies indicates to the existence of ultrafast optical nonlinearity with a characteristic time response that is substantially shorter than the time $\hbar/I$ of movement of an electron in an atom (where I is the ionization potential and $\hbar$ is Planck's constant). Indeed, in such strong fields, an atom is rapidly ionized, and, as shown in Refs [3 – 5], only an ionized atom can be responsible for fast nonlinearity resulting in the generation of high-energy photons.

As is known [6], the rate of field ionization of an atom depends very sharply on the laser-field amplitude. For this reason, even in the case of a smooth envelope of the laser pulse, ionization of an atom occurs during several periods of electromagnetic oscillations at the leading edge of the laser pulse. It is during this comparatively short time interval that the nonlinear polarization of atoms becomes strongest, and specific features of the microscopic dynamics should be most distinctly manifested in the emission spectra of a medium. It was shown in Ref. [5] that bursts of high-order harmonics of several femtosecond duration can be generated even with the help of normal femtosecond pulses of 50 fs or longer duration.

The aim of this paper is to study the generation of attosecond pulses using rapidly ionized atoms in fields of ultrashort optical pulses. In Section 2, we present classical and quantum mechanical descriptions of the nonlinear response of an atom in a rapidly increasing field of an ultrashort pulse and show the principal possibility of generating attosecond pulses. Section 3 is devoted to the study of the space – time dynamics of the nonlinear optical process of generation of the ultrashort pulse in an extended gas. In Section 4, we demonstrate the possibility of controlling the frequency spectrum emitted by an atom and, hence, the parameters of attosecond pulses with the help of the polarization structure of the incident optical radiation. Finally, in Section 5 we study the effect of the magnetic component of laser radiation, which plays an extremely important role especially for ultrashort pulses when the strengths of fields acting on an atom can be substantially higher than the atomic field strength.

2. Nonlinear response of an atom

Because the nonlinear response of an atom in a strong optical field is predominantly governed by its ionization, the radiation in this case should only be bremsstrahlung, and it appears when an electron is released from the intra-atomic potential or in collisions of electrons accelerated by the laser field with atoms and ions. In the case of a rarefied gas, when the interaction of released electrons with neighboring atoms during the propagation of the ultrashort pulse can be neglected, the so-called recollisions with parent ions become very important [3, 4]. A classical analysis of the movement of free electrons in a linearly polarized optical field with the given amplitude shows that about half these particles return to the parent ions and interact (collide) with them at least once. If the average oscillatory energy $U_p = e^2E_0^2/4m\omega_0^2$ acquired by a freely moving electron is large compared to the ionization potential of an atom, the recollisions can be accompanied by emission of photons with frequencies which are considerably higher than characteristic atomic frequencies, resulting in the appearance of a specific high-frequency plateau in the spectrum of nonlinear polarization response of the atom. The nature of this plateau is explained using the known properties of bremsstrahlung of a charged particle scattered from an attractive Coulomb center. The maximum frequency of the emitted photon should obviously correspond to the maximum possible energy of electrons in recollisions.

This simple rule is expressed by the well-known formula for the plateau cut-off frequency $\hbar\omega_{max} \approx 3.2U_p + I$ and is confirmed by numerous experimental observations. In the case of a linearly polarized laser pulse, due to the repetition of ionization events with the period of the optical field, which occur with various atoms in each physically small volume, emission of an ensemble of atoms corresponding to the plateau is phased in time and represents a set of the higher odd harmonics of the laser-field frequency. However, it is clear that for pulses with a smooth envelope, because of the sharp dependence of the ionization rate on the electric-field amplitude, the effective energy of the oscillatory movement of electrons is limited by the field value at which rapid ionization of the atom occurs. Typical values of this threshold field are of the order of $0.1E_a$ ($E_a \approx 5.19 \times 10^9$ V cm^{-1} is the typical value of the atomic electric field). This restriction, as was shown in Refs [7, 8], can be overcome by using laser pulses of 10 fs duration. This allows one to increase substantially the

conversion efficiency of optical radiation frequencies, to advance further to the shorter wavelengths and to shorten the generated radiation pulses, which is important for the development of tunable sources of ultrashort pulses in the UV and soft X-ray frequency regions.

This idea was most successfully realized in a recent paper [9], where upon excitation of helium atoms by 26-fs, 800-nm pulses from a Ti:sapphire laser, the generation of ultrashort coherent X-ray pulses at 2.7 nm, in the 'water window', was achieved for the first time.

A number of advantages of using ultrashort laser pulses [8, 10] can be easily understood from the consideration of classical trajectories of electrons responsible for the generation of high-energy photons in recollisions. In the case of a field with a constant amplitude, only those electrons are engaged in the rescattering process whose drift velocity is directed to the parent ion reciprocal, which takes place for electrons leaving atoms during time intervals when the absolute value of the electric-field strength decreases from its maximum to zero. Electrons that have escaped atoms when the field phase $\varphi \approx 108^\circ$ ($E = E_0 \sin \omega_0 t$) have maximum energy at the instant of returning to the parent ion. If the field amplitude increases with time, the trajectories prove to be returning for the fraction of electrons with $\varphi < 90^\circ$ as well. As an example, Fig. 1 presents the dependence of the kinetic energy of an electron reencountering the parent ion on the initial phase φ and the rate of the field amplitude increase for the case when the leading edge of the laser pulse is characterized by an exponential rise with the increment β. The energy of electrons is expressed in units of U_p and the increment β, in units of the laser field frequency ω_0.

One can see from Fig. 1 that as the rise rate of the leading edge of the laser pulse increases, an increasing number of electrons become involved in the generation of bremsstrahlung in recollisions. In this case, their energy before collisions with parent ions can be substantially higher than for $\beta = 0$. For $\beta \geqslant 0.28$, the trajectories of all electrons that have escaped atoms become returning. If the ionization probability of an atom before a change in the field sign is close to unity, a localized electron wave packet is formed, which is accelerated under the action of the field as a whole and collides, after the change of the field sign, at a high velocity with a parent ion, resulting in a powerful burst of emission of high-energy photons of duration less than fractions of the period of the laser field.

It is clear, however, that the process of burst generation depends on the time profile of the pulse, and, therefore one can imagine the opposite situation when the main bunch of released electrons does not return, resulting in a sharp decrease in the bremsstrahlung intensity. This can occur, for example, when the field passes the range of critical field strengths for the atom at the increasing part of the pulse but close to its center, when the amplitude of oscillations ceases to rise. In this case, a bunch of free electrons is formed mainly at $\varphi < 90^\circ$, while the value of β is close to 0 (Fig. 1). Thus, for a

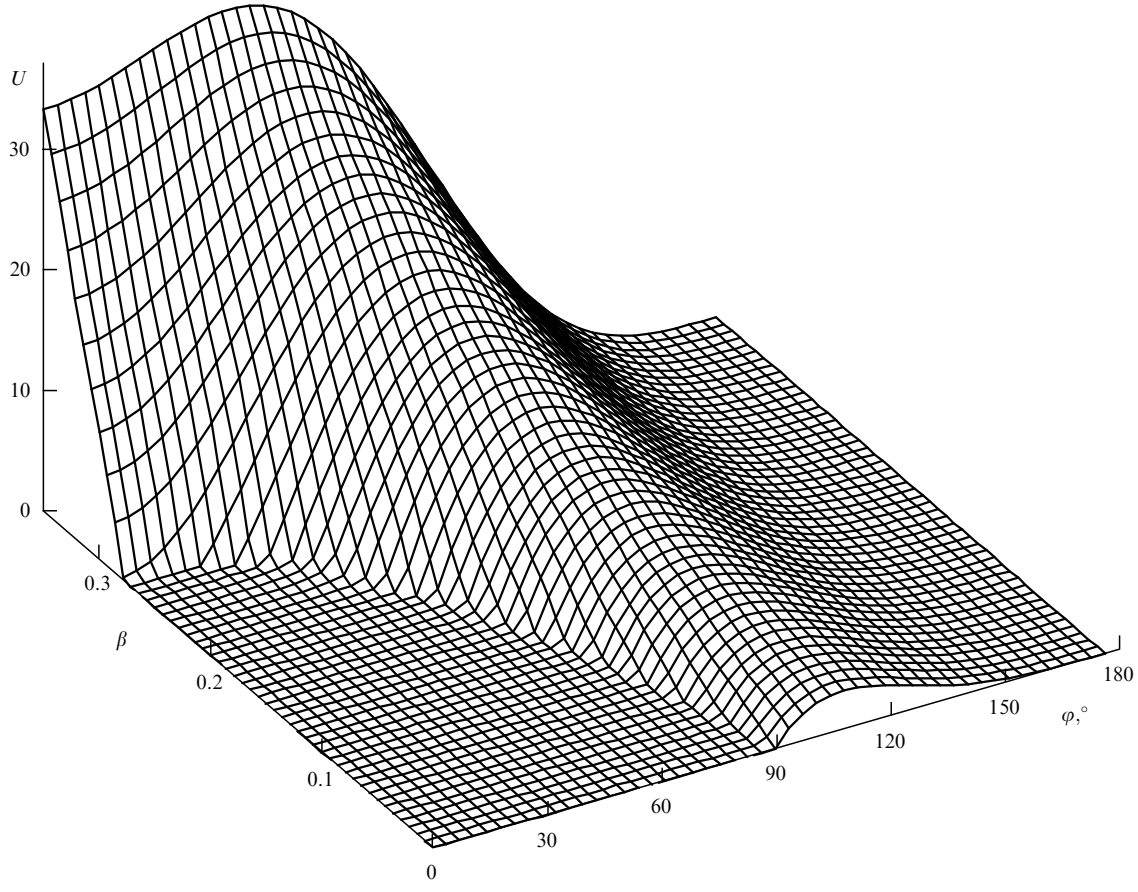

Figure 1. Dependence of the kinetic energy of an electron at the instant of returning to the parent ion on the initial phase φ of the field and the rise increment β of the leading edge of the exciting pulse.

certain class of profiles of few-optical-cycle laser pulses, the generation of attosecond pulses is forbidden.

It follows from the preceding consideration based on analysis of classical trajectories of electrons that the efficiency of the reciprocal-collision mechanism of generation of high-energy photons and the parameters of the generated emission depend substantially not only on the intensity and frequency of the incident radiation but also on the duration and shape of the exciting-field pulse. A comprehensive analysis of the effect of these and other important factors (polarization of the incident radiation, the effect of its magnetic component, etc.) requires detailed quantum mechanical calculations. The results of such calculations are given below. All calculations were performed in the one-electron approximation, the interaction of an electron with an ion core being described by a smoothed Coulomb potential

$$V(r) = -V_0\left(a^2 + r^2\right)^{-1/2}. \tag{1}$$

The results of the classical consideration are confirmed by the numerical solution of the Schrödinger equation presented in Fig. 2 in the form of the time dependence of the polarization response

$$R(t) = \int |\Psi|^2 \left(\frac{\partial V}{\partial x}\right) \mathrm{d}x \tag{2}$$

of a one-dimensional atom with the intra-atomic potential (1) ($V_0 = a = 1$) in the electric field of the optical pulse containing two oscillation periods (see Section 3). One can see from this figure that the atom is in fact completely ionized for a time less than first half-period of the field. Then, the electron wave packet formed in such a way, being free, acquires energy in the laser field and returns to the ion at the end of the next half-period ($t \sim 120$ in Fig. 2), giving rise to a burst of bremsstrahlung in collision with the ion. The duration of the burst of high-energy photons is of the order of 0.1 of the optical-field period, which corresponds to several hundred attoseconds for lasers in the near infrared region.

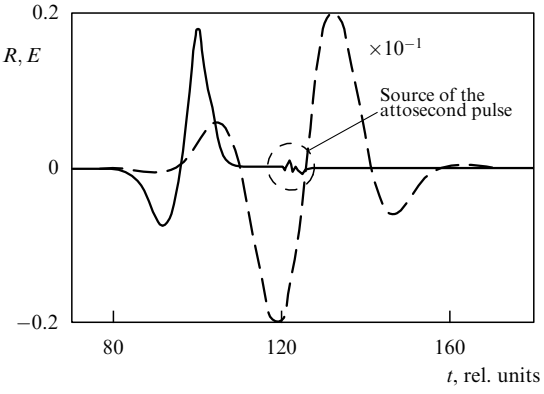

Figure 2. Time dependences of the electric field of the exciting pulse (the dashed line) and the atomic polarization response (the solid line).

3. Dynamics of generation of attosecond pulses

As the gas density increases, the emission spectra begin to exhibit new features related to collective processes in the matter, which can become dominant. In this case, the optimum length of the nonlinear interaction providing the maximum efficiency of generation of attosecond pulses in the given frequency interval is important, in particular, in the most interesting soft X-ray region. A detailed analysis of problems of interest to us was performed with the help of numerical calculations within the framework of the self-consistent model we developed earlier for studies of the high-order harmonics generation in fields of powerful femtosecond pulses [5]. Note that the main feature of the model in the case of a few-optical-cycle pulse is the abandonment of the slowly varying complex amplitude approximation and the use of the real electric field and real polarization of the medium. This is caused not only by the large spectral width of the pulse itself, but also by the impossibility of describing the nonlinear response of the medium only as a function of the amplitude of oscillations of the optical field.

Consider a one-dimensional pulse with the electric field linearly polarized along the x-axis, which propagates along the z-axis in a medium consisting of identical quantum particles. Our model uses Maxwell's equations for the real electric field $E(z,t)$ of the wave, which are written in the simplest form of the reduced (by neglecting the reflected wave) one-dimensional scalar wave equation and the nonstationary Schrödinger equation for the wave function of an electron in the potential $V(x)$ and in the laser-wave field, the interaction with the latter being described in the dipole approximation:

$$\frac{\partial^2 E}{\partial z\,\partial t} = -\frac{1}{2}(E + R), \tag{3}$$

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = -\frac{1}{2}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi + xE(z,t)\Psi. \tag{4}$$

Because the dynamics of a quantum particle in this problem are mainly determined by the action of the exciting linearly polarized electric field, we will restrict ourselves to the one-dimensional Schrödinger equation, neglecting the transverse quantum diffusion, which does not play an important role in this case. Thus, the effective dimensionality of the model can be defined as $1 + 1$. For convenient presentation of the results, we divided the polarization response of the medium [the right-hand side of Eqn (3)] into two terms, by separating the component $R(t)$ [see Eqn (2)], which is responsible for the short-wavelength radiation bursts we are interested in. In equations (3) and (4), $t \to t - z/c$ is the time measured from the leading edge of the pulse; the coordinate z along the propagation direction of the laser radiation is measured in units $me^4c/\omega_p\hbar^3$, where $\omega_p^2 = 4\pi e^2 N/m$ is the characteristic plasma frequency corresponding to the gas density. The time, the intra-atomic coordinate, the potential, and the electric field of the pulse are normalised to the corresponding atomic values: $t_a = \hbar^3/me^4$, $x_a = \hbar^2/me^2$, $V_a = me^4/\hbar^2$ and $E_a = m^2e^5/\hbar^4$.

Despite the brevity of its formulation, this system of equations is universal and can be used for the description of various nonlinear optical phenomena. By choosing the potential $V(x)$ in the proper form, the nonlinear response of almost any optical system, from a two-level to a spatially periodical system with band structure of the levels, can be simulated. In particular this model proves to be convenient for studies of excitation and ionization of atoms and ions in the laser field.

Let us take the atomic potential in the form (1) with $V_0 = a = 1$ (in this case, the bond energy is approximately

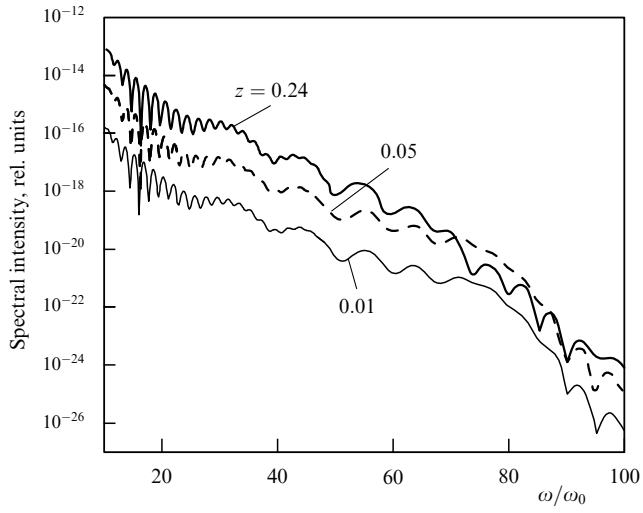

Figure 3. Spectrum of the field for different propagation length.

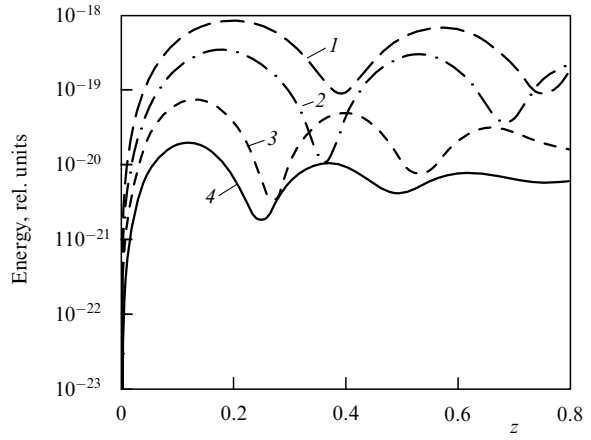

Figure 4. Dependence of the energy of high-frequency components of the field on the propagation length in the spectral ranges: (*1*) (55 – 60) ω_0, (*2*) (60 – 65) ω_0, (*3*) (65 – 70) ω_0 and (*4*) (75 – 80) ω_0.

equal to 0.67 a.u.) and consider the interaction of the ultrashort laser pulse, containing a small number of optical oscillations, with a gas of quantum particles.

Figures 2 – 4 present the results for the case of an incident laser pulse with a carrier frequency of 0.2 a.u. and a Gaussian envelope with a field amplitude of 2.1 a.u. (Fig. 2, dashed line). Figure 2 shows the time dependence of the polarization response of an atom located at the gas boundary ($z = 0$), when the field acting on the atom corresponds to its vacuum distribution. The spectrum of the fields excited at the medium boundary and along the propagation route is shown in Fig. 3. One can see that different components of the spectrum undergo different variations during the propagation of the driving pulse in a gas, these differences being most substantial in the high-frequency region. This is most distinctly seen in Fig. 4, where the energies contained in certain spectral intervals are presented as functions of the propagation length. One can see that these dependences are oscillatory and gradually tend to a certain quasi-stationary level, specific for each frequency interval.

As was noted above, the source of bursts of the polarization response of an atom in the field of the ultrashort ionizing pulse is localized within a narrow time interval of duration much shorter than the optical period (Fig. 2), which can be used for the generation of attosecond pulses. We studied the space – time dynamics of the excited bursts by the wavelet techniques [11], using the Morlet wavelet

$$W(t, t_0) = \exp(\mathrm{i}\, t) \exp\left[-\frac{(t - t_0)^2}{\sigma^2} \right]$$

as a basis (here, σ is a fixed parameter which should be greater than 2π). Figure 5 shows the nonlinear dynamics of generation and propagation of an ultrashort 13.0 – 14.2-nm X-ray pulse shorter than 150 attoseconds excited by a laser at a wavelength of 800 nm.

4. Polarization control of the emission spectrum

An important feature of the atomic nonlinearity in superstrong fields is its strong dependence on the polarization of the optical field. It is well known, for example, that the high-order harmonics are absent in the nonlinear response of an atom in a circularly polarized field. If the generation of high-order harmonics is mainly determined by bremsstrahlung in recollisions, the efficiency of this process should drastically decrease even in the presence of a weak circularly polarized component against the background of a strong linearly

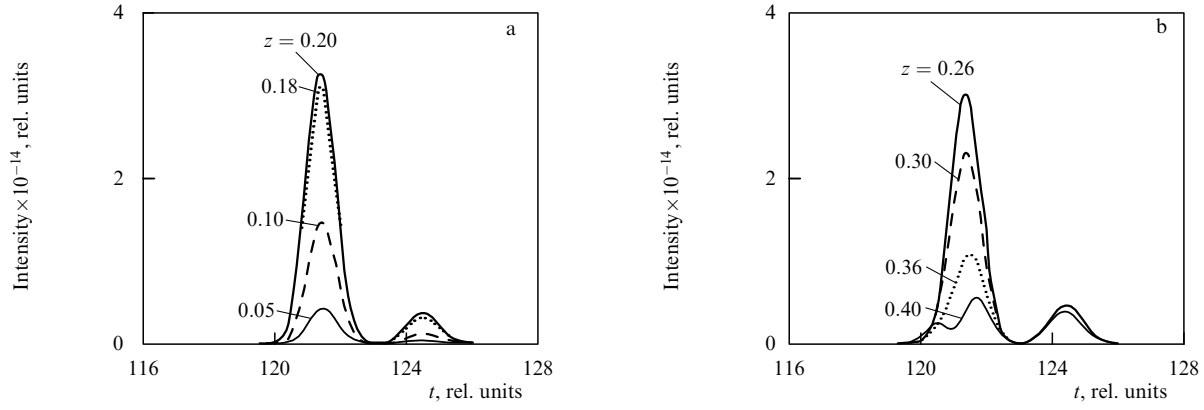

Figure 5. Time profile of the attosecond pulse at different points along the propagation route. The pulse contains spectral components in the interval from $55\omega_0$ to $60\omega_0$.

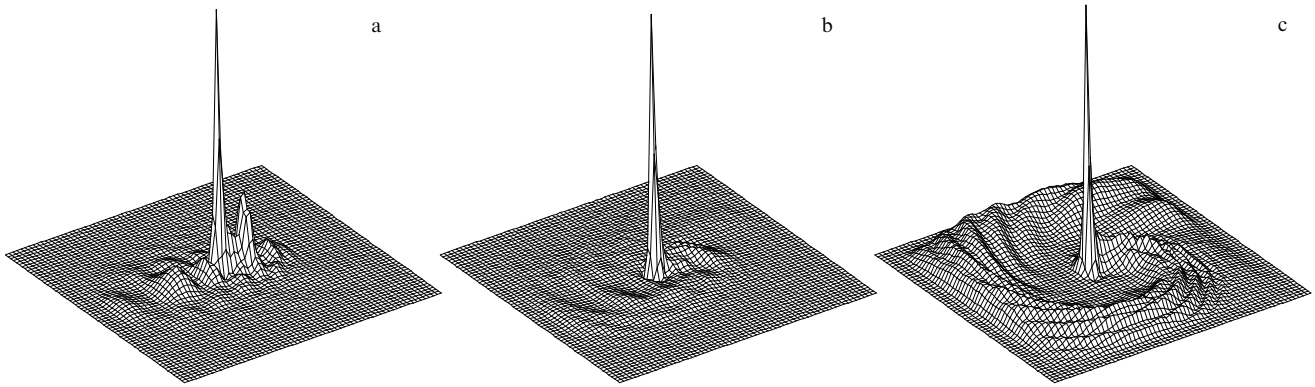

Figure 6. Spatial distribution of the electron probability for (a) linear, (b) elliptical, and (c) circular polarizations of the laser field (here, $\omega_0 = 0.4$ and $E_y = 0.7$).

polarized field. This is explained by the deviation of propagation of electrons outside the atom from a linear trajectory caused by the action of the circular component and the disappearance of recollisions. The high sensitivity of the reciprocal-collision mechanism of the high-order harmonics generation to polarization of the incident optical radiation allows one to control efficiently the parameters of the generated emission. Thus, the use of exciting radiation with time-modulated elliptical polarization permits, in principle, the generation of high-order harmonic radiation pulses of subfemtosecond duration [12]. Below, we analyse the dependence of the nonlinear response of an atom to polarization of the incident radiation within the framework of two-dimensional quantum mechanical model [13, 14].

We assume that the electric field of the laser field is elliptically polarized in the (x, y) plane and the time dependence of both components is characterized by the envelope $f(t)$:

$$\mathbf{E}(t) = f(t)\left(\mathbf{e}_x E_x \cos\omega_0 t + \mathbf{e}_y E_y \sin\omega_0 t\right). \tag{5}$$

If the interaction of an atom with the field is described in the dipole approximation, the movement of the atomic electron under the action of field (5) occurs in the plane coincident with its plane of polarization. This allows us to describe quantum mechanically the dynamics of the atom within the framework of the two-dimensional model. In this case, the Schrödinger equation has the form (in a.u.)

$$\mathrm{i}\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\Psi + V(x,y)\Psi + f(t)\left(xE_x\cos\omega_0 t + yE_y\sin\omega_0 t\right)\Psi. \tag{6}$$

We solved Eqn (6) numerically using the spectral splitting method and the fast Fourier transform. The interaction of an electron with the ion core was described with the help of a two-dimensional smoothed Coulomb potential (1).

Figure 6 shows the spatial distribution of the electron probability $|\Psi(x,y)|^2$ at the instant $t = 2T$, when

$$f(t) = \begin{cases} \sin^2\dfrac{\omega_0 t}{4}, & t \leqslant T \\ 1, & t > T \end{cases} \qquad \left(T = \frac{2\pi}{\omega_0}\right),$$

for linear ($E_x = 0$, $E_y = 0.7$), elliptical ($E_x = 0.3$, $E_y = 0.7$), and circular ($E_x = E_y = 0.7$) polarizations of the laser field. The calculations were performed for $\omega_0 = 0.4$ and $a = 0.1$ (in this case, the ionization potential is $I \approx 1.3$). One can see from Fig. 6a that in the case of linear polarization of the laser field, a substantial fraction of the wave packet released from the atomic potential during the first half-period of the optical field due to tunnel ionization, after the change in the field sign, returns to the parent ion and is scattered from it. As a result, the spectrum of the atomic nonlinear response exhibits a high-frequency tail slowly decreasing up to frequencies exceeding the frequency of the incident radiation (Fig. 7) by several orders of magnitude. In the case of elliptical polarization (Fig. 6b), the perpendicular component of the electric field deviates the trajectory of the central part of the wave packet from a straight line. Under such conditions, the interaction of the electron wave packet with the ion decreases and exists only due to its transverse quantum spreading. The further increase in ellipticity of the incident radiation further decreases role of recollisions in the formation of the spectrum of the nonlinear response of the atom (Fig. 7). In the case of circular polarization (Fig. 6c), the spatial distribution of the probability of released electrons has the form of a spiral, which unwinds in time from the atomic center, i.e., recollisions are completely absent. The high-frequency part of the

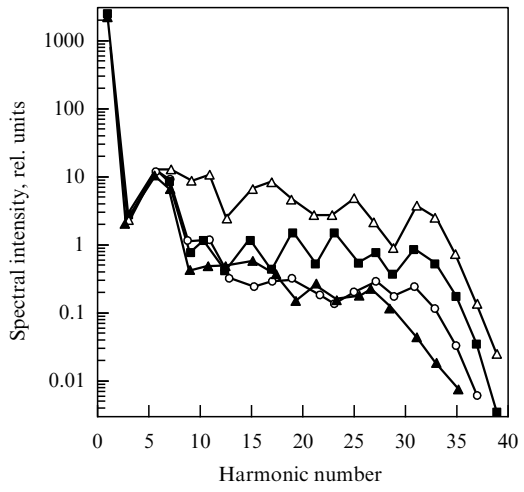

Figure 7. Intensities of harmonics in the spectrum of nonlinear atomic response for different ellipticities of the incident radiation (here, $\omega_0 = 0.2$ and $E_y = 0.5$): △ — $E_x = 0$; ■ — $E_x = 0.10$; ○ — $E_x = 0.15$; ▲ — $E_x = 0.20$.

response spectrum disappears, respectively, and only the features which are related to bremsstrahlung of electrons leaving the atomic potential remain. Thus, the use of light of different polarizations allows one to increase or decrease individual features of the nonlinear response of an atom and, hence, to control the parameters of the generated emission and selectively probe the microprocesses in an atom in the presence of a superstrong laser field.

5. Effect of the magnetic component of laser field

It follows from the preceding that the use of shorter powerful laser pulses provides, in principle, better conditions for excitation of subfemtosecond X-ray emission bursts. However, it should be taken into account that in light fields with intensities $I \geqslant 3 \times 10^{16}$ W cm^{-2}, electrons are accelerated up to velocities $v \geqslant 0.1c$ (where c is the speed of light). Under such conditions, their motion is appreciably affected by the Lorentz force caused by the magnetic field of the laser wave, which deviates their trajectories from linear. This raises the question of how this circumstance affects the efficiency of the reciprocal-collision mechanism of generation of high-energy photons described above. We consider this question with the help of a direct numerical solution of the Schrödinger equation [15].

The degree of influence of the magnetic component of the laser radiation field on the formation of the high-frequency component of the polarization response of the *electron + ion* system in recollisions substantially depends on the width of the electron wave packet propagating near the parent ion. This influence can be considerable, if the average drift velocity ($\overline{v_z}$) of the electron wave packet in the propagation direction of the laser wave is greater or of the order of its spreading rate (v_{sp}) in this direction. It follows from the solution of the classical equation of motion for an electron in the field of the plane linearly polarized monochromatic electromagnetic wave that for $v_z \ll c$, the drift velocity of an electron along the z-axis averaged over the field period is equal to

$$\overline{v_z} = \frac{c}{4}\left(\frac{v_{osc}}{c}\right)^2 (1 + 2\cos^2\varphi)\,, \tag{7}$$

where $v_{osc} = E_0/\omega_0$ is the velocity of the oscillatory movement of the electron along the electric field of the wave (in a.u.). The spreading rate of the electron wave packet is approximately equal to $1/\Delta$ in a.u. (Δ is the initial width of the wave packet), which is equal to $v_{sp} \sim 10^8$ cm s^{-1} in dimensional units. This gives an estimate of the critical electric field strength $E_0^{cr} \sim 10\omega_0$. Note that the effect of the magnetic component of the incident radiation increases as its frequency decreases and in the visible spectral range it becomes substantial for the field strengths of the order atomic.

Because the classical trajectories of electron movement in the field of a linearly polarized electromagnetic wave lie in the plane perpendicular to the magnetic field, the quantum mechanical calculations may be restricted to the two-dimensional model. Such an approach allows one, on the one hand, to take into account the effect of the magnetic field of the wave and the spreading of the electron wave packet, and on the other hand, it considerably reduces the time of computer calculations as compared to more realistic three-dimensional calculations. Let us choose the direction of the x-axis of the Cartesian coordinate system along the electric field of the incident linearly polarized wave and that of the z-axis, along the propagation direction of the wave.

To take into account the magnetic field of the wave, its is necessary to abandon the commonly used dipole approximation and to write the Schrödinger equation in the more general form:

$$\mathrm{i}\frac{\partial \Psi}{\partial t} = V(r)\Psi + \frac{1}{2}\left(\mathbf{p} + \frac{\mathbf{A}}{c}\right)^2 \Psi\,. \tag{8}$$

Because the spreading rate of the electron wave packet is much lower than the speed of light, its width during one period of the laser field remains much smaller than the wavelength of the incident radiation. This allows us to retain in the expansion of the vector potential over the coordinate z only the linear term

$$\mathbf{A}(z,t) = \mathbf{e}_x A(z,t) \approx \mathbf{e}_x\left[A_0(t) + B(t)z\right], \tag{9}$$

where $A_0(t) = A(0,t)$, $B(t) = \partial A(z,t)/\partial z|_{z=0}$. Since $\mathbf{A} = \mathbf{A}(t - z/c)$, then $B(t) = -(1/c)(\mathrm{d}A_0/\mathrm{d}t)$.

By neglecting in $\partial\Psi/\partial x$ the small term proportional to z and performing the unitary transformation

$$\widetilde{\Psi} = \exp\left[-\mathrm{i}v_z z + \frac{\mathrm{i}}{2}\int_{-\infty}^{t}(v_x^2 + v_z^2)\,\mathrm{d}t\right]\Psi\,, \tag{10}$$

where

$$v_x = \frac{A_0}{c}, \quad v_z = -\frac{1}{c^2}\int_{-\infty}^{t} A_0 B\,\mathrm{d}t = \frac{v_x^2}{2c}\,,$$

we obtain the Schrödinger equation in the form

$$\mathrm{i}\frac{\partial \widetilde{\Psi}}{\partial t} = V\widetilde{\Psi} - \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\widetilde{\Psi} - \mathrm{i}v_x\frac{\partial \widetilde{\Psi}}{\partial x} - \mathrm{i}v_z\frac{\partial \widetilde{\Psi}}{\partial z}\,. \tag{11}$$

Equation (11) was solved numerically by the method of spectral splitting using the fast Fourier transform. Potential (1) was used as the intra-atomic potential $V(x,z)$.

The results presented below were obtained for values of the potential parameters $a = 0.3$ and $V_0 = 0.712$, the energy of the ground state being equal to the ionization potential of the hydrogen atom ($\varepsilon_0 = -0.5$). The electric field at the leading edge of the laser pulse was written in the form

$$E(\tau) = E_0 f(\tau)\sin\omega_0\tau\,, \tag{12}$$

where $\tau = t - z/c$, $f(\tau) = \exp(2\omega_0\tau/\pi) - 1$. Below, the results of numerical calculations for $E_0 = 0.36$ and $\omega_0 = 0.114$ are presented (which correspond to the second harmonic of the Ti:sapphire laser). For the values of parameters chosen, ionization of an atom occurs with a probability close to unity during the first half of the optical cycle, which allows us to restrict our calculations to the time interval $0 \leqslant t \leqslant T$.

Figure 8 presents fragments of the evolution of the spatial distribution of the electron probability $|\Psi(x,z)|^2$ under the action of the laser field. The initial conditions correspond to the ground state in potential (1). The instant pictures correspond to different stages of the movement of the electron wave packet: (a) $t = 0.45T$, the movement after release from the atomic potential by overcoming the

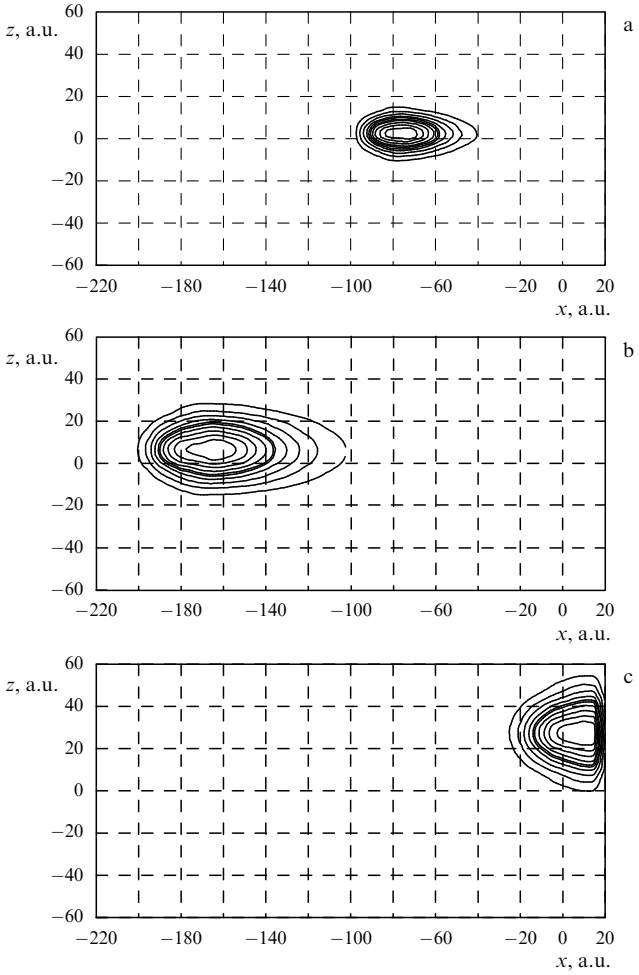

Figure 8. Dynamics of the electron wave packet in the laser field.

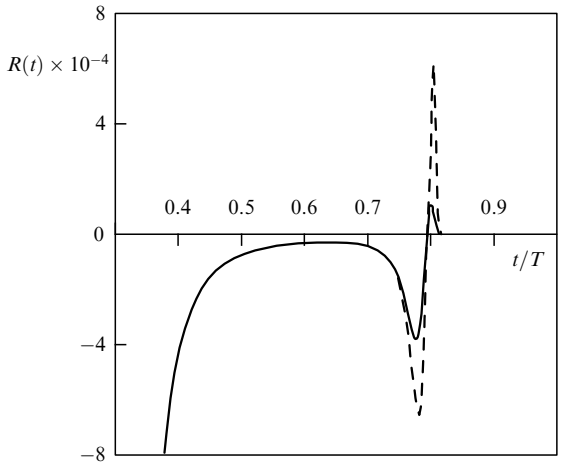

Figure 9. Time dependence of the atomic polarization response, both with account of the magnetic component of the light pulse (the solid line) and without (the dashed line).

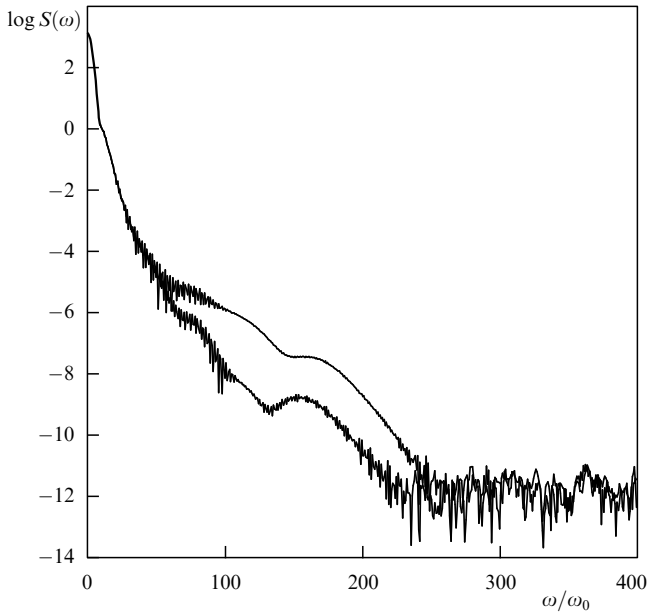

Figure 10. Spectral dependence of the atomic polarization response, both with account of the magnetic component of the light pulse (the bottom curve) and without (the upper curve).

Coulomb barrier; (b) $t = 0.675T$, the movement near the turning point; (c) $t = 0.8T$, the return to the parent ion. The isolines are constructed with a step equal to 0.1 of the maximum value. One can see that under the action of the magnetic field the center of the wave packet shifts, before a collision, from the coordinate origin in the direction of propagation of the laser wave by a distance greater than 20 Bohr radii, which is approximately twice as large as its width in this direction.

Figure 9 shows a fragment of the time dependence of the polarization response $R(t)$ corresponding to the stage of backward movement of the electron wave packet. For comparison, the dashed curve shows the time dependence obtained in the absence of a magnetic field. One can see that the high-frequency burst responsible for the generation of high-energy photons has a duration of $0.1T$, which is of the order of 100 as. The drift of the electron wave packet in the magnetic field of the laser wave results in substantial suppression of this burst, which is reflected in the spectrum of generated emission.

Figure 10 shows spectra of the polarization response $R(t)$ calculated both with account of the magnetic field (the lower curve) and without (the upper curve). The low-frequency part of the spectrum is mainly formed by bremsstrahlung of electrons escaping atoms and is almost independent of the magnetic field of the wave. The high-frequency region of the spectrum is mainly formed by bremsstrahlung of electrons upon recollisions, and a decrease in the efficiency of this mechanism due to the deviation of electrons from linear trajectories in the magnetic field results in a substantial (several tens times) decrease in the intensity of high-order harmonics.

The process studied by us distinctly exhibits both classical and quantum features of the dynamics of the electron wave packet. On the one hand, the movement of the packet center obeys the laws of classical mechanics with a good accuracy. Its trajectory proves to be very close to the classical trajectory of an electron in the field (12) with an initial phase $\varphi \approx 60^\circ$. This, in particular, allows us to analyse the applicability of the nonrelativistic approximation by solving the classical equations of motion. Comparison of the above classical trajectory of an electron with the trajectory obtained from the solution of the relativistic equation shows that they begin to differ noticeably for $t > 0.8T$, whereas a collision occurs at

$t \approx 0.79T$. This suggests that the movement of the wave packet under the conditions considered is weakly relativistic, which justifies the use of the Schrödinger equation. On the other hand, quantum diffusion plays an important role in the process of generation of high-energy photons in recollisions. The role of diffusion proves to be twofold: although it reduces the electron density in the interaction region, at the same time it partially compensates for the action of the factors (the magnetic field of the wave, the ellipticity of polarization) that cause deviations of electron trajectories. As a result, although before a collision the trajectory of the electron wave packet can deviate from linear by a distance greatly exceeding the atomic size, a complete break of the generation of high-frequency photons does not occur.

6. Conclusions

In this paper, we have suggested and theoretically studied the method of generation of attosecond pulses based on the transformation of powerful femtosecond laser pulses on rapidly ionized atoms. The problems of generation of attosecond radiation are directly included in the general problem of nonlinear optical interactions of the pulses containing a small number of field periods. The theoretical and experimental studies of this problem are still in the initial stages. The model of the interaction of the optical field with a gas of quantum particles suggested and studied in this paper is sufficiently versatile to be applied to studies of various nonlinear optical phenomena in variety of media.

Acknowledgements. This work was supported by the State Scientific and Technical Program 'Physics of Quantum and Wave Processes' and the Russian Foundation for Basic Research (grants 97-02-17525 and 98-02-17015).

References

1. Nisoli M et al. *Opt. Lett.* **22** 522 (1997)
2. Akhmanov S A, Vysloukh V A, Chirkin A S *Optika Femtosekundnykh Lazernykh Impul'sov* (Optics of Femtosecond Laser Pulses) (Moscow: Nauka, 1988) [Translated into English (New York; AIP, 1992)]
3. Kulander K C, Shafer K J, Krause J L, in *Super-Intense Laser-Atom Physics* (NATO AST Series, Series B: Physics, Vol. 316, Eds B Piraux, A L' Huillier, K Rzazewski) (New York: Plenum Press, 1993) p. 95
4. Corkum P B *Phys. Rev. Lett.* **71** 1994 (1993)
5. Vanin E V et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **58** 964 (1993) [*JETP Lett.* **58** 900 (1993)]
6. Delone N B, Krainov V P *Atom v Sil'nom Svetovom Pole* (Atoms in Strong Light Fields) (Moscow: Energoatomizdat, 1984)
7. Zhou J et al. *Phys. Rev. Lett.* **76** 752 (1996) [Translated into English (Springer Series in Chemical Physics, Vol. 28) (Berlin: Springer – Verlag, 1985)]
8. Sergeev A M, Kim A V, Vanin E V, in *Generation, Amplification, and Measurement of Ultrashort Laser Pulses III* (Proc. SPIE , Vol. 2701, Eds W E White, D H Reitze) (Bellingham, WA: SPIE, 1996) p. 235
9. Chang Z et al. *Phys. Rev. Lett.* **79** 2967 (1997)
10. Kim A V et al., in *Applications of High Field and Short Wavelength Sources VII* (OSA Technical Digest Series, Vol. 7) (Washington, DC: Optical Society of America, 1997) p. 223
11. Astaf'eva N M *Usp. Fiz. Nauk* **166** 1145 (1996) [*Phys. Usp.* **39** 1085 (1996)]
12. Corkum P B, Burnett N H, Ivanov M Y *Opt. Lett.* **19** 1870 (1994)
13. Babin A A et al. *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **39** 713 (1996)
14. Ryabikin M Yu et al. in *Abst. of the VII Int. Conf. on Multiphoton Processes ICOMP VII* (Garmisch – Partenkirchen, 30 September – 4 October, 1996)
15. Kim A V et al., in *Post-deadline report WP10 of XVI Int. Conf. on Coherent and Nonlinear Optics ICONO'98* (Moscow, 29 June – 3 July, 1998) (unpublished)

PACS number: **39.90.+d, 42.50.–p**

Stabilization of atoms in a strong laser field

M V Fedorov

1. Introduction

The phenomenon of stabilization of atoms in a strong laser field was first predicted and described theoretically [1, 2] and only recently confirmed experimentally [3, 4]. The idea of stabilization of atoms does not correspond to the simple intuitive notion of an increase in the ionization rate of atoms with increasing intensity of the laser field. By definition, stabilization means that, depending on the intensity I or the strength ε of the field, beginning from some critical values of I_c or ε_c, the ionization rate Γ or the total ionization probability w_i during the pulse ceases to increase (i.e., is saturated at the level $w_i \ll 1$) or even becomes a decreasing function of I or ε. In this case, the ionization time t_i of an atom or the residual probability of its 'non-ionization' $w_{res} = 1 - w_i$ also either becomes constant or starts to increase with I or ε (Fig. 1). The choice between these types of behavior of the functions $w_{res}(\varepsilon)$, $\Gamma(\varepsilon)$, and $w_i(\varepsilon)$ in a strong field and also the determination of the critical fields I_c and ε_c is the object of contemporary studies of the physics of stabilization of atoms.

There are several models of stabilization of atoms in a strong field, which differ in both the physical nature and conditions of their realization. Below, we will briefly describe two of these models: the high-frequency stabilization (or the Kramers – Henneberger stabilization [1]) and the interference stabilization of atoms [2]. However, before proceeding to an analysis of them, we briefly consider the experimental data available.

2. Experiments

Actually, several attempts have been made to observe the stabilization of atoms induced by a strong field. However, it seems that only two of them [3, 4] can be considered fully successful. In both cases, the electron yield (proportional to the ionization probability w_i per pulse) was measured as a function of the pulse duration for a fixed fluence, i.e., for the product of the peak intensity I_{max} of the field with the pulse duration τ equal to $F = I_{max}\tau = \text{const}$; or as a function of F for different but fixed laser pulse durations τ. The objects of the studies were excited barium [3] and neon [4] atoms. Despite substantial differences in the setting up and interpretation of experiments [3, 4], their results are quite similar as a whole; therefore, we will describe only experiment [3].

The Ba atoms were prepared in the 27D state using two-stage excitation and were then ionized by a short powerful laser pulse. Figure 2a shows the electron yield measured as a function of the pulse duration $N_e(\tau)$ for $F = \text{const} = 7.8\ \text{J cm}^{-2}$. A plateau, where $N_e(\tau) \approx \text{const}$, corresponds to the electron yield calculated in the first order of the perturbation theory using Fermi's 'golden' rule (FGR), according to which $N_e(\tau) \propto I_{max}\tau \equiv F = \text{const}$. The region