tions [12]:

$$P_{\beta}(w) = A\left(\frac{w}{D}\right)^{\beta} \exp\left[-\alpha\left(\frac{w}{D}\right)^{1+\beta}\right], \qquad (3)$$

with

$$\alpha = \left[\frac{1}{D}\,\Gamma\left(\frac{2+\beta}{1+\beta}\right)\right]^{1+\beta},$$

where $\Gamma(x)$ is the Gamma function, and β is the Brody parameter. When the Brody parameter $\beta = 0$ ($\beta = 1$), the Brody distribution reduces to the Poisson (Wigner) distribution. This distribution is only heuristic and does not have a theoretical basis as a measure of underlying chaos in the system. However, in the absence of a distribution which does have a theoretical basis, it is useful since it depends only on one parameter. We obtain the best fit of our data at 2.5 T with $\beta = 0.24$ and $D = 15$ meV; at fields of ≈ 0.5 T their values are 0.05 and 10 meV, respectively. Both β and D increase with the field up to ≈ 5 T and then saturate.

In summary, we have found statistical correlations in the magnetoexcitonic spectra of GaAs QW's which can be regarded as a hallmark of quantum chaos. The separation of the energy levels obeys a Brody distribution, which interpolates between a Wigner and a Poisson distribution. The departure from a pure Wigner distribution is due to the existence of excitonic levels which belong to different irreducible representations. Those can be energetically degenerate, thus the probability of zero energy spacing grows, introducing a Poisson contribution to the distribution. Further studies, using tilted magnetic fields with respect to the growth axis of the quantum well, are being performed to investigate the effect of reducing the symmetry of the system in the energy level distribution. Additionally, the application of an external electric field can also be used to lower the symmetry.

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Image of local density of states in differential conductance fluctuations in the resonance tunneling between disordered metals

V I Fal'ko

The discussion of the local density of states (LDOS) fluctuations in disordered metals was started in the theory more than a decade ago [1, 2], and a lot is already known [3] about their statistics in the metallic regime (i.e., at $p_{\mathrm{F}} l \gg 1$). Random point to point, or parametric dependences of this quantity discussed in the literature are the result of the interference of multiply scattered electron waves in a disordered metal. Recently, it was demonstrated that the amplitude of LDOS fluctuations and their statistics are mainly governed by statistical properties of the wave functions of a diffusive electron in a disordered metal [4 – 6], and particularly by correlations between the individual wave functions.

The direct manifestation of the density of states fluctuations in the transport experiments were discussed, at first, in the context of non-resonant tunneling between two disordered metals [7]. Later, this idea was extended [8] to the studies of resonant tunneling processes involving a resonance level in the barrier, and the contribution of LDOS fluctuations to the conductance of such a device was discussed in the linear response regime. In recent vertical transport experiments on small-area double-barrier semiconductor structures [9 – 13], resonant tunneling between two heavily doped semiconductors (which can be regarded as disordered metals) through a single impurity level created below the lowest quantum well sub-band by a fluctuation in the density of charged donors was observed and identified, so that more attention should now be paid to a quantitative analysis to provide a basis for a quantitative comparison with the existing experimental data. In the present paper, we report the results of such an analysis.

Under the experimental conditions of Refs [9 – 13], the linear response regime was hardly relevant, since, at a zero bias, the energy of a discrete impurity level, E_0 does not initially coincide with the chemical potential μ_{L} in the bulk electrodes coming to the resonance only after the bias voltage reaches the threshold value $V_0(E_0)$. Being essentially nonlinear, the current – voltage $I(V)$ characteristics of such a device can be divided into three typical intervals [10 – 14]: below the threshold, where $I \approx 0$; the threshold regime $V = V_0(E_0) \pm \Gamma/ae$, where $I(V)$ takes a step after the resonant level crosses the Fermi level μ_{L} in the emitter; and the interval of a plateau, $V_0(E_0) < V < V_1(E_1)$, where the current remains almost constant until the next impurity level E_1 is lowered enough to contribute to the transport. In most of the samples studied in Refs [10 – 13], the emitter barrier is much stronger than the collector barrier, so that in the theoretical analysis one can neglect the influence of the Coulomb blockade effect of the resonant impurity level (which plays a crucial role if the barrier configuration is perfectly symmetric [15]). If so, the width of the resonance in the conductance Γ is dominated by the electron escape from impurity to collector, $\Gamma = \Gamma_{\mathrm{R}} + \Gamma_{\mathrm{L}} \approx \Gamma_{\mathrm{R}}$, whereas the value of the current step is mainly determined by the tunneling rate Γ_{L} through the thick barrier on the emitter side. In this approximation, the current can be represented in the form [16, 8]

$$I(V) = \frac{e}{h}\int_{-\infty}^{\infty} \frac{\Gamma_{\rm R}(\epsilon)\Gamma_{\rm L}(\epsilon)\left[f_{\rm L}(\epsilon) - f_{\rm R}(\epsilon)\right]\,{\rm d}\epsilon}{\left[\epsilon - E_0(V)\right]^2 + (1/4)\Gamma^2(\epsilon)}\,,$$

where $f_{\rm L(R)}(\epsilon) = \left\{1 + \exp[(\epsilon - \mu_{\rm L(R)})/T]\right\}^{-1}$. Being averaged over disorder, the $I(V)$-characteristics at the threshold can be described by the height of the resonance conductance peak at the voltage V_0 providing $\mu_{\rm L} = E_0(V)$,

$$\left\langle \frac{{\rm d}I}{{\rm d}V}\right\rangle_{\max} \approx G_\Gamma = \frac{4ae^2}{h}\,\frac{\Gamma_{\rm L}}{\Gamma}\,, \tag{1}$$

and its width at the half-maximum, $V_\Gamma \approx \Gamma/ea$. The factor $a < 1$ in Eqn (1) produces an actual distribution of the potential drop across the structure. In the plateau regime (at $T < \mu_{\rm L} - E_0$) the average current $\langle I\rangle$ saturates at

$$\big\langle I(\mu_{\rm L} - E_0 > \Gamma)\big\rangle \to 2\pi e\,\frac{\Gamma_{\rm L}}{h} = \frac{\pi}{2}\,G_\Gamma V_\Gamma\,, \tag{2}$$

so that the disorder average of $\langle {\rm d}I/{\rm d}V\rangle$ tends to take a zero value.

The latter statement is made to stress that in the regime of bias voltages of $E_1 > \mu_{\rm L} > E_0 > \mu_{\rm R}$, the differential conductance of the device is dominated by an irregular sample-specific energy dependence of the tunneling coupling between an impurity and the continuum of the electron states, e.g., in a disordered emitter 'deep below' the Fermi level $\mu_{\rm L}$,

$$\Gamma_{\rm L}(\epsilon) = 2\int \frac{{\rm d}\mathbf{p}\,{\rm d}\mathbf{p}'}{(2\pi)^{2d}}\,{\rm Im}\left[G_\epsilon^{\rm A}(\mathbf{p},\mathbf{p}')\right] t(\mathbf{p})t^*(\mathbf{p}')\,, \tag{3}$$

where $t(\mathbf{p})$ is a formal tunneling matrix element between the impurity and bulk electrode, and ${\rm Im}\left[G^{\rm A}\right] = \left[G^{\rm A} - G^{\rm R}\right]/2$ are the exact retarded (advanced) Green functions of the electron in the bulk taken for a fixed configuration of disorder. Here, we focus only on fluctuations in the emitter, not only for quantitative reasons resulting from the assumed asymmetry of a structure, $\Gamma_{\rm L} \ll \Gamma_{\rm R}$, but also because at realistic voltages $V > V_0$ [10, 12, 13] the electron–electron collisions and emission of plasmons and optical phonons in the collector are fast enough to wash out the quantum interference effects coming from it.

When speaking about the differential conductance $G(V)$ in the plateau regime, its irregular oscillations around the zero average get a dominant contribution from the finest energetic scale resolved using a fixed spectrometer, which is Γ. As a result, the correlation parameters of the differential conductance fluctuations are strictly bound to the energetic width of the resonant impurity level: the correlation voltage can be estimated as Γ/e, and the correlation magnetic field is that which provides a magnetic flux quantum per area L_Γ^2, where $L_\Gamma = \sqrt{Dh/\Gamma}$ is the diffusion length in a disordered metal corresponding to the life-time h/Γ of an electron in the resonant impurity state. Qualitatively, the whole region of the size L_Γ around the resonance impurity is equally important for forming an individual portrait of $G(V,B)$, which is exactly that part of the system which can be tested by a diffusive electron during its typical lifetime at the impurity centre. As a result, the short-range features of the tunneling matrix element fall out of the theory, and the differential conductance fluctuations simply coincide with the energetic derivative of the local density of states fluctuations,

$$G(V,B) \equiv \frac{{\rm d}I}{{\rm d}V} \propto \frac{{\rm d}\delta\nu(\epsilon,B)}{{\rm d}\epsilon}\,.$$

The amplitude of irregular oscillations of ${\rm d}I/{\rm d}V$ found in the calculations below is also related to the resonance width. The variance (i.e., the mean square) of the differential conductance at $V > V_0 + \Gamma/e$ normalized by the height of the main resonance peak at $V = V_0$ is inversely proportional to the conductance $g(L_\Gamma)$ of a piece of an electrode with dimensions L_Γ^d measured in units of e^2/h which can be interpreted on the basis of the statistics of single-particle wave functions of a disordered metal. This is the result of the diagrammatic perturbation theory calculations and can be explained qualitatively in the following way. The value of the current in the plateau regime is determined by the sum of local densities (at the spectrometer position) of the wave functions $|\psi_\epsilon(r_0)|^2$ which lie in an energy interval Γ around E_0 in a piece of metal with characteristic dimensions L_Γ. These are the states which can contribute resonantly to the tunneling current. The average value of this sum is proportional to the total number of states $N(\Gamma)$ and to the typical density of a single state, $\big\langle|\psi_\epsilon(r_0)|^2\big\rangle \sim 1/L_\Gamma^d$. Being achieved over a step with width $V_\Gamma = \Gamma/ae$, such a value of the current plateau results in the height of the main differential conductance peak G_Γ which is proportional to $N(\Gamma)\big\langle|\psi_\epsilon(r_0)|^2\big\rangle/V_\Gamma$. On the other hand, $\psi_\epsilon(r_0)$ taken from an individual state is a random variable with dominantly Gaussian statistics in the metallic regime [5], so that ${\rm var}\big(|\psi_\epsilon(r_0)|^2\big) \sim \big\langle|\psi_\epsilon(r_0)|^2\big\rangle^2$. The variance of the sum of a large number, $N(\Gamma) \gg 1$, of random additives $|\psi_\epsilon(r_0)|^2$ is of the order of $N(\Gamma)\big\langle|\psi_\epsilon(r_0)|^2\big\rangle^2$, and being individual for each next energy interval Γ, this fluctuation is responsible for the fluctuation in the differential conductance with variance $N(\Gamma)\big\langle|\psi_\epsilon(r_0)|^2\big\rangle^2/V_\Gamma^2$. Following Thouless [17], we estimate $N(\Gamma) \sim g(L_\Gamma)$, which gives the estimate mentioned at the top of this paragraph, $\langle\delta G^2\rangle/G_\Gamma^2 \approx g^{-1}(L_\Gamma)$. It is interesting to note that these fluctuations should be almost insensitive to the temperature variations, since they are generated by the electron states deep below the Fermi level, and their amplitude is dominated just by the energetic width of the 'spectrometer' Γ.

To be described quantitatively, a random differential conductance dependence on the bias voltage and external magnetic field should be characterized by the disorder-averaged correlation function of current derivatives ${\rm d}I/{\rm d}V$ measured at different voltages or at slightly different magnetic fields,

$$\big\langle G(V,B)G(V+\Delta V, B+\Delta B)\big\rangle = \langle\delta G^2\rangle K_d(\Delta V,\Delta B)\,. \tag{4}$$

To get the analytical form of both the variance and correlation functions of differential conductance fluctuations, we employed the diagrammatic perturbation theory technique [18]. In the case when the electrodes are 3D metals (they extend in all directions over distances greater than $L_\Gamma = \sqrt{hD/\Gamma}$), which is the case in the vertical tunneling devices grown without any undoped spacer in front of the tunneling barrier, the result of the perturbation theory analysis has the form of Eqn (4) with variance

$$\langle\delta G^2\rangle_3 = \frac{(2\pi)^{3/2}\beta^{-1}}{16}\,\frac{\sqrt{\Gamma/hD}}{\nu hD}\,\frac{G_\Gamma^2}{[1+\hbar\gamma/\Gamma]^{3/2}}\,, \tag{5}$$

where the coefficient β should be specified for the case of a negligible ($\beta = 1$) and strong ($\beta = 2$) spin-orbit coupling, and the correlation properties are described by the function

$$K_3 = \frac{(2-Y)\sqrt{1+Y}}{\sqrt{2}\,Y^3}\,, \qquad \text{where} \qquad Y = \sqrt{1+\left(\frac{\Delta V}{\tilde{V}_\Gamma}\right)^2}\,.$$

In Equation (5), $\beta = 1$ for the limit of a zero magnetic field, $\beta = 2$ when $BL_\Gamma^2 > \Phi_0$, and $\tilde{V}_\Gamma = V_\Gamma + \hbar\gamma/(ae)$ is slightly modified, as compared to the width of the main resonance V_Γ by the decoherence rate γ of a floating-up 'hole' below the Fermi level created in the emitter after the tunneling event. The value of the variance of fluctuations in Eqn (5) is normalized by the value G_Γ from Eqn (1) in order to show how is it parametrically suppressed, as compared to the height of the main peak in $\mathrm{d}I/\mathrm{d}V$, by the factor $\delta G/G_\Gamma \sim g^{-1/2}(L_\Gamma)$, where $g(L_\Gamma)$ is the conductance of a piece of electrode with size L_Γ in all directions measured in units of e^2/h [the second multiplier in Eqn (5)].

A similar estimate is applicable to the case of a planar emitter. The analytical result for that case can be written as

$$\left\langle \delta G^2 \right\rangle_2 = \frac{\pi\theta\beta^{-1}}{4vhD}\,\frac{G_\Gamma^2}{[1+\hbar\gamma/\Gamma]^2}\,, \qquad K_2 = \frac{1-(\Delta V/\tilde{V}_\Gamma)^2}{\left[1+(\Delta V/\tilde{V}_\Gamma)^2\right]^2}\,. \tag{6}$$

In Eqn (6), the factor θ distinguishes between two configurations of the device: $\theta = 2$ for a 'horizontal' tunneling through a lithographically processed barrier in a 2D electron system, and $\theta = 1$ for a 'vertical' tunneling from a 2D layer accumulated in front of the double-barrier structure in a device grown with a wide spacer [10].

The correlation properties of random differential conductance oscillations under the variation of a magnetic field oriented along the current direction can be calculated in a standard way [19], and we arrive at the correlation function of differential conductances, $K_d(\Delta B)$ in the form of

$$K_2 = \sum_{n=0}^{\infty} \frac{2X^{-2}}{[n+1/2+1/X]^3} = -X^{-2}\psi^{(2)}\left(\frac{1}{X}+\frac{1}{2}\right),$$

$$K_3 = \sum_{n=0}^{\infty} \frac{(3/2)X^{-3/2}}{[n+1/2+1/X]^{5/2}}\,, \qquad X = \frac{2eD\Delta B}{c(\Gamma+\hbar\gamma)} = \frac{2L_\Gamma^2\Delta B}{\Phi_0}\,, \tag{7}$$

where $\psi^{(2)}(z)$ is the second order derivative from the psi-function. At small $X \ll 1$, the correlation function $K_d(\Delta H)$ in Eqn (7) can be approximated by

$$K_3 \approx 1 - \frac{5}{32}X^2\,, \qquad K_2 \approx 1 - \frac{1}{4}X^2\,.$$

The characteristic correlation field ΔB_c (half-width of the correlation function at half maximum) found from this is

$$\Delta B_c^{(d)} \approx \frac{c[\Gamma+\hbar\gamma]}{eD} \times \begin{cases} 1.8\,, & d=3\,, \\ 1.3\,, & d=2\,. \end{cases} \tag{8}$$

To complete the picture, we should also show the analytical form of the variance and correlation functions for the quasi-1D geometry, that is, for a wire with cross sectional dimensions less than L_Γ. The calculation for the Q1D geometry is standard [19], and we arrive at

$$\left\langle \delta G^2 \right\rangle = \frac{3(2\pi)^{3/2}}{32}\,\frac{1}{Sv\sqrt{hD\Gamma}}\,\frac{G_\Gamma^2}{[1+\hbar\gamma/\Gamma]^{5/2}}\,, \tag{9}$$

$$K(\Delta V, 0) = \frac{(4-2Y-Y^2)\sqrt{1+Y}}{\sqrt{2}\,Y^5}\,, \qquad Y = \sqrt{1+\left(\frac{\Delta V}{\tilde{V}_\Gamma}\right)^2}\,,$$

which, again, is determined by the conductance of a piece of wire with length L_Γ. The correlation function of differential conductance fluctuations as a function of the magnetic field can be found in a similar way. The calculation can easily be extended to a magnetic field tilted through an arbitrary angle φ with respect to the wire axis, and for cylindrical wires it yields

$$K(0,\Delta B) = \frac{1}{[1+X_1^2]^{5/2}}\,, \qquad X_1 = \frac{\sqrt{\pi\chi(\varphi)}}{2}\,\frac{e\Delta B}{hc}\,rL_\Gamma\,. \tag{10}$$

In this equation, $\chi(\varphi) = 1 + \sin^2\varphi$, so that the correlation parameter B_c for the field oriented along the wire axis and perpendicular to it may differ only by the factor of $\sqrt{2}$, $B_c^{\parallel}/B_c^{\perp} = \sqrt{2}$, which is very much in contrast to what one should expect in the case of a 2D emitter. Note that for a quasi-1D cylindrical wire, $B_c^{\parallel} = 0.64\Phi_0/rL_\Gamma$.

It is useful to note that power spectra of fluctuations of $\mathrm{d}I/\mathrm{d}V$ in 3D and quasi-1D geometries as a function of the voltage (which coincide with the Fourier transform of $K(\Delta V, 0)$ with respect to V) would give a different value from that obtained from the half-width of the correlation function, since the latter does not have a Lorenzean form in those cases. In particular, the correlation voltage that one would find from the power spectrum analysis coincides with V_Γ, whereas the correlation voltage to be extracted from the half-height of the correlation function is $0.65V_\Gamma$ in the 3D case and $0.39V_\Gamma$ in the quasi-1D case.

The results of Eqns (5), (6), and (8) can be extended to the differential conductance fluctuations in a classically strong magnetic field, $\omega_c\tau \geqslant 1$. To achieve such a generalization, it is enough [20] to replace the isotropic diffusion coefficient D in expressions for the diffuson P^d by a diagonal tensor $\mathrm{diag}(D, D_\perp, D_\perp)$ taking into account that the diffusion across the magnetic field direction is suppressed by the cyclotron motion, $D_\perp = D/(1+(\omega_c\tau)^2)$. When the magnetic field is oriented perpendicular to the tunneling barrier, the effect of skipping orbits [21] does not affect the boundary condition to the equations on the diffuson (in contrast to the case of the conductance fluctuations in metallic wires [22]), so that we find that the variance of $\mathrm{d}I/\mathrm{d}V$ and the correlation parameter ΔB_c increase with the magnetic field as

$$\frac{\left\langle(\delta G)^2\right\rangle_B}{\left\langle(\delta G)^2\right\rangle_{B=0}} \approx \frac{\Delta B_c(B)}{\Delta B_c(0)} \approx 1 + (\omega_c\tau)^2\,. \tag{11}$$

The correlation voltage and the form of the correlation function $K_d(\Delta V)$ remain unchanged, since they are determined solely by the spectrometer width independently of the diffusion coefficient.

To summarize, the differential conductance of a system with resonant tunneling from a disordered metal via a single impurity level was analyzed in the regime of a current plateau.

In the systems of all dimensions we studied here, it fluctuates around a zero average with a mean square value which scales with the height of the main resonance peak, G_Γ and is inversely proportional to the conductance $g(L_\Gamma)$ (measured in quantum units) of a piece of disordered electrode with typical dimensions $L_\Gamma \sim \sqrt{hD/\Gamma}$ determined by the width of the resonance itself:

$$\left\langle \left(\frac{\mathrm{d}I}{\mathrm{d}V} \right)^2 \right\rangle \sim \frac{G_\Gamma^2}{g(L_\Gamma)} .$$

The value of the correlation magnetic field of fluctuations is also related to the length L_Γ, $\Delta B_c \sim \phi_0 / L_\Gamma^2$ and the correlation properties of the pattern of $\mathrm{d}I/\mathrm{d}V$ with respect to the voltage variations are found in an analytical form both in two and three dimensions as a function of the voltage scaled by the width of the main resonance peak. Both the amplitude of fluctuations and the correlation parameter ΔB_c are expected to increase with the magnetic field.

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Resonant tunneling through a single-electron transistor

J König, H Schoeller, G Schön

1. Introduction

Electron transport through mesoscopic metallic islands and quantum dots is strongly influenced by the large charging energy, $E_C = e^2/2C$, associated with the low capacitance C of the system [1–3]. In the prototype of these systems, the 'single-electron transistor', a small island is coupled via tunnel junctions to leads and via a capacitor to a gate voltage source. At low temperatures, $T \ll E_C$, a variety of single-electron phenomena have been observed, including a Coulomb blockade and oscillations of the conductance as a function of the gate voltage.

The detailed features of the transport properties depend on the properties of the island. We consider here two opposite limits. In the first, the island contains a continuum of states, and the tunnel junctions are 'wide' with a large number of transverse channels. This is typically realized in metallic grains. If the dimensionless tunneling conductance of the junctions between the island and the lead electrodes,

$$\alpha_t \equiv \frac{R_K}{4\pi^2 R_t} \tag{1}$$

is low, on a scale given by the quantum resistance $R_K = h/e^2 \simeq 25.8\ \mathrm{k\Omega}$, the island charge is well-defined.

In the second limit, we consider the extreme case of an island containing one spin-degenerate level in the interesting energy range. This accounts for Coulomb blockade phenomena in zero-dimensional systems, such as double-barrier resonant-tunneling structures [4, 5], split-gate quantum-dot devices [6–8], quantum point-contacts with single-charge trap states [9], and ultra-small metallic tunnel junctions [10] with particles of diameter below 10 nm. In these islands the discrete level spectrum can be resolved, with a level spacing δ which may exceed T and eV. The coupling between the island and the leads is then characterized by the intrinsic level broadening in the non-interacting case Γ.

For $\alpha_t \ll 1$ in metallic islands or $\Gamma \ll T$ in quantum dots, sequential single-electron tunneling can be studied using perturbation theory [1, 3, 11–15]. On the other hand, recent experiments with strong tunneling show deviations from the classical description. In the metallic case, a broadening of the conductance peaks much larger than temperature has been observed [16, 17], demonstrating the effect of quantum fluctuations and higher-order coherent processes. Several theoretical papers [18–24] dealt with the problem of higher-order processes. This includes 'inelastic co-tunneling' [25, 24], where, in a second-order process in α_t, electrons tunnel via a virtual state of the island. (The term 'inelastic' indicates that with overwhelming probability different electron states are involved in the different steps of the correlated processes.) An extension of this process, which gains importance near resonances, is 'inelastic resonant tunneling' [20, 23], a process where electrons tunnel an arbitrary number of times between the reservoirs and the islands.

The quantum dot is described by the Anderson impurity model where the level is coupled via tunneling barriers to electron reservoirs. A strong on-site Coulomb repulsion suppresses double occupancy of the dot level. From the