Phenomena of growth of fractal systems

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Usp. Fiz. Nauk **159**, 391–393 (October 1989)

T. Vičzek. *Fractal Growth Phenomena.* World Scientific, Singapore; New Jersey; London; New York, 1989. 355 pp.

Fractal systems form their own world of objects and phenomena which, in contrast to continuous systems, have a tattered structure. Since nature provides for us a large number of processes and objects with a fractal structure this world is extensive and varied. It cannot be examined in isolation from the history of studying fractal systems. While concepts concerning fractal systems have entered into mathematics many decades ago, physicists have realized the value of these ideas quite recently. The reference point for bringing fractal concepts into physics can be justly deemed to be the publication of the remarkable books by B. Mandelbrot[1,2] concerning the fractal geometry of nature. The use of these concepts led to an understanding of a new group of properties of disordered systems, gave additional information concening them, and has established connections between what might appear to be different physical objects and phenomena. During a short period of time a number of new directions in physics has arisen, which include both the investigation of certain sets of physical systems, and also the development of effective methods of analyzing them.

The rapid progress in the development of fractal concepts in physics had also its own negative consequences associated with a distorted interpretation of some physical results due to the over-estimation of the role played by the fractal properties of objects. For example, the percolation cluster and the fractal aggregate (fractal cluster) are both clusters with fractal structure. However, their physical similarity ends with this statement, and therefore bringing these objects together on the basis of their fractal nature does not lead to a better understanding of the physics of either one of them.

In order to establish a correct understanding of the place occupied by fractal ideas in modern physics it is necessary to analyze the achievements obtained in this field by the scientists who were the ones that created it. In this respect of great value is the book under review by the Hungarian scientist T. Vičzek "Fractal Growth Phenomena" which has a number of valuable features. First of all, its author made a significant contribution to the development of the field under discussion. He obtained in this field a number of fundamental results with respect to computer simulation of fractal objects and phenomena, and on the analysis and description of processes of growth of different systems under conditions of diffusion limited aggregation. Therefore the author can better than many others evaluate the position which should be assigned to individual elements of this direction to physics. Secondly, the author certainly has to be commended for his ability to present the direction in physics under discussion in the form of a logically closed outline where certain results are a logical consequence or a logical extension of other results. Thirdly, the book is successful also because its contents are presented in a relatively simple form, so that the material of the book can be utilized not only by specialists engaged in these problems. This is very significant since the problems under discussion have relevance to a wide circle of physical systems and phenomena.

The book is divided into three parts. The first part of the book "Fractals" explains the fractal properties of mathematical and physical objects, and provides examples of different symmetrical systems with fractal properties. This part of the book is particularly successful as a textbook. In particular it provides a set of "deterministic" fractals—geometrical figures which are widely used as simple fractal models. Among them are Cantor sets, Peano curve, Sierpinski gasket, Julia set, Mandelbrot set, etc. These figures are introduced as illustrations of the basic properties of fractal systems. Moreover, the first part of the book analyzes different types of fractal systems and outlines their properties. Methods of determining fractal dimensions of systems are described.

The second part of the book "Models of Cluster Growth" analyzes the existing models of the formation and evolution of clusters consisting of rigidly coupled solid particles. One of the sections is devoted to an analysis of models describing a percolation cluster. The next section examines models of growth and evolution of a cluster, when this process is limited by diffusion of individual particles joining the cluster. Realistic DLA models (DLA–diffusion limited aggregation) are presented, which describe both the growth of a cluster in bulk, and on a substrate. The properties of the clusters being formed are analyzed. One of the fundamental properties of a DLA cluster was established by the author of the book and consists of the anisotropy of fractal properties, so that fractal dimensions, corresponding to radial and tangential directions of the cluster are different. It should be noted that DLA clusters occupy a central position in the physics of growth of fractal systems, since the mathematical description of the growth of such a cluster corresponds to a description of a wide circle of physical processes and phenomena. Other models, to which individual sections of the book are devoted, include the Eden model or the balistic model, when the aggregating particles or fragments move along rectalinear trajectories, and also the cluster-cluster aggregation (CCA) model, when a large cluster is formed as a result of joining of clusters of smaller size.

Along with a description of modifications of the approaches being discussed and of the properties of the objects being formed the book examines the results of experiments

 0038-5670/89/100941-04$01.80

in the analysis of physical systems corresponding to these models, and also the special features of the experimental methods.

The third part of the book—"Formation of fractal objects"—is devoted to concrete physical systems and phenomena which are described by the models mentioned above. The first section of this part analyzes computer methods of investigating the systems and processes under discussion. Different approaches are presented for simulating diffusion-limited growth processes. Considerable attention is devoted to the so-called Laplacian patterns whose growth is described by the Laplace equation with appropriate boundary conditions. The DLA cluster is one of the types of Laplacian patterns. In another limiting case these objects have a nonfractal morphology and, in particular, hexagonal symmetric snowflakes can be obtained as a result of evolution of such systems subject to a certain relationship between their parameters. A significant contribution to the development of the theory of Laplacian patterns was made by the author of this book.

Laplacian patterns are of physical interest in two respects. On the one hand, they enable one to trace the transition from fractal systems to nonfractal ones. On the other hand, they describe a broad range of physical processes and phenomena which, in addition to DLA clusters, include such phenomena as dielectric breakdown; formation of hydrodynamic structures—the so-called "viscous fingers" formed in forcing a liquid or a gas through a more viscous liquid; the electrochemical deposition of an element on an electrode in the course of electrolysis; the motion of the boundary between the crystalline and amorphous structures on heating a surface; the process of "chemical dissolution" associated with the chemical solution of a porous medium under the action of a liquid penetrating into its pores; the process of "drying-out" of a porous medium, and also a number of other physical, chemical and biological phenomena, occurring at an interphase boundary. Some special features of the experimental investigation of some of these phenomena are analyzed in the last section.

The book is very valuable. By successfully combining simplicity with completeness of presentation, it in a simple form presents to the reader new physical concepts, referring to systems and phenomena with fractal structure, and also to related physical phenomena. This book will be useful for specialists in different fields of physics, where the use of fractal ideas and concepts can turn out to be fruitful. It is of methodological interest for specialists in computer simulation of systems and processes. The book is accessible to a general physicist-reader, who will experience an aesthetic pleasure in becoming acquainted with the modern physical ideas presented in it. In recommending that readers should become acquainted with this book I consider that it can be used in modern courses on physics for students in various physical specialties.

[1] B. B. Mandelbrot. Fractals: Form, Chance and Dimension. Freeman, San Francisco, 1977.
[2] B. B. Mandelbrot. The Fractal Geometry of Nature. Freeman, San Francisco, 1982.

Translated by G. M. Volkoff