Superstrings, or beyond standard ideas[1)]

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Basic features of the superstring picture of the microworld are discussed. This picture suggests that there is a certain nonlocal object—the string—in many-dimensional ($D = 10, 26, 506$) space-time on the scale of the Planck length (10^{-33} cm). Transition to larger scales gives rise to the localization and compactification of the theory. The result is an effective supersymmetric low-energy theory combining all fundamental interactions, including gravity.

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INTRODUCTION

The aim of this brief paper is to present in a reasonably accessible form the captivating ideas that have appeared in high-energy physics in recent years and have given rise to real hope that a unified theory of all the fundamental interactions will be constructed. Our account lays no claim to originality or completeness.[1,2] The branch of science that we shall consider is rapidly evolving and is beginning to attract the attention of an increasing number of physicists and mathematicians. The fundamental object in this new theory is an extended entity—the string—for which the *action* has the property of supersymmetry. The theory is therefore called superstring theory. It is based on non-standard ideas and makes use of the mathematical formalism of topology, differential geometry, and so on, which is complicated and new to physicists. It has not been my aim to provide a review of mathematical methods used in superstring theory nor, as far as possible, to examine technical aspects of the problems being discussed. All this can be found in the already extensive literature on superstrings and in existing reviews.[3] My goal has been to present the basic concepts and the objects treated in superstring theory and to consider the tempting prospects that are now before us. Despite the complexity and abstract nature of the mathematical description, the extensive panorama that is being revealed to us offers great scope to imagination and fantasy, and questions are being asked that, not long ago, would have been regarded as, at least, premature. It follows that, even if the specific contemporary models that are now being proposed are eventually discarded, the breadth of the formulation of the problem and the non-standard nature of the ideas involved in the theories will greatly enrich our conceptual and technical arsenal, and therefore deserve investigation.

1. THE PANORAMA OF HIGH-ENERGY PHYSICS

The second part of our title is "beyond standard ideas." What do we understand by standard ideas and why should we wish to go beyond them? The phrase "standard model" is now understood to be a description of strong, weak, and electromagnetic interactions at the quark-lepton level within the framework of a local gauge theory based on the group

$$G_{\text{stand}} = \mathrm{SU_c}(3) \times \mathrm{SU_L}(2) \times \mathrm{U_Y}(1). \tag{1}$$

Strong interactions are described by quantum chromodynamics with the color gauge group $\mathrm{SU_c}(3)$. Electroweak interactions are described by the Glashow-Weinberg-Salam model with the $\mathrm{SU_L}(2)\times \mathrm{U_Y}(1)$ group. All existing experimental data are in good agreement with the standard model.

The standard scheme is sometimes supplemented by the requirement of supersymmetry. However, despite the attractiveness of supersymmetric models, they have not yet received experimental confirmation and must be regarded as hypotheses.

The next step along the path toward the unification of the forces of nature is provided by grand unification theories. These are also based on the idea of local gauge invariance and examine the symmetry groups SU(5), SO(10), E_6, and so on. The idea of grand unification has not been confirmed experimentally, but there are no facts that contradict it either.

Despite the advances that have been made, the standard model and the grand unification theories leave us with many unresolved problems and a considerable arbitrariness in the choice of the parameters. The following are some of the unanswered questions:[2]

—why are fermions chiral particles?

—how many generations of quarks and leptons are there?

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—what determines the Higgs sector?

—why SU(3)×SU(2)×U(1)?

We also do not understand:

—how to include gravitation in the overall scheme,

—should the theory be finite?

—why is the cosmological constant $\Lambda \simeq 0$?

The search for answers to these questions takes us to the re-examination of the basis of quantum field theory. A critical approach then gives rise to much more fundamental questions, to which we are directed by the most recent advances in the theory and, above all, superstring theory, namely:

—are infinities unavoidable?

—is local quantum field theory always valid?

—why four dimensions?

The answers to these questions lie outside the scope of standard ideas. And so we ask: where are we now?

Experiment:

—there is no sign of proton decay

—there is no sign of monopoles

—there is no sign of axions

—there is no sign of supersymmetry.

Conclusion: there are no reliable manifestations of anything that cannot be explained by the standard model.

Theory: radical changes and new ideas.

According to M. J. Duff,[2] going beyond the standard model now means going outside:

—four dimensions

—the Planck scale

—the limits of the imaginable.

We now have to face the paradoxical question: what is the dimensionality of the space in which we live?! The obvious answer is: $D = 4$. In superstring theory, the answer is less obvious, but logically better founded: $D = 10$, at least. In the boson variant of the theory, $D = 26$. On the other hand, if we adopt the Kaluza-Klein standpoint, then, as will be clear later, we find that $D = 506$. It seems that these are three equivalent variants of the mathematical description of a single physical reality. The reconciliation between experimental and theoretical points of view relies on the fact that the multidimensional superstring theory is fully valid at energies that are inaccessible to direct observation.

The modern panorama of high-energy physics is illustrated schematically in Fig. 1.

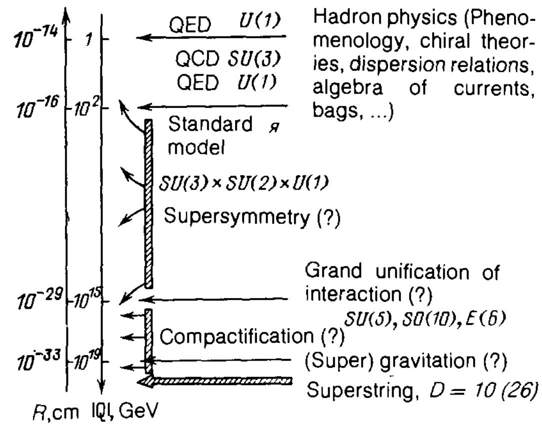

FIG. 1

2. WHAT ARE SUPERSTRINGS?

As already noted, the fundamental object studied in superstring theory is a nonlocal extended entity with characteristic size of the order of the Planck length, i.e., we discard local field theory at distances less than 10^{-33} cm and replace it with nonlocal theory. The basis for this theory is provided by classical, relativistic, extended objects—the strings. String dynamics is discussed in the next Section.

String theory already has a twenty-year history. Strings first arose in hadron physics as the dynamic basis for the Veneziano and dual-resonance models.[4] In quantum chromodynamics, strings bind quarks together, forming hadrons. From the very start, string theory encountered serious problems: rigorous quantization was found to be possible only for critical space-time dimensions. For the bosonic string, $D_{\text{crit}} = 26$ and, for the fermionic string, $D_{\text{crit}} = 10$. The particle spectrum then contains tachyons and massless spin 1 and 2 particles. These states are not present in hadron physics and must be removed.

The shortcomings of string theory are turned into advantages when they are examined from a totally different point of view. In the late 1970s, Scherk and Schwarz[5] recognized that string theory could serve as a basis for the unification of all the fundamental interactions, including gravitation. The high dimensionality of space-time is then perceived not as a curiosity, but literally. We observe here a re-emergence of the fifty-year-old ideas of Kaluza and Klein on a new basis. The "extra" massless states are now identified with Young-Mills and gravity fields, and tachyons are absent from this new theory because of supersymmetry.

In quantization, a string is an infinite sequence of normal modes, i.e., a sequence of mass states in quantum field theory. The mass splitting Δm^2 is then proportional to the tension T in the string. In superstring theory, $T \simeq (10^{19}\ \text{GeV})^2$, in contrast to hadronic physics, for which $T \sim (1\ \text{GeV})^2$.

Strings are open or closed. Open strings used for the lowest massless states contain spin 1 particles, i.e., Young-Mills fields, whereas closed strings contain spin 2 particles, i.e., gravitons. This route in string theory eventually leads to a quantum theory that unifies gravity and Young-Mills fields that are the conveyers of all interactions.

At distances much greater than the Planck length (10^{-33} cm) or at energies much greater than the Planck mass (10^{19} GeV), the massive states are "split off", and we have an effective point field theory (supergravity and Young-Mills supersymmetric theory) with fixed parameters and particle composition. The observed particles (quarks, leptons, gauge bosons, ...) should then be found among the massless excitations ($m \ll 10^{19}$ GeV). At distances below 10^{-33} cm (or at energies in excess of 10^{19} GeV), massive states are present and accomplish the modification of the general theory of relativity at these scales. It turns out that

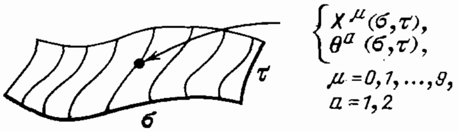

FIG. 2

the GTR action is only the first term in the expansion of the effective superstring theory.

3. SUPERSTRING DYNAMICS

Just as a point traces out a world line in space-time, a moving string sweeps out a world surface. This two-dimensional surface can be inscribed into a space of any dimension. However, anticipating quantization, let us examine the ten-dimensional space. Moreover, since the string is supersymmetric, we supplement the space with Grassman generators, i.e., we consider a superspace in which the number of Grassman generators depends on the type of supersymmetry. The world suface (Fig. 2) depends on the two parameters σ and τ, which can be interpreted as the length along the string and the proper time. A point on the surface has the superspace coordinates $X^{\mu}(\sigma,\tau)$ and $\theta^{\alpha}(\sigma,\tau)$.

The string action is a generalization of action for a point particle. In the latter case, $S=$ length of world line $=\int ds$. For the string, $S=$ area of world surface $=\int d\Sigma$. The parametrically invariant action of the boson string is

$$S=-\frac{T}{2}\int d\sigma\, d\tau \eta_{\mu\nu}(-g)^{1/2} g^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X^{\nu}, \qquad (1)$$

where T is the tension in the string, $\eta_{\mu\nu}$ is the metric in ten-dimensional space (Minkowski space), and $g^{\alpha\beta}$ is the metric in σ, τ space. It is readily seen that this is a generalization of the well-known action for a point particle in quantum mechanics:

$$S\sim\int\frac{\dot{x}^2}{2}\,dt.$$

In the case of the superstring, $\partial_{\alpha}X^{\mu}$ is replaced with $\Pi^{\mu}_{\alpha}=\partial_{\alpha}X^{\mu}-i\bar{\theta}\gamma^{\mu}\partial_{\alpha}\theta$, so that action becomes invariant under the supersymmetry transformation, and there are also further terms that are cubic and quartic in the coordinates:

$$S=-\frac{T}{2}\int d\sigma\, d\tau \eta_{\mu\nu}(-g)^{1/2} g^{\alpha\beta}\Pi^{\mu}_{\alpha}\Pi^{\nu}_{\beta}$$
$$-iT\int d\sigma\, d\tau \eta_{\mu\nu}e^{\alpha\beta}(\partial_{\alpha}X^{\mu}\bar{\theta}\gamma^{\nu}\partial_{\beta}\theta-i\bar{\theta}\gamma^{\nu}\partial_{\alpha}\theta\bar{\theta}\gamma^{\nu}\partial_{\beta}\theta). \qquad (1')$$

If we look upon X^{μ} and θ^{α} as fields specified on a two-dimensional surface, we can treat the theory with action (1′) as two-dimensional GTR with matter fields X^{μ} and θ^{α}. This shows the close analogy between string theory and two-dimensional nonlinear σ-models that are being so intensively investigated at present.

The string configuration can be very complicated (Fig. 3) and is determined by solutions of the equations of motion, subject to appropriate boundary conditions.

4. TYPES OF THEORY

It is common to distinguish between the following types of superstring theory.[6]

4.1. Type I

This comprises open unoriented strings with $N=1$ supersymmetry and gauge charges at the ends. In the mathematical description, the string ends have associated with them the matrices of the fundamental representation of the gauge group. The strings are then realized as both singlet and nonsinglet representations, and the consistent quantum theory of unoriented strings admits only the classical groups SO(n) and USp(n). This scheme for including the internal symmetry group is called the Chan-Paton scheme and arose in hadronic physics in the interpretation of mesons as open strings with quarks at the ends. It subsequently became clear that the requirement of cancelation of anomalies and divergences ensures that only the group SO(32) remains. When they interact, open strings form closed configurations that are singlets under the internal symmetry group.

During quantization, open strings generate Young-Mills fields, whereas closed strings generate gravity fields. In the low-energy limit, i.e., when $p\ll T^{1/2}$, type I superstrings lead to the $D=10$ supersymmetric Young-Mills theory and $N=1$ supergravity, where the gauge interaction constant g and the gravitational interaction constant $K=G^{1/2}$ (G is Newton's constant) are related by

$$K=g^2T. \qquad (2)$$

Gauge and gravitational interactions are intimately related to one another by their origin. The superstring is thus a revival of the Einstein idea of a unified theory combining electromagnetic (gauge) and gravitational interactions.

4.2. Type II

This type comprises closed oriented strings with $N=2$ supersymmetry. There is no internal symmetry group. In the

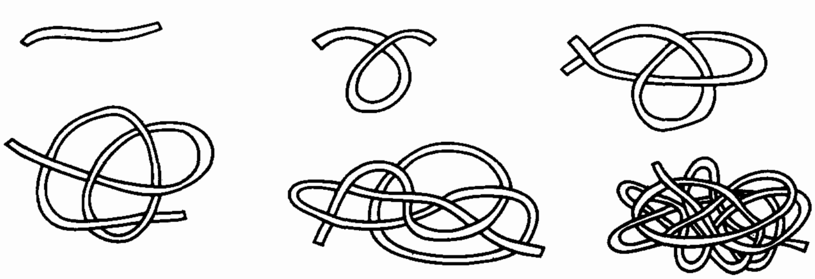

FIG. 3

low-energy limit, $p \ll T^{1/2}$, the result is the $D = 10$, $N = 2$ supergravity theory.

It is common to distinguish between type IIA and IIB theories. In the former, fermions have different chirality but, in the second, they have the same chirality. The IIB theory leads to an effective low-energy theory with chiral fermions, as demanded by phenomenology.

A further superstring theory, called heterotic or hybrid,[7] has recently appeared. It has given rise to great expectations.

4.3. Heterotic string

This is a closed oriented string. It is called heterotic because it is a hybrid of the 26-dimensional bosonic string and the 10-dimensional fermionic string of type IIB. The heterotic string has the properties of type I and II strings: it has the same $N = 1$ Poincaré superalgebra as the type I string, and the same topological properties as the type II string. Like the other type II closed strings, the heterotic string can be obtained by the compactification of the $D = 26$ bosonic string to the 16-dimensional torus T^{16}. In this scheme, the lattice of roots of the group is identified with the lattice of discrete momenta associated with internal dimensions compactified to the torus, and fermions appear as the solitons of the bosonic theory. Following the Kaluza-Klein ideology, this leads us to 16 gauge bosons of the $[U(1)]$ group.[16] However, the string can wind itself on the torus following topologically nontrivial configurations—the solitons. These form a further 480 gauge bosons. We thus have a total of 496. The heterotic string thus gives rise to a gauge group despite the fact that the string itself is closed. It is clear that the rank of the group is 16 and its dimension 496. $SO(32)$ and $E_8 \times E_8$ are gauge groups of this kind, where E_8 is the maximal exclusive group in the Cartan classification.

The critical dimension of space for the bosonic string has thus received the interpretation $26 = 10 + \text{rank } G$. In consistent Kaluza-Klein ideology, the space corresponding to this situation has the dimensionality $506 = 10 + \dim G$. The gauge symmetry group appears here as a result of the compactification of the 496 dimensions.[2]

For the heterotic string, as for the type I string, the Young-Mills fields are intimately related to gravity by their origin. However, in contrast to type I strings, the relation between the gauge constant g and the gravitational constant K is different:

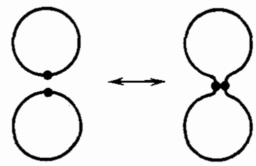

FIG. 4

$$K^2 = \frac{g^2}{T}, \tag{3}$$

and this is preserved during compactification, as can be seen by dimensional analysis.

The different types of self-consistent superstring theory are listed in the following table.[2]

5. INTERACTION BETWEEN SUPERSTRINGS

Although strings are nonlocal objects, the interaction between them is local in character. We do not have, at present, a gauge-invariant covariant formalism for the description of the second-quantized string.[2] The light-front gauge is employed. The strings are then described by functionals of the lateral coordinates which, in turn, are specified on the world surface of the string and the conjugate momenta p^+. Open strings are denoted by $\Phi[\mathbf{x}(\sigma),\theta(\sigma),\bar{\theta}(\sigma),p^+]$ and closed strings by $\Psi[\mathbf{x}(\sigma),\theta(\sigma),\bar{\theta}(\sigma),p^+]$, where Φ is the associated representation matrix and Ψ is the gauge group singlet. These functionals are generalizations of local fields that depend on the space-time point.

String interactions are described by local cubic terms of the form

$$\frac{\delta^2}{\delta x^2(\sigma)} \Psi^3$$

for closed strings and, similarly, $\mathrm{Tr}\Phi^3$ for open strings and $(\mathrm{Tr}\ \Phi)\Psi$ for transitions from one to the other. Figure 4 shows an example of an interaction that leads to the merging of two closed strings. These interactions generate contact interactions between the normal modes of string excitation, i.e., local quantum fields. This example is a generalization of the three-graviton interaction in GTR. Local interactions of the form $\mathrm{Tr}\Phi^4$ are also possible for open strings.

String interactions can be represented in terms of the string Feynman diagrams, except that the lines representing particles are replaced by surfaces. Figure 5 shows an exam-

TABLE I. Types of superstring theory

Type	Spinor	String	Massless states
I[SO(32)]	Weyl-Majorana	Open + closed	$N = 1$, SO(32) Young-Mills theory + $N = 1$ supergravity
IIA	Majorana	Closed	$N = 2$ nonchiral supergravity
IIB	Weyl	Closed	$N = 2$ chiral supergravity
Heterotic [SO(32)]	Weyl-Majorana	Closed	$N = 1$, SO(32) Young-Mills theory + $N = 1$ supergravity
Heterotic ($E_8 \times E_8$)	Weyl-Majorna	Closed	$N = 1$, $E_8 \times E_8$ Young-Mills theory + $N = 1$ supergravity

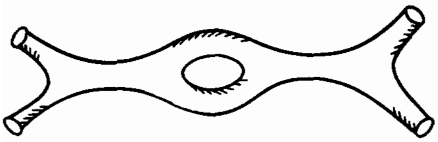

FIG. 5

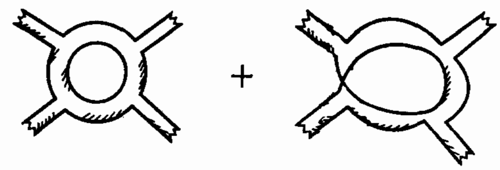

FIG. 7

ple of a diagram of this kind for interactions between closed strings.

We note the absence of high-order contact interactions. The superpotential contains only cubic terms, just as in the case of the supersymmetric local theories. All contact interactions arise in Einstein gravity as "low-energy" effective theory for $p \ll T^{1/2}$, by analogy with the way in which the four-fermion interaction arises in the Weinberg-Salam model when $p \ll M_W$.

6. SUPERSTRING PERTURBATION THEORY

The next step is to construct a perturbation theory. As they interact, strings may become scattered, create new strings, and emit point particles. In the effective local theory, this corresponds to all the possible interactions between local fields.

6.1. Tree diagrams

As an example, let us consider the scattering of two gravitons by a string. In the lowest-order approximation, the scattering amplitude is given by (Fig. 6)

$$T_4 = \text{(Einstein's Supergravity)} \times \frac{\Gamma\left(1-\frac{s}{T}\right)\Gamma\left(1-\frac{t}{T}\right)\Gamma\left(1-\frac{u}{T}\right)}{\Gamma\left(1+\frac{s}{T}\right)\Gamma\left(1+\frac{t}{T}\right)\Gamma\left(1+\frac{u}{T}\right)}, \tag{4}$$

where s, t, u are the usual Mandelstam variables, T is the tension in the string, and Γ is the Euler gamma-function.

The amplitude (4) has the necessary property of crossing symmetry and gives the Einstein supergravity in the low-energy limit ($T \to \infty$).

6.2. 1-loop diagrams

Here, we encounter divergences for the first time, and there are topological differences between open and closed strings. Crossings must be taken into account for open strings. For example, in the case of scattering of two open type I strings, there are two diagrams (Fig. 7). As in the case of local theories, in the first of these, we have a trace over the internal symmetry group. The second diagram does not contain this trace. This means that the contribution to the scattering amplitude for the groups $SO(n)$ and $USp(n)$ is proportional to $n \mp 32$, respectively. Each diagram diverges, so that the condition for the cancelation of divergences determines the single possible symmetry group $SO(32)$. The same symmetry group arises when we examine divergences in the effective point theory, obtained from the type I superstring in the limit as ${}^8T \to \infty$. Strictly speaking, two possibilities arise: $SO(32)$ and $E_8 \times E_8$, but, as we have already noted, exclusive groups cannot be realized on a type I superstring. The group $E_8 \times E_8$ corresponds to the heterotic string.

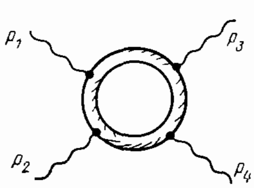

FIG. 6

Figure 8 shows the two-string heterotic scattering diagram for a type II string. These diagrams are topologically equivalent and finite.

6.3. *N*-loop diagrams

N-loop diagrams are constructed in accordance with the same rules as single-loop diagrams. Hopes for finite amplitudes here rest on supersymmetry. We now turn to the analysis of these questions.

7. DIVERGENCES

Consider closed strings. The diagrams shown in Fig. 8 are finite because they are topologically equivalent to "tadpole"-type diagrams (Fig. 9).

The resulting tadpole corresponds to the propagation of the so-called "dilaton," i.e., a massless scalar particle. In early string theories, tadpoles diverged, but, in superstring theories, supersymmetry ensures that they are equal to zero ($\bullet\!\sim\sim = 0$). This shows the difference as compared with point field theories. Actually, in such theories, the analog of Fig. 9 is shown in Fig. 10 and does not lead to a finite theory.

The cancelation of divergences in superstring theories is thus based not simply on nonlocality, but is related to the topology and supersymmetry of Feynman diagrams.

The situation is very similar (Fig. 11) in the multiloop case. Here, the theory is finite if

 $= 0.$

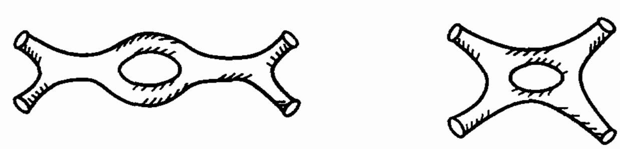

FIG. 8

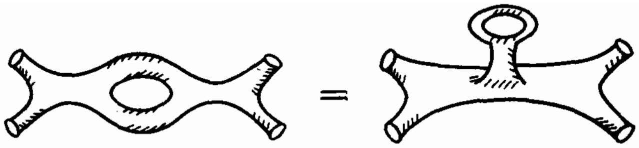

FIG. 9

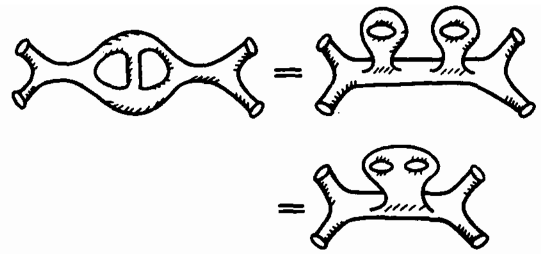

FIG. 11

However, a general proof of the absence of divergences in all orders of perturbation theory has not yet been produced. Another approach to this problem is based on the use of the functional integral. However, in contrast to integration over random local fields, the integration in this case is performed over random surfaces.

8. ANOMALIES

The problem of divergences turns out to be closely related to another well-known problem in quantum field theory, i.e., the problem of anomalies. As in local theories, the interaction between physical and nonphysical (longitudinal) modes of gauge fields can lead to anomalies in superstring theory. Hexagonal diagrams (Fig. 12) are anomalous in ten-dimensional space. They are the analogs of the triangular anomaly in four-dimensional space. The Young-Mills fields or gravitons are the external fields in this case, and the loop corresponds to a string propagator, or the propagators associated with spin 1/2 or 3/2 fermions and antisymmetric tensorial fields of the effective local theory. Gauge, gravitational, or mixed anomalies are obtained, depending on the type of external field.

The remarkable fact is the cancelation of all the anomalies in the case of the internal symmetry groups[9] SO(32) and $E_8\times E_8$. Thus, these two symmetry groups appear unavoidably in superstring theory. There are three sources of this, namely, (1) cancelation of divergences, (2) cancelation of anomalies, and (3) reduction from the 26-dimensional bosonic string to the 10-dimensional heterotic superstring.

9. LOW-ENERGY EFFECTIVE THEORY

Let us consider the effective local field theory in 10-dimensional space, which arises from the superstring when $p \ll T^{1/2}$. In principle, this theory can be produced by performing an expansion of $T^{-1/2}$ into a series in terms of energy. As already noted, the field composition is determined by the spectrum of the massless normal modes. This is the supergravity multiplet $(g^{\mu\nu}, B^{\mu\nu}, \Phi, \Psi_\mu^+, \lambda^-)$, which contains a graviton, antisymmetric tensor fields, a scalar, a gravitino, i.e., Rarita-Schwinger fields with positive chirality, and a Majorana spinor with negative chirality, as well as the Young-Mills supermultiplet (A_μ, X^+), which contains a vector field and a Majorana fermion of positive chirality. Here, we have enumerated the physical degrees of freedom without auxiliary fields.

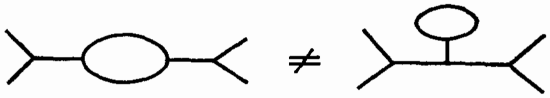

FIG. 10

Action has the form

$$S = -\int d^{10}x\,(-g)^{1/2}\Big[\frac{1}{2k^2}R + \frac{1}{k^2}\Big(\frac{\partial\Phi}{\Phi}\Big)^2 + \frac{1}{4g^2\Phi}F^{\alpha}_{\mu\nu}F^{\mu\nu\alpha} + \frac{3k^2}{2g^4\Phi^2}H_{\mu\nu\rho}H^{\mu\nu\rho} + \dots\Big]. \quad (5)$$

We note that this expression does not contain free parameters, and the constant g is absorbed in the normalization of the field Φ. The notation is as follows: $F^{ab}_{\mu\nu}$ are the Young-Mills fields, $a,b, = 1,2,\dots,\dim G$, $R^{mn}_{\mu\nu}$ is the ten-dimensional curvature, $m,n = 1,2,\dots,10$, and $H_{\mu\nu\rho}$ is the intensity of the field B:

$$H = dB - \frac{1}{30}\,\omega^0_{3Y} + \omega^0_{3L}. \quad (6)$$

The ellipsis in (5) corresponds to fermion terms and higher-order terms (R^2, etc.).

Particular attention must be paid to the so-called Chern-Simons terms ω^0_{3Y} and ω^0_{3L} in (6): $d\omega^0_{3Y} = F\widetilde{F}$, $d\omega^0_{3L} = R\widetilde{R}$. These are absent from the minimal local field theory, but are necessary for the cancelation of quantum anomalies. In superstring theory, the Chern–Simons terms are automatically present in the required form.

10. SPONTANEOUS COMPACTIFICATION

Interest in superstring theories rose very substantially when Green and Schwarz[8,9] discovered the remarkable cancelation of anomalies and divergences in the effective local theories for the groups SO(32) and $E_8\times E_8$. The gauge group was predicted for the first time on the basis of the internal properties of quantum theory! We shall see later that the group E_8 is preferable from the phenomenological

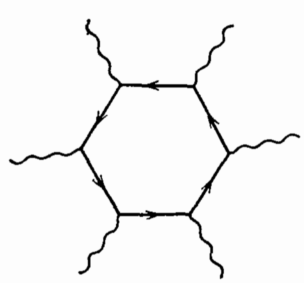

FIG. 12

point of view because it contains the known grand unification group $E_8 \supset E_6 \supset SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$.

The necessary modification of the tensor $H_{\mu\nu\rho}$, discussed in the last Section, turns out to be a no less important fact. It is closely related to the compactification of the "extra" spatial dimensions. Witten *et al.*[10] were the first to discover that the additional Chern-Simons terms $R \times \tilde{R}$ and the higher-order derivatives in the equation of motion led to the spontaneous compactification of six spatial dimensions. They found solutions of the form $M_{10} = M_4 \times K$, where M_n is the n-dimensional Minkowski space and K is a specific compact space, called the Calabi-Yao manifold. This manifold does not have continuous symmetries, so that the additional Kaluza-Klein vector fields do not appear on compactification. On the other hand, gravitational spin connectivities in six-dimensional space turn out to be equal to the Young-Mills potentials, so that dH is equal to zero.[6]

It transpires that the topological properties of the Calabi-Yao manifolds determine, in many respects, the physics in the effective four-dimensional theory.

11. DIMENSIONAL REDUCTION

The solutions of the equations of string field theory should, in principle, determine the structure of space-time and lead to the compactification of the six spatial dimensions. However, this is a very complex problem and, so far, only relatively simple variants of compactification have been obtained. At the same time, attempts to solve the problem within the framework of the effective low-energy theory have not always been valid because the discarded terms in the expansion of the supersymmetric action can be significant.

All the same, there are certain topological restrictions that do not depend on the particular variant of compactification. For example, it follows from (6) that the necessary condition for the single-valued global determination of the intensity H is[11]

$$\int_{M_4} \left(\mathrm{tr}\, R\tilde{R} - \frac{1}{30}\,\mathrm{Tr}\, F\tilde{F}\right) dv = 0, \tag{7}$$

where the integral is evaluated over an arbitrary closed four-dimensional manifold. The restriction (7) guarantees the absence of anomalies in the compactified theory and relates the topological properties of space with the value of the Young-Mills fields in the "extra" dimensions. Actually, since $R^{mn}_{\mu\nu} \neq 0$ in the compact space, it follows from (7) that $F^{ab}_{\mu\nu} \neq 0$ for a subgroup H of SO(32) or $E_8 \times E_8$. As a result, the symmetry group of the compactified theory is broken for some subgroup G. The spectrum of massless particles of the effective four-dimensional theory generates the representations of the group G. Schematically:

$$D = 10 \xrightarrow{R \neq 0} D = 4,$$
$$\left.\begin{matrix} \mathrm{SO}\ (32), \\ E_8 \times E_8 \end{matrix}\right\} \xrightarrow{F \neq 0} G.$$

Thus, the topological properties of space determine the internal symmetry group! In the case of the Calabi-Yao compactification, both the holonomy group $\mathscr{H} = SU(3)$ and (7) signify that

$$R^{mn}_{\mu\nu} = F^{mn}_{\mu\nu} \subset \mathrm{SU}\ (3).$$

This, in turn, leads to the breaking of, for example, the group $E_8 \times E_8$ down to $E_8 \times E_6$. The group E_6 may be looked upon as a group of the (super)theory of grand unification, and the unbroken group E_8 refers to the so-called shadow world.

The properties of the theory depend on the form of the Calabi-Yao manifolds, of which there are about 10 000 known, but, so far, no example has been found that satisfies all the necessary conditions. It is possible that some internal restrictions will be found, as was the case for the gauge symmetry groups.

12. PHENOMENOLOGICAL CONSEQUENCES

The phenomenological consequences of superstring theory have not yet been extensively worked out. They depend, to a considerable extent, on the specific mechanism of compactification. In the transition to the effective grand unification theory, the situation is not very different from the models that have already been studied with the exception of certain restrictions.

The effective theory that appears after the $E_8 \times E_8$ compactification of the heterotic string onto the Calabi-Yao space has the following properties:[1,2]

—the $N = 1$ supersymmetry remains unbroken

—one of the E_8 groups is broken to E_6 as a result of the holonomy group $\mathscr{H} = SU(3)$

—the second E_8 remains unbroken and is related to the "shadow" world or the "hidden" sector which interacts with our world only by gravitation or by the exchange of particles with mass of the order of M_{Pl}

—fermions belong to the 27-plet group E_6; apart from quarks and leptons, there are also some exotic particles;

—the number N_F of generations is fixed and given by $N_F = |X|/2$, where X is the so-called Euler characteristic—a topologic invariant of the manifold

—"realistic" Calabi-Yao spaces, where $N_F = 3, 4, \ldots$, are not simply connected, e.g., they contain holes; the E_6 symmetry is then broken by ring vortices embedded in the manifold

$$E_6 \to SU(3) \times SU(2) \times U(1) \times \underline{U(1) \times U(1)},$$
$$\to SU(3) \times SU(2) \times U(1) \times \underline{SU(2)},$$

but missing the groups SO(10) and SU(5); this results in additional, as compared with the standard model, symmetries that should be seen experimentally

—there are no correcting parameters in the Higgs sector; all the Yukawa coupling constants of the low-energy theory are determined by the topology and differ from standard theories of grand unification; $\sin^2\theta_W$ turns out to be standard on the unification scale

—the proton is (practically) stable

—supersymmetry can be broken by the "gluon condensate" in the shadow world without the appearance of the cosmological constant; in the classical theory, $\Lambda = 0$.

Specific predictions are, so far, very scanty and not too different from the grand unification theories. Moreover, it follows from superstring theories that:

—there are additional Z and W bosons due to the additional U(1) and SU(2) symmetries

—there are fractional electric (e/k) and correspondingly multiple magnetic ($2\pi/ek$) charges, where k is an integer; these are stable particles of mass of the order of M_{Pl}.

13. FINITE SUPERSYMMETRIC THEORIES

The possible finiteness of superstring theories gives rise to the hope that a self-consistent quantum-field theory, including gravitation, will be constructed. It may subsequently lead to a finite grand unification theory, as well. However, as we have already said, there is as yet no unambiguous way of obtaining the effective low-energy theory. This problem can also be tackled from the low-energy side by trying to construct a finite model of a local quantum field theory.

The first such model was obtained[12] by the compactification of the type I superstring to the six-dimensional torus T^6. It is found to be the maximally extended $N = 4$ supersymmetric Young-Mills theory, which is finite in all orders of perturbation theory.[13]

The extension of the class of point theories has been achieved within the framework of the $N = 2$ supersymmetric models. Here, the divergences arise only in one loop by virtue of the so-called renormalization theorems. Having achieved the cancelation of single-loop divergences by a suitable choice of the set of hypermultiplets, it is possible to obtain a finite theory.[14] However, such theories are not very acceptable from the phenomenological point of view because they contain mirror partners of ordinary particles that have not been seen experimentally.

Ermusher *et al.*[15] have recently put forward a method for constructing finite $N = 1$ supersymmetric Young-Mills theories. This class of theories is sufficiently extensive and is not exhausted by the $N = 4$ or $N = 2$ supersymmetric models. The set of matter fields in these theories satisfies the single restriction

$$\sum_R T(R) = 3C_G; \qquad \text{(A)}$$

where C_G is the quadratic Casimir operator of the gauge group and $T(R)\delta^{ab} = \mathrm{Sp}R^aR^b$, where R^a is the matrix (generally speaking) of the irreducible representation of matter fields.

Finite $N = 1$ supersymmetric grand unification theories constructed by this method have been isolated out of all the unified theories. They offer, first, a fixed number of fields (number of generations) and, second, there is a rigid relation between the amplitudes for the different processes by virtue of the presence of a unique, independent coupling constant.

14. PROBLEMS AND PROSPECTS

The evolution of the superstring picture has demonstrated that this theory is a very promising generalization of point field theories. This path may well lead us to a self-consistent quantum field theory of all the fundamental interactions. However, the contemporary understanding of the problem is very approximate and there are many obstacles to further advances. The existing five superstring theories probably do not exhaust all the possibilities, but even this number is too large if we desire to find a unique quantum theory that is internally self-consistent and has distinctive mathematical properties.

It is still not clear what determines the low-energy parameters of the theory: why is it that the observed dimension of space is $D = 4$? Why is $SU(3)\times SU(2)\times U(1)$ the symmetry group of the standard theory? What is the compactification radius?

One of the main questions that have to be answered is: what is the physical principle underlying the superstring theory? It is possible that there is a generalization of the GTR equivalence principle in the space of all the string configurations that will lead to the geometric description of the superstrings. A related problem is the understanding of how geometrical properties determine the physics of space-time.

It is likely that we shall find the answers to many of the above questions in the near future, and it will then become clear whether superstring theory is the long-awaited "theory of everything", or whether the bluebird has flown once again.

[1)] Paper presented at the Twenty-Third session of the Theoretical Physics Section of the Scientific Council of the Joint Institute for Nuclear Research, November 1985.

[2)] Different variants of this formalism have been put foward very recently (see, for example, Ref. 16).

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