Lepton mixing and neutrino oscillations

S. M. Bilen'kiĭ and B. M. Pontecorvo

Joint Institute for Nuclear Research
Usp. Fiz. Nauk **123**, 181–215 (October 1977)

The phenomenon of neutrino oscillations is reviewed. The case of mixing of two neutrinos (both Majorana and Dirac neutrinos) of masses m_1 and m_2 is considered in detail. It is shown that the mixing hypothesis is consistent with the existing data if $|m_1^2-m_2^2|\lesssim 1$ eV^2 (for maximal mixing). A discussion is given of possible experiments to search for oscillations of neutrinos from reactors, meson factories, and high-energy accelerators. Oscillations can be observed in these experiments if $|m_1^2-m_2^2|\gtrsim 10^{-2}$ eV^2. The prospects of searches for neutrino oscillations in experiments using cosmic neutrinos, and in particular solar neutrinos, are discussed in detail. Such experiments have a unique sensitivity as tests of the mixing hypothesis (in the case of solar neutrinos, mixing effects would be observed if $|m_1^2-m_2^2|\gtrsim 10^{-12}$ eV^2). The "solar neutrino paradox" is discussed in detail from the point of view of neutrino mixing. Neutrino oscillations are also considered in the case when there exist in nature $N>2$ types of neutrinos. Schemes involving the mixing of heavy leptons are discussed. It is shown in the framework of a specific scheme involving right-handed currents that for heavy-lepton masses of the order of several GeV the probabilities of processes such as $\mu\to e\gamma$ and $\mu\to 3e$ can be close to the existing experimental upper limits. The probabilities of these processes are practically zero when only the neutrinos are mixed.

PACS numbers: 14.60.Gh, 13.10.+q, 13.15.+g, 96.60.Kx

CONTENTS

INTRODUCTION

This review is concerned with the problem of neutrino oscillations. Qualitative discussions of neutrino oscillations were given many years ago.[1] Neutrino oscillations were introduced in these papers in analogy with the well-known oscillations of neutral kaons. In recent years, there have appeared many papers in which neutrino oscillations were considered from very different points of view. We shall give a rather detailed survey of these papers. We begin with a brief discussion of the basic results of the theory of neutrino oscillations.

It is usually assumed (and this is in accord with the existing data) that the electronic and muonic lepton numbers are strictly conserved. Of course, there can be no neutrino oscillations in this case. Oscillations can occur if, in addition to the ordinary weak interaction, there exists an interaction that does not conserve the lepton numbers. In this case,[1] the neutrino masses are non-zero[1)] and the state vectors of the ordinary electronic and muonic neutrinos ν_e and ν_μ are superpositions of the state vectors of neutrinos with definite masses (ν_1 and ν_2). If some weak process leads to the production of a beam of muonic neutrinos, for example, then at some distance from the point of production this beam becomes a coherent superposition of ν_μ and ν_e (there occur oscillations $\nu_\mu \rightleftarrows \nu_e$).

This picture is analogous to that of the oscillations in the system of neutral kaons. The strong interaction in the kaon case is analogous to the weak interaction in the neutrino case, and the analog of the strangeness-changing weak interaction for the neutrino case is the (super-weak?) interaction which violates the lepton numbers; the ν_e and ν_μ are analogous to the K^0 and $\overline{K}^0$ (these particles are not described by stationary states), and the ν_1 and ν_2 are analogous to the K_L and K_S (these particles have definite masses and are therefore described by stationary states).

We note also the following differences between oscillations of neutral kaons and possible neutrino oscillations:

1) The mass difference between the K_L and K_S is much smaller than the kaon mass; the mass difference between the neutrinos ν_1 and ν_2 may be comparable with their masses.

2) The K_L and K_S are unstable; the instability of the heavier neutrino can be neglected.

[1)]We shall use the term "neutrino" for neutral leptons whose mass is less than, say, the mass of the electron; all other hypothetical neutral leptons will be called heavy neutral leptons. We also note that a system of units in which $\hbar=c=1$ is employed throughout this paper.

 © 1978 American Institute of Physics

3) Unlike oscillations of neutral kaons, neutrino oscillations may not have the maximum amplitude.

4) Kaons are bosons, but neutrinos are fermions; consequently, kaon oscillations are possible for two states, whereas neutrino oscillations are possible for a minimum of four states.

The first quantitative theory of oscillations was developed in Ref. 2. In constructing the Hamiltonian that violates the lepton numbers in that paper, the basis states were taken to be the two-component neutrino states (the four states for the two types of neutrinos). The neutrinos with definite masses are Majorana neutrinos in this theory.

Recent work on neutrino oscillations is closely related to the unified theory of the weak and electromagnetic interactions due to Weinberg and Salam.[3] It was assumed in Refs. 4 and 5 that the operators ν_e and ν_μ which appear in the charged lepton current are orthogonal combinations of the Dirac neutrino fields with definite non-zero masses. Thus, in addition to the usual hypothesis about the quark-lepton analogy, the authors of these papers also make the new hypothesis that the leptons, like the quarks, are mixed. This theory leads to oscillations $\nu_e \rightleftarrows \nu_\mu$.[5,6] The theory of Ref. 2 is also a scheme with lepton mixing, but its field operators ν_e and ν_μ are orthogonal combinations of the field operators for the massive Majorana neutrinos. The positive neutrino oscillations $\nu_e \rightleftarrows \nu_\mu$ and $\bar{\nu}_e \rightleftarrows \bar{\nu}_\mu$ are described by the same expressions in the two schemes. These expressions contain two parameters—the mixing angle θ and the difference $M^2 = |m_1^2 - m_2^2|$ between the squares of the neutrino masses.

No special searches for neutrino oscillations have been made in the neutrino experiments that have been carried out so far. Nevertheless, by making certain assumptions about the value of the mixing angle θ, we can estimate an upper limit for the parameter M^2 from the data obtained in these experiments. Assuming that the mixing is maximal ($\theta = \pi/4$), we find $M^2 \lesssim 1$ eV^2.

Many experiments to search for oscillations using neutrinos from high-energy accelerators, meson factories, and reactors are either in progress or in preparation at the present time. These experiments will provide a test of the hypothesis of neutrino mixing if $M^2 \gtrsim 10^{-2}$ (eV)^2. If $M^2 \ll 10^{-2}$ eV^2, neutrino oscillations can be observed in cosmic-ray experiments, and in particular in experiments using solar neutrinos.[1] Experiments using solar neutrinos can be used to observe oscillations if $M^2 \gtrsim 10^{-12}$ eV^2.

The neutral current of the Weinberg-Salam theory contains no non-diagonal terms in the lepton fields in schemes involving orthogonal mixing of the leptons. However, non-symmetrical neutral lepton currents effectively occur in higher orders of perturbation theory. This situation is completely analogous to the case of the hadron currents. In analogy with processes such as $K_L \to \mu^+\mu^-$ and $K^+ \to \pi^+\bar{\nu}\nu$, theories with neutrino mixing in principle admit processes such as $\mu \to e\gamma$ and $\mu \to 3e$, in which the lepton numbers are not conserved. However, calculations show that the probabilities of such processes, both in a theory with mixing of Majorana neutrinos and in a theory with mixing of Dirac neutrinos, are tens of orders of magnitude below the experimental upper limits. Thus the search for neutrino oscillations is the only possible method of testing these theories. The high sensitivity of experiments to study neutrino oscillation is due to the interference character of the oscillations phenomenon. These experiments can provide information on the matrix element of the interaction that does not conserve the lepton charges, whereas experiments to measure decay probabilities and reaction cross sections can provide information on the square of the matrix element.

Theories involving mixing of an arbitrary number $N \geqslant 2$ of neutrinos (both Majorana and Dirac neutrinos) with different masses have been considered.[7–9] Experimental searches for neutrino oscillations provide the most sensitive method of testing the mixing hypothesis in this case. If some weak process leads to the production of ν_e, for example, then in a theory with N types of neutrinos the minimum average intensity of ν_e at some distance from the source may be $1/N$ of that expected in the absence of oscillations of intensity. If the energy of the initial ν_e is below the threshold for meson production (as in the case of solar or reactor neutrinos, for example), the $\nu_\mu, \ldots$ appearing as a result of the oscillations are "sterile." As we emphasized above, if there exist in nature only neutrinos, then the probabilities of processes such as $\mu \to e\gamma$ and $\mu \to 3e$, which do not conserve the lepton numbers, are many orders of magnitude below the corresponding experimental upper limit in the mixing schemes. The situation is fundamentally different if there exist heavy leptons with masses of the order of several GeV and if the fields of the heavy leptons appear in the weak current in a mixed form. In this case, the probabilities of processes such as $\mu \to e\gamma$ and $\mu \to 3e$ can be close to the existing experimental upper limits. Searches for $\mu \to e\gamma$ and $\mu \to 3e$ and other processes which are "forbidden" by the lepton-number conservation laws, even at a level close to the existing level, are of great interest.

The plan of this review is as follows. Chapter 2 contains a detailed discussion of the various weak-interaction theories involving mixing of the fields of two neutrinos with non-zero masses. This section also gives the currently available data on tests of the lepton-number conservation laws. Chapter 3 contains a detailed discussion of neutrino oscillations. In Chap. 4, we then consider the possible experiments to search for neutrino oscillations. In connection with the problem of oscillations, we discuss cosmic neutrinos, and in particular solar neutrinos, in Chap. 5. The general case of $N \geqslant 2$ types of neutrinos is considered in Chap. 6. In Chap. 7 we discuss the decay $\mu \to e\gamma$ in schemes involving lepton mixing. Finally, in the concluding section (Chap. 8) we review the "ideology" of lepton mixing.

2. WEAK-INTERACTION THEORIES WITH NEUTRINO MIXING

a) The electronic and muonic lepton numbers

In this section we consider theories in which neutrino oscillations occur. All these theories are based on the

TABLE I. Lepton numbers of the particles.

	ν_e	e^-	ν_μ	μ^-	Hadrons, photon
L_e	1	1	0	0	0
L_μ	0	0	1	1	0

assumption that, in addition to the ordinary weak interaction, there exists an interaction which does not conserve the lepton numbers. In accordance with the existing data, it is also assumed that this last interaction is weaker than the ordinary weak interaction.

The Hamiltonian of the ordinary weak interaction has the form

$$\mathcal{H}_W = \mathcal{H}_W^C + \mathcal{H}_W^0. \tag{1}$$

The first term of this expression is equal to

$$\mathcal{H}_W^C = \frac{G}{\sqrt{2}} 4 j_\alpha \bar{j}_\alpha; \tag{2}$$

here

$$j_\alpha = (\bar{\nu}_{eL}\gamma_\alpha e_L) + (\bar{\nu}_{\mu L}\gamma_\alpha \mu_L) + j_\alpha^h \tag{3}$$

is the charged weak current (j_α^h is the hardonic part of the current), $\psi_L = [(1+\gamma_5)/2]\psi$ is the left-handed component of the operator ψ, $G \approx 10^{-5}/M^2$ is the weak-interaction constant (M is the proton mass), and $\bar{j}_\alpha = j_\alpha^* (\eta_\alpha)$ ($\eta_4 = 1$, $\eta_i = -1$). The second term of the expression (1) is the contribution of the neutral current. Its structure is unimportant for our purposes.

The interaction (2) conserves the individual sums of the electronic lepton number L_e and the muonic lepton number L_μ [2]:

$$\sum_i L_e^{(i)} = \text{const}, \tag{4}$$

$$\sum_i L_\mu^{(i)} = \text{const}. \tag{5}$$

The values of the lepton numbers of various particles are given in Table I. The corresponding antiparticles have the opposite values of the lepton numbers.

There are several types of experiments in which the lepton-number conservation laws are tested. In each experiment, it is attempted to observe nonconservation of the lepton numbers. In Table II we list a number of processes for which searches have been made and which were not detected, and we give the corresponding upper limits on the ratio $W(\mathrm{I})/W(\mathrm{II})$ of the probabilities of the processes which are forbidden (I) and allowed (II) by the lepton-number conservation laws. This table gives the latest data on: a) possible nonconservation of the electronic lepton number; b) possible nonconservation of the muonic lepton number; c) possible nonconservation of both lepton numbers but conservation of their sum; d) possible nonconservation of both lepton numbers.

[2] It is well known that other formulations of the lepton-number conservation law are also possible. The scheme of Refs. 10 and 11 is economic and attractive. In this scheme, there is only one conserved leptonic charge. Here the e^- and μ^- possess opposite values of the lepton number, and the neutrinos are described by a single four-component Dirac spinor.

TABLE II. The ratio $W(\mathrm{I})/W(\mathrm{II})$ of the probabilities of the processes forbidden (I) and allowed (II) by lepton-number conservation.

Process forbidden by lepton-number conservation (I)	Observed process allowed by lepton-number conservation (II)	$\frac{W(\mathrm{I})}{W(\mathrm{II})}$ *)	Confidence limit, %	Ref.
a) Neutrinoless double β decay $^{82}Se \to {}^{82}Kr + e^- + e^-$	Double β decay $^{82}Se \to {}^{82}Kr + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$	$<0,09$	68	12, 13
b) $\nu_\mu + N \to \mu^+ + \ldots$	$\nu_\mu + N \to \mu^- + \ldots$	$<5 \cdot 10^{-3}$	95	14
c) $\mu^+ \to e^+ + \gamma$	$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$	$<2.2 \cdot 10^{-8}$	90	15
$\mu^+ \to e^+ + e^+ + e^-$	$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$	$<1.9 \cdot 10^{-9}$	90	16
$\mu^- + Cu \to e^- + \ldots$	$\mu^- + Cu \to \nu_\mu + \ldots$	$<1.6 \cdot 10^{-8}$	90	17
$\nu_\mu + N \to e^- + \ldots$	$\nu_\mu + N \to \mu^- + \ldots$	$<3 \cdot 10^{-3}$	95	18
d) $\mu^- + Cu \to e^+ + \ldots$	$\mu^- + Cu \to \nu_\mu + \ldots$	$<2.6 \cdot 10^{-8}$	90	17

*) In the literature one often finds a parameter α, which qualitatively characterizes the maximum relative amplitude for a process forbidden by lepton-number conservation; crudely speaking, $\alpha = \sqrt{W(\mathrm{I})W(\mathrm{II})}$ for all cases listed in the table, apart from the double β decay of ^{82}Se. In this case, $\alpha \approx 10^{-3}\sqrt{W(\mathrm{I})/W(\mathrm{II})}$, where the factor 10^{-3} takes into account the fact that the phase space for the neutrinoless double β decay is 10^6 times as large as for "ordinary" double β decay.

It can be seen that the existing experimental data are consistent with (4) and (5).

We shall assume that there exists an interaction which is weaker than the interaction (1) and which does not conserve the lepton numbers. We shall see in what follows that neutrino beams exhibit oscillations in this case. Experiments to search for oscillations therefore provide tests of the hypothesis that there exists an interaction that does not conserve the lepton numbers. These experiments are much more sensitive than experiments to look for the neutrinoless double β decay $\mu \to e\gamma$ [3] and other similar processes.

b) Majorana neutrinos

The first theory of neutrino oscillations was developed in Ref. 2. This theory is based on the theory of the two-component neutrino.

According to the theory of the two-component neutrino, the Hamiltonian can include only the left-handed components of the neutrino fields,

$$\nu_{eL} = \frac{1+\gamma_5}{2}\nu_e, \quad \nu_{\mu L} = \frac{1+\gamma_5}{2}\nu_\mu, \tag{6}$$

and the right-handed components of the antineutrino fields,

$$\begin{aligned} \nu_{eR}^c &= \frac{1-\gamma_5}{2}\nu_e^c = (\nu_{eL})^c, \\ \nu_{\mu R}^c &= \frac{1-\gamma_5}{2}\nu_\mu^c = (\nu_{\mu L})^c; \end{aligned} \tag{7}$$

here

$$\nu_{e,\mu}^c = C\bar{\nu}_{e,\mu} \tag{8}$$

is the charge-conjugate spinor. The matrix C obeys the equations

[3] This statement may be incorrect if there exist heavy leptons (see Sec. 7).

$$\left.\begin{aligned} C^+C&=1,\\ C\gamma_\alpha^T C^{-1}&=-\gamma_\alpha,\\ C^T&=-C.\end{aligned}\right\} \qquad (9)$$

The Hamiltonian which is quadratic in the neutrino fields and which does not conserve the lepton numbers of the interaction has the following general form[4]:

$$\mathscr{H}=m_{\bar{e}e}\bar{\nu}^c_{eR}\nu_{eL}+m_{\bar{\mu}\mu}\bar{\nu}^c_{\mu R}\nu_{\mu L}+m_{\bar{\mu}e}(\bar{\nu}^c_{\mu R}\nu_{eL}+\bar{\nu}^c_{eR}\nu_{\mu L})+\text{h.c.}, \qquad (10)$$

where the parameters $m_{\bar{e}e}$, $m_{\bar{\mu}\mu}$, and $m_{\bar{\mu}e}$ have the dimension of mass.

The Hamiltonian (10) can be written in the more compact form

$$\mathscr{H}=\bar{\nu}^c_R M\nu_L+\bar{\nu}_L M^+\nu^c_R; \qquad (11)$$

here

$$\nu_L=\begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\end{pmatrix},\quad \nu^c_R=\begin{pmatrix}\nu^c_{eR}\\ \nu^c_{\mu R}\end{pmatrix}, \qquad (12)$$

and

$$M=\begin{pmatrix}m_{\bar{e}e} & m_{\bar{\mu}e}\\ m_{\bar{\mu}e} & m_{\bar{\mu}\mu}\end{pmatrix}. \qquad (13)$$

If the interaction (10) is invariant under the CP transformation, then the parameters $m_{\bar{e}e}$, $m_{\bar{\mu}\mu}$, and $m_{\bar{\mu}e}$ are real.[5]

In this case, the interaction Hamiltonian can be written in the form

$$\mathscr{H}=\bar{\nu}^c_R M(\nu_L+\nu^c_R)+\bar{\nu}_L M(\nu^c_R+\nu_L)=\bar{\chi}M\chi; \qquad (14)$$

here

$$\chi=\nu_L+\nu^c_R=\begin{pmatrix}\nu_{eL}+\nu^c_{eR}\\ \nu_{\mu L}+\nu^c_{\mu R}\end{pmatrix}=\begin{pmatrix}\chi_e\\ \chi_\mu\end{pmatrix}. \qquad (15)$$

We have

$$\chi^c=C\bar{\chi}=\chi. \qquad (16)$$

Thus χ_e and χ_μ are Majorana neutrino fields. Clearly, the Hamiltonian (14) contains the Majorana fields because we have introduced an interaction that does not conserve the lepton numbers.

The matrix M can easily be diagonalized. We have

$$M=UM_0U^T, \qquad (17)$$

where U is an orthogonal matrix ($U^TU=1$) and

$$M_0=\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix}. \qquad (18)$$

Substituting (17) in (14), we obtain the following expression for the Hamiltonian:

$$\mathscr{H}=\bar{\varphi}M_0\varphi=\sum_{\sigma=1,2}m_\sigma\bar{\varphi}_\sigma\varphi_\sigma; \qquad (19)$$

here

$$\varphi=U^T\chi. \qquad (20)$$

Thus φ_1 and φ_2 are Majorana neutrino fields with masses m_1 and m_2, respectively. It follows from (20) that

$$\chi=U\varphi. \qquad (21)$$

Hence the fields ν_{eL} and $\nu_{\mu L}$ in the Hamiltonian of the ordinary weak interactions are related to the Majorana neutrino fields by the equations

$$\begin{aligned}\nu_{eL}&=\sum_{\sigma=1,2}U_{1\sigma}\varphi_{\sigma L},\\ \nu_{\mu L}&=\sum_{\sigma=1,2}U_{2\sigma}\varphi_{\sigma L}.\end{aligned} \qquad (22)$$

Thus, if our assumptions are valid, the weak-interaction Hamiltonian contains orthogonal superpositions of Majorana neutrino fields with non-zero masses m_1 and m_2. As we shall see later, this case gives rise to neutrino oscillations.

The orthogonal matrix U has the general form

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}. \qquad (23)$$

Using (20) and (23), we have

$$\begin{aligned}\varphi_1&=\cos\theta\chi_1-\sin\theta\chi_2,\\ \varphi_2&=\sin\theta\chi_1+\cos\theta\chi_2\end{aligned} \qquad (24)$$

and

$$\begin{aligned}\nu_{eL}&=\cos\theta\varphi_{1L}+\sin\theta\varphi_{2L},\\ \nu_{\mu L}&=-\sin\theta\varphi_{1L}+\cos\theta\varphi_{2L}.\end{aligned} \qquad (25)$$

It can be seen from these expressions that the angle θ characterizes the mixing of the Majorana fields φ_1 and φ_2. We now obtain relations which connect the masses m_1 and m_2 and the mixing angle θ to the quantities $m_{\bar{e}e}$, $m_{\bar{\mu}\mu}$, and $m_{\bar{\mu}e}$. Using (17), we find

$$m_{\bar{e}e}=\cos^2\theta m_1+\sin^2\theta m_2, \qquad (26)$$

$$m_{\bar{\mu}\mu}=\sin^2\theta m_1+\cos^2\theta m_2, \qquad (27)$$

$$m_{\bar{\mu}e}=\sin\theta\cos\theta(-m_1+m_2). \qquad (28)$$

These relations give

$$\text{tg}\,2\theta=\frac{2m_{\bar{\mu}e}}{m_{\bar{\mu}\mu}-m_{\bar{e}e}}, \qquad (29)$$

$$m_{1,2}=\frac{1}{2}\left(m_{\bar{e}e}+m_{\bar{\mu}\mu}\pm\sqrt{(m_{\bar{e}e}-m_{\bar{\mu}\mu})^2+4m^2_{\bar{\mu}e}}\right). \qquad (30)$$

As we shall see in Chap. 3, oscillations occur whenever $\theta\neq 0$ and $m_1\neq m_2$. It follows from (27) and (28) that a necessary condition for this is that $m_{\bar{\mu}e}$ and at least

---

[4]It is easy to see that $(\bar{\nu}^c_{\mu R}\nu_{eL}-\bar{\nu}^c_{eR}\nu_{\mu L})=0$. In fact,

$$\bar{\nu}^c_{\mu R}\nu_{eL}=-\nu_\mu C^{-1}\frac{1+\gamma_5}{2}\nu_e=-\nu_e\frac{1+\gamma_5^T}{2}C^{-1}\nu_\mu=\bar{\nu}^c_{eR}\nu_{\mu L}.$$

[5]In fact,

$$\begin{aligned}U_{PC}\bar{\nu}^c_R(x)M\nu_L(x)U^{-1}_{PC}&=U_{PC}\bar{\nu}^c(x)M\frac{1+\gamma_5}{2}\nu(x)U^{-1}_{PC}\\ &=\bar{\nu}(x')\gamma_4 M\frac{1+\gamma_5}{2}\gamma_4\nu^c(x')=\bar{\nu}_L(x')M\nu^c_R(x'),\end{aligned}$$

where U_{PC} is the operator of the PC transformation, and $x'=(-\mathbf{x},ix_0)$. The condition of CP invariance, $U_{PC}\mathscr{H}(x)U^{-1}_{PC}=\mathscr{H}(x')$, therefore gives $M^+=M$. Moreover, since $M^T=M$, we have $M^*=M$.

one of the two parameters $m_{\bar{\nu}_e}$ and $m_{\bar{\nu}_\mu}$ is non-zero. Equation (27) implies that if $m_{\bar{\nu}_e} = m_{\bar{\nu}_\mu}$ and $m_{\bar{\nu}_\mu e} \neq 0$, then $\theta = \pi/4$ (maximal mixing). In this case, we have the relations

$$\nu_{eL} = \frac{1}{\sqrt{2}}(\varphi_{1L} + \varphi_{2L}), \quad \nu_{\mu L} = \frac{1}{\sqrt{2}}(-\varphi_{1L} + \varphi_{2L}), \tag{31}$$

which are analogous to the relations between the wave functions of the K^0 and $\bar{K}^0$ mesons and those of the K_1 and K_2 mesons. We note that the mixing is also maximal in the case when $m_{\bar{\nu}_e}, m_{\bar{\nu}_\mu} \ll m_{\bar{\nu}_\mu e}$.

Let us consider the foregoing scheme from a somewhat different point of view. Our discussion was based on the theory of the two-component neutrino. It is well known that this theory is equivalent to a theory of the Majorana neutrino with zero mass. We can therefore base the disucssion on the theory of the Majorana neutrino. In essence, the introduction of the interaction (10), which does not conserve the lepton numbers, is equivalent to a mixing of massive Majorana neutrino fields. We note that the Majorana neutrinos can differ only in their masses in such a scheme. There is then no need to introduce the concept of lepton number. We shall see that for sufficiently small particle masses this scheme is consistent with the existing experimental data. Its most important consequence is the existence of neutrino oscillations.[2]

c) Dirac neutrinos

We now turn to a discussion of the theory proposed in Refs. 4 and 5. This theory is based on the quark-lepton analogy. We recall that the Cabibbo hadronic current is given by the expression

$$(j_\alpha^h)_C = \bar{u}_L \gamma_\alpha d'_L; \tag{32}$$

here

$$d' = d\cos\theta_C + s\sin\theta_C \tag{33}$$

(θ_C is the Cabibbo angle). In (32) and (33), the symbols u, d, and s denote the field operators for u quarks ($Q = \frac{2}{3}$, $T_3 = \frac{1}{2}$, $S = 0$), d quarks ($Q = -\frac{1}{3}$, $T_3 = \frac{1}{2}$, $S = 0$), and s quarks ($Q = -\frac{1}{3}$, $T = 0$, $S = -1$), respectively. It is well known that all the existing data, including those obtained in experiments on deep inelastic neutrino-nucleon interactions, are consistent with (32).

The Cabibbo hadronic current has the same V − A structure as the lepton current (both currents contain only the left-handed field components). However, if we confine ourselves to Eq. (32) for the hadronic current, there is a significant difference between the weak lepton and hadronic currents. This is so mainly because there are four leptons, while the expression (32) contains only three quark fields. To eliminate this asymmetry, a fourth quark with a new quantum number C (charm) equal to unity was introduced in Ref. 19.[6]

A significant step forward in constructing a theory of the weak interaction was made in Ref. 21a. This work was based on the four-quark hypothesis. The authors assumed that, in addition to the Cabibbo current (32), the charged hadronic current contains a term

$$(j_\alpha^h)_{GIM} = \bar{c}_L \gamma_\alpha s'_L; \tag{34}$$

here

$$s' = -d\sin\theta_C + s\cos\theta_C, \tag{35}$$

and c is the field operator for c quarks ($Q = \frac{2}{3}$, $T = 0$, $S = 0$, $C = 1$). (This is now known as the standard theory.)

If we take the charged hadronic current in the form

$$j_\alpha^h = (j_\alpha^h)_C + (j_\alpha^h)_{GIM}, \tag{36}$$

then, as was shown in Ref. 21a, the neutral weak current of the Weinberg-Salam gauge theories of the weak and electromagnetic interactions contains no strangeness-changing terms. Since the charged current (36) changes strangeness, a strangeness-changing neutral current appears in higher orders of perturbation theory. The standard theory with the current (36) contains a compensating mechanism[21a] which makes it possible to obtain agreement between the calculated values of the K_L-K_S mass difference and the probabilities for processes such as $K^+ \to \pi^+ \bar{\nu}\nu$ and $K_L \to \mu^+\mu^-$ and their experimental values. We note that the cancellation of the diagrams which induce the strangeness-changing neutral current occurs because the d and s quark fields appear in the charged current in the orthogonal combinations (33) and (35).

Let us now return to a discussion of the leptons. By comparing the current (36) of the standard theory with the usual lepton current (Eq. (3)), we can see that there is a significant difference between these expressions. The distinction is that the hadronic current contains orthogonal combinations of the d and s quark fields, while the current (3) contains the electronic and muonic neutrino fields (the hadronic current does not conserve strangeness, while the current (3) conserves the lepton numbers).

To eliminate this distinction, let us assume[4,5] that there exist two neutrinos with non-zero masses (which we shall denote by ν_1 and ν_2) and that the operators ν_e and ν_μ in Eq. (3) for the lepton current are orthogonal superpositions[7]

$$\nu_e = \nu_1\cos\theta + \nu_2\sin\theta, \quad \nu_\mu = -\nu_1\sin\theta + \nu_2\cos\theta, \tag{37}$$

where ν_1 and ν_2 are Dirac neutrino fields with masses m_1 and m_2, and θ is a mixing angle.[8] *A priori*, there is no reason to assume that the mixing angle θ is equal

[6]This problem is discussed in greater detail in the review of Ref. 20. Particles possessing the new quantum number have now been observed in neutrino experiments and in e^+e^- colliding-beam experiments (see e.g., the Proc. of the 18th Conf. on High Energy Physics, Tbilisi, 1976).

[7]It became known to us quite recently that lepton and hadron mixing had already been proposed[21b] in 1963.

[8]We note that the lepton mixing angle is denoted by θ and the neutrino masses by m_1 and m_2, both in the case of mixing of Majorana fields (see Sec. 2b) and in the case of mixing of Dirac fields.

to the Cabibbo angle θ_C. We note only that the two values $\theta=0$ and $\theta=\pi/4$ of the mixing angle have a special significance. The case $\theta=0$ (no mixing) corresponds to the usual theory with strictly conserved electronic and muonic lepton numbers. The case $\theta=\pi/4$ corresponds to maximal mixing, i.e., the maximum amplitude for the oscillations (see Chap. 3).

Thus we have a theory in which the weak currents of the leptons and quarks are completely analogous. In essence, the scheme involves no lepton number which distinguishes the two types of neutrino ν_e and ν_μ; the two neutrinos ν_1 and ν_2 are distinguished from one another only by their masses (like the d and s quarks).

It follows from (37) that the neutral current of the Weinberg-Salam theory has no asymmetric terms (processes $\mu\to e\gamma$, $\mu\to 3e$, etc.) to first order in G. In higher orders of perturbation theory in our scheme, such processes become possible (in analogy with the processes $K_L\to 2\mu$, $K\to\pi\bar{\nu}\nu$, etc.). However, calculations show[22] that the probabilities of these processes are extremely small in the two-neutrino scheme, owing to a compensation mechanism analogous to the GIM mechanism[21] (see Chap. 7 for further details).

The principal distinction between the theory considered here and the usual theory with two conserved lepton numbers is the possible existence of neutrino oscillations in the former.[5,6]

d) Comparison of the theories with neutrino mixing

We conclude this part of our review with some remarks concerning the comparison of the theory with mixing of Majorana neutrinos (the M theory) considered in Chap. 2b and the theory with mixing of Dirac neutrinos (the D theory) considered in Chap. 2c.

1) In the schemes discussed above in Chaps. 2b and 2c, the neutrino masses are non-zero and the neutrino field operators appear in the Hamiltonian in the form of orthogonal superpositions. The difference between these theories is that the neutrinos are Majorana particles in the M theory and Dirac particles in the D theory. The Hamiltonian (10) of the M theory does not conserve the muonic and electronic lepton numbers L_μ and L_e. The Hamiltonian of the D theory conserves the sum L_e+L_μ. As we shall see later, neutrino oscillations are described by identical expressions in the two theories. Unlike the D theory, the M theory admits neutrinoless double β decay and other processes in which L_e+L_μ is not conserved. However, the probabilities of such processes are very small, and the existing experimental data enable us to obtain only (very weak) bounds[23] on the parameters of the theory.

We note that the neutrinos of the D scheme are described in the same way as all the other particles (charged leptons and quarks), whereas the neutrinos of the M scheme occupy a special place among the set of fundamental particles. In the M theory, each type of neutrino is described by two states; in the D theory, there are four states for each type of neutrino. In this sense, there is no lepton-quark analogy in the M theory. In particular, it is easy to see[2] that the M theory is equivalent to a scheme[10,11] with only a single lepton charge, and this charge is not conserved (see the footnote[2)]).

2) The mass term in the Hamiltonian of the D theory has the form

$$\mathscr{H}_1=m_1\bar{\nu}_1\nu_1+m_2\bar{\nu}_2\nu_2. \tag{38}$$

Let us express ν_1 and ν_2 in terms of ν_e and ν_μ (see (37)) and substitute the results in Eq. (38). This gives

$$\mathscr{H}_1=m_{ee}\bar{\nu}_e\nu_e+m_{\mu\mu}\bar{\nu}_\mu\nu_\mu+\mathscr{H}; \tag{39}$$

here

$$\mathscr{H}=m_{\mu e}(\bar{\nu}_\mu\nu_e+\bar{\nu}_e\nu_\mu), \tag{40}$$

and

$$\left.\begin{aligned} m_{ee}&=m_1\cos^2\theta+m_2\sin^2\theta,\\ m_{\mu\mu}&=m_1\sin^2\theta+m_2\cos^2\theta,\\ m_{\mu e}&=\sin\theta\cos\theta(-m_1+m_2). \end{aligned}\right\} \tag{41}$$

Clearly, m_{ee} and $m_{\mu\mu}$ are the bare masses of the electronic and muonic neutrinos. The Hamiltonian $\mathscr{H}$ does not conserve L_e and L_μ individually, but it conserves L_e+L_μ. Thus, if we assume that the masses of the electronic and muonic neutrinos are non-zero and that, in addition to the ordinary weak interaction, there exists an interaction (40) which does not conserve the lepton numbers, we obtain the D theory.

The masses m_1 and m_2 and the mixing angle θ are obviously related to the quantities m_{ee}, $m_{\mu\mu}$, and $m_{\mu e}$ by equations analogous to (29) and (30):

$$\begin{aligned} \operatorname{tg}2\theta&=\frac{2m_{\mu e}}{m_{\mu\mu}-m_{ee}},\\ m_{1,2}&=\frac{1}{2}\left(m_{ee}+m_{\mu\mu}\pm\sqrt{(m_{ee}-m_{\mu\mu})^2+4m_{\mu e}^2}\right). \end{aligned} \tag{42}$$

We have considered the simplest theories with mixing of two neutrino fields with different masses. Mixing of $N\geq 2$ types of neutrinos was studied in Refs. 7–9. The results obtained in these papers will be discussed briefly in Chap. 5.

3. NEUTRINO OSCILLATIONS

The theories considered above lead to neutrino oscillations. We shall now discuss this phenomenon.

We shall write $|\nu_e\rangle$ and $|\nu_\mu\rangle$ for the state vectors of the electronic and muonic neutrino (the neutrinos which take part in the ordinary weak interaction) with momentum $\mathbf{p}$ and helicity -1. It follows from (22) and (37) that

$$|\nu_l\rangle=\sum_{\sigma=1,2}U_{l\sigma}|\nu_\sigma\rangle\qquad(l=e,\mu), \tag{43}$$

where $|\nu_\sigma\rangle$ $(\sigma=1,2)$ is the state vector of a neutrino with mass m_σ, momentum $\mathbf{p}$, and helicity -1 (we are assuming that $p\gg m_\sigma$). The orthogonal matrix U has the form (23). The vectors $|\nu_\sigma\rangle$ describe Majorana neutrinos in the theory of Ref. 2 and Dirac neutrinos in the theory of Refs. 4 and 5. We have

$$H|\nu_\sigma\rangle=E_\sigma|\nu_\sigma\rangle, \tag{44}$$

where H is the complete Hamiltonian, and

$$E_\sigma = \sqrt{m_\sigma^2 + \mathbf{p}^2}.$$

In addition,

$$|\nu_\sigma\rangle = \sum_{l=e,\mu} U_{l\sigma} |\nu_l\rangle. \tag{45}$$

Consider the behavior of a neutrino beam produced as a result of an ordinary weak process. At the initial instant of time, such a beam is described by the vector $|\nu_l\rangle$. At time t, the state vector of the beam is given by the expression

$$|\nu_l\rangle_t = e^{-iHt} |\nu_l\rangle = \sum_{\sigma=1,2} U_{l,\sigma} e^{-iE_\sigma t} |\nu_\sigma\rangle. \tag{46}$$

We note that the neutrinos are assumed to be stable particles here. In the schemes which we are considering, however, the heavier neutrino (say, the ν_1) can decay into a ν_2 and a photon. The probability of this decay, calculated according to the theory with neutrino mixing, is given by the expression[22]

$$\Gamma(\nu_1 \to \nu_2 + \gamma) = \frac{9}{16} \frac{G^2}{128\pi^4} \alpha m_1^5 \sin^2\theta \cos^2\theta \left(\frac{m_\mu^2}{M_W^2}\right)^2;$$

here m_μ and M_W are the masses of the muon and the intermediate boson, respectively (for simplicity, we have assumed that $m_1 \gg m_2$). It is easy to see from this expression that the neutrino lifetime is much greater than the age of the Universe. Thus the instability of the neutrino can be neglected.[9)]

Let us now return to our discussion of the behavior of a neutrino beam. It can be seen from (46) that in the theories considered here such a beam is described not by a stationary state, as in the usual theories, but by a superposition of stationary states. This is obviously due to the fact that the vector $|\nu_l\rangle$ is not an eigenvector of the Hamiltonian H.

Let us expand the state vector (46) in terms of the vectors $|\nu_{l'}\rangle$. This gives

$$|\nu_l\rangle = \sum_{l'=e,\mu} a_{\nu_{l'};\nu_l}(t) |\nu_{l'}\rangle, \tag{47}$$

where

$$a_{\nu_{l'};\nu_l}(t) = \sum_{\sigma=1,2} U_{l\sigma} e^{-iE_\sigma t} U_{l'\sigma} \tag{48}$$

is the probability amplitude for observing $\nu_{l'}$ at time t after the production of the ν_l. We have

$$a_{\nu_{l'};\nu_l}(0) = \sum_{\sigma=1,2} U_{l\sigma} U_{l'\sigma} = \delta_{l'l}.$$

Clearly, if $m_1 \neq m_2$ and $U_{l\sigma} \neq \delta_{l\sigma'}$, then $a_{\nu_\mu,\nu_e}(t) = a_{\nu_e,\nu_\mu}(t) \neq 0$ (i.e., the oscillations $\nu_e \rightleftarrows \nu_\mu$ occur).

The probability of the "transition" $\nu_l \to \nu_{l'}$ is given by the expression

$$w_{\nu_{l'};\nu_l}(t) = w_{\nu_l;\nu_{l'}}(t) = \sum_{\sigma,\sigma'} U_{l\sigma} U_{l'\sigma} U_{l\sigma'} U_{l'\sigma'} \cos(E_\sigma - E_{\sigma'}) t. \tag{49}$$

It is easy to see that the quantities $w_{\nu_{l'};\nu_l}(t)$ satisfy the relation

$$\sum_{l'} w_{\nu_{l'};\nu_l}(t) = 1.$$

Moreover, in the case $p \gg m_1, m_2$ in which we are interested, we have

$$E_1 - E_2 = \frac{m_1^2 - m_2^2}{2p}. \tag{50}$$

Using (23), (49), and (50), we obtain

$$w_{\nu_e;\nu_e}(R) = w_{\nu_\mu;\nu_\mu}(R) = 1 - \frac{1}{2}\sin^2 2\theta \left(1 - \cos 2\pi \frac{R}{L}\right), \tag{51}$$

$$w_{\nu_e;\nu_\mu}(R) = w_{\nu_\mu;\nu_e}(R) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos 2\pi \frac{R}{L}\right); \tag{52}$$

here $w_{\nu_{l'};\nu_l}(R)$ is the probability of observing $\nu_{l'}$ at a distance R from the source of ν_l, and

$$L = 4\pi \frac{p}{|m_1^2 - m_2^2|} \tag{53}$$

is the wavelength of the oscillations.[24] We note that the quantities $\sin^2 2\theta$ and L are related to the parameters $m_{\bar{e}e}$, $m_{\bar{\mu}\mu}$, and $m_{\bar{\mu}e}$ of the theory of Ref. 2 by the equations

$$\sin^2 2\theta = \frac{4m_{\bar{\mu}e}^2}{(m_{\bar{e}e} - m_{\bar{\mu}\mu})^2 + 4m_{\bar{\mu}e}^2}, \qquad L = \frac{4\pi p}{(m_{\bar{e}e} + m_{\bar{\mu}\mu})\sqrt{(m_{\bar{e}e} - m_{\bar{\mu}\mu})^2 + 4m_{\bar{\mu}e}^2}}. \tag{54}$$

Clearly, the relations between these quantities and the parameters of the theory of Refs. 5 and 6 can be obtained by making the following substitutions in (54):

$$m_{\bar{e}e} \to m_{ee}, \quad m_{\bar{\mu}\mu} \to m_{\mu\mu}, \quad m_{\bar{\mu}e} \to m_{\mu e}.$$

We note also that the basic relations of this section (Eqs. (47)–(49)) also hold in the general case of N types of neutrinos. In this case, U is an orthogonal $N \times N$ matrix (if CP invariance holds), and the summation over σ goes from 1 to N.

4. POSSIBLE EXPERIMENTAL TESTS OF THE NEUTRINO MIXING HYPOTHESIS

a) General considerations

In this section, we shall make use of Eqs. (51) and (52), obtained for the case of two types of neutrinos. The general case of N types of neutrinos will be discussed briefly in Chap. 6.

First of all, we note that the observation of neutrino mixing effects (which we shall also call oscillation effects) includes both the observation of the sinusoidal term in the neutrino intensity and the confirmation that the constant term in $w_{\nu_l;\nu_l}(R)$ is different from unity or that the constant term in $w_{\nu_{l'};\nu_l}(R)$ $(l' \neq l)$ is different from zero (see (51) and (52)). To observe the sinusoidal term, it is necessary that this term does not vanish after averaging over the dimensions of the neutrino source, the time of the measurement, the dimensions of the detector, and the spectrum of neutrino momenta. In particular, it is obvious that a necessary (but not suf-

---

9) We note that if the lifetime of the ν_1 were less than the age of the Universe, the decay $\nu_1 \to \nu_2 + \gamma$ would lead to the production of neutrinos described by a stationary state.

TABLE III. Values of the parameter M^2_{min} (the parameter characterizing the sensitivity of experiments to search for oscillations) for various neutrino sources*).

Neutrino source	p_{min}, MeV	R_{max}, m	M^2_{min}, eV^2
Reactor	1	10^2	$3\cdot10^{-2}$
Meson factory	10	10^2	$3\cdot10^{-1}$
High-energy accelerator	10^3	10^4	$3\cdot10^{-1}$
The Sun	$2\cdot10^{-1}$	$1\cdot5\cdot10^{11}$	$4\cdot10^{-12}$

*)The inequalities (55) and (59) are highly qualitative. It should also be borne in mind that the values of p_{min} and R_{max} (and hence M^2_{min}) are provisional and reflect our apprasial of the experimental situation.

ficient) condition for the observation of the sinusoidal term is that the dimensions of the neutrino source are small in comparison with the wavelength of the oscillations and that the dispersion in the time of neutrino emission is small in comparison with the period of the oscillations.

Now it can be seen from Eqs. (51) and (52) that if the wavelength L of the oscillations is much greater than the distance R between the source and the detector, then

$$w_{\nu_{l'};\nu_l}(R)\approx 0 \quad (l'\neq l) \text{ and } w_{\nu_l;\nu_l}(R)\approx 1.$$

Thus any effects due to oscillations can be observed whenever

$$L\leqslant R. \tag{55}$$

Let us introduce the quantity

$$M^2=|m_1^2-m_2^2|. \tag{56}$$

Equation (53) for the wavelength of the oscillations can be rewritten in the form

$$L=4\pi\frac{p}{M^2}. \tag{57}$$

Substituting the numerical value of $\hbar c$ in (57), we obtain

$$L=2.5\,\frac{p(\mathrm{MeV})}{M^2(\mathrm{eV}^2)}\ \mathrm{m}. \tag{58}$$

If, for example, $M^2=1$ eV^2, then at neutrino momenta equal to 1 MeV, 10 MeV, and 1 GeV the wavelength of the oscillations is equal to 2.5 m, 25 m, and 2.5 km, respectively.

It can be seen from (55) and (57) that the oscillation effects can be observed when

$$M^2\geqslant 4\pi\frac{p}{R}. \tag{59}$$

Thus experiments to search for neutrino oscillations should be carried out at the "minimum" possible neutrino momentum p_{min} in a given experiment and the "maximum" possible distance R_{max} between the neutrino source and detector. In order to characterize the sensitivity of various experiments to search for oscillations, we introduce the parameter

$$M^2_{min}=4\pi\frac{p_{min}}{R_{max}}. \tag{60}$$

Neutrino oscillation effects can in principle be observed whenever $M^2\gtrsim M^2_{min}$.

In Table III we show the values of the parameter M^2_{min} for various neutrino sources.[25]

It should be stressed, however, that this table is merely illustrative. In addition, we remark that Table III is very "conservative." For example, it was proposed in Ref. 26 to look for oscillations in an experiment to observe neutrinos produced at the Batavia accelerator by means of a detector situated in Canada. In this experiment, $R\approx10^3$ km (to be compared with $R_{max}\approx10$ km in Table III).

b) Limits on the parameter M^2 obtained from the existing data

We now consider what limits on the parameter $M^2=|m_1^2-m_2^2|$ can be obtained from the currently available data. It should be emphasized that these data do not come from experiments designed to search for oscillations.

In Ref. 27 measurements of the cross section for the process

$$\bar\nu_e+p\to e^++n \tag{61}$$

induced by antineutrinos from a reactor gave the result

$$\frac{\sigma_{exp}}{\sigma_{theor}}=0.88\pm0.13\ ; \tag{62}$$

here σ_{exp} is the experimentally measured cross section for the process (61), and σ_{theor} is the cross section expected from the V – A theory. For simplicity, let us consider the case of maximal mixing ($\theta=\pi/4$). For monoenergetic antineutrinos, we find in this case from (51) that

$$\frac{I_{\bar\nu_e;\bar\nu_e}(R,p)}{I^0_{\bar\nu_e;\bar\nu_e}(R,p)}=\frac{1}{2}\left(1+\cos2\pi\frac{R}{L}\right), \tag{63}$$

where $I_{\bar\nu_e;\bar\nu_e}$ is the $\bar\nu_e$ intensity, and $I^0_{\bar\nu_e;\bar\nu_e}$ is the expected intensity in the absence of oscillations. In the experiment of Ref. 27, $R\sim10$ m and the effective momenta of $\bar\nu_e$ from the reactor are several MeV. Using (62) and (63) and taking into account the form of the antineutrino spectrum, we find the estimate

$$M^2\leqslant 1\ \mathrm{eV}^2. \tag{64}$$

We can also estimate an upper limit on the parameter M^2 by making use of experimental data obtained with a high-energy neutrino beam. If oscillations occur, a beam of "muonic" neutrinos is a mixture of both ν_μ and ν_e. Using (51) and (52), we find that the ratio of the ν_e and ν_μ intensities is givenby the expression

$$\frac{I_{\nu_e;\nu_\mu}(R,p)}{I_{\nu_\mu;\nu_\mu}(R,p)}\approx\frac{1}{2}\sin^2 2\theta\left(1-\cos2\pi\frac{R}{L}\right). \tag{65}$$

Data obtained in the Gargamelle bubble chamber using neutrinos from the 30-GeV CERN accelerator were re-

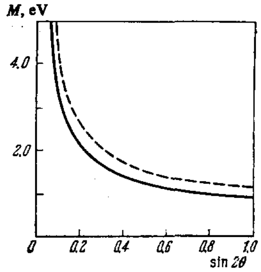

FIG. 1. The upper limit on the parameter $M=\sqrt{|m_0^2-m_2^2|}$ for various values of the mixing angle θ (m_1 and m_2 are the masses of the neutrinos ν_1 and ν_2[18]). The upper and lower curves correspond to the confidence levels 95 and 68%, respectively.

cently analyzed in Ref. 18. Taking into account a possible background from K_{e3} decays, the authors of this paper showed that the intensity of electronic neutrinos which might arise from oscillations does not exceed 0.3% of the ν_μ intensity (at the 95% confidence level). Figure 1 shows the upper limit on the parameter M as a function of $\sin 2\theta$ (the upper and lower curves correspond to the 95% and 68% confidence levels).

We conclude with a few words about an experiment[28,29] involving solar neutrinos. As we shall see in Chap. 5, neutrino oscillations are by no means ruled out by the currently available data. This means that no upper limits on the parameter M^2 can be obtained from these experimental data at the present time. The possibility of obtaining a lower limit on M^2 from this experiment will be discussed in Chap. 5.

c) Possible experiments

We now turn to the question of what special types of experiments can be used to study neutrino oscillations.

We recall that (see (51))

$$I_{\nu_l;\nu_l}(R,p)=\left[1-\frac{1}{2}\sin^2 2\theta\left(1-\cos 2\pi\frac{R}{L}\right)\right]I^0_{\nu_l}(R,p), \qquad (66)$$

where $I_{\nu_l;\nu_l}(R,p)$ is the intensity of ν_l ($l=e,\mu$) with momentum p at distance R from the source of ν_l, and $I^0_{\nu_l}(R,p)$ is the expected intensity of ν_l in the absence of oscillations. Similarly, we have (see (52))

$$I_{\nu_{l'};\nu_l}(R,p)=\frac{1}{2}\sin^2 2\theta\left(1-\cos 2\pi\frac{R}{L}\right)I^0_{\nu_l}(R,p) \quad (l'\neq l;\ l,\ l'=e,\ \mu); \qquad (67)$$

here $I_{\nu_{l'};\nu_l}(R,p)$ is the intensity of $\nu_{l'}$ at distance R from the source of ν_l.

One of the possible methods of looking for neutrino oscillation effects is to compare the averaged (see below) intensity of neutrinos of a given type with the intensity expected in the absence of oscillations. In this case, the sinusoidal term in Eqs. (66) and (67) vanishes after averaging over the neutrino spectrum, the region in which the neutrinos are produced, etc. Such experiments can be carried out by detecting either neutrinos of the same type as the emitted neutrinos or those of the opposite type.

We find that the average intensities are given by

$$\bar{I}_{\nu_l;\nu_l}=\delta_{\nu_l;\nu_l}\bar{I}^0_{\nu_l}, \qquad (68)$$

$$\bar{I}_{\nu_{l'};\nu_l}=\delta_{\nu_{l'};\nu_l}\bar{I}^0_{\nu_l} \quad (l'\neq l); \qquad (69)$$

here

$$\delta_{\nu_l;\nu_l}=1-\frac{1}{2}\sin^2 2\theta, \qquad (70)$$

$$\delta_{\nu_{l'};\nu_l}=\frac{1}{2}\sin^2 2\theta \quad (l'\neq l). \qquad (71)$$

For maximal mixing ($\theta=\pi/4$), the coefficient $\delta_{\nu_l;\nu_l}$ takes the minimum value

$$(\delta_{\nu_l;\nu_l})_{\min}=\frac{1}{2}, \qquad (72)$$

while $\delta_{\nu_{l'};\nu_l}$ takes the maximum value

$$(\delta_{\nu_{l'};\nu_l})_{\max}=\frac{1}{2} \quad (l'\neq l). \qquad (73)$$

The relation $\bar{I}_{\bar\nu_e;\bar\nu_e}=\delta_{\bar\nu_e;\bar\nu_e}\bar{I}^0_{\bar\nu_e}$ can be tested in experiments using low-energy (~1 MeV) antineutrinos obtained from a reactor. If it is found that the observed intensity $\bar{I}_{\bar\nu_e;\bar\nu_e}$ is less than the expected intensity $\bar{I}^0_{\bar\nu_e}$, this will indicate that neutrino mixing occurs. In this case, the mixing angle θ can be determined from Eq. (70). If, on the other hand, the intensity $\bar{I}_{\bar\nu_e;\bar\nu_e}$ is equal to $\bar{I}^0_{\bar\nu_e}$, we can of course not conclude that there is no mixing; this result might occur not only because $\theta=0$, but also because the wavelength of the oscillation is much greater than the distance from the source to the detector. It has been proposed to carry out experiments to search for electrons in beams of neutrinos obtained from the decays of pions and kaons (at sufficiently "large" distances from the source to the detector), using high-energy accelerators and meson factories. If these experiments detect electrons which cannot be explained by various "trivial" backgrounds (K_{e3}, π^0, etc.), this would indicate the presence of neutrino mixing and it would be possible to determine the mixing angle from Eq. (71).[10] If, on the other hand, no electrons are observed, then no conclusions can be drawn about the absence of oscillations and it is only possible to obtain limits on the parameter M^2 (see, e.g., Fig. 1).

As we have already pointed out, oscillations can be observed in experiments using neutrinos from reactors and accelerators if $M^2 \gtrsim 10^{-2}$ eV^2. If $M^2 \ll 10^{-2}$ eV^2, the best prospects for observing neutrino oscillations might lie in detecting cosmic neutrinos, and in particular solar neutrinos. A detailed discussion of the problem of solar neutrinos is given in Chap. 5.

The experiments discussed so far can at best be used to establish the presence of neutrino mixing and to determine the mixing angle θ. If the mixing effect is actually found in such experiments, we would be led to the very interesting problem of obtaining information about the parameter $M^2=|m_1^2-m_2^2|$. To do this, it would be necessary to observe the sinusoidal term in either (66) or (67). This would make it possible to determine the wavelength of the oscillations and hence M^2.[11]

[10] If there exist in nature $N>2$ types of neutrinos, then neutrino mixing is described by an $N\times N$ matrix and Eq. (70) is replaced by Eq. (87) (see Sec. 6). Our discussion in this subsection is limited to the simplest case of two neutrinos.

[11] In experiments using neutrinos from accelerators or reactors, it is perfectly feasible and not too difficult to determine the wavelength of the oscillations (if the observed and expected intensities are found to be different). In the case of experiments using solar neutrinos, however, the problem of determining the wavelength of the oscillations is incomparably more difficult than that of establishing the presence of mixing (see Sec. 6 for the details).

We first discuss the possibility of performing such experiments using antineutrinos from a reactor.[25] We shall assume that the wavelength of the oscillations is much greater than the dimensions of the reactor. Since the wavelength of the oscillations depends on the antineutrino momentum, the spectrum $N(R,p)$ of antineutrinos at a distance R from the source is different from the spectrum expected in the absence of oscillations. Information about the spectrum $N(R,p)$ can be obtained by measuring the spectrum of positrons in the process $\bar{\nu}_e+p\to e^+ +n$. On the other hand, the spectrum of antineutrinos in the region in which they are produced can be determined sufficiently accurately from measurements of the spectrum of electrons emitted by the fission products under saturation conditions. A comparison of these spectra would make it possible to determine $L(p)$.

Information about the wavelength of the oscillations can also be obtained by measuring the antineutrino intensity, averaged over a relatively narrow part of the spectrum, at various distances from the reactor.

Next we discuss possible experiments using meson factories and high-energy accelerators that could provide a determination of the sinusoidal term in (66) and (67). We shall assume that the effective dimensions of the neutrino source are much smaller than the wavelength of the oscillations. A good method would be to measure the ratio $r(p)$ of the number of electrons to the number of muons as a function of the momentum p in neutrino processes at a definite distance from the source of ν_μ. If the intensity of ν_e in the region in which neutrinos are produced can be neglected, then we find from (66) and (67) that the ratio $r(p)$ is given by

$$r(p)=\frac{(1/2)\sin^2\theta\,[1-\cos a\,(1/p)]}{1-(1/2)\sin^2\theta\,[1-\cos a\,(1/p)]}, \tag{74}$$

where

$$a=\frac{1}{2}R\,|m_1^2-m_2^2|. \tag{75}$$

It would also be useful to measure the ratio $r(p)$ at various distances R. We note that a measurement of the neutrino intensity, averaged over a sufficiently narrow part of the spectrum, at various distances R may also make it possible to obtain information about the parameter M^2.

Let us now list the advantages and disadvantages of experiments to look for oscillations using meson factories (proton energies in the range 500–800 MeV) and high-energy accelerators (proton energies of hundreds of GeV). The advantages of experiments using meson factories are as follows[12]: 1) the small dimensions of the region in which the neutrinos are produced; 2) the small wavelengths of the oscillations (small neutrino momenta); 3) the fact that the appearance of electrons is practically direct evidence for the presence of oscillations (there is not enough energy for π^0-meson production). A disadvantage of such experiments is the fact that it is impossible to make a direct measurement of $I^0_{\nu_\mu}$ (the sterility of the ν_μ with energies below the muon production threshold).

The advantages of experiments to look for oscillations using high-energy accelerators are: 1) the large distances between the neutrino source and detector; 2) the possibility of directly measuring $I^0_{\nu_\mu}$; 3) the possibility of producing heavy charged leptons as a result of the oscillations (if there exist new types of neutrinos in the weak current in addition to the charged heavy leptons and if all the neutrino fields are mixed). The disadvantages of such experiments are: 1) the large wavelength of the oscillations (large neutrino momenta); 2) the background of π^0 mesons.

We conclude with a few remarks about some specific experimental projects to search for and study neutrino oscillations that are known to us. Such experiments using reactors are planned in at least three laboratories: at the I. V. Kurchatov Institute of Atomic Energy,[30b] at the Laue-Langevin Institute in Grenoble,[31] and in the University of California at Irvine.[32] An experiment to search for oscillations will be carried out using the Brookhaven accelerator.[33] The originality of this experiment is that it is proposed to use the Brookhaven accelerator as a meson factory (with a proton energy of only 800 MeV and a proton beam intensity of 10^{14} p/sec).

Finally, we mention a paper[26] in which it is proposed to place a neutrino detector in Canada, at a distance ~1000 km from the accelerator at Fermilab (Batavia). This experiment is of definite interest not only in connection with the problem of neutrino oscillations, but also because it makes it possible in principle to measure a distance between two points of the Earth comparable with the Earth's radius.

5. THE SUN AND COSMIC NEUTRINOS

a) The Brookhaven experiment

In this section, we consider various possible methods of searching for oscillations in experiments using cosmic neutrinos. We begin with solar neutrinos. It can be seen from Table III that experiments using solar neutrinos provide an extremely sensitive method of searching for neutrino oscillations. This is so because the energies of solar neutrinos are small and the distance between the Earth and the Sun is great.

Over a period of many years, Davis *et al.* (see Ref. 28 and the references cited therein) have been carrying out an experiment to search for solar neutrinos. This experiment makes use of the chlorine-argon method.[34] In particular, a study is made of the reaction

$$\nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}. \tag{76}$$

The ^{37}Ar produced in this process undergoes K-capture with a period of 35.1 days. The detector contains 3.8×10^5 liters of C_2Cl_4 (2.2×10^{30} atoms of ^{37}Cl) and is situated at a depth of 4400 m below sea level in a gold mine

[12] We have in mind beams of ν_μ obtained at meson factories from the decays of pions in flight (and not from stopped pions), in particular from the decays of pions with an energy of several hundred MeV captured by a magnetic trap (see Ref. 30a).

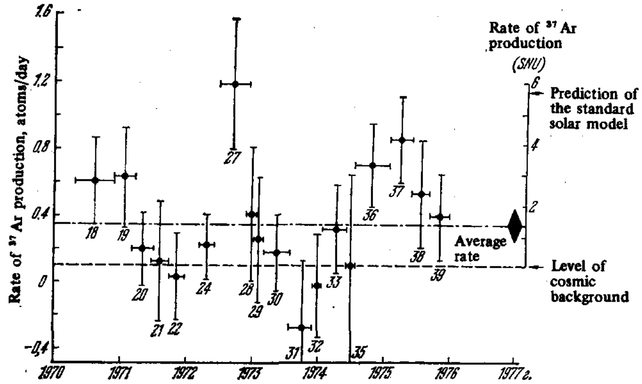

FIG. 2. The rate of ^{37}Ar production in the Brookhaven chlorine-argon experiment to detect solar neutrinos.[28,29] The solar neutrino unit is

$$1\ \mathrm{SNU} = 10^{-36}\frac{\text{events of } \nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}}{\text{atoms of } {}^{37}\mathrm{Cl} \cdot \mathrm{sec}}.$$

The axis at the left gives the rate of ^{37}Ar production per day within the entire volume of the detector ($3.8 \times 10^5 C_2Cl_4$).

in South Dakota (U.S.A.) The argon is extracted from the detector by passing helium through it, and it is then separated from the helium and put in a proportional counter of small dimensions, in which practically every ^{37}Ar K-capture event is detected. We shall say a few words about the advantages of the chlorine-argon method:

1) It is a simple matter to extract several atoms of ^{37}Ar from a large quantity of C_2Cl_4 (we stress that, despite this "simplicity," the experiment of Davis *et al.* requires heroic efforts).

2) C_2Cl_4 is an inexpensive, relatively non-toxic, and non-inflammable liquid.

3) The K-capture of ^{37}Ar is accompanied by an energy release equal to 2.8 keV; this makes it possible to use a proportional counter having a small background as the detector.[35] 13) The low background is achieved not only by measuring the amplitudes of the pulses in the proportional counter, but also by measuring the shape of the pulses. The signal corresponding to decay of ^{37}Ar is characterized by a rapid growth, owing to the fact that the corresponding ionization is well localized (in contrast with the ionization of the background processes).[36] At the present time, the effective background of the counter in the Brookhaven experiment is 0.5 counts in 35 days within the range 1.5–5 keV and with discrimination in the form of the pulse. Each exposure and period of measuring the radioactivity in the counter lasted several months during a period of five years (1970–1975). An exposure now lasts about a month.

b) Results of the experiment

The results obtained in the experiment of Davis *et al.* are shown in Fig. 2. The vertical axis at the left gives the number of ^{37}Ar atoms produced per day within the entire volume of the detector. The vertical axis at the right gives the same quantity in so-called solar neutrino units SNU (where 1 SNU $= 10^{-36}$(events of $\nu_e + {}^{37}\mathrm{Cl} \to e^- + {}^{37}\mathrm{Ar}$)/(atoms of ^{37}Cl$\cdot$ sec)), in which the theoretical results are usually given. The value calculated on the basis of the standard solar model (see Refs. 29 and 37) is indicated by an arrow at the right. The expected background level of cosmic rays and the average rate of ^{37}Ar production obtained by averaging over all the measurements are also shown in Fig. 2.

The experimental results[28] are given in Table IV. Expressing the average rate of ^{37}Ar production given in Table IV in solar neutrino units, we have

$$1.3 \pm 0.4\ \mathrm{SNU}$$
(1970–1975)

This value should be compared with the quantity

$$\int \varphi(E)\,\sigma(E)\,dE = 6 \pm 2\ \mathrm{SNU}, \tag{77}$$

calculated (see Refs. 29 and 37) on the basis of the standard solar model. In this expression, $\varphi(E)$ is the flux of solar neutrinos of energy E, and $\sigma(E)$ is the cross section for the reaction (76).

c) Conclusions

What conclusions can be drawn from these data? On the basis of the measurements that have been made, Davis *et al.* are of the opinion that in view of the uncertainties due to the background processes and other uncertainties this result (1.3 ± 0.4 SNU) can give only an upper limit of 1.7 SNU on the flux of solar neutrinos at the 68% confidence level.[28] This conservative conclusion of the authors can only be welcomed. However, it should be borne in mind that for a number of years the authors were of the opinion that there was a "solar neutrino paradox," i.e., that the flux of solar neutrinos was *much* less than the expected flux.

It seems to us that Davis *et al.* observed solar neutrinos and that the observed rate of ^{37}Ar production was comparable with the expected rate, although probably smaller.

The expected rate of ^{37}Ar production by solar neutrinos was calculated[37] on the basis of the generally

TABLE IV. Results of the Brookhaven experiment to search for solar neutrinos.

	Atoms of ^{37}Ar/day
Rate of ^{37}Ar production, averaged over all exposures from April 1970 to Feb 1976	0.32 ± 0.08
Rate of ^{37}Ar production by cosmic μ and ν_μ	0.08 ± 0.02
Rate of ^{37}Ar production (after subtraction of the cosmic background) which can be attributed to solar neutrinos	0.24 ± 0.09

13) Proportional counters with a high gas amplification factor were in fact first built in connection with the chlorine-argon method.

TABLE V. Thermonuclear reactions of the hydrogen cycle.

$$\left.\begin{matrix} p\,(p,\ e^+\nu_e) \\ (99.75\%) \\ p\,(e^-p,\ \nu_e) \\ (0.25\%) \end{matrix}\right\}\ {}^2\mathrm{H}\,(p,\ \gamma)^3\,\mathrm{He}\left\{\begin{matrix} ({}^3\mathrm{He},\ 2p)\,{}^4\mathrm{He}\ (86\%), \\ ({}^4\mathrm{He},\ \gamma)^7\,\mathrm{Be}\left\{\begin{matrix} (e^-,\ \nu_e)\,{}^7\mathrm{Li}\,(\mathrm{p},\ {}^4\mathrm{He})\,{}^4\mathrm{He}\ (14\%), \\ (\mathrm{p},\ \gamma)\,{}^8\mathrm{B}\rightarrow\nu_e+e^++{}^8\mathrm{Be}^*\ (\sim 0.02\%) \\ \downarrow \\ {}^4\mathrm{He}+{}^4\mathrm{He} \end{matrix}\right. \end{matrix}\right.$$

accepted hypothesis that the energy from the Sun comes from the thermonuclear reactions of the hydrogen cycle (Table V). It is assumed here that the original chemical composition of the Sun was uniform throughout the Sun up to its surface. It is also assumed that the nuclear data are known with sufficient accuracy.

Table VI gives the maximum energies of the neutrinos produced in the various reactions, the corresponding neutrino fluxes, and the contribution of each of the reactions to the rate of ^{37}Ar production in the reaction $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$.

The threshold for the reaction $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ is 0.81 MeV, which is much larger than the maximum energy of the neutrinos from the principal thermonuclear reaction $p + p \rightarrow d + e^+ + \nu_e$ that takes place in the Sun.[14] It can be seen from Table VI that the main contribution to the rate of ^{37}Ar production comes from the high-energy neutrinos from the decay of ^8B (this is so because the cross section for the reaction (76) rises rapidly with energy). The expected flux of neutrinos from the decay of ^8B accounts for a very small fraction of the total flux of solar neutrinos but has a strong dependence on the parameters of the model; in particular, there is a very strong dependence on the temperature of the central regions of the Sun. Despite this fact, it is widely believed (see, e.g., Ref. 29) that the quoted error in the calculated value of the rate of ^{37}Ar production by solar neutrinos (6 ± 2 SNU) is not underestimated.

d) The "solar neutrino paradox" and oscillations

We recall that if neutrino oscillations occur, then the flux of solar neutrinos for maximal mixing ($\theta = \pi/4$) in the case of two types of neutrinos can be $\frac{1}{2}$ of the flux expected in the absence of oscillations, in accordance with (66) and (68). If the mixing angle is different from $\pi/4$, the neutrino flux can be anywhere in the range between the expected flux and $\frac{1}{2}$ of the expected flux. We shall show in Chap. 6 that if there are $N > 2$ types of neutrinos then the minimum flux of solar neutrinos can be $1/N$ of the flux expected in the absence of oscillations.

If the "solar neutrino paradox" is genuine (i.e., if the signal in the experiment of Davis *et al.* is actually smaller than the calculated value and if this calculation is reliable), then the resolution of this paradox on the basis of the hypothesis of neutrino oscillations is more natural than any other hypothesis that has been proposed, either in the domain of elementary-particle physics or in the domain of astrophysics. These hypotheses are

[14]We have already pointed out the strong advantages of the chlorine-argon method. The relatively high threshold of the reaction $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$ is of course a drawback of this method.

TABLE VI. Contributions of various reactions in the Sun to the expected rate of ^{37}Ar production in the process $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$.

Neutrino source	Maximum energy of ν_e, MeV	Neutrino flux, $cm^{-2}\,sec^{-1}$	Expected rate of reaction $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$, SNU
$p + p \rightarrow d + e^+ + \nu_e$	0.42	6.10^{10}	0
$p + e^- + p \rightarrow d + \nu_e$	1.44 (monochr.)	$1.5 \cdot 10^8$	~0.3
^7Be (K-capture)	0.86(90%), 0.38(10%)	$4.5 \cdot 10^9$	~1
^8B (β decay)	14	$5.4 \cdot 10^6$	~4.5
			6 ± 2

listed in Ref. 29, where appropriate references can be found. They include the hypothesis that neutrinos are unstable particles that decay in flight between the Sun and the Earth,[38] as well as the following exotic astrophysical hypotheses: the energy of the Sun does not have a thermonuclear character; there is a black hole inside the Sun; the Sun is not in an equilibrium state, and its luminosity due to the extremely slow process of diffusion of photons from the central regions to the surface is much greater than its "internal luminosity," information about which can be obtained almost instantaneously by experiments to detect solar neutrinos; the Sun acquired an appreciable mass from outside at some time in the past, and the compositions of the inner and outer regions of the Sun are completely different (so that calculations based on the assumption that the Sun is homogeneous are incorrect); and others.

From the standpoint of contemporary elementary-particle physics, lepton mixing is a highly probable and attractive hypothesis. If we accept that the flux of solar neutrinos is actually too small, then oscillations provide a reasonable and non-exotic explanation of this fact, which is also consistent with current ideas about the lepton-quark analogy. It is pertinent to mention that, unlike the proposals for resolving the "neutrino paradox" enumerated above, the idea of neutrino oscillations was not invented especially to explain the experimental results of Davis *et al.*

In conclusion, we note that if lepton mixing turned out to be the resolution of the "solar neutrino paradox," then we would be able to infer that:

1) The value of the mixing angle is close to the maximum value $\pi/4$.

2) The wavelength of the oscillation, $L(\bar{p}) = 4\pi\bar{p}/|m_1^2 - m_2^2|$, is smaller than the distance from the Earth to the Sun, i.e., $|m_1^2 - m_2^2| > 10^{-11}$ eV^2.[15]

e) The problem of oscillations and future experiments using solar neutrinos

To test the hypothesis of neutrino mixing, we must determine the coefficient $\delta_{\nu_e;\nu_e}$ in Eq. (68). This obvi-

[15]To obtain this limit, we must take into account the fact that the effective momentum of the neutrinos in the experiment of Davis *et al.* was ~10 MeV.

ously requires a sufficiently accurate knowledge of the intensity $\bar{I}^0_{\nu_e}$ expected in the absence of oscillations. As we have already emphasized, the use of detectors based on the chlorine-argon method entails an error in the expected signal which is rather large (see (77)), and it is quite possible that this error is underestimated (owing to the strong dependence on the parameters of the model for the Sun). If the detector could detect the low-energy (≤ 0.4 MeV) neutrinos from the reaction $p+p\to d+e^+ +\nu_e$, which account for most of the solar neutrinos (see Table VI), it would then be possible to make a reliable prediction[37] of the intensity expected in the absence of oscillations. In fact, the total flux I_{ν_e} of solar neutrinos is related to the luminosity $L_\odot$ by the equation

$$I_{\nu_e}=\frac{2L_\odot}{4\pi R^2 Q},$$

where R is the distance from the Sun to the Earth. This relation can easily be derived by considering the fact that the various thermonuclear processes in the Sun lead ultimately to the conversion of four protons into ^{4}He, $2e^+$, and $2\nu_e$ with an energy release $Q\approx 25$ MeV.

Thus the problem of determining the coefficient $\delta_{\nu_e;\nu_e}$ in Eq. (68) may be solved if a new detector capable of detecting the low-energy neutrinos from the reaction $p+p\to d+e^+ +\nu_e$ is constructed. Such a detector may be based on the Ga-Ge radiochemical method which is now being developed.[28,39] The reaction[16)] $\nu_e + {}^{71}\text{Ga}\to e^- + {}^{71}\text{Ge}$ has a threshold of 230 keV. The half-life of ^{71}Ge (K-capture) is 11 days. The chemical problems of extracting the germanium from the gallium and the problems of detecting the ^{71}Ge K-capture events may be solved. In order to build the detector, it would be necessary to have several tons of gallium, a quantity which exceeds its world-wide annual production. Nevertheless, it appears that such an experiment will be performed.

In conclusion, we would like to make the following two remarks:

1) Other methods of detecting solar neutrinos have recently been proposed. We shall not discuss these methods here, and we cite only the appropriate references.[40]

2) Preparations are now being made[41] for an experiment to detect solar neutrinos by the chlorine-argon method at the Baksan neutrino station of the Insitute for Nuclear Research, USSR Academy of Sciences. The quantity of C_2Cl_4 in this experiment will be approximately five times as great as the quantity of C_2Cl_4 in the experiment of Davis *et al.*

f) Experiments using solar neutrinos and the prospects of determining the wavelength of the oscillations

We have already pointed out that if experiments using reactors or accelerators show that either $\delta_{\nu_e;\nu_e}\neq 1$ or $\delta_{\nu_e;\nu_\mu}\neq 0$ (see (68) and (69)), i.e., if neutrino mixing occurs, then experiments of comparable difficulty designed to determine the wavelength of the oscillations and hence also the difference between the squares of the neutrino masses will undoubtedly be carried out. The alternative is to carry out experiments using solar neutrinos. The problem of determining the wavelength of the oscillation in experiments using solar neutrinos is incomparably more difficult than the problem of determining the coefficient $\delta_{\nu_e;\nu_e}$.

In this subsection, we discuss possible experiments using solar neutrinos, from which it might be possible to obtain information about the wavelength of the oscillations.

To begin with, let us assume that the detector makes it possible to detect neutrinos with a definite momentum p. In this case, the intensity of ν_e is given by Eq. (66) (we shall confine our discussion to the simplest case of two types of neutrinos). If the wavelength $L(p)$ of the oscillations is much smaller than the dimensions $r\sim 10^5$ km of the region of the Sun in which the detected neutrinos are effectively produced, then the sinusoidal term in the neutrino intensity vanishes in this case because of the averaging, and only the above-mentioned mixing effects ($\delta_{\nu_e;\nu_e}\neq 1$) can be observed at the surface of the Earth.

It was pointed out by I. Ya. Pomeranchuk that variations in the intensity of solar neutrinos may occur under different conditions, as a result of the fact that the distance $R(t)$ between the Earth and the Sun varies with time. A variation can exist in principle if the wavelength $L(p)$ of the oscillations is greater than r and if there are variations in the distance between the Sun and the Earth comparable with $L(p)$, i.e., if $r<L(p)\lesssim \Delta R$, where $\Delta R\sim 5\times 10^6$ km is the maximum variation of R (the difference between the semiaxes of the Earth's orbit). It is an extremely difficult problem to observe time-dependent variations. This is so because even the maximum relative variation of the distance R ($\Delta R/R \approx 0.03$) is usually much less than the relative dispersion in the momenta of the neutrinos that are actually detected (there do not exist detectors which could detect neutrinos with a strictly defined momentum). Thus the sinusoidal term generally vanishes after averaging over the neutrino momentum.

Does there nevertheless exist some possible method of observing time-dependent variations in the intensity of solar neutrinos? It should be borne in mind that the reactions taking place in the Sun which lead to neutrino production include some which produce monoenergetic neutrinos (see Table VI). The problem of detecting time time-dependent variations would become somewhat simpler if there were some method of detecting the "neutrino lines" from such reactions (or coming mainly from such reactions).[17)]

We note that variations in the intensity of neutrinos of momentum p would have a period of the order of a hun-

[16)] This reaction for detecting solar neutrinos was proposed by V. A. Kuz'min.[39b]

[17)] There exists a very remote, but possible, radiochemical method of detecting the neutrinos produced mainly in the reaction $p+e^- +p\to d+\nu_e$ in the Sun. This method is based on detection of the process $\nu_e + {}^7\text{Li}\to e^- + {}^7\text{Be}$ (see Ref. 40a).

dred days if $L(p) \approx \Delta R \approx 5\times10^6$ km, and of the order of ten days if $L(p) \approx 5\times10^5$ km. Variations with a period less than 10 days cannot occur. This is so because the thermal energy of the particles in the active region of the Sun is $kT \sim 1$ keV and the neutrinos produced in the reactions $e^- + {}^7\text{Be} \to \nu_e + {}^7\text{Li}$ and $p + e^- + p \to d + \nu_e$ are not monoenergetic. For neutrinos of energy ~1 MeV, the relative dispersion in the momentum, $\Delta p/p \sim 10^{-3}$, is much larger than the relative daily variation in the distance between the Earth and the Sun, so that the sinusoidal term vanishes after averaging over the neutrino spectrum.

Thus there exists in principle a possible method of measuring the wavelengths of the oscillations of neutrinos with energy ~1 MeV, provided that they lie within the range between 5×10^5 km and 5×10^6 km; such information can be obtained by studying time-dependent oscillations in the intensity of solar neutrinos with a period in the range between ~10 days and ~100 days (corresponding to $M^2 = |m_1^2 - m_2^2|$ in the range between 0.5×10^{-8} eV^2 and $\sim 0.5\times10^{-9}$ eV^2).

From an experimental point of view, however, the prospects for exclusive detection of neutrino lines seems very remote.

In this connection, let us return to the discussion of the chlorine-argon method. We see from Table VI that ~15% of the expected signal is due to the process ${}^7\text{Be} + e^- \to {}^7\text{Li} + \nu_e$, in which neutrinos of energy 0.86 MeV are produced. This means that, if the accuracy of the measurements is increased, even the ordinary chlorine-argon method can be used to observe variations due to the sinusoidal term (for M^2 in the range somewhat below that indicated above, the half-life of ${}^{37}\text{Ar}$ is ~1 month).

We have already pointed out that the relative variation in the distance between the Earth and the Sun is much less than the characteristic relative dispersion in the momenta of the solar neutrinos. It would be desirable to make use of this dispersion to obtain information about the wavelength of the oscillation.[18] If solar neutrinos are measured by detectors with various thresholds, it might be possible to do this by analyzing the entire set of data.

We conclude this subsection with the following remarks.

We have already mentioned many times that the sinusoidal term in the intensity of solar neutrinos (see Eq. (66)) generally vanishes after taking all possible averages and that the averaged observed intensity of ν_e can vary in the range between $\frac{1}{2}$ and 1 times the expected intensity in the case of two types of neutrinos. In the general case of $N \geq 2$ types of neutrinos, the averaged intensity can vary in the range between $1/N$ and 1 times the expected intensity.

We note that the observed intensity of solar neutrinos may be much smaller than the expected intensity, even in the case of two types of neutrinos. In fact, suppose that the wavelength of the oscillations is comparable with the distance between the Earth and the Sun. In this case, the sinusoidal term in Eq. (66) for the neutrino intensity may "survive" after averaging over the momentum. Thus we have another possible interpretation of the small signal from solar neutrinos observed in the chlorine-argon experiment[28] (although this interpretation is connected with a special value for the wavelength of the oscillation).

[18] See also Ref. 42a, where it was proposed to measure the spectrum of solar neutrinos from β decay of ${}^8\text{B}$ and to compare it with the expected spectrum.

g) A note concerning the coherence of neutrino beams

We shall discuss here the coherence condition for a neutrino beam considered in Ref. 42b. It was pointed out in this paper that the dimensions of a packet of solar neutrinos may be very small. This effect is due to collisions between nuclei and is analogous to the well-known broadening of spectral lines. According to Ref. 42b, the dimensions d of a packet of solar neutrinos are given by

$$d \approx 10^{-6} \text{ cm}. \tag{78}$$

Clearly, the coherence condition is satisfied if

$$R\,\Delta\beta \leqslant d, \tag{79}$$

where R is the distance between the Earth and the Sun, and

$$\Delta\beta = \frac{|m_1^2 - m_2^2|}{2p^2} = 2\pi\frac{1}{pL}$$

is the difference between the velocities of neutrinos with momentum p and masses m_1 and m_2. Using the estimate (78), we can rewrite the condition (79) for a neutrino with momentum ~1 MeV in the form

$$L \geqslant 10^{-4}R. \tag{80}$$

If, for example, $M^2 = |m_1^2 - m_2^2| \approx 1$ eV^2, then $L \approx 2.5$ m, and the condition (80) is certainly not satisfied (this is the case that was considered in Ref. 42b).

However, as we have already stressed many times, a necessary (but not sufficient) condition for the sinusoidal term to "survive" is that the dimensions of the source are small in comparison with the wavelength of the oscillation:

$$r \leqslant L; \tag{81}$$

here r gives the dimensions of the region of the Sun from which the neutrinos are effectively emitted. Since $r \approx 10^{-3}R$, the inequalities (80) and (81) are essentially equivalent, and we conclude that no further restrictions (other than (81)) are imposed by the coherence condition (79).

h) Oscillations and cosmic neutrinos

If the phenomenon of neutrino oscillations occurs, it might also play a major role in experiments using cosmic neutrinos.[19] Let us give some examples.

[19] The importance of studying cosmic neutrinos was first emphasized in Ref. 42c.

1) At the underground neutrino installation of the Institute for Nuclear Research of the USSR Academy of Sciences, preparations are being made for an experiment[43] to detect high-energy muonic neutrinos originating from the decays of mesons produced in collisions of cosmic-ray protons with nuclei of nitrogen and oxygen in the atmosphere. The energy spectra and other characteristics of these neutrinos were calculated in Ref. 44. The high-energy muons produced by the interactions of ν_μ with the Earth will be detected by 8 arrays of scintillation detectors, each of area 1500 m^2, connected in coincidence and providing information about the direction from which the muons arrive. This arrangement will detect both muons incident on the detector "from above" and muons incident "from below," i.e., muons produced by neutrinos incident on the opposite side of the Earth and passing through the Earth. The average neutrino momentum is 5–10 GeV in such experiments, while the distance from the neutrino source to the detector is $R \approx 10^4$ km. It is possible to test the hypothesis of neutrino mixing by comparing the measured and expected intensities of ν_μ and making use of Eq. (66). In principle, the sensitivity of these experiments is sufficiently high.[43] It can easily be seen that $M^2_{\min} \sim 10^{-3}$ eV^2 (see (60) for the definition of the parameter $M^2_{\min}$). Thus these experiments have a sensitivity (as regards the search for neutrino mixing) intermediate between the sensitivity of experiments using artificial neutrinos (from reactors or accelerators) and that of experiments using solar neutrinos. However, the statistics that can be attained in these experiments are very poor ($\lesssim 100$ events/yr); moreover, the quantities required for the interpretation of these experiments, such as the expected intensity and the spectrum of ν_μ, can be calculated only with a poor accuracy.

2) A project (the DUMAND project[45,46]) for studying cosmic neutrinos by means of a detector of extremely large effective mass ($>10^7$ tons of water in the ocean) is now being widely discussed. It is proposed to submerge the detecting apparatus in the ocean at a depth of ~5 km. Owing to its large luminosity, this arrangement can also be used to study neutrino oscillations.

3) If there are more than two types of neutrinos, oscillations of cosmic neutrinos can give rise to new types of neutrinos whose fields appear in the weak-interaction Hamiltonian together with the fields of the heavy leptons. Such a mechanism can explain[47] the inconsistency between the events observed at an underground Indian laboratory[48] and the data obtained at the Batavia accelerator, provided that the wavelength of the oscillations is ~100 km (~100 km is the typical distance traversed by high-energy neutrinos produced in the atmosphere and detected underground).

6. OSCILLATIONS IN THE GENERAL CASE OF N TYPE NEUTRINOS

a) The case of left-handed fields

We have so far been considering neutrino oscillations in the case of two types of neutrinos. It is also of interest to consider oscillations in the general case of an arbitrary number $N \geq 2$ of types of neutrinos. This case was analyzed in Refs. 7–9. Here we shall give a brief account of the results obtained in those papers.

The theoretical relations (22)[2] have the following generalizations in the case of N types of neutrinos:

$$\nu_{lL} = \sum_{\sigma=1}^{N} U_{l\sigma}\varphi_{\sigma L}; \tag{82}$$

here ν_l ($l = e, \mu, M, \ldots$) is the field operator of the (electronic, muonic, etc.) neutrino that takes part in the ordinary weak interaction, φ_σ ($\sigma = 1, \ldots, N$) is the field operator of a Majorana neutrino with mass m_σ, and $U_{l\sigma}$ is an orthogonal $N\times N$ matrix (for the case in which CP invariance holds). Similarly, the theoretical relations (37)[4,5] are replaced by[9]

$$\nu_{lL} = \sum_{\sigma=1}^{N} U_{l\sigma}\nu_{\sigma L}, \tag{83}$$

where ν_σ ($\sigma = 1, \ldots, N$) is the field operator of a Dirac neutrino with mass m_σ. If the weak process leads to the production of ν_l, then the probability of observing a $\nu_{l'}$ at distance R from the point of production of the ν_l is given by the expression

$$w_{\nu_{l'};\nu_l}(R) = \sum_{\sigma=1}^{N} U^2_{l\sigma}U^2_{l'\sigma} + \sum_{\sigma\neq\sigma'} U_{l\sigma}U_{l'\sigma}U_{l\sigma'}U_{l'\sigma'} \cos 2\pi \frac{R}{L_{\sigma\sigma'}} \quad (l, l' = e, \mu, M, \ldots), \tag{84}$$

where

$$L_{\sigma\sigma'} = 4\pi \frac{p}{|m^2_\sigma - m^2_{\sigma'}|}. \tag{85}$$

If the wavelength of the oscillations are less than or of order R and if the terms $\cos(2\pi R/L_{\sigma\sigma'})$ in (84) vanish after averaging, then the averaged intensity $\bar{I}_{\nu_l;\nu_l}$ of neutrinos ν_l is related to the intensity $\bar{I}^0_{\nu_l}$ expected in the absence of oscillations by the equation

$$\bar{I}_{\nu_l;\nu_l} = \delta_{\nu_l;\nu_l}\bar{I}^0_{\nu_l}, \tag{86}$$

where

$$\delta_{\nu_l;\nu_l} = \sum_{\sigma=1}^{N} U^4_{l\sigma}. \tag{87}$$

Using the orthogonality of the matrix U, it is easily shown that

$$(\delta_{\nu_l;\nu_l})_{\min} = \frac{1}{N}. \tag{88}$$

The minimum value of $\delta_{\nu_l;\nu_l}$ is attained when

$$U^2_{l1} = U^2_{l2} = \ldots = U^2_{lN} = \frac{1}{N}. \tag{89}$$

Thus, in the case of N types of neutrinos, the oscillations can lead to an intensity of solar neutrinos equal to $1/N$ of the expected intensity.[7–9]

Equation (84) also implies that the averaged intensity of $\nu_{l'}$ ($l' \neq l$; $l, l' = e, \mu, M, \ldots$) is related to the intensity $\bar{I}^0_{\nu_l}$ by the equation

$$\bar{I}_{\nu_{l'};\nu_l} = \delta_{\nu_{l'};\nu_l}\bar{I}^0_{\nu_l}; \tag{90}$$

TABLE VII. Neutrino mixing schemes.

Number of types of neutrinos	Number of neutrino states	Bare masses of neutrinos	Particles with definite masses	Oscillations in ν_e beam	$\delta\nu_e;\nu_e$	Ref.
1. Two (ν_e, ν_μ)	4	0	2 Marjorana neutrinos	$\nu_e \rightleftharpoons \nu_\mu$	1/2	[2]
2. Two (ν_e, ν_μ)	8	$\neq 0$	2 Dirac neutrinos	$\nu_e \rightleftharpoons \nu_\mu$	1/2	[5, 6]
3. Two (ν_e, ν_μ)	8	$\neq 0$	4 Majorana neutrinos	$\nu_e \rightleftharpoons \nu_\mu$, $\nu_e \rightleftharpoons \bar{\nu}_{eL}$, $\nu_e \rightleftharpoons \bar{\nu}_{\mu L}$	1/4	[8]
1a. $N>2(\nu_e, \nu_\mu, \nu_M, \ldots)$	$2N$	0	N Majorana neutrinos	$\nu_e \rightleftharpoons \nu_\mu$, $\nu_e \rightleftharpoons \nu_M, \ldots$	$1/N$	[7]
2a. $N>2(\nu_e, \nu_\mu, \nu_M, \ldots)$	$4N$	$\neq 0$	N Dirac neutrinos	$\nu_e \rightleftharpoons \nu_\mu$, $\nu_e \rightleftharpoons \nu_M$,	$1/N$	[7, 9]
3a. $N>2(\nu_e, \nu_\mu, \nu_M, \ldots)$	$4N$	$\neq 0$	$2N$ Majorana neutrinos	$\nu_e \rightleftharpoons \nu_\mu$, $\nu_e \rightleftharpoons \nu_M$, $\nu_e \rightleftharpoons \bar{\nu}_{eL}$, $\nu_e \rightleftharpoons \bar{\nu}_{\mu L}$, $\nu_e \rightleftharpoons \nu_{ML}, \ldots$	$1/2N$	[8]

here

$$\delta_{\nu_{l'};\nu_l} = \sum_{\sigma=1}^{N} U_{l\sigma}^2 U_{l'\sigma}^2 \qquad (l' \neq l). \tag{91}$$

If $\delta_{\nu_l;\nu_l}$ takes the minimum value $1/N$, we find from (89) that

$$(\delta_{\nu_{l'};\nu_l})_{\max} = \frac{1}{N} \sum_{\sigma} U_{l'\sigma}^2 = \frac{1}{N} \qquad (l' \neq l). \tag{92}$$

Thus, if the averaged probability of the "transition" $\nu_l \to \nu_l$ (with fixed l) is equal to $1/N$, then the probability of the "transition" $\nu_l \to \nu_{l'}$ $(l' \neq l)$ is also equal to $1/N$. We note that if, for $N=2$, we have

$$\delta_{\nu_e;\nu_e} = \frac{1}{2},$$

then $\theta = \pi/4$ and

$$\delta_{\nu_\mu;\nu_\mu} = \frac{1}{2}. \tag{93}$$

In the case $N>2$, if the coefficient $\delta_{\nu_l;\nu_l}$ takes the minimum value $1/N$, then for $l' \neq l$ we have[42]

$$\delta_{\nu_{l'};\nu_{l'}} > \frac{1}{N}. \tag{94}$$

In fact, if $N \neq 2n$ (where n is an integer), the elements of the orthogonal matrix U cannot satisfy the conditions (89) for different values of l simultaneously.[20)]

b) The general case of left-handed and right-handed neutrino fields

If we assume that the Hamiltonian that does not conserve the lepton charges contains both left-handed and right-handed components of the neutrino fields and that the bare neutrino masses are non-zero, then the eigenstates of the total Hamiltonian are the states of $2N$ Majorana neutrinos[8] (the weak-interaction Hamiltonian contains N types of four-component neutrinos). In this case, a ν_e beam exhibits oscillations of the types $\nu_e \rightleftharpoons \nu_\mu$, $\nu_e \rightleftharpoons \bar{\nu}_{eL}$, $\nu_e \rightleftharpoons \bar{\nu}_{\mu L}, \ldots$ (where $\bar{\nu}_{eL}$, $\bar{\nu}_{\mu L}, \ldots$ are antineutrinos with negative helicity). The minimum value of the parameter $\delta_{\nu_e;\nu_e}$ in such a theory is given by

$$(\delta_{\nu_e;\nu_e})_{\min} = \frac{1}{2N}. \tag{95}$$

We note that this scheme becomes attractive if the weak-interaction Hamiltonian contains both left-handed and right-handed currents. This possibility has been widely discussed in the literature (see, e.g., Refs. 49 and 50).

c) Concluding remarks

1) In Table VII we give the values of $(\delta_{\nu_l;\nu_l})_{\min}$ for all the neutrino mixing schemes which we have discussed.

2) The SLAC experiments[51] yielded very convincing data in favor of the existence of a heavy charged lepton of mass ~1.8 GeV. In view of this discovery, the possibility that there exist in nature $N>2$ types of neutrinos seems very natural (in analogy with the e and μ, each new charged lepton may correspond to a new type of neutrino).

3) All the theories considered above are based on the assumption that the neutrino masses $m_1, m_2, \ldots$ are non-zero and that the Hamiltonian of the ordinary weak interaction contains the fields of these neutrinos in a mixed form. Clearly, the experiments which give limits on the masses of the muonic and electronic neutrinos may also yield limits[5,42b] on the masses m_1, $m_2, \ldots$. The best upper limit on the mass of the electronic neutrino has been obtained from experiments on β decay of ^{3}H. Recent measurements gave the result[52]

[20)] For example, Eq. (88) is satisfied for $N=3$ if the square of each of the elements of the corresponding row is equal to $\frac{1}{3}$. It is obvious from the orthogonality of the matrix U that the elements of either of the other two rows cannot satisfy this condition.

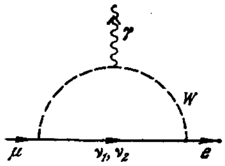

FIG. 3. Diagrams for the process $\mu \to e\gamma$ with virtual neutrinos ν_1 and ν_2.

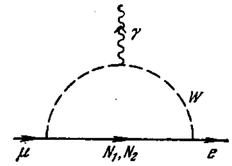

FIG. 4. Diagrams for the process $\mu \to e\gamma$ with virtual heavy neutral leptons N_1 and N_2.

$$m_{\nu_e} < 35 \text{ eV}. \qquad (96)$$

Experiments on $K_{3\mu}$ decay gave the following upper limit on the mass of the muonic neutrino[53] 21):

$$m_{\nu_\mu} < 0.65 \text{ MeV}. \qquad (97)$$

Consider the case of two neutrinos. Assuming that the mixing angle is close to $\pi/4$, we find from (95) that

$$\frac{1}{2} \sum_{i=1}^{2} m_i^2 < (35 \text{ eV})^2.$$

Taking into account the fact that $|m_1^2 - m_2^2| \lesssim 1 \text{ eV}^2$ (see (64)), we have

$$m_1, m_2 < 35 \text{ eV}. \qquad (98)$$

7. THE DECAY $\mu \to e\gamma$ AND NEUTRINO OSCILLATIONS

a) The case of neutrinos

The theories involving neutrino mixing which we have considered here can in principle lead to not only neutrino oscillations, but also decays such as $\mu \to e\gamma$ and $\mu \to 3e$, which are forbidden in the ordinary theory. These processes occur in higher orders of perturbation theory (see the diagrams of Fig. 3). The ratio R_μ of the probability for the process $\mu^+ \to e^+\gamma$ to the probability for the process $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$, which was first calculated in a theory with mixing in Ref. 22, is given by the expression

$$R_\mu = \frac{\Gamma(\mu^+ \to e^+\gamma)}{\Gamma(\mu^+ \to e^+\nu_e\bar{\nu}_\mu)} = \frac{3}{32}\,\frac{\alpha}{\pi}\left(\frac{m_1^2 - m_2^2}{M_W^2}\right)^2 \times \sin^2\theta\cos^2\theta, \qquad (99)$$

where M_W is the mass of the charged intermediate boson ($M_W \geq 37$ GeV). It is easy to see that the value of R_μ is many orders of magnitude below the experimental upper limit even if m_1 and m_2 are taken to be 35 eV and 0.65 MeV (the upper limits on the masses of the electronic and muonic neutrinos). In this case, we have

$$R_\mu < 3 \cdot 10^{-26}, \qquad (100)$$

whereas the experiments of Refs. 15a and 54 give

$$R_\mu^{\text{exp}} < 2.2 \cdot 10^{-8}. \qquad (101)$$

If the mixing angle is not small, we can make use of the limit $|m_1^2 - m_2^2| \lesssim 1 \text{ eV}^2$ obtained by considering oscillations (see (64)). The ratio R_μ is many orders of magnitude smaller than (100) in this case. It can be shown that that the ratio R_μ is also many orders of magnitude smaller than the experimental limit (101) in the general case of N types of neutrinos with non-zero masses and mixing. Thus, if neutrinos are the only neutral leptons which exist, there is only one way of searching for a violation of the lepton-number conservation laws, namely the study of neutrino oscillations.22)

b) Heavy leptons

The situation may be radically different[55] 23) if there exist heavy leptons. (We note that mixing in a scheme involving heavy leptons was discussed in Ref. 57.) Let us consider the decay $\mu \to e\gamma$ in a scheme with right-handed currents. As an example, we assume that in addition to the left-handed doublets of the standard theory

$$\binom{\nu_e}{e}_L, \quad \binom{\nu_\mu}{\mu}_L \qquad (102)$$

there exist right-handed doublets

$$\binom{N_e}{e}_R, \quad \binom{N_\mu}{\mu}_R, \qquad (103)$$

where $e_R = (1 - \gamma_5)e/2$, etc., and

$$N_e = N_1\cos\theta' + N_2\sin\theta', \quad N_\mu = -N_1\sin\theta' + N_2\cos\theta'; \qquad (104)$$

here N_1 and N_2 are the field operators of heavy neutral leptons with masses M_1 and M_2, respectively ($M_1, M_2 \geq M_K$, where M_K is the kaon mass), and θ' is a mixing angle. Clearly, the charged lepton current has the form

$$j_\alpha^l = j_\alpha + j_\alpha', \qquad (105)$$

where the first term is given by the expression (37), and

$$j_\alpha' = \bar{N}_{eR}\gamma_\alpha e_R + \bar{N}_{\mu R}\gamma_\alpha\mu_R. \qquad (106)$$

The decay $\mu \to e\gamma$ is described by the diagrams of Fig. 3, as well as additional diagrams containing virtual heavy leptons N_1 and N_2 (Fig. 4). Neglecting the contribution

---

21)It should be mentioned that cosmological arguments can be used to obtain limits on the mass of the ν_μ which are four orders of magnitude better than (97).[53b]

22)The physical reason for the sensitivity of the method of oscillations is that this method permits a measurement of the amplitude for the process instead of the square of the amplitude, as in the usual methods of measuring probabilities of processes. A good illustration of the sensitivity of the method of oscillations is provided by the measurement of the mass difference between the K_L and K_S mesons. This is the only effect of second order in the weak interaction whose magnitude can be measured in a sufficiently simple way and with great accuracy.

23)After Ref. 55 was described in the present review, we became aware of many papers of similar content, in which processes such as $\mu \to e\gamma$ and $\mu \to 3e$ were considered on the basis of the hypothesis of heavy-lepton mixing. We cite here only those papers[56] that were known to us by April 10, 1977, and we consider the scheme of Ref. 55 as an example.

from the diagram of Fig. 3, we find that the ratio R_μ is given by the expression

$$R_\mu = \frac{3}{32}\frac{\alpha}{\pi}\left(\frac{M_1^2 - M_2^2}{M_W^2}\right)^2 \sin^2\theta' \cos^2\theta'. \qquad (107)$$

Using (101) and (107), we obtain $(|M_1^2 - M_2^2|/M_W^2)^{1/2} < 1.4\times10^{-1}$ in the case of maximal mixing. If $M_W = 60$ GeV, we have $(|M_1^2 - M_2^2|)^{1/2} < 8.5$ GeV. This case also leads to the result $|M_1 - M_2| < 8.5$ GeV.[24)]

For values of $|M_1^2 - M_2^2|^{1/2}$ equal to 1, 2, 3, and 4 GeV, we find that the ratio R_μ has the values 4.2×10^{-12}, 6.7×10^{-11}, 3.4×10^{-10}, and 1.1×10^{-9}, respectively.

Thus, if there exist neutral leptons with masses of the order of several GeV and if mixing occurs, the probability for the decay $\mu\to e\gamma$ may be close to the upper limit obtained in the experiments of Refs. 15a and 54. We note that this decay can be observed at the APEC installation[58] if $R_\mu \geqslant 10^{-11}$.

c) The decay $\mu\to e\gamma$ and neutrino oscillations

Let us consider the relation between the phenomenon of neutrino oscillations and the decay $\mu\to e\gamma$ (and similar processes). The observation of such effects would certainly indicate that lepton mixing occurs. In this general sense, the observation of one of these effects would make the existence of the other more probable (in particular, it would then be probable that the neutrino masses are non-zero). However, there is no direct relation between neutrino oscillations and processes of the type $\mu\to e\gamma$. In the first place, it is possible that neutrino oscillations can be observed, while the decay $\mu\to e\gamma$ is practically unobservable. This is the situation which we have discussed in the first six sections of this review (the case in which there exist only neutrinos and no heavy neutral leptons). Secondly, it may happen that the decay $\mu\to e\gamma$ is completely observable (for example, if heavy neutral leptons exist) but that neutrino oscillations are practically unobservable because the mixing angle or the neutrino mass difference is small. Thirdly, it is possible that the $\mu\to e\gamma$ decay probability is sufficiently large (for example, if heavy charged leptons exist and the Hamiltonian contains non-symmetric neutral currents) but that neutrino oscillations are completely absent (these two phenomena are completely unrelated).

8. CONCLUSIONS

a) The fundamental hypothesis which constitutes the basis of the physical phenomena discussed here is the hypothesis of lepton mixing. The basic consequences of this hypothesis depend on what leptons exist in nature. In the case of only four leptons (the muon, the electron, and two neutrinos), the main observable physical consequences of the mixing hypothesis are neutrino oscillations. The probabilities of processes such as $\mu\to e\gamma$ and $\mu\to 3e$, while non-zero in principle, are many orders of magnitude below the quantities that are accessible to measurements.

b) If neutrino mixing is to be possible, we must assume that the neutrino masses are non-zero. We note that non-zero neutrino masses do not contradict any of the currently known physical principles (like gauge invariance, according to which the photon mass is equal to zero). In this sense, the question of the neutrino masses is completely open (not only in the "trivial" sense that the experimental upper limits on the masses of the ν_μ and ν_e are quite large).

c) The question of how many leptons exist is also open at the present time. If there are more than two neutrinos and all the neutrino fields are mixed, oscillations between all the types of neutrino become possible. In this case, the probabilities of decays such as $\mu\to e\gamma$ and $\mu\to 3e$ are also practically equal to zero.

d) The situation is different if heavy leptons exist and their fields appear in the Hamiltonian in a mixed form. If the masses of the heavy leptons are of the order of several GeV, the probabilities of processes such as $\mu\to e\gamma$ and $\mu\to 3e$ may be close to the experimental upper limits. There is in general no unique relation between the probabilities of these processes and the parameters that characterize the neutrino oscillations.

e) We have seen that two types of theories are possible: those with Majorana and those with Dirac neutrinos. Majorana fields are attractive because of their economy (in the case of two types of neutrinos, there are a total of four states). If the neutrinos are described by Dirac fields, these particles are described in the same way as all the other leptons and quarks. This is very attractive from the point of view of the quark-lepton analogy. The picture of oscillations is exactly the same in these two different theories, and it is practically impossible to distinguish them.

f) If the neutrino fields are mixed, leptonic charge becomes meaningless at first sight. However, orthogonal mixing is in essence equivalent to the conservation of the leptonic charges if the neutrino masses are sufficiently small (which is actually the case).

To first order in the weak-interaction constant, the amplitudes for processes due to non-symmetric neutral currents, such as $\mu\to e\gamma$ and $\mu\to 3e$, are equal to zero. These processes occur in higher orders, however, with amplitudes proportional to the difference between the squares of the neutrino masses. Thus it is the small neutrino masses that guarantee effective conservation of the lepton numbers, and we have already seen that if there exist heavy leptons with masses of the order of a GeV, then processes such as $\mu\to e\gamma$ and $\mu\to 3e$ occur.

g) Neutrino oscillations have so far not been observed. An analysis of all neutrino experiments which have been carried out using accelerators and reactors (which, however, have not been designed to search for oscillations) makes it possible to obtain a limit on the difference be-

[24)] The lepton-quark analogy manifests itself here in the fact that an upper limit on the mass difference between the heavy leptons can be obtained from the upper limit on the $\mu\to e\gamma$ decay probability in the same way that an upper limit on the mass of the charmed quark is obtained from the data on the K_L–K_S mass difference and the rare decays of kaons.

tween the squares of the neutrino masses ($|m_1^2 - m_2^2| < 1$ (eV)^2 for maximal mixing). On the other hand, the only experiment which has so far been carried out using solar neutrinos gave a weaker detected signal than what was expected—the well-known "solar neutrino paradox." If such a paradox actually exists, neutrino oscillations provide its most natural resolution.

h) The need to perform experiments to search for neutrino oscillations and for the decays $\mu \to e\gamma$ and $\mu \to 3e$ and other similar processes at some level better than the current one is completely obvious.

In conclusion, it is a pleasure to express our deep gratitude to G. T. Zatsepin, S. M. Korenchenko, G. V. Mitsel'makher, L. B. Okun', S. Petkov, M. I. Podgoretskiĭ, A. A. Pomanskiĭ, and A. E. Chudakov for useful discussions.

[1]B. M. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)]; see also: Zh. Eksp. Teor. Fiz. 33, 549 (1957) [Sov. Phys. JETP 6, 429 (1958)]; 34, 247 (1958) [**7**, 172 (1958)].
[2]V. Gribov and B. Pontecorvo, Phys. Lett. B28, 493 (1969).
[3]S. Weinberg, *ibid.* **19**, 1264 (1967). A. Salam, in: Proc. of the Eighth Nobel Symposium, Wiley, New York, 1968.
[4]S. Eliezer and D. A. Ross, Phys. Rev. D10, 3088 (1974).
[5]S. M. Bilenky and B. Pontecorvo, Phys. Lett. B61, 248 (1976).
[6]S. Eliezer and A. Swift, Nucl. Phys. B105, 45 (1976).
[7]B. Pontecorvo, Pis'ma Zh. Eksp. Teor. Fiz. 13, 281 (1971) [JETP Lett. 13, 199 (1971)].
[8]S. M. Bilenky and B. Pontecorvo, Lett. Nuovo Cimento **17**, 569 (1976).
[9]H. Fritzsch and P. Minkowsky, Phys. Lett. B62, 72 (1976).
[10]Ya. B. Zel'dovich, Dokl. Akad. Nauk SSSR **86**, 505 (1952).
[11]E. J. Konopinski and M. Mahmoud, Phys. Rev. 92, 1045 (1953).
[12]B. T. Cleveland, W. R. Leo, C. S. Wu, L. R. Kasday, A. M. Rushton, P. J. Gollon, and J. D. Ullman, Phys. Rev. Lett. 35, 757 (1975).
[13]T. Kirsten and H. W. Müller, Earth and Planet Sci. Lett. 6, 271 (1969). B. Srinivasan, E. C. Alexander, F. D. Beaty, D. E. Sinclair, and O. K. Mannel, Econ. Geol. 68, 252 (1973).
[14]K. Borer, B. Hahn, H. Hofer, H. Haspar, F. Krenen, and T. G. Seiler, in: Proc. of the 14th Intern. Conf. on High Energy Physics, Vienna, 1968.
[15]a) S. Parker, H. L. Anderson, and C. Rey, Phys. Rev. B133, 768 (1964). b) S. M. Korenchenko, B. F. Kostin, G. V. Mitsel'makher, K. G. Nekrasov, and V. S. Smirnov, Yad. Fiz. 13, 341 (1971) [Sov. J. Nucl. Phys. 13, 190 (1971)].
[16]S. M. Korenchenko, B. F. Kostin, G. V. Mitsel'makher, K. G. Nekrasov, and V. S. Smirnov, in: Trudy XVIII Mezhdunarodnoĭ konferentsii po fizike vysokikh énergiĭ (Proc. of the 18th Intern. Conf. on High Energy Physics) (Tbilisi, 1976), Vol. II, JINR, Dubna, 1977, V 175.
[17]D. A. Bryman, M. Blecher, K. Gotow, and R. J. Powers, Phys. Rev. Lett. **28**, 1469 (1972).
[18]E. Belotti, D. Cavalli, E. Fiorini, and M. Rollier, Lett. Nuovo Cimento **17**, 553 (1976).
[19]J. D. Bjorken and S. L. Glashow, Phys. Lett. **11**, 255 (1964).
[20]L. B. Okun', in: Nekotorye problemy sovremennoĭ teorii yadra (Some Problems of Modern Nuclear Theory), Part II, Moscow Engineering Physics Inst., Moscow, 1974.
[21]a) S. L. Glashow, J. Illiopulos, and L. Maiani, Phys. Rev. D2, 1285 (1970). b) M. Nakagawa, H. Okonogi, S. Takata, and A. Tojoda, Prog. Theor. Phys. 30, 727 (1963).
[22]S. T. Petkov, Yad. Fiz. 25, 641 (1977) [Sov. J. Nucl. Phys. 25, 340 (1977)].
[23]M. Shchepkin, Yad. Fiz. 18, 153 (1973) [Sov. J. Nucl. Phys. 18, 79 (1974)].
[24]B. Pontecorvo, in: Old and New Problems in Elementary Particles, a Volume Dedicated to G. Bernardini, Academic Press, New York, 1968, p. 252.
[25]S. M. Bilen'kii and B. Pontecorvo, cited in Ref. 16, V 122; Preprint E2-10032, JINR, Dubna, 1976.
[26]A. K. Mann and H. Primakoff, Phys. Rev. D15, 655 (1977).
[27]F. A. Nezrik and F. Reines, *ibid.* **142**, 852 (1966).
[28]R. Davis and D. Evans, in: Trudy seminara po aktivnym protsessam na Solntse i probleme solnechnykh neĭtrino (Proc. of the Seminar on Active Processes in the Sun and the Problem of Solar Neutrinos), Physico-technical Inst., USSR Academy of Sciences, Leningrad, 1976, p. 83.
[29]J. N. Bahcall and R. Davis, Science **191**, 264 (1976).
[30]a) V. M. Lobashev and O. V. Sedryuk, Nucl. Instrum. Methods 136, 61 (1976). b) A. A. Borovoĭ and A. A. Mikaélyan, in: Trudy konferentsii "Neĭtrino-77" (Proc. of the Conf. "Neutrino-77"), Inst. for Nuclear Research, USSR Academy of Sciences, Moscow, 1977.
[31]Cited in: E. Egelman *et al.*, Preprint, Harvard Univ., January 17, 1977.
[32]F. Reines, Preprint UCI-10, P19-106, Univ of California, Irvine, 1976.
[33]E. Egelman, B. Gordon, W. Kozanecki, W. Loomis, J. LoSecco, C. Rubbia, A. Sessoms, J. Strait, L. Sulak, C. Tao, R. Wilson, and M. Yudis, cited in Ref. 31.
[34]B. Pontecorvo, Report PD-205, Chalk River Laboratory, 1946.
[35]B. Pontecorvo, Helv. Phys. Acta 23, Suppl. III, 97 (1950).
[36]B. Pontecorvo, in: Mezhdunarodnyĭ seminar po fizike neĭtrino i neĭtrinnoĭ astrofizike (Intern. Seminar on Neutrino Physics and Neutrino Astrophysics) (Moscow, September 9–12, 1968). Nauka, Moscow, 1968. A. A. Pomanskiĭ and S. A. Severnyĭ, Prib. Tekh. Eksp., No. 3, **51** (1968). D. Davis, J. C. Evans, V. Radeka, and L. Rogers, in: Proc. of the Conf. "Neutrino-72," Vol. 1, Balatonfüred, Hungary, 1972, p. 5.
[37]J. N. Bahcall, *ibid.*, p. 29.
[38]N. Bahcall, N. Cabibbo, and A. Yahil, Phys. Rev. Lett. 28, 316 (1972).
[39]a) A. A. Pomansky and A. I. Sevastjanov, in: Proc. of the Conf. "Neutrino-75,", Vol. 2, 1975, p. 383. b) V. A. Kuz'min, Zh. Eksp. Teor. Fiz. 49, 1532 (1965) [Sov. Phys. JETP 22, 1051 (1966)].
[40]a) V. Kuz'min and G. Zatsepin, in: Proc. of the Intern. Cosmic Ray Conf. of MU-NU, London, 1965, p. 36. J. Bahcall, Phys. Rev. Lett. **23**, 251 (1969). b) M. S. Freedman *et al.*, Science **193**, 117 (1976). R. S. Raghavan, Phys. Rev. Lett. 37, 259 (1976). R. D. Scott, Nature (London) 264, 729 (1976).
[41]I. R. Barabanov *et al.*, cited in Ref. 39a, Vol. 2, p. 387.
[42]a) B. M. Pontecorvo, Usp. Fiz. Nauk **104**, 3 (1971) [Sov. Phys. Usp. **14**, 235 (1971)]. b) S. Nussinov, Phys. Lett. **B63**, 201 (1976). c) M. A. Markov and I. M. Zheleznykh, Nucl. Plys. **27**, 385 (1961).
[43]A. E. Chudakov, Cosmnews, No. 2 (1977).
[44]L. V. Volkova and G. T. Zatsepin, Yad. Fiz. **14**, 211 (1971) [Sov. J. Nucl. Phys. **14**, 117 (1972)].
[45]A. Roberts, H. Blood, J. Learned, and F. Reines, Preprint Conf. 76/59-exp., Fermilab, 1976.
[46]G. A. Askar'yan and B. A. Dolgoshein, Preprint No. 160, Physics Inst., USSR Academy of Sciences, Moscow, 1976.
[47]K. Sarma and L. Wolfenstein, Preprint COO-3066-59, 1975.
[48]M. R. Krishuaaswamy *et al.*, Phys. Lett. B57, 105 (1975).
[49]H. Fritzsch, M. Gell-Mann, and P. Minkowsky, *ibid.* B59, 256 (1975). A. De Rujula, M. Georgi, and S. L. Glashow, Phys. Rev. D12, 3589 (1975). See also: E. M. Lipmanov,

Zh. Eksp. Teor. Fiz. **43**, 893 (1963) [Sov. Phys. JETP **16**, 634 (1963)]. S. S. Gershtein, L. G. Landsberg, and V. N. Folomeshkin, in : Trudy seminara po $\mu-e$-probleme (Proc. of the Seminar on the $\mu-e$ Problem), Nauka, Moscow, 1974, p. 432.
[50]F. Gursey and P. Sikivie, Phys. Rev. Lett. **36**, 755 (1976).
[51]M. L. Perl *et al.*, Phys. Lett. **B63**, 466 (1976). G. J. Feldman *et al.*, Phys. Rev. Lett. **38**, 117 (1977).
[52]E. F. Tret'yakov *et al.*, Preprint No. 15, ITEP, Moscow, 1976.
[53]a) A. Clark *et al.*, Phys. Rev. D9, 533 (1974). b) S. S. Gershtein and Ya. B. Zel'dovich, Pis'ma Zh. Eksp. Teor. Fiz. 4, 174 (1966) [JETP Lett. 4, 120 (1966)]. G. Marx and A. Szalay, in: Neutrino-72, Vol. 1, 1972, p. 191. R. Cowsic and T. Clelland, Phys. Rev. Lett. **29**, 669 (1972).
[54]S. M. Korenchenko, B. F. Kostin, G. V. Mitsel'makher, K. G. Nekrasov, and V. S. Smirnov, Yad. Fiz. **13**, 341 (1971) [Sov. J. Nucl. Phys. **13**, 190 (1971)].
[55]S. M. Bilenky, S. T. Petcov, and B. Pontecorvo, Phys. Lett. **B67**, 309 (1977).
[56]E. P. Shabalin, Preprint No. 9, ITEP, Moscow, 1977. V. Barger and D. V. Napopulos, Preprint 1977 COO-583, 1977. P. T. Cheng and L. F. Li, Phys. Rev. Lett. **38**, 381 (1977). J. D. Bjorken and S. Weinberg, *ibid.* **38**, 622 (1977). S. B. Treiman, F. Wilczek, and A. Zee, Preprint, Princeton Univ., 1977. W. J. Marciano and A. I. Sanda, Preprint COO-2232B-116, 1977. B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Phys. Rev. Lett. **38**, 937 (1977). R. Decker and J. Restieau, Preprint UCL-IPT-77/04. W. J. Marciano and A. I. Sanda, Preprint COO-2232B-122, 1977. A. Pais, Preprint COO-2232B-118, 1977.
[57]É. N. Lipmanov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 363 (1976) [JETP Lett. **23**, 327 (1976)].
[58]S. M. Korenchenko, G. V. Mitsel'makher, and K. G. Nekrasov, Preprint R13-9542, JINR, Dubna, 1976.

Translated by N. M. Queen