agreement with the observed value of 3.9 K. The boundaries between the collinear and noncollinear phases are also in satisfactory agreement with experiments. An increase in the Néel temperature under the effect of a magnetic field also corresponds to the predictions of the model. Notice that an ordinary antiferromagnet is characterized by the opposite effect of the field on the transition temperature. In the classical model [6] under consideration, the width of the interval of fields in which the fluctuations stabilize the phase with a moment $M_{\text{sat}}/3$ tends to zero with decreasing temperature. The allowance for quantum fluctuations should lead to a nonzero interval at the zero temperature. The estimate of the width of this interval [1] caused by quantum fluctuations, which is shown in Fig. 7 by a vertical bar near the ordinate axis, is also in agreement with the extrapolation of the experimental dependences $H_{c1}(T)$ and $H_{c2}(T)$ to the zero temperature. One discrepancy with the predictions of the model is the nonzero value of the field H_{c1} at temperatures immediately adjoining T_N from below. It should be noted that the nonzero value of H_{c1} is predicted on the basis of the Heisenberg model [7]. The high-field phase boundary between the canted antiferromagnetic and paramagnetic phases also demonstrates an unordinary fluctuation behavior. In the region of temperatures exceeding 2 K, the specific-heat peak in the $C(H)$ dependence is observed in a field that is lower than the field of tending to saturation in the magnetization curve, determined from the falloff of the derivative $\mathrm{d}M/\mathrm{d}H$. The positions of the singularities in the $C(H, T)$ curves and in the $\mathrm{d}M/\mathrm{d}H$ field dependences are shown in the phase diagram in Fig. 7 near the high-field boundary of the ordered phase. The discrepancy at $T = 3$ K constitutes about 1 T. The scenario of the two-step transition to the saturated phase for a two-dimensional antiferromagnet on a triangular lattice, which was predicted in Ref. [2], is related to fluctuations: in the lower critical field, the long-range order of spin components perpendicular to the magnetic field disappears. In the interval between the two upper critical fields, the correlation between the transverse spin components falls according to a power law, with the sample remaining unsaturated. In the upper critical field, the correlations begin decreasing according to an exponential law, and the transverse component disappears.

Thus, the $RbFe(MoO_4)_2$ crystal represents a model system corresponding to a classical two-dimensional antiferromagnet on a triangular lattice. The character of the phase diagram and the existence of a magnetization plateau demonstrate good agreement with the results of a theoretical simulation of this system in terms of the classical two-dimensional XY model.

Some aspects of three-dimensional (i.e., interlayer) ordering were beyond the scope of discussion in this report; on a qualitative level they can be considered [10] based on an analysis of the interlayer interaction and related phases, which was performed in the theoretical work [14].

References

1. Chubokov A V, Golosov D I *J. Phys. Condens. Matter* **3** 69 (1991)
2. Korshunov S E *J. Phys. C* **19** 5927 (1986)
3. Anderson P W *Mater. Res. Bull.* **8** 153 (1973)
4. White S R, Chernyshev A L *Phys. Rev. Lett.* **99** 127004 (2007)
5. Nakatsuji S et al. *Science* **309** 1697 (2005)
6. Lee D H et al. *Phys. Rev. B* **33** 450 (1986)
7. Kawamura H, Miyashita S *J. Phys. Soc. Jpn.* **54** 4530 (1985)
8. Zhitomirsky M E, Petrenko O A, Prozorova L A *Phys. Rev. B* **52** 3511 (1995)
9. Smirnov A I et al. *Phys. Rev. B* **75** 134412 (2007)
10. Svistov L E et al. *Phys. Rev. B* **74** 024412 (2006)
11. Svistov L E et al. *Phys. Rev. B* **67** 094434 (2003)
12. Svistov L E et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **81** 133 (2005) [*JETP Lett.* **81** 102 (2005)]
13. Kenzelmann M et al. *Phys. Rev. Lett.* **98** 267205 (2007)
14. Gekht R S, Bondarenko I N *Zh. Eksp. Teor. Fiz.* **111** 627 (1997) [*JETP* **84** 345 (1997)]

PACS numbers: 67.30.er, **75.45.+j**, **76.60.–k**
DOI: 10.3367/UFNe.0180.201008m.0884

Spin superfluidity and magnons Bose–Einstein condensation

Yu M Bunkov

The prehistory of the discovery of magnetic superfluidity goes back to the mid-1970s, when two students at Moscow Institute of Physics and Technology (MFTI in *Russ. abbr.*), Boris Dumesh and Yuriy Bunkov, started studying, under the guidance of Academician Andrei Stanislavovich Borovik-Romanov, antiferromagnetic crystals with a dynamic frequency shift. The experiments were mainly performed using $MnCO_3$ and $CsMnF_3$. In these antiferromagnets, the hyperfine field of manganese atoms gives rise to a strong polarization of ^{55}Mn nuclei, such that their frequency of precession becomes about 600 MHz. This frequency is comparable to the frequency of the low-frequency line of antiferromagnetic resonance in a weak external magnetic field. As a result, modes of coupled electron–nucleus oscillations are formed, whose frequency depends on the magnitude of interaction, viz. on the projection of the nuclear magnetic moment onto the magnetization axis of the atoms. The frequency shift of the quasi-NMR of ^{55}Mn nuclei can reach several hundred megahertz at a temperature on the order of 1 K, as is shown in Fig. 1, and decrease upon heating or upon deflection of the magnetization vector of the nuclear subsystem. This results in a strong nonlinearity of the nuclear magnetic resonance (NMR) — the frequency of the precession depends on the angle of deflection of the nuclear magnetization vector. Under these conditions, the effective mechanism of formation of a spin echo is the mechanism of frequency modulation rather than the Hahn echo mechanism. The results of successful investigations of this echo formation mechanism by the researchers of our group were reported in Refs [1–3].

The antiferromagnetic resonance can also be excited parametrically, by the modulation of the external magnetic field at a doubled frequency. It also proved possible to parametrically excite an NMR mode. A new formation mechanism of an echo was discovered, in which the echo was excited by a single resonance pulse and then by a single pulse of parametric pumping [4]. This mechanism of echo

Yu M Bunkov Institut Néel, Grenoble, France
E-mail: Yuriy.bunkov@grenoble.cnrs.fr

Uspekhi Fizicheskikh Nauk **180** (8) 884 – 889 (2010)
DOI: 10.3367/UFNr.0180.201008m.0884
Translated by S N Gorin; edited by A Radzig

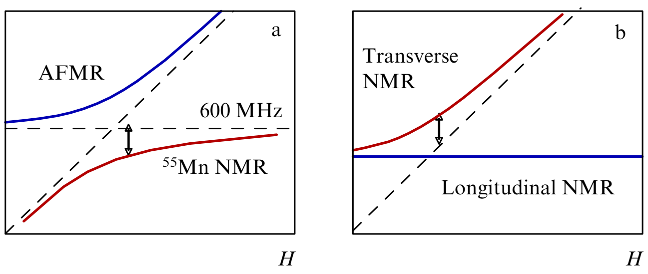

Figure 1. (a) Schematic of a spectrum of nuclear magnetic resonance (NMR) and antiferromagnetic resonance (AFMR) in $MnCO_3$. (b) Spectrum of NMR modes in superfluid ^{3}He. Arrows show the change in the NMR frequency upon nuclear magnetization deflection.

formation proved to be linear, i.e., the signal amplitude was linear in the amplitudes of both the first and second pulses [5, 6]. On the basis of the parametric mechanism of echo formation, information-processing units can be developed. All the results of our investigations into systems with a dynamic frequency shift were summed up in review paper [7].

Late in the 1970s, the superfluidity of ^{3}He was discovered, and the leading laboratories all over the world started extensive investigations of this new superfluid substance. The dynamic properties of the NMR in superfluid ^{3}He are similar to those of the systems studied in our work. A dynamic frequency shift is also observed in ^{3}He, which depends on the angle of deflection of the nuclei, as is shown schematically in Fig. 1b. Therefore, it was of interest to apply our methods of nonlinear NMR to the investigations of superfluid ^{3}He. At that time, the Institute for Physical Problems (Moscow) was fruitfully collaborating with the Low-Temperature Laboratory of the Helsinki University of Technology, where, under the guidance of Prof. Olli V Lounasmaa, investigations of superfluid ^{3}He were performed by the linear NMR methods.

Professor Lounasmaa suggested that P L Kapitza send the author of this paper to Helsinki to conduct experiments on nonlinear NMR. However, P L Kapitza and A S Borovik-Romanov decided that the idea of these investigations was so good that it was expedient to carry them out by themselves, at the Institute for Physical Problems, rather than give them up to foreign laboratories. But to conduct these studies it was necessary to construct a domestic cryostat for nuclear demagnetization and reach temperatures as low as 1 mK. To design such a cryostat, a special group was formed under the guidance of Yu M Bunkov, which included two students, V V Dmitriev and Yu M Mukharskii, and a mechanic S M Elagin.

P L Kapitza gave the green light for the fulfillment of our orders in the mechanical shop of the Institute. The construction of the cryostat for nuclear demagnetization took four years; in 1984, we obtained superfluid ^{3}He.

By that time, a number of NMR investigations of ^{3}He had been carried out at large angles of magnetization deflection, mainly at Cornell University and at Bell Laboratories, both in the United States [8]. It turned out that the induction signal in ^{3}He-A falls off quite rapidly, whereas in ^{3}He-B the longitudinal relaxation strongly depends on the magnetic field gradient, and a long-lived tail of the induction signal is observed in it. Attempts were undertaken to explain the rapid relaxation in ^{3}He-A by the magnetization transfer from the zone of sensitivity of the NMR coils by superfluid spin current, and the existence of a long-lived tail of the induction signal in ^{3}He-B, by standing spin waves. The dependence of the relaxation on the magnetic field gradient was considered at that time to be mysterious [8].

For our first experiment with superfluid ^{3}He we designed an almost closed chamber in the hope of confirming the presence of spin superfluidity in ^{3}He-A. However, the signal decayed in the closed chamber as rapidly [9] as in a chamber open at both ends, although there was no way for the magnetization to be escaped in our case! An explanation for this effect was found on the basis of the Fomin theory of the instability of homogeneous precession of magnetization in the superfluid A phase of ^{3}He [10]. Thus, the interpretation of the results of the preceding experiments as the observation of spin superfluidity in ^{3}He-A was refuted.

An unexpected result was obtained in our experiments with ^{3}He-B in the same closed chamber. We revealed that the NMR induction signal first falls off because of the inhomogeneity of the magnetic field, and then it spontaneously restores its amplitude to almost the initial magnitude [11], and that this effect is observed even at a very large inhomogeneity of the magnetic field. The results of this experiment were explained theoretically by I A Fomin as a redistribution of the magnetization deflected by the superfluid spin current [12]. In this case, a homogeneously precessing domain (HPD) is formed, in which magnetization deflected by an angle of more than 104° precesses in a spatially uniform manner. The matter is that the gradient of the precession phase creates a spin supercurrent which flows until the gradient of the precession disappears, but in ^{3}He-B this is possible only at angles of deflection exceeding 104°, at which there appears a dynamic frequency shift of a dipole–dipole nature. It is precisely this shift that compensates for the inhomogeneity of the external magnetic field. In such a way the first effect arising owing to the existence of a superfluid spin current was discovered. For this discovery, A S Borovik-Romanov, Yu M Bunkov, V V Dmitriev, Yu M Mukharskii, and I A Fomin were awarded the State Prize of the Russian Federation in 1993. A detailed analysis of the history of the discovery of spin superfluidity and its investigations can be found in Refs [13, 14].

Figure 2 displays a stroboscopic digital record of the NMR induction signal of ^{3}He in a magnetic field with strong gradient, when the inhomogeneity of the magnetic field on the scale of the chamber dimensions reaches 600 Hz. It is seen that the induction signal rapidly dephases, in approximately 2 ms. Then, a transfer of the deflected magnetization occurs in the course of 10 ms into the region of the chamber with a lower magnetic field, and an HPD is formed in the subsequent 10 ms. By applying the Holstein–Primakoff transformation, the deflected and precessing magnetization can be interpreted as the production of a gas of long-lived quasiparticles—magnons. For magnons, the gradient of a magnetic field plays the same role as a gravitational field for atoms. The field gradient and the walls of the chamber create a trap in which magnons can condense in the presence of an appropriate interaction between them. The fact that condensation occurs is seen from the spectroscopic analysis of the induction signal presented in Fig. 3. In the case of the excitation of magnons, the spectral width of the signal equals 600 Hz, which corresponds to an inhomogeneity of the magnetic field on the scale of the chamber dimensions. In 30 ms, the line collapses to a width of 0.5 Hz. This corresponds to a 1000-fold narrowing of the magnon spectrum. Such condensation had

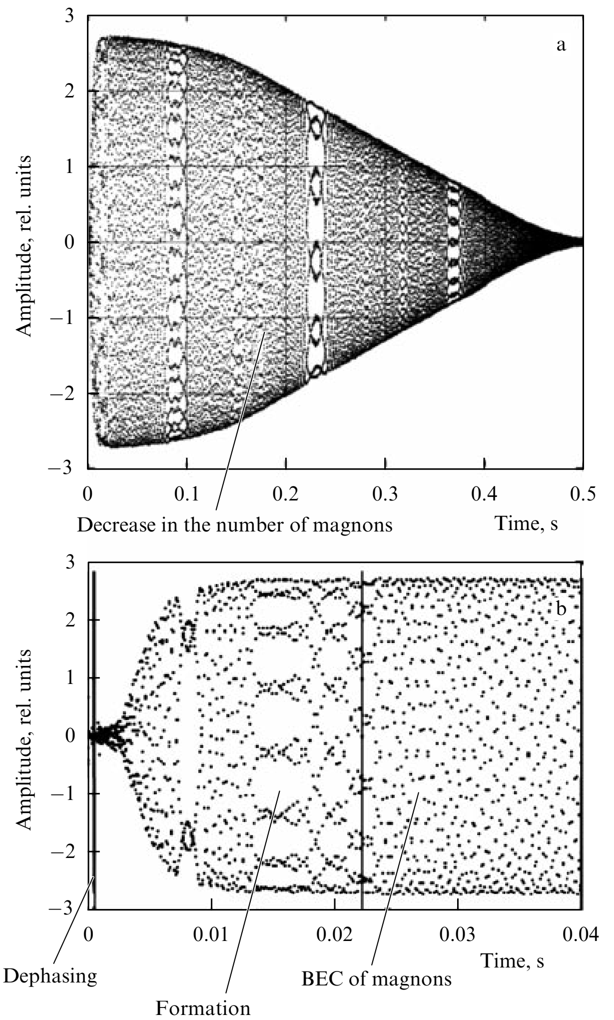

Figure 2. (a) Stroboscopic record of the induction signal. (b) The initial portion of the signal.

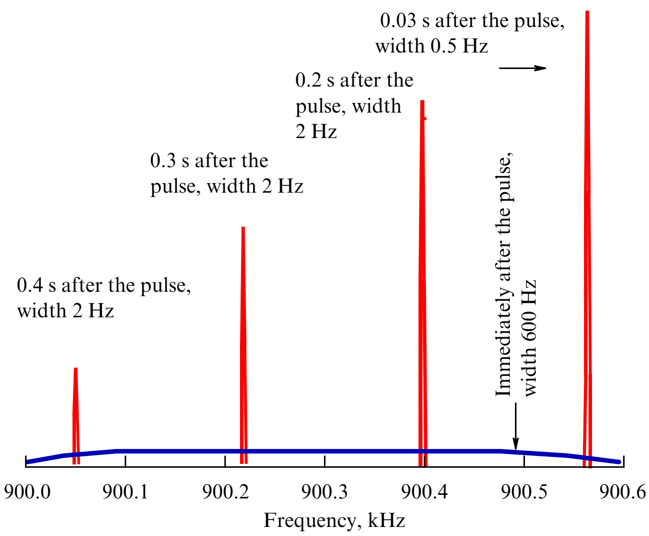

Figure 3. Spectral width of the NMR signal immediately after the arrival of a pulse and after the formation of a magnon Bose–Einstein condensate.

never been observed in an atomic Bose–Einstein condensate! A broadening of the 0.5 Hz signal arises because of the relaxation of the number of magnons.

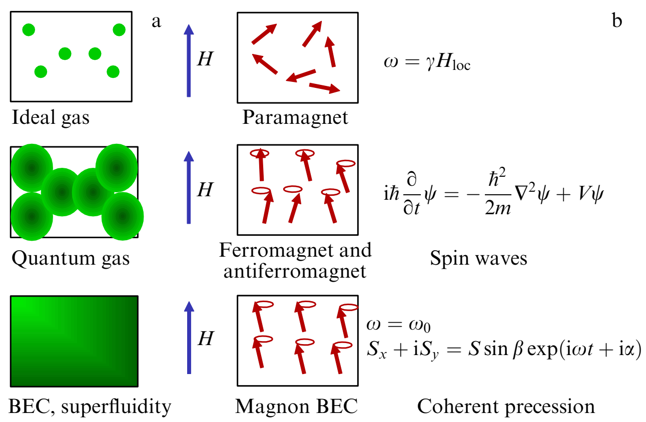

Figure 4. A schematic of the states of (a) an atomic gas, and (b) a gas of magnons: ω is the frequency of precession in a local field $H_{\rm loc}$, γ is the gyromagnetic ratio, and ω_0 is the common precession frequency.

As a result of such a weak relaxation, the signal of the Bose–Einstein condensate (BEC) is observed for a time of about 1 s. Let us recall that the atoms in the trap are also evaporated, so that the atomic BEC also lives for approximately 1 s. However, we can excite additional magnons which compensated for their natural loss; therefore, the condensate of magnons, in contrast to the atomic BEC, could exist continuously.

Figure 4 schematically depicts various states of the atomic gas and analogous states of the gas of magnons. It is necessary to distinguish the magnetically ordered state, in which spin wave modes are formed, from the state with a coherent precession, in which all magnons are described by a single wave function, just as the BEC of atoms. It is precisely the first state that was observed in experiments with a long-lived induction signal with a small amplitude [15, 16]. In these experiments, a standing spin wave was observed on the scale of the chamber sizes, whose parameters exactly corresponded to the modes of spin waves that were investigated in detail in Ref. [17]. The formation of standing spin wave modes is due to the gradient energy and boundary conditions at the walls of the chamber. In contrast, the Bose–Einstein condensation of magnons occurs due to the interaction between magnons. In this case, the NMR signal corresponds to the signal of a single oscillator whose frequency depends on the amplitude. It is this dependence that is seen well in Fig. 3. In Refs [15, 16], no dependence of the signal frequency on the amplitude was observed; therefore, these investigations cannot be considered the observation of a BEC in the form of either an HPD or a Q-ball whose properties will be considered below.

To describe the process of the Bose–Einstein condensation of magnons, we shall use the Gross–Pitaevskii equations and shall search for the solution in the form of a wave function Ψ of the homogeneous precession:

$$\Psi = \sqrt{\frac{2S}{\hbar}}\,\sin\frac{\beta}{2}\,\exp\left(i\omega t + i\alpha\right),$$

$$S_x + iS_y = S\sin\beta\exp\left(i\omega t + i\alpha\right),$$

which satisfy the conditions

$$\frac{\delta F}{\delta\Psi^*} = 0\,, \tag{1}$$

$$F = \int d^3r\left(\frac{|\nabla\Psi|^2}{2m_{\rm M}} + \left(\omega_{\rm L}(z) - \omega\right)|\Psi|^2 + F_{\rm D}\right).$$

Here, S is the magnetization, S_x and S_y are its projections onto the corresponding coordinate axes, β is the angle of deflection of magnetization, and ω and α are the frequency and phase of the magnon precession, respectively.

In the last equation, the first term on the right-hand side is the gradient energy that is responsible for the formation of spin waves and superfluid spin current, and m_M is the mass of the magnon; the second term stands for the spectroscopic energy, where $\omega_L(z)$ is the Larmor frequency (the potential in the external magnetic field), ω is the magnon precession frequency (the chemical potential), and F_D is the dipole–dipole energy of interaction of the magnon with the field of magnons. For superfluid ^{3}He-B, where the orbital moment is directed along the magnetic field and the magnetization vector is deflected by an angle β, the dipole–dipole energy F_D is equal to zero for $\beta < 104°$, and to

$$F_D = \frac{8}{15}\chi\Omega_L^2\left(\frac{|\Psi|^2}{S} - \frac{5}{4}\right)^2 \qquad (2)$$

for $\beta > 104°$. Here, χ is the magnetic susceptibility, and Ω_L is the Leggett frequency characterizing the intensity of the dipole–dipole interaction. Figure 5 shows the sum of the dipole and spectroscopic energies as a function of the angle of the magnetization deflection at different values of $\Delta\omega$ — the difference between the precession frequency and the local Larmor frequency, i.e., the magnitude of the dynamic frequency shift. It is convenient to measure this shift as a percentage of the maximum possible shift, which in ^{3}He-B is equal to $\omega_d = \Omega_L^2/(2\omega_L)$. The magnons condense at a minimum energy, which arises at the angles of deflection on the order of 104°, viz. at an NMR frequency exceeding the Larmor value.

The following problem is the determination of the frequency of the nonlinear NMR. In the case of pulsed NMR, the total number of magnons produced in the experimental chamber is specified. The magnetic field gradient leads to the appearance of a gradient of the precession phase and that, in turn, gives rise to the gradient of the magnetic part of the order parameter, viz. to a superfluid magnetization transfer. This process terminates after an equilibrium distribution of magnons is reached,

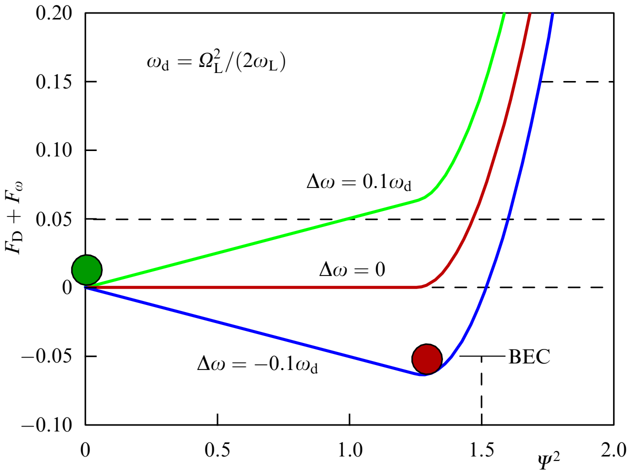

Figure 5. Spectroscopic and dipole energies as functions of the magnon density at the positive and negative difference between the NMR and Larmor frequencies. BEC arises at the minimum of the energy upon deflecting the magnetization by an angle of 104°.

which corresponds to the minimum of the dipole and spectroscopic energies over the entire chamber. In this case, the system is divided into two domains. In one of them, the magnetization vector is directed along the field; in the second, a homogeneously precessing domain (HPD) arises. The dimensions of the domains are determined by the total number of magnons, and the precession frequency is determined by the Larmor frequency at the boundaries of the domains. In other words, a Bose–Einstein condensation of magnons generated by an rf pulse in the case of a pulsed NMR occurs at a minimum magnetic field. In the case of continuous NMR, the rf field specifies the frequency of the magnon precession. The equilibrium distribution of magnons corresponds to the formation of a domain with a precessing magnetization in that region of the chamber where the Larmor frequency is lower than the frequency of the rf field. In this case, the rf field specifies the chemical potential of the system, and the number of magnons is fitted to this potential. The natural relaxation of magnons in the second case is compensated for by the production of new magnons in the rf field. Thus, contrary to atomic BEC which lives in a trap for only a rather short time (on the order of 1 s), the magnon Bose–Einstein condensate can be maintained for an infinitely long time [18].

This feature makes it possible to carry out a whole series of experiments with magnetic superfluidity in a channel that connects two Bose–Einstein condensates. The scheme of the experiment is demonstrated in Fig. 6. Two independent NMR spectrometers reliably shielded from one another are mounted in two chambers connected by a channel. Each spectrometer generates an HPD with a frequency and phase equal to those of the rf pumping. Condensate also fills the channel between the chambers, which we can observe with the help of miniature coils mounted in the channel. Each spectrometer measured the magnitude of the NMR signal absorption, which corresponded to the rate of magnon relaxation. When a phase difference was established between the spectrometers, a superfluid spin current began flowing through the channel, which transferred the magnetization from one chamber to another. Correspondingly, this current also transferred the Zeeman energy. As a result, the absorption signal in one chamber increased, and in the other decreased, which made it possible to measure the magnitude of the spin current. At a sufficiently high current, we managed to obtain a situation where the Zeeman energy coming into one of the chambers became so large that the absorption signal changed its sign. The BEC began emitting an rf field! Thus, we constructed a transformer based on superfluid spin current [19, 20].

Figure 6b depicts the scheme of the experimental chambers in the channel between which a constriction with an orifice diameter of 0.48 mm was arranged. The coherence length for the spin superfluidity depends on the difference between the NMR and Larmor frequencies and can reach 1 mm. By varying this difference in frequencies, we could observe a classical Josephson effect (signal *3* in Fig. 6c), a nonlinear Josephson effect (signals *2* and *4*), and a phase slippage (signal *1*) [21, 22].

Numerous other effects that confirm the magnetic coherency of the HPDs have been observed. For example, there were revealed and investigated Goldstone modes of HPDs oscillations, such as a torsional mode [23], and a surface mode [24]. A quantum vortex in a spin supercurrent was also created and studied [25]. All the results of these

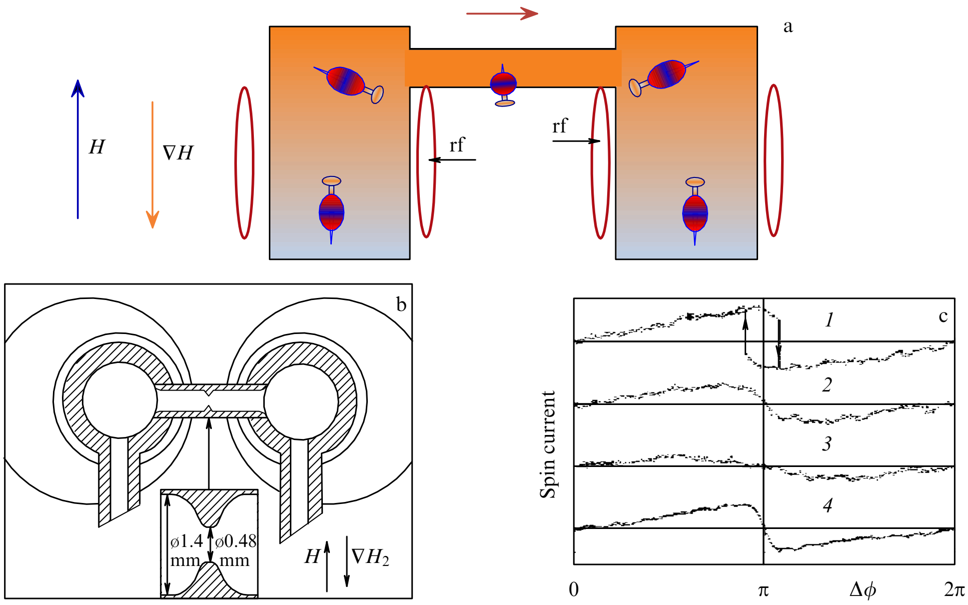

Figure 6. (a) Schematic of the experiment with two Bose–Einstein condensates connected by a channel. In the presence of a phase difference, a DC spin current flows between them; in the case of a difference in frequencies, the current increases and reaches a critical value, after which a phase slippage occurs; (b) a constriction in the channel, at which the Josephson effect was observed (c).

experiments demonstrate that an HPD is a state with a magnon Bose–Einstein condensation.

If it is assumed that only the antiferromagnetic part of the order parameter is responsible for the formation of BEC in superfluid ^{3}He, then why has the coherent state of magnons not previously been discovered in solid magnets? The reason lies in the different types of instability of the homogeneous precession and its decay into spin waves with a nonzero wave vector **k**. It turned out that in the superfluid ^{3}He as well, for temperatures $T < 0.4T_c$, where T_c is the superfluid transition temperature, an instability of the homogeneous precession also develops [26]. This instability is now explained by two mechanisms: the interaction of the precessing magnetization with the walls of the chamber [27–29], and the anisotropy of the velocity of spin waves [30]. In both cases, the instability manifests itself when, with decreasing temperature, the damping of spin waves decreases and they begin swinging parametrically. In this case, there arises a characteristic exponentially increasing curve of the decay of the homogeneous precession that was observed in Ref. [26].

In superfluid ^{3}He at lower temperatures, a new long-lived induction signal with a small amplitude was revealed, whose duration may be equal to minutes or even hours [31, 32]. After long and contradictory investigations, this signal was explained as being due to the emission of a new state of BEC produced in a trap created by the texture of the orbital part of the order parameter [33]. Here, we are dealing with the model of interaction of two quantum fields. One field, without a charge, is the field of the ^{3}He orbital moment; the other is the spin field carrying a charge. The spin field is concentrated in the minimum of the orbital field; as a result, the minimum of the orbital field decreases even greater. A situation described in the quantum theory of fields as a Q-ball arises [34]. Using very weak rf pumping, we managed to excite magnons corresponding not only to the ground state but also to excited states of the Q-ball. This was illustrated most vividly in recent experiments with a rotating cryostat for nuclear demagnetization in Helsinki, in which the profile of the orbital field was varied and, correspondingly, the frequencies of the excited states were changed [35]. After switching off the pumping, the magnons go into the ground state which emits an induction signal.

The BEC of magnons in superfluid ^{3}He is not limited to the two above-considered states. Usually, the orbital field in free ^{3}He-A is oriented transversely to the magnetic field, and the dipole–dipole energy leads to a homogeneous precession instability. In paper [36], it was predicted that the magnon BEC in ^{3}He-A can be realized if the orbital field can be aligned along the magnetic field. Recently, it was revealed that if ^{3}He-A is placed in an aerogel squeezed along the field, the anisotropy of the aerogel results in the orientation of the orbital moment along the field as well [37]. Under these conditions, a homogeneous precession [38] and the formation of BEC in ^{3}He-A [39, 40] were observed.

In addition, when the orbital moment in ^{3}He-B is oriented perpendicularly to the field, in the range of large angles of magnetization deflection a minimum of dipole energy is formed, in which the BEC can form, as was predicted in Ref. [41]. Quite recently, the formation of this BEC was revealed in Grenoble. It should be noted that all the above-mentioned types of BEC are formed not only in traps of different types but also in circumstances where various dipole–dipole interactions occur. BEC that is formed in the case of a strong counterflow of a superfluid and normal liquids in ^{3}He-B should also be mentioned [42]. Thus, to date five different states of magnon BECs in superfluid ^{3}He have been revealed. Notice also that the spin waves with a nonzero wave vector **k** can also form BEC; this was recently demonstrated in experiments with iron yttrium garnet [43]. In more detail, the properties of magnon Bose–Einstein condensates in the ^{3}He superfluid phases are considered in the reviews [44–46].

Finally, let us return to NMR in magnets with a dynamic frequency shift, which we considered at the beginning of this paper. The dependence of the precession frequency on the angle β of deflection of the nuclear magnetization in them is described by the formula $\omega = \omega_0 - \Delta \cos\beta$. Here, ω_0 is the NMR frequency in the limit of high temperatures, and Δ is the dynamic frequency shift. Correspondingly, the energy of the hyperfine interaction varies as $F \sim -\Delta \sin\beta$, i.e., is a concave function. Consequently, under appropriate conditions BEC of magnons can occur in these magnets. In our experiments of the 1970s, a strange echo signal was observed, whose frequency corresponded to the exciting pulse frequency lying between ω_0 and $\omega_0 - \Delta$, rather than to the frequency $\omega_0 - \Delta$ of the linear NMR. We called this effect the capture echo. The capture echo is likely to have been the first observation of the magnon BEC, but this requires additional verification. At present, we are studying the Bose–Einstein condensation of magnons in solid magnets with the support of the Ministry of Education and Science of the Russian Federation (Federal Target Program 'Scientific and Pedagogical Personnel of Innovative Russia', project No. 02.740.11.5217).

Acknowledgments. In conclusion, I wish to emphasize that all our achievements are due to the support of the scientific school of the Institute for Physical Problems, RAS and to that atmosphere of confidence and benevolence which was so ably created by Andrei Stanislavovich Borovik-Romanov.

References

1. Bun'kov Yu M, Dumesh B S, Kurkin M I *Pis'ma Zh. Eksp. Teor. Fiz.* **19** 216 (1974) [*JETP Lett.* **19** 132 (1974)]
2. Bun'kov Yu M, Dumesh B S *Zh. Eksp. Teor. Fiz.* **68** 1161 (1975) [*Sov. Phys. JETP* **41** 576 (1975)]
3. Borovik-Romanov A S, Bunkov Yu M, Dumesh B S *Physica B+C* **86** 1301 (1977)
4. Bun'kov Yu M *Pis'ma Zh. Eksp. Teor. Fiz.* **23** 271 (1976) [*JETP Lett.* **23** 244 (1976)]
5. Bun'kov Yu M, Gladkov S O *Zh. Eksp. Teor. Fiz.* **71** 2181 (1977) [*Sov. Phys. JETP* **46** 1141 (1977)]
6. Bun'kov Yu M, Maksimchuk T V *Zh. Eksp. Teor. Fiz.* **79** 1408 (1980) [*Sov. Phys. JETP* **52** 711 (1980)]
7. Borovik-Romanov A S et al. *Usp. Fiz. Nauk* **142** 537 (1984) [*Sov. Phys. Usp.* **27** 235 (1984)]
8. Lee D M, Richardson R C, in *The Physics of Liquid and Solid Helium* Vol. 2 (Eds K H Bennemann, J B Ketterson) (New York: Wiley, 1978)
9. Borovik-Romanov A S, Bun'kov Yu M, Dmitriev V V, Mukharskii Yu M *Pis'ma Zh. Eksp. Teor. Fiz.* **39** 390 (1984) [*JETP Lett.* **39** 469 (1984)]
10. Fomin I A *Pis'ma Zh. Eksp. Teor. Fiz.* **39** 387 (1984) [*JETP Lett.* **39** 466 (1984)]
11. Borovik-Romanov A S, Bun'kov Yu M, Dmitriev V V, Mukharskii Yu M *Pis'ma Zh. Eksp. Teor. Fiz.* **40** 256 (1984) [*JETP Lett.* **40** 1033 (1984)]
12. Fomin I A *Pis'ma Zh. Eksp. Teor. Fiz.* **40** 260 (1984) [*JETP Lett.* **40** 1037 (1984)]
13. Bunkov Yu M, in *Low Temperature Physics* (Ed. A S Borovik-Romanov) (Moscow: MIR Publ., 1985) p. 132
14. Bunkov Yu M *Prog. Low Temp. Phys.* **14** 69 (1995)
15. Giannetta R W, Smith E N, Lee D M *J. Low Temp. Phys.* **45** 295 (1981)
16. Corruccini L R, Osheroff D D *Phys. Rev. B* **17** 126 (1978)
17. Masuhara N et al. *Phys. Rev. Lett.* **53** 1168 (1984)
18. Borovik-Romanov A S et al. *Zh. Eksp. Teor. Fiz.* **88** 2025 (1985) [*Sov. Phys. JETP* **61** 1199 (1985)]
19. Borovik-Romanov A S et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **45** 98 (1987) [*JETP Lett.* **45** 124 (1987)]
20. Borovik-Romanov A S et al. *Phys. Rev. Lett.* **62** 1631 (1989)
21. Borovik-Romanov A S et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **47** 400 (1988) [*JETP Lett.* **47** 478 (1988)]
22. Borovik-Romanov A S et al. *J. Physique* **49** (C-8) 2067 (1988)
23. Bun'kov Yu M, Dmitriev V V, Mukharskii Yu M *Pis'ma Zh. Eksp. Teor. Fiz.* **43** 131 (1986) [*JETP Lett.* **43** 168 (1986)]
24. Bunkov Yu M, Dmitriev V V, Mukharskiy Yu M *Physica B* **178** 196 (1992)
25. Borovik-Romanov A S et al. *Physica B* **165** 649 (1990)
26. Bunkov Yu M et al. *Europhys. Lett.* **8** 645 (1989)
27. Bunkov Yu M, Lvov V S, Volovik G E *Pis'ma Zh. Eksp. Teor. Fiz.* **83** 624 (2006) [*JETP Lett.* **83** 530 (2006)]
28. Bunkov Yu M, Lvov V S, Volovik G E *Pis'ma Zh. Eksp. Teor. Fiz.* **84** 349 (2006) [*JETP Lett.* **84** 289 (2006)]
29. Bunkov Y M, Golo V L *J. Low Temp. Phys.* **137** 625 (2004)
30. Surovtsev E V, Fomin I A *Pis'ma Zh. Eksp. Teor. Fiz.* **90** 232 (2009) [*JETP Lett.* **90** 211 (2009)]
31. Bunkov Yu M et al. *Phys. Rev. Lett.* **68** 600 (1992)
32. Bunkov Yu M et al. *Physica B* **194–196** 827 (1994)
33. Bunkov Yu M, Volovik G E *Phys. Rev. Lett.* **98** 265302 (2007)
34. Coleman S *Nucl. Phys. B* **262** 263 (1985)
35. Bunkov Yu M et al., arXiv:1002.1674
36. Bunkov Yu M, Volovik G E *Europhys. Lett.* **21** 837 (1993)
37. Kunimatsu T et al. *Pis'ma Zh. Eksp. Teor. Fiz.* **86** 244 (2007) [*JETP Lett.* **86** 216 (2007)]
38. Sato T et al. *Phys. Rev. Lett.* **101** 055301 (2008)
39. Bunkov Yu M, Volovik G E *Pis'ma Zh. Eksp. Teor. Fiz.* **89** 356 (2009) [*JETP Lett.* **89** 306 (2009)]
40. Hunger P, Bunkov Y M, Collin E, Godfrin H *J. Low Temp. Phys.* **158** 129 (2010)
41. Bun'kov Yu M, Volovik G E *Zh. Eksp. Teor. Fiz.* **103** 1619 (1993) [*JETP* **76** 794 (1993)]
42. Dmitriev V V et al. *Phys. Rev. B* **59** 165 (1999)
43. Malomed B A et al. *Phys. Rev. B* **81** 024418 (2010)
44. Volovik G E *J. Low Temp. Phys.* **153** 266 (2008)
45. Bunkov Yu M, Volovik G E *J. Phys. Condens. Matter* **22** 164210 (2010)
46. Bunkov Yu M, Volovik G E, arXiv:1003.4889

PACS numbers: 03.75.Hh, 75.30.Ds, **75.45.+j**
DOI: 10.3367/UFNe.0180.201008n.0890

Kinetics and Bose–Einstein condensation of parametrically driven magnons at room temperature

O Dzyapko, V E Demidov, S O Demokritov

The origin of the ferromagnetic state is the quantum-mechanical exchange interaction between spins of individual atoms, which aligns the spins in parallel to each other. The paramagnet–ferromagnet transition is documented by a divergence of the coherence length describing the correlation between the longitudinal components of the spins located far from each other. The fluctuations above the ground state of a ferromagnet with totally parallel spins are usually described by means of quantized low-energy spin-wave excitations, which are called magnons. Magnons in thermal equilibrium do not show coherence effects because at nonzero temperatures the transverse spin components remain uncorrelated even in a ferromagnetic phase. In fact, they are usually considered to form a gas of elementary excitations (quasi-

O Dzyapko, V E Demidov, S O Demokritov Institut für Angewandte Physik, Westfälische Wilhelms-Universität Münster, Deutschland
E-mail: demokrit@wwu.de

Uspekhi Fizicheskikh Nauk **180** (8) 890–894 (2010)
DOI: 10.3367/UFNr.0180.201008n.0890
Translated by S O Demokritov; edited by A Radzig