PACS numbers: **03.75.–b**, 05.70.Fh, 64.60.Ht
DOI: 10.1070/PU2008v051n06ABEH006544
DOI: 10.3367/UFNr.0178.200806h.0633

Formation kinetics of the Bose condensate and long-range order

Yu M Kagan

The rapidly developing area of research related to ultracold gases has opened up the unique possibility of studying the formation kinetics of a Bose condensate and long-range order. The isolation of a gas from the walls in magnetic and electric traps and the possibility of observing the intrinsic real-time evolution of the system are the decisive factors in this case. Although the first theoretical papers in this field appeared in the early 1990s, it was not until 2007 that the first experimental research on the time evolution of long-range order was reported in the literature [1 – 2]. A vigorous study of this phenomenon was pursued between these dates, and this report is concerned with the analysis of the data and existing notions in this area.

The capability of rapid cooling by cutting off the Maxwellian tails enables studying the evolution starting from the points in time when all correlation properties of a gas are purely classical and there is not the slightest trace of a condensate. In this case, the kinetics proceed with conservation of the total energy and the number of particles in the system. As it turns out, the evolution comprises four stages.

During the first stage, which is described by the Boltzmann equation, a particle flux forms in the energy space, directed towards lower energies. When the particles that constitute the condensate in equilibrium occur in the energy range where the kinetic energy is lower than the interparticle interaction energy, the formation of collective correlations sets in and the kinetic equation is no loner valid (the number of particles that fall into this energy range, which is commonly termed the coherence interval, is comparable with the total number of particles). But even before this, the evolution goes through a stage during which all occupation numbers of individual modes become much greater than unity. As shown in Refs [3, 4], the system is then adequately described by the classical Bose field, which obeys the nonlinear Schrödinger equation in the form of the Gross – Pitaevskii equation. The solution of this equation leads to an important result: in the coherence interval, the fluctuations of density are suppressed and the single-particle density matrix depends only on phase fluctuations. At this stage, a special quasicondensate state emerges, which is equivalent to the genuine condensate in local properties, but has no long-range order. An instantaneous picture of the gas actually demonstrates the division of the system into finite-size quasicondensate domains. Each domain has a specific phase in the absence of phase correlation between different domains.

This picture underlay the prediction that the evolution during the third stage should be accompanied by the emergence of a vorticity structure. This prediction was borne out by the direct numerical solution of the nonlinear Schrödinger equation [5], which demonstrated the emergence of a vorticity ball and its temporal evolution.

The final stage is characterized by the damping of nonequilibrium regular-phase fluctuations and the relaxation of the vorticity structure. This occurs with an increase of quasicondensate domains in size, which is effectively equivalent to an increase in the density-matrix decay distance, thereby determining the evolution of the long-range order scale [3, 4] (see also Ref. [6]). The long-range order settling time τ_L increases with the domain size L: $\tau_L \sim L^n$, where $n = 1-2$, depending on parameter ratios.

The report presents a comparison with the theory and a comprehensive analysis of the experimental results found in Refs [1, 2], especially of the temporal evolution of long-range order formation [1]. The analysis relies on the theory elaborated for the analog of the Hanbury – Brown – Twiss effect for particles in the 'two sources, one detector' setup in the evolution of a nonequilibrium system involving a classical-to-quantum transformation of correlations [7]. Experimental data are qualitatively and quantitatively compared with theoretical predictions.

References

1. Ritter S et al. *Phys. Rev. Lett.* **98** 090402 (2007)
2. Hugbart M et al. *Phys. Rev. A* **75** 011602(R) (2007)
3. Kagan Yu, Svistunov B V *Phys. Rev. Lett.* **79** 3331 (1997)
4. Kagan Yu, Svistunov B V *Pis'ma Zh. Eksp. Teor. Fiz.* **67** 495 (1998) [*JETP Lett.* **67** 521 (1998)]
5. Berloff N G, Svistunov B V *Phys. Rev. A* **66** 013603 (2002)
6. Kagan Yu, Svistunov B V *Zh. Eksp. Teor. Fiz.* **105** 353 (1994) [*JETP* **78** 187 (1994)]
7. Kagan Yu (2008) (in press)

PACS numbers: 05.30.Fk, **71.10.–w**, 74.20.Fg
DOI: 10.1070/PU2008v051n06ABEH006548
DOI: 10.3367/UFNr.0178.200806i.0633

Superfluid Fermi liquid in a unitary regime

L P Pitaevskii

1. Introduction

When choosing the subject of my presentation at this session dedicated to the 100th anniversary of the birth of Landau, I wanted to speak about something that would have surprised Landau. I believe that the recently prepared physical object — a universal superfluid Fermi liquid — meets this requirement in the best way possible.

As is well known, Landau did not regard the microscopic theory of fluids as a problem worth being occupied with. I quote a well-known passage from *Statistical Physics* [1]: "In contrast to gases and solids, liquids do not permit calculating the thermodynamic quantities or at least their temperature dependences in the general form. The reason lies with the strong interaction between the molecules of a liquid and, at the same time, the absence of the smallness of oscillations, which imparts simplicity to the thermal motion in solids. Because of the high intensity of molecular interaction, the knowledge of a specific interaction law, which is different for different liquids, becomes significant for calculating thermodynamic quantities."

This statement is perfectly correct for all liquids existing in nature. However, progress in experimental techniques has recently enabled preparing liquids with properties independent of any quantities that characterize the interaction. This situation emerges because the interatomic interaction in these bodies is, in a sense, infinitely strong. The case in point is ultracold gases near the so-called Feshbach resonances.

First of all, we pose the question: what is the word liquid taken to imply? We accept a natural definition: a liquid is a fluid body with a strong interaction between its particles. We emphasize that fluidity implies the absence of strict periodicity, of a crystalline long-range order.

The liquids of interest to us are made from gases whose atoms obey the Fermi statistics. The gas is dilute in the sense that the average interatomic distance $n^{-1/3}$, where n is the atomic number density, is much greater than the characteristic range r_0 of interatomic forces:

$$r_0 \ll n^{-1/3} . \tag{1}$$

Condition (1) is always satisfied for the objects under consideration. However, the fulfillment of this condition does not yet signify that we are dealing with a gas in the sense that the interaction is weak. Let the temperature be sufficiently low, such that the gas is degenerate, $T \leqslant E_{\mathrm{F}}$.[1] It is then valid to say that all body properties depend on one parameter f, the amplitude of the scattering of atoms with the orbital momentum $l = 0$ by each other. The interaction is weak, i.e., the body is indeed a gas, if the amplitude is small in comparison with the interatomic distances:

$$|f| \ll n^{-1/3} . \tag{2}$$

The quantities r_0 and $|f|$ are typically of the same order of magnitude and conditions (1) and (2) are practically equivalent. However, this is not the case when a system of two atoms has an energy level close to zero. According to the general scattering theory, the scattering amplitude is then expressed in the form (see, e.g., Landau and Lifshitz [2]) $f(k) = -(a^{-1} + \mathrm{i}k)^{-1}$, where k is the wave vector and $a = -f(0)$ is the scattering length, a constant that characterizes the scattering completely. When $a > 0$, the system of two atoms has a bound state with the negative energy $\epsilon = -\hbar^2/ma^2$. When $a < 0$, the system is said to have a virtual level. If $|a|$ is high enough, $|a| \geqslant k^{-1} \sim n^{-1/3}$, the interaction weakness condition (2) is certainly violated and we are by definition dealing with a liquid, although a dilute liquid in the sense of condition (1). In this case, its properties are characterized by the sole parameter a. When $|a| \gg k^{-1}$, the scattering amplitude reaches its 'unitary limit' $f \approx \mathrm{i}/k$. The length a then drops out of the theory and we are dealing with a universal liquid, whose properties do not depend on the interaction at all. Of course, the picture under discussion implies the possibility of changing the scattering amplitude. This opportunity arises in the presence of Feshbach resonances, in the vicinity of which the position of the energy level of the system of two atoms depends on the magnetic field [3]. The scattering length as a function of the magnetic field can be represented as

$$a = a_{\mathrm{bg}}\left(1 - \frac{\Delta_B}{B - B_0}\right) . \tag{3}$$

Near the resonance $B \approx B_0$, the scattering length is large and the system is a universal liquid.

We qualitatively consider the properties of the system at $T = 0$ in different ranges of the scattering length a. When this length is positive and relatively small, $r_0 \ll a \ll n^{-1/3}$, the system of two atoms has a bound state and the atoms combine to form molecules with a binding energy ϵ. The system is a Bose gas consisting of weakly bound diatomic molecules, or dimers. It is significant that the dimer – dimer scattering length a_{dd} is positive, i.e., these molecules experience mutual repulsion. Calculating a_{dd} is an intricate problem, which was solved in [4]. It turned out that $a_{\mathrm{dd}} = 0.6a$. Therefore, in this regime, the system is a weakly nonideal superfluid Bose gas described by the Bogolyubov theory [6], with the obvious change $m \to 2m$, $a \to 0.6a$.

The question of the lifetime of this system is of paramount importance for the entire area of physics involved. This lifetime is limited by transitions from the weakly bound level to deep molecular levels in molecular collisions accompanied by the release of a large amount of energy. The molecule number loss in these inelastic processes is described by the equation $\dot{n}_{\mathrm{d}} = -\alpha_{\mathrm{dd}} n_{\mathrm{d}}^2$. The dependence of the recombination coefficient α_{dd} on a was also studied in Ref. [4]. It turned out that $\alpha_{\mathrm{dd}} \propto a^{-2.25}$. Therefore, the system becomes more stable with an increase in the scattering length, i.e., as the resonance is approached. This paradoxical result stems from the Fermi nature of atoms or, to be more precise, from the fact that fermions with parallel spins cannot reside at the same point. In a Bose gas, which was also studied in experiments, the lifetime decreases sharply as the resonance is approached. This is the reason why only the Fermi liquid can actually be investigated in the unitary mode. The experimentally measured dependence of α_{dd} on a is depicted in Fig. 1. It is in satisfactory agreement with the theory.

We now consider the opposite limit case, where the scattering length is negative and small in modulus, $a < 0$, $r_0 \ll |a| \ll n^{-1/3}$, as is the case on the opposite side of the resonance. The system is then a weakly nonideal Fermi gas with attraction between the atoms. According to the theoretical concepts of Bardeen – Cooper – Schrieffer and Bogolyubov, the occurrence of a Fermi surface gives rise to Cooper pairs in this case. As a result, a gap appears in the fermion energy spectrum and the system becomes superfluid.

In the immediate vicinity of the resonance, the system is a universal unitary Fermi liquid. Because the system is superfluid in both limit cases considered, it is reasonable to assume that it is superfluid in all of the interval of a values. (Different arguments are presented below.) Of course, the system is then assumed to be stable in the unitary mode. This assumption is supported by the wealth of experimental data and theoretical calculations.

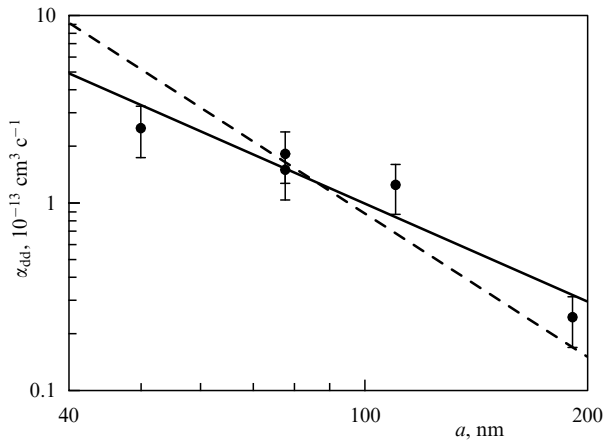

Figure 1. The dimer recombination coefficient α_{dd} as a function of the scattering length a (borrowed from Ref. [5]). The slope of the dashed line corresponds to the theoretical dependence $\alpha_{\mathrm{dd}} \propto a^{-2.5}$.

[1] In all formulas, we set $k_{\mathrm{B}} = 1$.

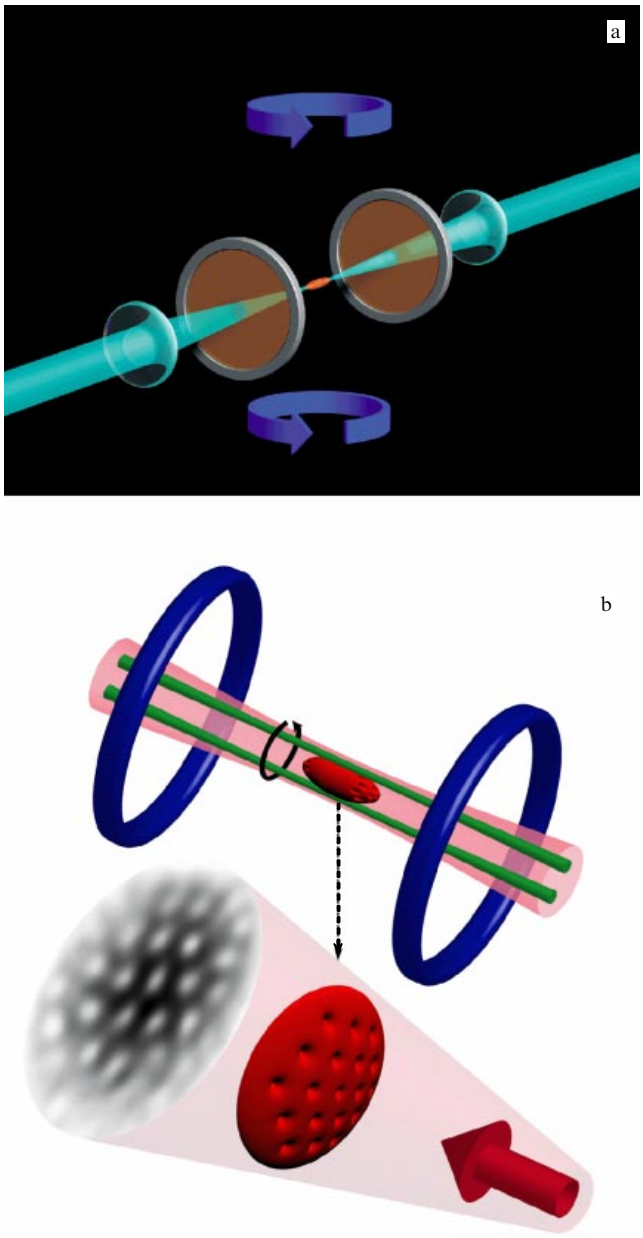

Figure 2. (a) Schematic of the facility employed at Duke University to investigate the properties of a Fermi gas in an optical trap near a Feshbach resonance (borrowed from [7]). (b) Schematic of the facility used at MIT for investigating the rotation of a superfluid Fermi gas (borrowed from [8]). Two laser beams aligned with the axis set the gas in rotation. Separately shown is the scheme for observing the vortices from the shadowgraph of the expanding fermionic cloud.

Prior to discussing these results, I briefly describe the typical experimental arrangement using the example of a facility at Duke University [7] (Fig. 2a). Two types of Fermi atoms were actually used in the experiments, ^{6}Li and ^{40}K isotopes. The isotope choice was dictated by the presence of a Feshbach resonance in a convenient range of the magnetic field and the occurrence of spectral lines in a convenient wavelength range. The atoms are confined in an optical trap formed by a focused laser beam. The chosen light frequency is somewhat lower than the absorption line frequency, and therefore the atoms are 'attracted' to the intensity peak. Because the intensity near the focus decreases rapidly in the radial direction and slowly in the axial direction, the sample was elongated and cigar-shaped. Solenoids induce the magnetic field required to attain the resonance. Since the main objective of the experiments was to investigate superfluidity, two types of fermions were needed. In superconductivity theory, electrons with opposite values of spin projection are usually considered. In our case, atoms in different hyperfine structure states were used.

Experiments with fermions are arduous and the number of groups working with them is smaller than the number of groups investigating the Bose–Einstein condensation. The work is undertaken at the JILA (Joint Research Institute of the National Institute of Standards and Technology and the University of Colorado) (Boulder), Massachusetts Institute of Technology (MIT) (Boston), Duke University (Durham), and Rice University (Houston) in the USA, the École Normale Supérieure (Paris) in France, and the University of Innsbruck in Austria. It is a pleasure for me to mention that A Turlapov, one of the leading experimenters at Duke University, has returned to Nizhnii Novgorod and is making a facility there.

I give the typical parameters of recent experiments. The number of atoms in the trap is $N \sim 3 \times 10^6 - 10^7$ and the atom density at its center is $n \sim 2 \times 10^{12}$. Accordingly, the Fermi energy is $E_F \sim 200-500$ nK and the magnitude of the Fermi wave vector is $k_F \sim 0.3\ \mu\text{m}^{-1}$. The parameters of the trap are conveniently characterized by the frequencies of atomic oscillations in it. The radial frequency $\nu_\perp$ normally lies in the 60–300 Hz range and the longitudinal frequency $\nu_z \sim 20$ Hz. The lowest attainable temperature turns out to be under $0.06E_F$, i.e., of the order of 10 nK. As is evident from the subsequent discussion, it has been possible not only to conduct experiments at these prodigiously low temperatures but also to set up a thermodynamic temperature scale in this domain. I cannot enlarge on the techniques of gas cooling, and only mention that during the final stage, the gas is cooled due to the evaporation of the faster atoms from the trap, much like tea is cooled in a cup left on a table.

One of the most important experimental tasks was to ascertain that the system was superfluid. An immanent property of superfluidity is the existence of quantized vortices. The velocity circulation around a vortex in a Fermi liquid is $\Gamma = \pi\hbar/m$, two times smaller than in a Bose liquid. Accordingly, in the rotation with a sufficiently high angular velocity Ω, the number of vortices per unit area must be equal to $2\Omega m/(\pi\hbar)$. How can the liquid be set in rotation? MIT experimenters positioned a pair of thin laser beams along the trap axis, which were shifted from the axis (Fig. 2b) [8]. This 'mixer' rotated about the axis and entrained the liquid. At some instant, the trap was disengaged, the liquid expanded, and observations of the density distribution were made. The result is shown in Fig. 3. The vortex cores are observed as dark reduced-density domains. A simple calculation of the number of vortices confirms the theoretical value of the circulation given above.

We now consider the liquid precisely at the resonance point, when $a \to \pm\infty$. (It is pertinent to note that this is not a phase transition point.) We begin from the properties of a uniform liquid at $T = 0$. Apart from the density, there are no parameters at our disposal on which the thermodynamic functions may depend. Dimensionality considerations suggest, e.g., that the chemical potential of the liquid must be of the form

$$\mu(n) = \xi\mu^{\text{id}}(n)\,, \tag{4}$$

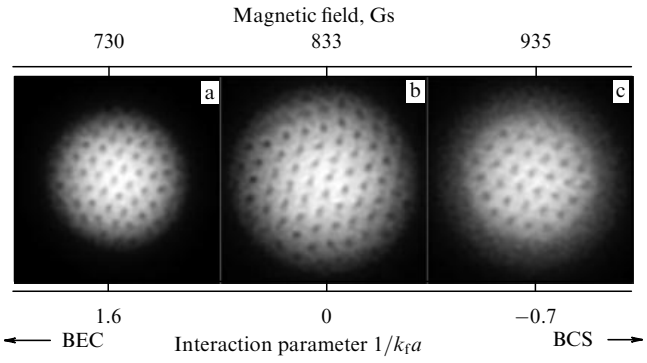

Figure 3. Quantized vortices in a rotating superfluid Fermi gas (borrowed from [8]): (a) corresponds to a dilute gas of dimers, (b) to a Fermi liquid in the vicinity of the unitarity point, and (c) to a dilute Fermi gas with a weak attraction between the atoms.

where $\mu^{\rm id}(n) = (3\pi^2 n)^{2/3}(\hbar^2/m)$ is the chemical potential of an ideal Fermi gas with the density n for $T = 0$ and ξ is a dimensionless coefficient independent of the kind of liquid. The theoretical task consists in the calculation of ξ and the experimental task involves its measurement. The first estimates of ξ were made proceeding from the Bardeen – Cooper – Schrieffer – Bogolyubov (BCSB) theory. This theory is a mean-field theory and, needless to say, is inapplicable near the unitarity point. But its ingenious generalization to the strong-coupling case has allowed obtaining formulas sound in both limit cases (see, e.g., [9]). At exactly the unitarity point, this theory yields $\xi = 0.59$. The most reliable result is provided by calculations involving the quantum Monte Carlo (QMC) technique: $\xi = 0.42$ [10]. It is noteworthy that the absence of a small parameter in the theory is substantially favorable to numerical calculations. It is not infrequent that the existence of such a parameter impairs convergence. An attempt has been made to apply the ε-expansion technique, which relies on the fact that $\xi = 0$ in a four-dimensional space [11]. The theory is constructed in the space of $D = 4 - \varepsilon$ dimensions under the assumption that ε is small, and the results are then extrapolated to $\varepsilon = 1$. This technique, which is highly beneficial in the theory of phase transitions, supposedly yields poor accuracy in this case. It is significant that the parameter $\xi < 1$. This signifies that the interaction at the unitarity point lowers the fluid pressure, i.e., is an effective attraction. It is therefore reasonable that it leads to fermion pairing and to superfluidity. A quantitative characteristic of the pairing is the gap Δ in the Fermi branch of the spectrum. Once again, the dimensionality considerations suggest that

$$\Delta(n) = \theta\mu^{\rm id}(n)\,. \tag{5}$$

QMC calculations yield $\theta = 0.5$ [10].

We now turn our attention to the experimental verification of the theory. The most direct method of determining ξ consists in the precise measurement of fluid density in the trap. In the semiclassical approximation, this distribution is given, in view of expression (4), by the equation $\xi\mu^{\rm id}\left[n(\mathbf{x})\right] + V(\mathbf{x}) = \text{const}$. Fitting to the observed distribution allows determining ξ. At Rice University, the value $\xi = 0.46$ was thus found for ^{6}Li [12]. Another method was applied by experimenters at JILA, who worked with ^{40}K. They measured the density distribution and calculated the potential energy $U_{\rm pot} = \int n(\mathbf{x})\,V(\mathbf{x})\,{\rm d}\mathbf{x}$ of the liquid, which is proportional to $\sqrt{\xi}$ [13]. By this means, they obtained the value $\xi = 0.46$. The proximity of the values for ^{6}Li and ^{40}K to the theoretical one confirms the universal nature of ξ. Reliable measurements of the gap Δ, in my opinion, have not been made to the present day.

Important information about the properties of the liquid may be obtained by investigating its oscillations in the trap. These oscillations are described by the Landau superfluid hydrodynamics [14]. (I emphasize that Landau believed from the outset that his equations applied both to Bose and Fermi superfluid liquids.) An especially simple result for the oscillation frequencies in a harmonic trap is obtained for a liquid with the polytropic equation of state $\mu(n) \propto n^\gamma$. We consider an important type of oscillation: axially symmetric radial oscillations whose frequency is $\omega = \sqrt{2(\gamma+1)}\,\omega_\perp$ [15]. According to this formula, in the molecular limit ($a > 0$, $na^3 \ll 1$), when $\mu \propto a_{\rm dd} n$, i.e., $\gamma = 1$, the frequency $\omega = 2\omega_\perp$. In the unitary limit and BCSB limit, $\gamma = 2/3$ as in an ideal Fermi gas and $\omega = \sqrt{10/3}\,\omega_\perp = 1.83\,\omega_\perp$. For intermediate values of a, the frequency cannot be calculated analytically, but it appears reasonable that the frequency for $a > 0$ is monotonically decreasing with increasing a. These were precisely the indications of the first experiments. Theories that have this property and rely on the mean-field approximation have also been proposed.

However, the situation is not that simple. For $na^3 \ll 1$, the theory permits rigorous calculations of not only the first term in μ but also a correction, which was first determined in [16]. This gives a correction to the frequency equal to [17]

$$\frac{\delta\omega}{\omega} = +0.72\sqrt{n(\mathbf{x}=0)\,a_{\rm dd}^3}\,. \tag{6}$$

The positive correction sign signifies that the frequency must initially increase with increasing a and only then decrease to attain the limit value $1.83\omega_\perp$. This reasoning was disputed on the grounds that molecular dimers are nevertheless not entirely bosons. However, correction (6) bears a clear physical meaning. It stems from the contribution to the energy made by zero-point phonon oscillations, whose occurrence in the superfluid liquid is beyond question. This is why it is anomalously large, of the order of the square root of the gas parameter na^3, while the 'normal' expansion is performed in this parameter. All this leads us to the statement that the author has been vigorously promoting, namely, that the monotonic behavior of the frequency would imply a catastrophe for the theory. Fortunately, the situation has recently been clarified. New experiments do yield above-$2\omega_\perp$ values of the frequency on the molecular side of the resonance. They are in good agreement with the calculations throughout the interval of a values performed by the QMC technique [19] (Fig. 4). We note that the disagreement with the data of previous experiments is attributable to the fact that the temperature in those experiments was not low enough. Meanwhile, correction (6) is temperature sensitive, because it is related to the excitation of relatively low energies $\hbar\omega \sim \mu$.

We now discuss the fluid properties at the unitarity point at finite temperatures. In this case, the temperature is assumed to be not too high, and therefore the wavelength of atoms in their thermal motion is long in comparison with the atomic size: $r_0 \ll \hbar/\sqrt{mT}$. We are actually dealing with temperatures of the order of $E_{\rm F}$.

The question of the temperature of transition to the superfluid state is all-important here. The most reliable data were obtained in [20] using the Monte Carlo technique:

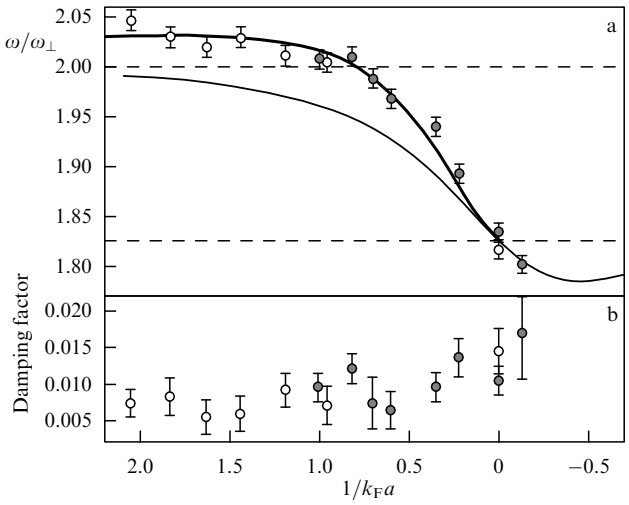

Figure 4. (a) Frequency of radial oscillations as a function of the scattering length. The upper solid curve represents the data calculated by the quantum Monte Carlo technique and the lower one is the result of calculations by the mean-field theory. The points stand for experimental values. The upper and lower dashed straight lines show the limit frequency values in a tenuous dimer gas and the unitarity point. (b) Measured values of oscillation damping. (Borrowed from [18].)

$T_c = 0.16\mu^{id}$. This result is in good agreement with experiment. It is noteworthy that the transition temperature is relatively low, and hence at temperatures somewhat higher than T_c, we face an interesting research object — a degenerate normal Fermi liquid in the unitary regime.

At finite temperatures, the equation of state cannot be written proceeding from only the dimensionality considerations. But these considerations lead to important similarity relations. For instance, the chemical potential is of the form $\mu(n, T) = \mu^{id}(n) f_\mu[T/\mu^{id}(n)]$; the entropy per atom can be written as $s(n, T) = f_S[T/\mu^{id}(n)]$. The last relation implies that under an adiabatic density variation, the temperature varies as $\propto n^{2/3}$, as in an ideal monoatomic gas.

For a fluid in the trap, these formulas lead to an important integral relation. Following the standard derivation of the virial theorem, it can be shown that

$$2U_{pot} = E\,, \tag{7}$$

where U_{pot} is the potential energy and E is the total energy, i.e., the sum of potential, internal, and hydrodynamic kinetic energies [21]. As indicated above, U_{pot} can be calculated directly from the measured density distribution. The total energy may be changed in a controllable way. For this, the trap potential was switched off for some 'heating time' t_{heat}. During this period, the liquid was free to expand. The sum of the kinetic and internal energies was conserved in the process. Then, the trap was turned on again and the system came to equilibrium, and its potential energy, which was measured anew, turned out to be higher. This ingenious method enabled the authors of [21] to verify relation (7) with high precision and thus confirm the similarity laws formulated above.

The total entropy $S = \int n(\mathbf{x})\, s(\mathbf{x})\, d\mathbf{x}$ of the system as a function of its energy E was measured in a similar experiment in [22]. In the experiment, the energy was varied and measured as described above; to measure the entropy, the magnetic field was adiabatically increased, taking the system away from resonance, where the interaction was insignificant. Measurements of the cloud dimension enabled calculating the entropy from the formulas for an ideal Fermi gas, which, due to the adiabaticity of the process, was equal to the entropy of the liquid before the increase in the magnetic field. It is noteworthy that the derivative $T = dE/dS$ directly yields the absolute temperature of the system. I believe that the capability of measuring the absolute temperature in the nanokelvin domain is a wonderful achievement by itself. Another way of measuring the absolute temperature is described below.

The aforesaid leaves no room for doubt that the theoretical notions about the properties of a 'unitary' superfluid liquid are amply borne out by experiments. I believe, however, that the significance of the issue calls for high-precision verification. Such a possibility does exist. For this, the fluid should be placed in a trap that is harmonic and isotropic with a high degree of accuracy. Then, we can state with certainty that the spherically symmetric cloud pulsations are precisely equal to $2\omega_h$ in frequency, where ω_h is the eigenfrequency of the trap, and do not attenuate [23]. This theorem is valid both below and above the superfluid transition point and applies to oscillations of arbitrary amplitude. It is a corollary of the hidden symmetry of the system at the unitarity point. (A similar situation occurs for oscillations of a dilute Bose gas in a cylindrical trap [24, 25].) The absence of damping signifies that the second viscosity ζ of the fluid is equal to zero above the transition point. Of the three second viscosity coefficients introduced by Khalatnikov [26], ζ_1 and ζ_2 turn out to be zero [27] in the superfluid phase.

So far, we have dealt with experiments in which the numbers of atoms in two spin states were equal. Recently, active work commenced to study polarized systems in which the number of atoms in one spin state (we conventionally speak of 'spin-up' atoms) is greater than in the other state. This question had already been discussed for superconductors. In [28] and [29], the existence of spatially inhomogeneous phases (LOFF phases) was predicted, in which the superconducting gap is a periodic function of coordinates [28, 29]. In superconductors, the population difference of the spin states may exist in ferromagnetic bodies or may be induced by an external magnetic field. In both cases, the magnetic field affects the orbital motion and destroys superconductivity.

In our neutral dilute systems, the spin relaxation time is quite long and the numbers of atoms in different states are practically arbitrary parameters, determined by the initial conditions. Theoretical calculations in [30] and the experiment in [31] show that the liquid in a trap at $T = 0$ near the unitarity domain breaks up into three phases. At the center is the superfluid phase with equal numbers of 'spin-up' and 'spin-down' atoms. It is surrounded by the partially polarized normal phase with unequal densities of the atoms of different polarization. At the periphery is the completely polarized phase, which consists of only the atoms of excess polarization. In this case, the existence of LOFF-type phases in some parameter value ranges is not ruled out.

The measurement data are depicted in Fig. 5. The system with $N_\uparrow = 5.9 \times 10^6$, $T/E_F = 0.03$ and the spin state population ratio $N_\downarrow/N_\uparrow = 0.39$ was investigated. Figure 5a shows the shadowgraph of the two-dimensional polarization distribution (the column density) $\delta n_a(x, z) \equiv \int dy[n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})]$ and Fig. 5b shows the 'weighted' distribution $\delta n_b(x, z) \equiv \int dy[0.76 n_\uparrow(\mathbf{r}) - 1.43 n_\downarrow(\mathbf{r})]$, which gives a higher-contrast picture. Figure 5c shows the curves $\delta n_a(0, z)$ and $\delta n_b(0, z)$, and Figs 5d and 5e the plots of integrated linear densities

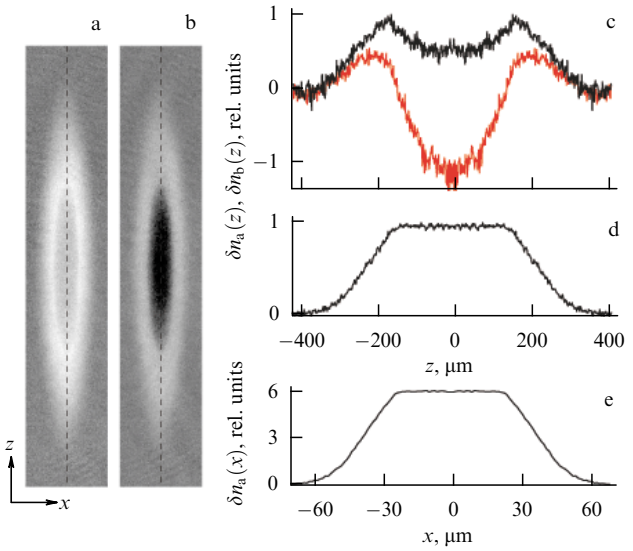

Figure 5. Polarization distribution in a system with unequal spin state populations (borrowed from [31]). (a) Shadowgraph of the two-dimensional polarization distribution $\delta n_a(x, z)$. (b) Weighted polarization distribution pattern $\delta n_b(x, z)$. (c) Plots of the functions $\delta n_a(0, z)$ (the upper curve) and $\delta n_b(0, z)$ (the lower curve). (d) Linear polarization density $\delta n_a(z)$ along the z axis. (e) Linear polarization density $\delta n_a(x)$ along the x axis.

$\delta n_a(z) \equiv \int \mathrm{d}x n_a(x, z)$ and $\delta n_a(x) \equiv \int \mathrm{d}z n_a(x, z)$. I emphasize that the measurements were made in the trap itself, without prior expansion of the fluid. Processing the measured two-dimensional distribution by the Abel transform enabled reconstructing the three-dimensional polarization distribution and confirmed the three-phase fluid structure. Measurements at different temperatures were also made. Worthy of note in this connection is the special role played by the completely polarized phase. Because slow fermions with parallel spins hardly interact with each other, this phase is an ideal Fermi gas. By measuring the density distribution of this phase and fitting it to formulas for the ideal gas, it is possible to determine the thermodynamic temperature of the system. The polarized phase plays the role of an ideal-gas thermometer contacting with other phases. It is significant in this case that the fermions of the polarized phase interact with the fermions of other phases, which ensures thermodynamic equilibrium. These temperature measurements permitted verifying the transition temperature calculated in Ref. [20]. The results under discussion are at some variance with the findings in [12], where a smaller number of atoms was considered. Conceivably, the surface tension at the phase boundaries plays a role under these conditions.

I mention several interesting possibilities for future investigations. One of them involves employing two types of fermions of different masses for which the Feshbach resonance exists [32]. Theory predicts unconventional properties for a superfluid liquid formed as a result of Cooper pairing of the fermions of different masses.

Another possibility is related to the vortex-free rotation of a Fermi liquid [33]. The vortex lattice shown in Fig. 3 is formed due to a strong fluid perturbation by the rotating mixers. If a trap asymmetric about the axis is simply set in rotation, there are grounds to believe that vortices would be formed only for a high rotation rate, when the fluid shape becomes unstable. At lower rotation rates, the fluid would break up into two phases. The center of the weakly deformed trap would be occupied by the superfluid liquid at rest, while the normal phase of the liquid would rotate in the usual way at the periphery. The existence of the normal phase at absolute zero kept by rotation from transiting into the superfluid state raises difficult theoretical issues.

A very rich area of research opens up when the fluid is placed in a periodic lattice produced by counterpropagating laser beams (see the author's review Ref. [34]). This research in the unitary domain is still in its infancy.

We see that the investigations of a near-resonance Fermi gas in a trap have opened up entirely new theoretical and experimental opportunities in condensed matter physics, reflecting the modern trend. Work to an increasing extent is shifting to the investigation of specially fabricated objects that do not exist in nature and have surprising new properties. In view of this, I believe, no exhaustion of our realm of physics is to be expected in the foreseeable future.

Acknowledgements. I express my appreciation to S Stringari for discussions, to J Thomas for providing the original of Fig. 2a, and to R Grimm for providing the original of Fig. 4.

References

1. Landau L D, Lifshitz E M *Statisticheskaya Fizika* (Statistical Physics) (Moscow: Gostekhteorizdat, 1951) p. 223 [Translated into English: (Oxford: Pergamon Press, 1969)]
2. Landau L D, Lifshitz E M *Kvantovaya Mekhanika: Nerelyativistskaya Teoriya* (Quantum Mechanics: Non-Relativistic Theory) (Moscow: Fizmatlit, 2001) [Translated into English (Oxford: Pergamon Press, 1977)]
3. Feshbach H *Ann. Phys.* (New York) **5** 357 (1958); **19** 287 (1962)
4. Petrov D S, Salomon C, Shlyapnikov G V *Phys. Rev. Lett.* **93** 090404 (2004)
5. Petrov D S, Salomon C, Shlyapnikov G V *J. Phys. B: At. Mol. Opt. Phys.* **38** S645 (2005)
6. Bogolyubov N N *Izv. Akad. Nauk SSSR, Ser. Fiz.* **11** 77 (1947)
7. Thomas J E, Gehm M E *Am. Scientist* **92** 238 (2004)
8. Zwierlein M W et al. *Nature* **435** 1047 (2005)
9. Nozières P, Schmitt-Rink S *J. Low Temp. Phys.* **59** 195 (1985)
10. Carlson J, Reddy S *Phys. Rev. Lett.* **95** 060401 (2005)
11. Nishida Y, Son D T *Phys. Rev. Lett.* **97** 050403 (2006)
12. Patridge G B et al. *Science* **311** 503 (2006)
13. Stewart J T et al. *Phys. Rev. Lett.* **97** 220406 (2006)
14. Landau L D *Zh. Eksp. Teor. Fiz.* **11** 592 (1941)
15. Cozzini M, Stringari S *Phys. Rev. Lett.* **91** 070401 (2003)
16. Lee T D, Huang K, Yang C N *Phys. Rev.* **106** 1135 (1957)
17. Pitaevskii L, Stringari S *Phys. Rev. Lett.* **81** 4541 (1998)
18. Altmeyer A et al. *Phys. Rev. Lett.* **98** 040401 (2007)
19. Astrakharchik G E et al. *Phys. Rev. Lett.* **95** 030404 (2005)
20. Burovski E et al. *New J. Phys.* **8** 153 (2006)
21. Thomas J E, Kinast J, Turlapov A *Phys. Rev. Lett.* **95** 120402 (2005)
22. Luo L et al. *Phys. Rev. Lett.* **98** 080402 (2007)
23. Castin Y *C.R. Physique* **5** 407 (2004)
24. Pitaevskii L P *Phys. Lett. A* **221** 14 (1996)
25. Kagan Yu, Surkov E L, Shlyapnikov G V *Phys. Rev. A* **54** R1753 (1996)
26. Khalatnikov I M *Teoriya Sverkhtekuchesti* (Theory of Superfluidity) (Moscow: Nauka, 1971)
27. Son D T *Phys. Rev. Lett.* **98** 020604 (2007)
28. Larkin A I, Ovchinnikov Yu N *Zh. Eksp. Teor. Fiz.* **47** 1136 (1964) [*Sov. Phys. JETP* **20** 762 (1965)]
29. Fulde P, Ferrell R A *Phys. Rev.* **135** A550 (1964)
30. Lobo C et al. *Phys. Rev. Lett.* **97** 200403 (2006)
31. Shin Y et al., arXiv:0709.3027
32. Petrov D S *Phys. Rev. A* **67** 010703 (2003)
33. Bausmerth I, Recati A, Stringari S *Phys. Rev. Lett.* **100** 070401 (2008); arXiv:0711.0653
34. Pitaevskii L P *Usp. Fiz. Nauk* **176** 345 (2006) [*Phys. Usp.* **49** 333 (2006)]