# Can a bubble in liquid helium contain half an electron? 

## L P Pitaevskii

## Abstract. Arguments are advanced to show that bubbles with a charge $e / 2$ cannot exist in liquid helium.

The September 2006 issue of the Uspekhi Fizicheskikh Nauk (Physics-Uspekhi) journal contained an interesting brief review by V P Bykov entitled "Fractional charge: a new trend in electronics" [1]. In this letter I would like to make a simple remark relating to the first part of the review, concerned with H Maris's striking idea that it is possible to split an electron-containing bubble in liquid helium into two bubbles, either containing, in a sense, half the electron.

I believe that the situation discussed by Maris is impossible. This is evident even from V A Rubakov's comment [2], whose reasoning I wholly share, published on the heels of Bykov's paper. Indeed, according to Ref. [2], measuring the charge of one of the bubbles yields either $e$ or zero. Let the outcome of the measurement be zero. So, the bubble contained no electron. However, a bubble cannot exist without an electron. Hence, there was no bubble, either. In this respect, an electron in a bubble is different from an electron in a given potential well which exists irrespective of whether the electron is present or not.

How should we regard Maris's reasoning? After all, it is reliant on quantum mechanics whose applicability to this system cannot be doubted. The fact is that Maris's theory is approximate, and this approximation fails when the waist between the bubbles in Fig. 3 (see p. 981 in Ref. [1]) becomes too small.

The conditions of validity of the theory may be crudely estimated in the following way. Consider the physical picture of the effect. The electron is a point object. How can it simultaneously 'support' all the bubble walls? The matter is that the electron in a bubble moves fast. The characteristic electron velocity is, according to the uncertainty principle, on the order of $\hbar /(m R)$, so that it runs all over the bubble in a time $t \sim\left(m R^{2}\right) / \hbar$, which is much shorter than the time an empty bubble takes to collapse. A simple estimate yields a quantity $\tau \sim\left(\rho R^{3} / \alpha\right)^{1 / 2}$ for the collapse time, where $\rho$ is the helium density. It is easy to verify that the inequality $t \ll \tau$ is valid. It is precisely this inequality that underlies the theory outlined in Ref. [1]. (For spherical bubbles, it was developed
by Careri, Fasoli, and Gaeta [3].) The rapidity of motion of the electron permits performing averaging over its motion for a given dimension and shape of the bubble and subsequently minimization with respect to these parameters. This approximation is similar to the Born-Oppenheimer approximation in the molecular theory, where the averaging is performed over the electron motion for a given nuclear arrangement.

However, the situation changes when the bubble begins to split up. The waist between the bubbles is a potential barrier for the electron. When the bubble waist is small enough, the electron may pierce from one half to the other only by way of tunneling. To do this requires a time on the order of $1 / w$, where $w$ is the tunneling probability per unit time. The theory a fortiori ceases to apply, when $1 / w$ comes to be on the order of $\tau$, yet before the bubble splits up. What will take place if the pressure is further increased? Nothing particularly interesting, I believe. The bubble will supposedly cease to be deformed and will collapse.

In summary, I should mention that my point of view coincides with the opinion of the authors of Ref. [4], which was cited in Ref. [2], if I have understood correctly the last paragraph of their work.

## References

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[^0]:    L P Pitaevskii P L Kapitza Institute for Physical Problems,
    Russian Academy of Sciences,
    ul. Kosygina 2, 119334 Moscow, Russian Federation
    Department of Physics, University of Trento,
    CNR-INFM Center on BEC Research, I-38050 Povo, Trento, Italy
    E-mail: lev@science.unitn.it
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