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Coexistence of ferromagnetism and nonuniform superconductivity

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(1) Superconductivity and ferromagnetism appear to be antagonists in relation to a magnetic field: a superconductor expels a magnetic field (the Meissner–Ochsenfeld effect), while a ferromagnet concentrates such a field. Hence, it is more appropriate to use the name 'antiferromagnet' in relation to superconductors than to substances commonly known as antiferromagnets. The first attempt to tackle the problem of the coexistence of these states was made by V L Ginzburg [1] in 1956, even before the Bardeen–Cooper–Schrieffer (BCS) microscopic theory appeared [2].

According to the work [1], coexistence is possible if the critical magnetic field H_c is higher than the magnetic induction I. From the microscopic point of view [2], the magnitude of H_c is determined in most cases by the effect that this magnetic field (and the induction) has on the orbital motion of pairs. Moreover, due to pairing with oppositely directed spins, Zeeman splitting also suppresses superconductivity (the paramagnetic effect), and it is this splitting that is predominant [3].

When the superconducting transition temperature T_c is much higher than the ferromagnetic transition temperature T_m, the magnetic state is nonuniform in the coexistence region [4]. (A discussion of the existing theoretical and experimental results can be found in the review [5].)

When $T_c \leqslant T_m$, there exists a narrow interval of values of the magnetization I, where the superconducting state proves to be nonuniform in the state coexistence conditions [6, 7].

Currently, a large number of works have appeared (e.g., see Refs [8, 9]) in which the coexistence of superconductivity and ferromagnetism has been observed in layered cuprate $RuSr_2GdCu_2O_8$ compounds, in which T_m is much higher than T_c ($T_m = 132$ K, and $T_c = 46$ K). Such a T_m-to-T_c ratio is unacceptable for the simple spherical Fermi surface which lies at the base of the model discussed by Larkin and Ovchinnikov [6] as well as by Fulde and Ferrell [7]. As is well known, the uniform superconducting state is insensitive to the shape of the Fermi surface [2, 3], and the existence of such a state requires that $\varepsilon_\sigma(\mathbf{p}) = \varepsilon_{-\sigma}(-\mathbf{p})$. The nesting condition $\varepsilon_\sigma(\mathbf{p}) = -\varepsilon_{-\sigma'}(\mathbf{p} + \mathbf{q})$ is preferable for electron–hole pairing (the insulating state), while the mirror nesting condition $\varepsilon_\sigma(\mathbf{K}/2 + \mathbf{p}) = \varepsilon_{-\sigma}(\mathbf{K}/2 - \mathbf{p})$ is preferable for the realization of superconducting pairing with a large total momentum $\mathbf{K}$ in the case of electron–electron repulsion. On the other hand, coexistence of ferromagnetism and a nonuniform superconducting state, with the magnetic-nesting condition
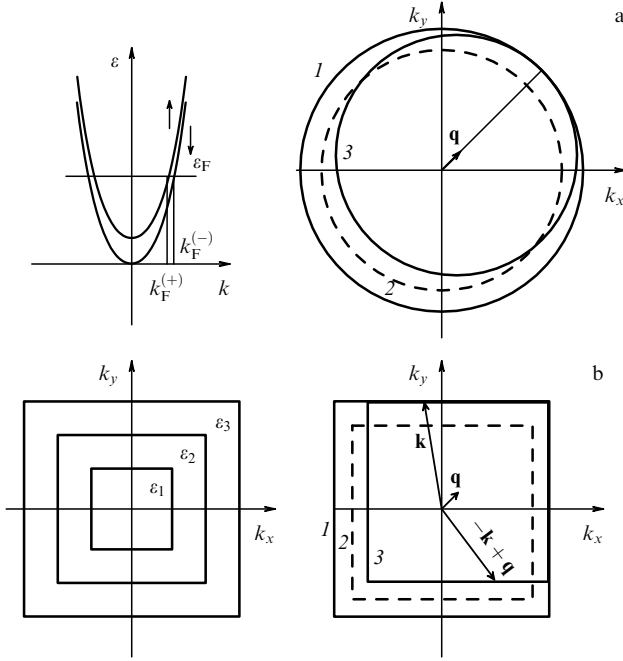
$$\varepsilon_\sigma(\mathbf{p}) = \varepsilon_{-\sigma}\left(-\mathbf{p} + \frac{\mathbf{n}I}{v_F}\right) \qquad (1)$$

being met for an electron dispersion law with spin σ in a selected direction $\mathbf{n}$, is possible for an arbitrarily large magnetization I (here v_F is the Fermi velocity). Thus, in the given situation the main mechanism by which magnetization suppresses superconductivity is the orbital mechanism examined by Ginzburg [1].

In the present work, we show that the processes of hopping to third-sphere centers, which were ignored in Refs [10, 11] but which exceed the processes of hopping to second-sphere centers, drastically change the situation concerning the state coexistence. Furthermore, we show that a superconducting state with a large total momentum of the pairs [12] can coexist with a ferromagnetic state at high enough magnetizations.

(2) We select a simple model that meets the magnetic-nesting condition (1), namely, a two-dimensional model of an electronic spectrum corresponding to the constant energy lines in the form of squares within a certain energy interval (on the order of the cutoff energy ω of the attractive interaction V) (Fig. 1). Assuming that ω ($\omega = \omega_{ph}$ for electron–phonon coupling) is small compared to the Fermi energy ε_F, we can write down the equation for the order parameter Δ [$\Delta(\mathbf{r}) = |\Delta| \exp(i\mathbf{qr})$, where $\mathbf{q}$ is the pair momentum] within the BCS theory at $T = 0$ in the form

$$\frac{1}{\lambda} = \int_0^\omega \frac{d\xi}{\sqrt{\xi^2 + |\Delta|^2}} \left\{ 1 - \frac{1}{2}\left[n(\varepsilon + I + Q) + n(\varepsilon + I - Q) \right.\right.$$
$$\left.\left. + n(\varepsilon - I - Q) + n(\varepsilon - I + Q) \right] \right\}, \qquad (2)$$

Figure 1. The dispersion law and the Fermi contours for systems with a parabolic spectrum (a), and for systems with magnetic nesting (b): *1*, $\varepsilon^{(-)}(k) = \varepsilon_F$; *2*, $\varepsilon^{(+)}(k) = \varepsilon_F$, and *3*, $\varepsilon^{(+)}(k+q) = \varepsilon_F$.

Figure 2. The order parameter δ as a function of the magnetization $\tilde{I}$.

where $\varepsilon = (\xi^2 + |\Delta|^2)^{1/2}$, $\lambda = VN$ (with N the density of states at the Fermi level), $n(\varepsilon) = (e^{\varepsilon/T} + 1)^{-1}$, and $Q = qv_F/2$.

Below we formally limit ourselves to the case with a single selected **q**. The complete solution can be found by summing over all the equivalent states with momentum **q** [6].

(3) In 1964, Larkin and Ovchinnikov [6] as well as Fulde and Ferrell [7] found that a superconducting phase may emerge in superconductors with a quadratic dispersion law for $I > I_c$, but that the order parameter $\Delta(\mathbf{r})$ of such a phase is nonuniform. The new phase, which became known as the FFLO phase, emerges as a result of a second-order phase transition and exists in a narrow interval of magnetic field strengths:

$$0.707 < \frac{I}{\Delta_0} < 0.754, \qquad Q_c = 0.897\Delta_0. \qquad (3)$$

Let us examine the nonuniform state by using equation (2). First we must analyze the most interesting case where the spectrum splitting caused by I is balanced by the condensate momentum in view of the magnetic-nesting condition (1):
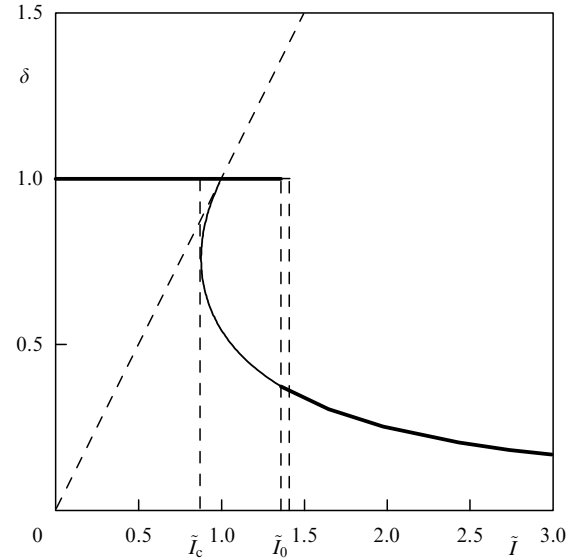
$$I = Q, \qquad 2I > \Delta. \qquad (4)$$

At temperature $T = 0$ only one term remains finite, $n(\varepsilon - I - Q) \equiv n(\varepsilon - 2I)$, and from Eqn (2) we obtain an equation for $\delta = \Delta/\Delta_0$:

$$\delta\left(\tilde{I} + \sqrt{\tilde{I}^2 - \delta^2}\right) = 1, \qquad \tilde{I} = \frac{2I}{\Delta_0}, \qquad (5)$$

or an equivalent equation

$$2\delta\tilde{I} = 1 + \delta^4. \qquad (6)$$

Clearly, equations (5) and (6) do not have a zero solution. The dependence of the order parameter δ on the magnetiza-

tion $\tilde{I}$ is depicted in Fig. 2. In the interval $\tilde{I}_c < \tilde{I} < 1$ ($\tilde{I}_c = 2 \times 3^{-3/4} \approx 0.87$, and $\tilde{I} > \delta$), the dependence is two-valued, while for $\tilde{I} > 1$ it becomes single-valued, and δ monotonically decreases as $\tilde{I}$ increases, but remains finite.

The existence of a solution at $\Delta \neq 0$ for $\tilde{I} > \delta$ is caused by the fact that the excitation energy of a pair with $Q = I$ under condition (1) vanishes on a line, instead of vanishing at separate points when the dispersion law is isotropic [6, 7]. The drop in Δ with increasing I results from the decrease in the length of the line of zeros [the line on which the condition $\varepsilon_\sigma(\mathbf{p}) = \varepsilon_{-\sigma}(-\mathbf{p} + \mathbf{n}I/v_F) = 0$ is met].

The energy difference of a nonuniform superconducting state and a normal state is calculated in the ordinary way:

$$U_s - U_n = -N \frac{\Delta^3\sqrt{4I^2 - \Delta^2}}{2\Delta_0^2} = -\frac{N\Delta_0^2}{2}\, \delta^3\sqrt{\tilde{I}^2 - \delta^2}. \quad (7)$$

This implies that in terms of energy efficiency the nonuniform superconducting state is preferable to the normal state when $\tilde{I} > \delta$. However, when $\tilde{I} < \sqrt{2}$, the energy (7) of the nonuniform state must be compared with the energy of a uniform superconducting state. The two are compared at values of $\tilde{I}_0$ and δ_0 satisfying a system of equations consisting of equation (6) and the equation

$$\delta_0^3\sqrt{\tilde{I}_0^2 - \delta_0^2} = 1 - \frac{\tilde{I}_0^2}{2}. \qquad (8)$$

Analysis shows that when $\delta \leqslant \delta_0$ and $\tilde{I} > \tilde{I}_0$, the solution of equation (6) can be written with a high degree of accuracy in the form

$$\delta = \frac{1}{2\tilde{I}}. \qquad (9)$$

Combining Eqn (9) for δ_0 and $\tilde{I}$ with Eqn (8), we arrive at an approximate equation

$$\tilde{I}_0^4 - 2\tilde{I}_0^2 + \frac{1}{4} = 0, \qquad (10)$$

which yields $\tilde{I}_0 = (1 + \sqrt{3}/2)^{1/2} \approx 1.36$, and $\delta_0 \approx 0.36$.

Thus, in a system with magnetic nesting for $I > 0.68\Delta_0$ there occurs a first-order phase transition from the super-conducting state with a uniform order parameter $\Delta = \Delta_0$ into a nonuniform state with $|\Delta| = 0.36\Delta_0$. As I increases, the order parameter monotonically decreases in accordance with formula (9), while the energy difference (7) for $\tilde{I} > \sqrt{2}$ is given by the expression

$$U_s - U_n \approx -N \frac{\Delta_0^2}{16\tilde{I}^2} \,. \tag{11}$$

What is interesting is that the energy difference (11) at $I = \Delta_0$ exceeds the corresponding maximum value for the FFLO phase by a factor of almost 100.

Now, let us examine the situation where $Q > I$ and $Q - I > \Delta$. It is in this interval of parameters that the FFLO phase is realized. Using equation (2), we arrive at the equation for Δ:

$$\left(Q + I + \sqrt{(Q+I)^2 - \Delta^2} \right)$$
$$\times \left(Q - I + \sqrt{(Q-I)^2 - \Delta^2} \right) = \Delta_0^2 \,. \tag{12}$$

Clearly, in contrast to expression (5), here we have a solution with $\Delta = 0$. However, an analysis of the solution to equation (12) is of no interest since the solution is energetically disadvantageous. Indeed, the energy difference

$$U_s - U_n = \frac{N\Delta^2}{2} a_+ a_- \left[1 - \sqrt{\left(1 - \frac{\Delta^2}{a_+^2}\right)\left(1 - \frac{\Delta^2}{a_-^2}\right)} \right],$$
$$a_\pm = Q \pm I \quad (13)$$

is always positive. The result obtained suggests that the optimum situation is achieved when $Q \leqslant I$ and $I - Q < \Delta$, i.e., when superconductivity remains amazingly stable under the action of the ferromagnetic exchange field up to very high magnetizations.

(4) The above calculations were carried out for an idealized situation, where the Fermi contour is a square and the spectrum satisfies condition (1). In real high-T_c cuprates (including those containing ferromagnetic layers, such as $RuSr_2GdCu_2O_8$), the energy spectrum can be approximated with high accuracy by the strong coupling approximation with allowance for three coordination spheres:

$$\varepsilon(k_x, k_y) = 2 - 2t(\cos \pi k_x + \cos \pi k_y)$$
$$- 4t_1 t \cos \pi k_x \cos \pi k_y - 2t_2 t(\cos 2\pi k_x + \cos 2\pi k_y) \,. \tag{14}$$

The typical values of the hopping integrals used in the literature are $t = 0.5$ eV and $t_1 = -0.3$, while the parameter t_2 varies within a sufficiently broad interval from 0 to 0.8. Such a choice of the dispersion law makes it possible to describe the experimentally examined shape of the Fermi contour at half-band filling. Rotation of the Fermi contour (compared to the case where only the nearest neighbors are taken into account) and the magnetic nesting (1) are largely determined by the overlap integral t_2 with the third-to-nearest neighbor, with the result that in what follows we do our calculations for different values of t_2. Shimahara and Hata [11] limited themselves only to second-to-nearest neighbors (i.e., $t_2 = 0$), while the value of t_1 was assumed to be double the commonly accepted one needed for ensuring the rotation of the Fermi contour and for

the contour's shape to resemble a square with rounded corners. We believe such a choice to be less realistic than achieving the same effect with the help of varying t_2. What is more, our approach makes it also possible to describe the experimentally examined corrugation of the Fermi contour. The presence of magnetization leads to a shift in the dispersion law (14) for one direction of spin by I.

We noted previously that stability of superconductivity is due to the long length of the line of zeros of the excitation energy $\varepsilon_\sigma(k) = \varepsilon_{-\sigma}(k+q) - I$. In the above simplest model, $\varepsilon_\sigma(k)$ and $\varepsilon_{-\sigma}(k+q) - I$ coincide perfectly on a section of the Fermi contour, while with a real dispersion law this happens only approximately. Nevertheless, this difference can be small within a broad energy interval. To qualitatively estimate the effect of the field I, we will not solve the self-consistency equation for Δ. Instead, we will limit ourselves to calculating the length L of the section of the Fermi contour on which the magnetic-nesting condition (1) is met approximately (to a given accuracy). In the following is the algorithm for calculating L: for the given occupancy n we use formula (14) to find the Fermi energy, the corresponding constant-energy curve (the Fermi contour) $k_y = F(k_x)$, and the length L_0 of this curve. Then, we calculate the energy for a particle with an opposite spin on the Fermi contour with allowance made for magnetic splitting I and the shift along the wave vector $\mathbf{q}$ (which takes into account the deviation of the pair momentum from zero), or $\varepsilon_{-\sigma}(k+q) - I$. We characterize the deviation from the magnetic-nesting condition by the parameter $\gamma = |\varepsilon_\sigma(k) - \varepsilon_{-\sigma}(k+q) - I|$, and for the final quantity we use the length L of the Fermi contour on which $\gamma < \gamma_0$, where γ_0 is a fixed (small) quantity. Actually, γ_0 is determined by a set of parameters of the problem (by the cutoff parameter, for one), but we will be interested only in some general regularities that are not detail-specific. Roughly, L/L_0 is the estimate of the expression in the braces in self-consistency equation (2). In the FFLO model, the ratio is close to zero, while in Eqn (2) its value is on the order of 0.5.

Let us first use the above algorithm to estimate how the difference between the real spectrum (14) and the ideal spectrum, which produces square constant-energy curves, affects the possibility of the coexistence of superconductivity and ferromagnetism. Figure 3 shows the dependences of L, calculated for the upper right quadrant of the momentum plane, on the wave vector q (it is assumed that the vector $\mathbf{q}$ is directed along a diagonal, $\mathbf{q} = (q,q)/\sqrt{2}$, as in the case considered above) at $I = 0.02$ eV and $\gamma_0 = 0.001$ eV for different values of t_2 and an occupancy $n = 1.14$. The inset to this figure shows the respective constant-energy curves. A characteristic feature of these curves is the presence of sharp maxima, and for a finite t_2 two peaks are observed. This fact is due to the complexity of the Fermi contour: at one peak, certain regions of the contour merge (after shifts by I and q), while at the other peak, other regions merge (see Fig. 4). States corresponding to the absolute maximum of $L(q)$ will be realized. As Fig. 3 shows, L_{max} increases with t_2 and reaches its maximum at $t_2 = 0.2$, which is due to the fact that at such a value of t_2 the shape of the Fermi contour is closest to a square as can be possible (there are flat sections; see the inset to Fig. 3). The magnitude of L_{max}/L_0 at its maximum is 0.65.

Figure 5 shows the dependences $L_{max}(I)$, which qualitatively resemble the functions $\Delta(I)$ (5), for different values of t_2 and occupancies equal to 1.14 (electron doping) and 0.86 (hole doping); the inset shows the corresponding dependences
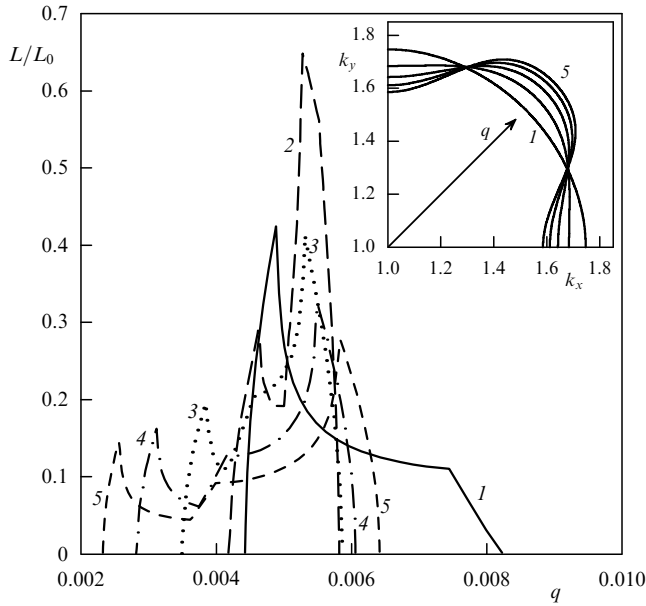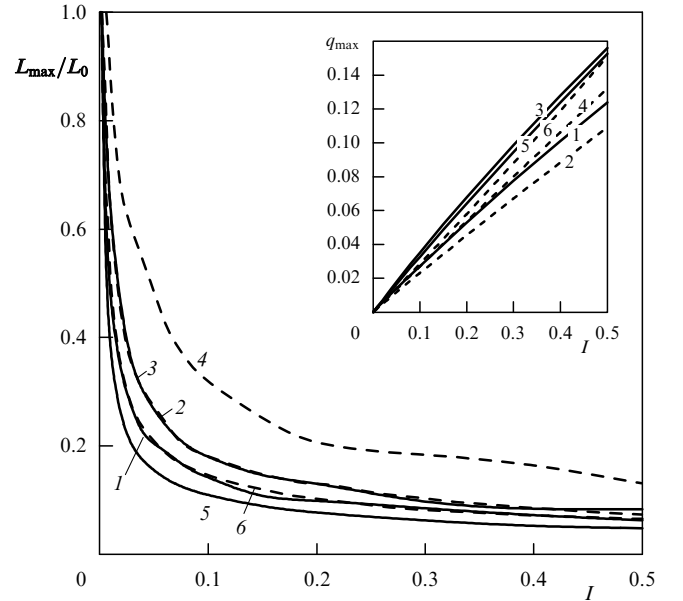
Figure 3. Dependence of the length L, over which the magnetic-nesting condition is approximately met, on the wave vector q at $I = 0.02$ eV and $\gamma = 0.001$ eV for different values of the parameter t_2: *1*, $t_2 = 0$; *2*, $t_2 = 0.2$; *3*, $t_2 = 0.4$; *4*, $t_2 = 0.6$, and *5*, $t_2 = 0.8$. The inset shows the respective Fermi contours.
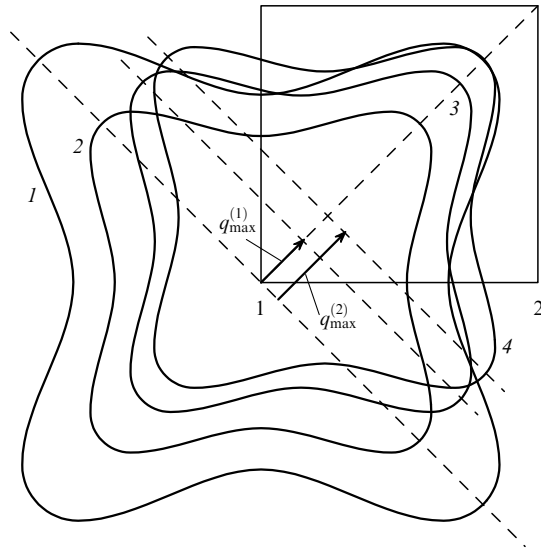

Figure 4. Illustration of the possibility of formation of two nonuniform states with $q_{max}^{(1)}$ and $q_{max}^{(2)}$: *1*, $\varepsilon^{(-)}(k) = \varepsilon_F$; *2*, $\varepsilon^{(+)}(k) = \varepsilon_F$; *3*, $\varepsilon^{(+)}(k + q_{max}^{(1)}) = \varepsilon_F$, and *4*, $\varepsilon^{(+)}(k + q_{max}^{(2)}) = \varepsilon_F$.

of the wave vector q_{max}. It should be emphasized that L_{max} has a larger value for electron filling, and up to $I \approx 0.2$ exceeds 0.2 for $t_2 = 0.2$. It would be interesting to establish whether this fact of preference for electron filling is absolute, i.e., whether it occurs at all values of t_2. Figure 6 shows the dependences $L_{max}(t_2)$ at $I = 0.05$ for occupancies equaling 0.86 and 1.14. Clearly, there is a narrow region of t_2 values in which hole filling is the optimum one, while for other values of t_2 the preference for electron doping is obvious.

Similar results can also be obtained for $\mathbf{q} = (0, q)$, but the absolute values of L_{max} prove to be somewhat smaller than in the case where the Fermi contours are shifted along the diagonal $\mathbf{q} = (q, q)/\sqrt{2}$.
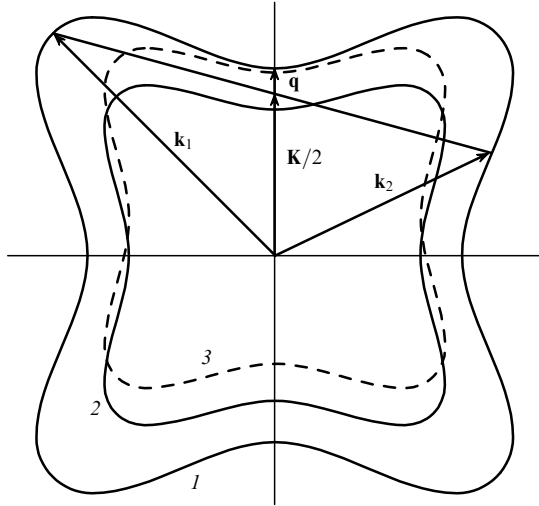

Figure 5. Dependence of the length L_{max} on the magnitude of the magnetic shift I for electron (*2*, *4*, and *6*) and hole (*1*, *3*, and *5*) doping at different values of t_2 and N: *1*, $t_2 = 0$ and $N = 0.43$; *2*, $t_2 = 0$ and $N = 0.57$; *3*, $t_2 = 0.2$ and $N = 0.43$; *4*, $t_2 = 0.2$ and $N = 0.57$; *5*, $t_2 = 0.6$ and $N = 0.43$, and *6*, $t_2 = 0.6$ and $N = 0.57$. The inset shows the respective optimum values of the wave vector $q_{max}(I)$.
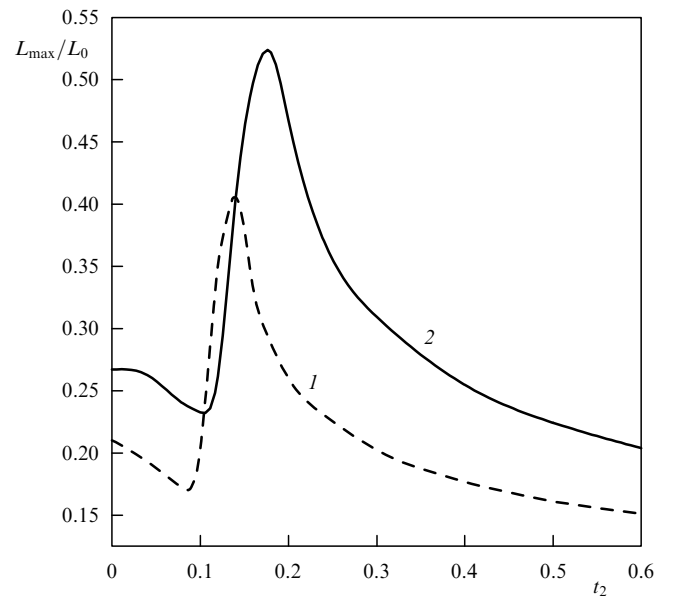

Figure 6. The dependence $L_{max}(t_2)$: *1*, for electron ($n = 1.14$) doping, and *2*, for hole ($n = 0.86$) doping ($I = 0.05$ eV).

(5) Another interesting problem is the possibility of coexistence of superconductivity and ferromagnetism for the case of pairing with a finite momentum $\mathbf{K}$ in the absence of ferromagnetism [12] (Fig. 7). Here what is known as the kinematic constraint region is the decisive factor. The region is formed by the intersection of the region $\varepsilon(\mathbf{k}) < \varepsilon_F$ and the region $\varepsilon(\mathbf{k} + \mathbf{K}) < \varepsilon_F$ (i.e., shifted by $\mathbf{K}$). In this case, one must check the kinematic constraint regions for their coincidence under a shift in momentum by $\mathbf{q}$, and a shift in energy by I. The optimum situation here is the case of coincidence along the directions of $\mathbf{q}$ and $\mathbf{K}$. Although the meaning of L_0 differs

Figure 7. Fermi contours in the case where there is pairing with a finite momentum: 1, $\varepsilon^{(-)}(k) = \varepsilon_{\mathrm{F}}$; 2, $\varepsilon^{(+)}(k) = \varepsilon_{\mathrm{F}}$, and 3, $\varepsilon^{(+)}(k+q) = \varepsilon_{\mathrm{F}}$.
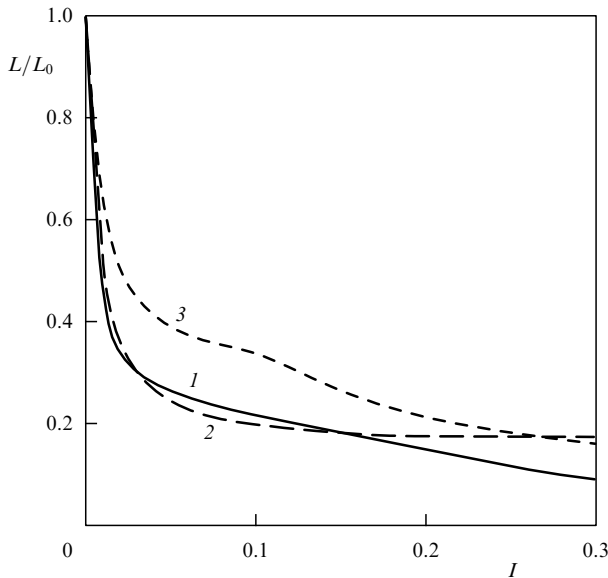
Figure 8. Dependence of the length L/L_0 of a part of the Fermi contour over which $\varepsilon(K/2 + k) = \varepsilon(K/2 - k)$, on the magnetization I for pairing with a finite momentum: 1, $n = 0.86$; 2, $n = 1.00$, and 3, $n = 1.14$.

from the one discussed earlier for $K = 0$ (in the sense that this length will enter the self-consistency equation in a different way), the ratio L/L_0 still characterizes the suppression of superconductivity by ferromagnetism. The kinematic constraint region occupies a section of the Fermi surface within a certain sector directed along $\mathbf{K} = (0, K)$. Hence, when calculating L (and L_0), we must limit ourselves to this sector. As a result, the value of L_0 proves to be smaller than the value obtained above at $K = 0$, while the ratio L/L_0 is larger. The dependences of L/L_0 on I for different occupancies are shown in Fig. 8. Comparing Figs 5 and 8, we see that the situation of pairing with a finite $\mathbf{K}$ is preferable from the point of view of coexistence of ferromagnetism and superconductivity.

(6) Lee and Kim [13] studied the effect that a change in magnetization (caused by variations in niobium concentration) has on the magnetic properties of $Ru_{1-x}Nb_xSr_2Eu_{1.5}Ce_{0.5}Cu_2O_z$ samples. As magnetization

decreases with increasing niobium concentration, a diamagnetic response appears when the sample is cooled in a magnetic field (the Meissner–Ochsenfeld effect). This result has been interpreted as the formation of a spontaneous vortex structure under growing magnetization [14]. Within the model examined in the present work, the emergence of a spontaneous vortex state is due to the appearance of a nonuniformity ($q \neq 0$) and is determined by condition (10). At lower magnetizations, a uniform vortex-free superconducting state, corresponding to ideal diamagnetism, proves to be preferable.

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Theory of magnetic contacts between clean superconductors

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The problem of the coexistence of superconductivity and ferromagnetism is one of the most interesting in the physics of condensed matter and over the years has attracted a lot of attention, beginning with Ginzburg's paper [1]. A special case is the problem of the coexistence and the mutual effect of spatially separated and adjacent superconducting and ferromagnetic phases. Such a statement of the problem includes research into contacts between superconductors and magnetic substances and, in particular, superconducting contacts through ferromagnetic interlayers. At present it is a well-known fact that under certain conditions such contacts constitute what is known as π-contacts. The possibility of forming a π-contact due to the magnetic properties of the interlayer was recognized by theoreticians more than a quarter of a century ago [2], while the first specific example