

Baryons with two heavy quarks

V V Kiselev, A K Likhoded

DOI: 10.1070/PU2002v045n05ABEH000958

Contents

1. Introduction	455
2. Spectroscopy of doubly heavy baryons: the potential approach	459
2.1 The nonrelativistic potential model; 2.2 Numerical results; 2.3 Discussion	
3. Nonrelativistic QCD sum rules: two-point correlators	470
3.1 Sum rules for baryons with two heavy quarks; 3.2 Numerical estimates; 3.3 Discussion	
4. Baryon production processes	479
4.1 The production of doubly heavy baryons in e^+e^- -annihilation; 4.2 Perturbative diquark fragmentation;	
4.3 Hadron production	
5. Lifetimes and decays of $\Xi_{QQ'}$ baryons	491
5.1 Operator expansion for heavy baryons; 5.2 Hadron matrix elements; 5.3 Numerical estimates; 5.4 Exclusive decays in NRQCD sum rules; 5.5 Discussion	
6. Conclusions	502
7. Appendices	504
7.1 Spectral density coefficients; 7.2 Transverse momentum distribution; 7.3 Spectator effects in baryon decays	
References	505

Abstract. Basic physical characteristics of doubly heavy baryons are examined, including spectroscopy (which is treated in the potential approach and within the QCD sum rules framework), production mechanisms for various interactions (the fragmentation model with preasymptotic twist corrections of higher order in the baryon transverse momentum), inclusive decays and lifetimes (operator expansion in the inverse powers of the heavy quark masses), and exclusive decays (in the QCD sum rules framework). The effective theory of heavy quarks is extended to systems with two heavy quarks and one light quark. The masses, decay widths and yields of doubly heavy baryons are calculated for the experimental facilities now being operated or planned. Prospects for the detection and observation of such baryons are discussed. The most interesting physical effects involving hadrons are analyzed and their impact on the theory of heavy quark dynamics is considered.

1. Introduction

After the high-precision studies of the neutral intermediate Z-boson at LEP (CERN) and observation of the t-quark at FNAL, the investigation of electroweak interactions in the sector of heavy quarks is one of the most important issues of elementary particle physics. Precisely in such studies within

the framework of the Standard Model [1] can there a full picture of effects related to the irreversibility of time at energies below the scale of electroweak symmetry violation be achieved.

Comparative analysis of the decay processes of hadrons containing heavy quarks, involving violations of combined CP-parity with respect to charge inversion (C) and mirror reflection of space (P), has become possible on the basis of very precise measurements owing to the commissioning of specialized facilities such as Belle (KEK) and BaBar (SLAC), as well as the upgraded detectors CDF and D0 at FNAL. The results of such experiments will, probably, permit the addition of an essential hitherto absent link to the Standard interaction model, namely, to give a complete description of the charged currents of three quark generations [2], which represents a most important problem at the same level as observation of the scalar Higgs particles providing for the mechanism of electroweak symmetry violation and the investigation of neutrino currents.

The problem of high-precision investigation of the electroweak properties of heavy quarks raises a profound theoretical issue, namely, the description of the dynamics of strong quark interactions bringing about the formation of bound states — hadrons: mesons and baryons, since the observable characteristics (for example, rare decays exhibiting effects of CP-symmetry violation) are related precisely to bound states and it is necessary to have clear and reliable ideas of the direct relationship between these characteristics and the interaction parameters of heavy quarks. Here, the fine effects of electroweak physics may be identified only in the case of high-precision description of the dominant contributions of quantum chromodynamics (QCD).

We are actually dealing with the general problem of describing quark confinement in QCD, which can be

V V Kiselev, A K Likhoded Russian State Research Center
“Institute for High Energy Physics”,
142284 Protvino, Moscow Region, Russian Federation
Tel. (7-0967) 71 37 80. Fax (7-0967) 74 28 24
E-mail: kiselev@th1.ihep.su

Received 23 February 2001, revised 10 October 2001
Uspekhi Fizicheskikh Nauk 172 (5) 497 – 550 (2002)
Translated by G Pontecorvo; edited by A Radzig

investigated fruitfully not only in spectroscopy and in production and decay processes of exotic hybrid and glueball states [3], but also in studying characteristics of hadrons with heavy quarks. In practice, the measured decay asymmetries of heavy hadrons and so on are expressed in the form of functions of such characteristics as, for example, charged heavy quark currents, and these functions depend parametrically on the hadron matrix elements of certain quark operators. The latter often cannot be determined directly from the broad variety of diverse experimental data, so that a detailed theoretical analysis is required of such matrix elements in QCD.

The more complete the set of bound heavy quark states studied, the broader the range of variability of the conditions in which QCD forces act on the heavy quarks and the more advanced must the theoretical methods for describing hadrons containing heavy quarks be, in order to obtain a consistent description of various quark systems. From this point of view, in our opinion, a new field of activity is presented by baryons containing two heavy quarks. Theoretical prediction of their characteristics seems to represent an interesting and topical problem.

Doubly heavy baryons extend in a natural manner the sequence of long-lived heavy hadrons both with a single heavy quark (D-, B-mesons and Λ_c , Σ_c , Ξ_c , Ω_c , Λ_b baryons) and with two heavy quarks (the B_c -meson) and may resemble heavy quarkonia $c\bar{c}$ and $b\bar{b}$ in the character of their strong interactions. From a practical point of view, one can expect doubly heavy baryons to be observed in modern experiments performed at hadron colliders of high luminosity (Tevatron, LHC), since their yield should be comparable to that of the doubly heavy meson containing quarks of differing flavors (the B_c -meson¹), and the methods for detecting rare decays of heavy particles have recently become extremely effective owing to the development of the technique of vertex detectors, which was successfully demonstrated by the first experimental observation of the B_c -meson by the CDF collaboration [5].

The construction of theoretical methods for describing QCD dynamics in the case of heavy quarks is based on a physically clear definition: a quark Q is heavy, if its mass m_Q very significantly exceeds the scale of quark confinement in a bound state, $\Lambda_{\text{QCD}}: m_Q \gg \Lambda_{\text{QCD}}$. Thus, in the problem of strong interactions of heavy quarks, i.e. in calculations of hadron matrix elements for the quark operators, there exists a small parameter $\Lambda_{\text{QCD}}/m_Q \ll 1$ which can serve for developing formal approximate methods. Thus, in hard subprocesses involving virtualities of the order of the heavy quark masses (for example, in heavy quark production processes) it is the QCD coupling constant that is small [$\alpha_s \propto \ln^{-1}(m_Q/\Lambda_{\text{QCD}})$], so that the standard technique of *perturbation theory* in the coupling constant can be applied.

Another fruitful method makes use of the *operator expansion* in inverse powers of the heavy quark mass. In this approach, calculation of the hadron matrix element for the quark operator results in summation of the matrix elements of operators, the properties of which imply the existence of a hierarchy with respect to the small parameter $\Lambda_{\text{QCD}}/m_Q \ll 1$, i.e. suppression of a number of contributions provided by powers of Λ_{QCD}/m_Q , since interaction in a bound state containing a heavy quark is characterized by energies close to Λ_{QCD} , i.e. by the inverse size of the hadron. This expansion

yields a complete description of hadron systems containing a single heavy quark.

In the presence of two heavy quarks in the hadron (side by side with the scale of nonperturbative interactions) there also exists such an energy characteristic as the momentum transfer in a Coulomb-like interaction, i.e. exhibiting a virtuality $\mu \sim \alpha_s m_Q$, and the relative velocity v of motion of the two heavy quarks in the hadron is determined by the relatively small QCD coupling constant: $v \sim \alpha_s$, where the coupling constant α_s is taken on the scale of virtualities peculiar to the Coulomb-like interaction. Thus, in heavy quarkonium $\bar{Q}Q'$, for example, it is the relative velocity v of the nonrelativistic quarks that can serve as the additional small parameter to be used in the operator expansion when calculating hadron matrix elements.

The following three methods for calculating the properties of bound states containing heavy quarks can be identified in accordance with the above-indicated approach:

operator expansion in the inverse mass of the heavy quark in QCD for calculation of the inclusive widths and lifetimes of heavy hadrons, where corrections to the leading term are given by external parameters [6];

QCD sum rules and nonrelativistic QCD for the two-point correlators of quark currents in spectroscopic calculations and for three-point correlators in estimating the form factors of exclusive decay modes [7];

potential models applied for calculating exclusive characteristics of hadrons with heavy quarks [8].

It must be noted that sum rules are also essentially based on operator expansion, but the external parameters they involve are only fundamental quantities, such as the masses of heavy quarks, the norm of the QCD coupling constant and quark–gluon vacuum condensates (unlike inclusive estimations by the operator expansion, where it is necessary to give, for example, the coupling energies of the heavy quark in the hadron, the mean momentum squared of the heavy quark, and so on).

Another observation consists in the fact that perturbation theory and renormgroup relations necessary for calculating the Wilson coefficients in the operator expansion are still important in calculations making use of expansions in the inverse powers of the heavy quark mass and in the relative velocity of heavy quarks in the hadron, since these Wilson coefficients serve as factors of the operators or matrix elements in the expansion.

It is important to stress that in the method of operator expansion for heavy quarks one can consider a certain actual operator, say, the weak decay current or the product of currents, like in the QCD sum rules, and its subsequent expansion. However, it turns out to be quite useful, also, to apply the method of effective field theory, in which the expansion of the heavy-quark QCD Lagrangian itself serves directly as the construction starting point. In this approach it is possible to single out the leading term in the effective Lagrangian and to interpret the subsequent terms of the expansion as perturbations. In this case the leading term depends on the character of the problem, i.e. on the actual convergence of the estimates of physical quantities calculated in the effective Lagrangian.

Thus, in the case of hadrons with a single heavy quark, heavy quark effective theory (HQET) was developed [9], in the leading term of which one can neglect the binding energy of the heavy quark in the hadron, in particular, its kinetic energy. It is important to note that, first, the effective

¹ For a review of the physics of B_c -mesons see Ref. [4].

Lagrangian in the HQET leading approximation exhibits symmetry: heavy quarks with identical velocities, equal to that of the hadron in which they are bound, can undergo permutation (symmetry in the heavy quark flavors), and the spin of the heavy quark is decoupled from the interaction with virtual gluons, since the current is determined by the quark velocity v_v (spin symmetry).

Second, the leading term of the effective Lagrangian leads to renormgroup behavior differing from complete QCD. Thus, for example, currents being conserved in complete QCD and, consequently, having zero anomalous dimensions become divergent upon transition to effective theory fields. The same is true for the correction terms of the effective Lagrangian: the respective Wilson coefficients in effective theory have nonzero anomalous dimensions. Thus, we arrive at a situation in which for formulating the theory it is necessary to set an infinite number of normalizing conditions for the Wilson coefficients, the conditions that are anomalous in a renormgroup respect.

Quite a clear physical reason underlies this problem: in the effective theory constructed for fields with small virtualities it is necessary to introduce a cutoff in the ultraviolet region on a scale of the order of the heavy quark mass, since in the case of large virtualities the assumptions made when developing the theory are erroneous. The effective theory having been obtained from complete QCD, one should consider, from a constructive point of view, the Lagrangians of complete QCD and of effective theory to be equal on a scale μ_{hard} of the order of the quark mass m_Q . This means that in this order in the QCD coupling constant it is necessary to calculate (with due account of the respective loop corrections) the effective action in QCD and to pass on in the latter to expansion in the inverse powers of the heavy quark masses. The Lagrangian obtained has to be equated to the effective Lagrangian calculated in effective theory in the same order in the coupling constant with the anomalous Wilson coefficients on the scale μ_{hard} , which results in matching conditions for the unknown integration constants of the renormgroup equations for the Wilson coefficients in effective theory. After having done the matching, we remove arbitrariness in the choice of finite renormalization terms in the effective Lagrangian, i.e. it is determined for μ below the scale μ_{hard} of matching with complete QCD. Thus, a consistent scheme of effective theory, HQET, has been developed for hadron systems containing a single heavy quark together with light ones.

The physical situation in the case of heavy quarkonia consisting of a heavy quark and a heavy antiquark is somewhat different. Indeed, the Coulomb interaction of nonrelativistic heavy quarks results in the quark kinetic energy turning out to be comparable to their potential energy, while it would be naive to expect the kinetic term $\mathbf{p}^2/2m_Q$ to have to be suppressed by the inverse mass of the heavy quark. However, since in the case of the Coulomb exchange $p \sim \alpha_s m_Q$, no such suppression of kinetic energy exists and the wave functions of the heavy quarkonia depend on the quark masses, i.e. on their flavors.

Within the formal approach of effective theory for nonrelativistic quarks in heavy quarkonium, the leading term of the Lagrangian is determined with due account of the contribution from kinetic energy, and we arrive at nonrelativistic QCD (NRQCD) [10]. Unlike HQET, the spin symmetry of the leading term in the effective Lagrangian is conserved in NRQCD, but no symmetry exists with respect to the flavors of heavy quarks, since the contribution from

kinetic energy depends explicitly on the quark masses. Like in HQET, the Wilson coefficients in the NRQCD effective Lagrangian must be matched with the complete QCD on a scale of the order of magnitude of the heavy quark mass, and, generally speaking, the Wilson coefficients have anomalous dimensions differing from the anomalous dimensions in HQET, since the kinetic energy results in a different, as compared with HQET, ultraviolet behavior of the quark operators.

Operator expansion also underlies the potential approach. Thus, static potential signifies expansion of the QCD effective action for two infinitely heavy sources j that are at a fixed distance r from each other, so that the expression $\Gamma(j) = -V(r) T$, where $T \rightarrow \infty$ is the time of action of the sources, holds valid for the effective action. In the case of real problems, a long time signifies that the virtualities of the external gluon fields interacting with the heavy quarks are much smaller than the inverse distances, i.e. $\mu \ll 1/r \sim m_Q v$. This usability condition of the potential approach may be expressed in terms of effective theory, which is called potential nonrelativistic QCD (pNRQCD) [11].

Construction of pNRQCD theory is based on matching, at $\mu \sim m_Q v$, the NRQCD and the effective actions with external supersoft fields in the multipole QCD expansion and with nonrelativistic quarks with a leading term determined both by the kinetic energy and the Wilson coefficient. The latter has the meaning of a static potential depending on the distance r between the quarks. Thus, the potential approach has the status of an operator expansion, while the static approximation for the potential is determined by the convergence of this expansion in pNRQCD.

Theoretical investigation of baryons with two heavy quarks is also interesting because to describe them it is necessary to develop and apply a complex approach that will combine the features of HQET, NRQCD and pNRQCD, since within the system both the interaction of the light quark with the heavy quarks and the interaction between the two heavy quarks are essential (Fig. 1).

In this review we deal with the problem of describing bound baryonic states $QQ'q = \Xi_{QQ'}$ with two heavy quarks Q, Q' and a light quark $q = u, d$ on the basis of factorization of interactions with virtualities that are determined by:

the scale of confinement, Λ_{QCD} , for nonperturbative interactions of the heavy quarks with the light one and of the quarks with vacuum quark – gluon condensates;

the size of the heavy diquark, $r_{\text{diq}} \sim 1/(m_{Q,Q'} v)$ (where $\text{diq} = QQ'$), composed of two nonrelativistic heavy quarks moving with a small relative velocity $v \ll 1$, for interactions between the heavy quarks;

the scale of hard gluon corrections at energies of the order of magnitude of the heavy quark masses, while the approximation considered the leading one is the approximation for which there exists a hierarchy of QCD interaction scales in $\Xi_{QQ'}$, i.e.

$$\Lambda_{\text{QCD}} \ll m_Q v \ll m_Q. \quad (1.1)$$

Within this approach, the doubly heavy diquark is perceived by the light quark as a local heavy source of the gluon field (antitriplet over the QCD color group), while the diquark actually represents a system of two nonrelativistic quarks in the weakly changing low-frequency field of the light quark. Thus, for the motion of the light quark and the diquark one can apply the effective HQET theory, whereas for the motion of the heavy quarks in the diquark one has to

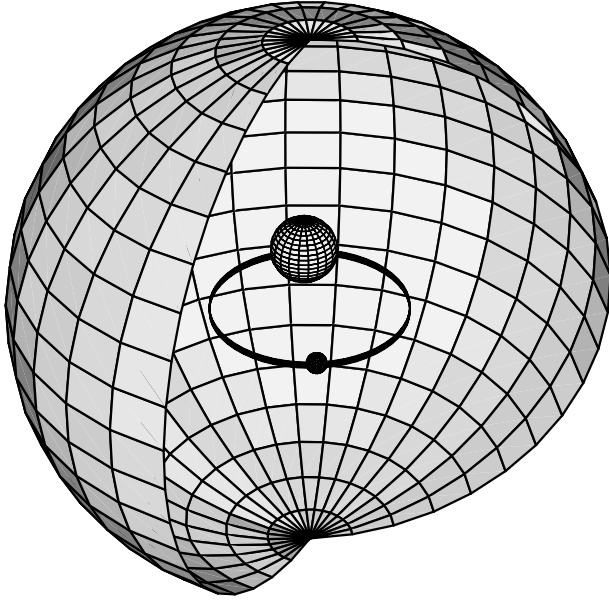


Figure 1. Character of strong interactions in the doubly heavy Ξ_{bc} baryon. The quark Compton lengths $\lambda_Q = 1/m_Q$, the size of the heavy diquark $r_{bc} \sim 1/(m_Q v)$, and the scale of nonperturbative confinement of the light quark, $r_{QCD} = 1/\Lambda_{QCD}$, are ordered as $\lambda_b \approx \lambda_c/3 \approx r_{bc}/9 \approx r_{QCD}/27$.

modify NRQCD and pNRQCD in order to deal with the nonrelativistic fields of the heavy quarks in the antitriplet, instead of the singlet, state.

On the basis of the quark – diquark picture of interactions we consider various physical aspects of baryons with two heavy quarks. In Section 2, the mass spectrum of $\Xi_{QQ'}$ baryons is constructed within the potential approach, and the characteristics are computed of the ground and excited levels both in the heavy diquark system and in the light quark – diquark system. We show that there exists an entire family of the $\Xi_{QQ'}$ levels including quasi-stable states for a heavy diquark composed of identical heavy quarks. In this case the Pauli exclusion principle dictates quite definite values of the total quark spin: the spin is unity for P -even coordinate wave functions, and zero for P -odd ones, since the antitriplet color state of the diquark is antisymmetric with respect to permutation of the color indices of the heavy quarks. With due account of the small size of the diquark, of the nonrelativistic motion of the heavy quarks and of the small ratio Λ_{QCD}/m_Q , the transition operators of the excited P -wave diquark to the ground S -wave state with the emission of a π -meson turn out to be suppressed, since in such a transition both the spin and the orbital states of the heavy diquark must change. We define the range of applicability of the quark – diquark approximation for the calculation of the $\Xi_{QQ'}$ mass spectra within the potential approach.

The two-point NRQCD sum rules for baryon currents with two heavy quarks and the stability criterion of the results following from the sum rules for masses and the $\Xi_{QQ'}$ baryon coupling constants are considered in Section 3. Reliable results can be obtained if one takes into account both the quark and gluon condensates and their product, i.e. the combined condensate of higher dimension. The masses and coupling constants have been calculated for the ground states of $\Xi_{QQ'}$ baryons and of doubly heavy baryons with strangeness $\Omega_{QQ'}$, and reliable predictions have been made for the mass splitting $M[\Xi_{QQ'}] - M[\Omega_{QQ'}]$. The anomalous dimen-

sions of baryon currents in NRQCD have been obtained in the two-loop approximation, which permits one to estimate the baryon coupling constants with baryon currents not only in NRQCD, but also in complete QCD.

The production mechanisms of $\Xi_{QQ'}$ baryons are dealt with in Section 4. At high energies, the inclusive production of doubly heavy baryons in e^+e^- -annihilation can be described (owing to factorization of interactions within the quark – diquark approach) as the sequential fragmentation of a heavy quark into a heavy diquark, and of the diquark into a baryon. Here, the virtualities in the first subprocess are determined by the heavy quark masses, and upon factorization of the soft motion of the heavy quarks in the diquark one can apply QCD perturbation theory to obtain the analytical form of the fragmentation functions into diquark states with differing spins and orbital quantum numbers. With due account of the difference between the diquark and heavy quarkonium in color structure, such calculations repeat the calculations of fragmentation into doubly heavy mesons with an accuracy up to a color factor. Assuming the existence of a local heavy diquark field in its interactions with the light quark and applying the approximation of dominant contribution from fast valence quarks for estimation of the baryon field components we have developed a QCD-motivated perturbation model for diquark fragmentation into a baryon and have deduced the analytical form of the fragmentation functions of vector and scalar diquarks into the $\Xi_{QQ'}$ baryon with spin $1/2$. Calculations have been done of exclusive baryon pair production cross sections in e^+e^- -annihilation in the vicinity of the threshold.

The analysis turns out to be more complicated for the production mechanism of $\Xi_{QQ'}$ baryons in hadron collisions in quark – antiquark annihilation (low energies, fixed-target experiments) and gluon – gluon fusion (high energies, hadron colliders) subprocesses. The enhanced complexity is due to the large number of diagrams in the leading approximation — in the fourth order in the QCD coupling constant. Applying the numerical method we show that at high energies of the parton subprocess and at transverse momenta greatly exceeding the baryon mass the complete set of diagrams in the given order of QCD perturbation theory results in factorization of the hadron production of the heavy quark and its subsequent fragmentation into a doubly heavy diquark with the universal fragmentation function computed analytically within the framework of perturbative QCD. This points to the applied method being self-consistent in perturbation theory.

The advantage of dealing with the complete set of diagrams, which is dictated by gauge invariance, consists in the possibility of computing not only the leading (in transverse momentum p_\perp) contribution from fragmentation exhibiting a $1/p_\perp^4$ dependence, but also the corrections — higher twists in the transverse momentum. Here, it turns out to be possible to obtain a certain estimate for the transverse momentum that separates the fragmentation and recombination regions (of higher twists), and it becomes clear that the statistics of events involving the production of doubly heavy $\Xi_{QQ'}$ baryons accumulates mainly at low transverse momenta, i.e. in the recombination region. Estimates have been made of the total and differential production cross sections for $\Xi_{QQ'}$ baryons in hadron experiments at various energies and, also, of the exclusive pair production cross section of doubly heavy diquarks in quark – antiquark annihilation.

The method of operator expansion in the inverse powers of the heavy quark masses is applied in Section 5 to a detailed analysis of the lifetimes and inclusive decay widths of $\Xi_{QQ'}$ baryons. The following three effects play an essential part in the analysis of the decay mechanisms of heavy quarks composing doubly heavy baryons:

there arise significant corrections to the spectator decay widths of heavy quarks, when account is taken of the motion of the quarks in a small-sized heavy diquark, and, consequently, the relative momenta of the quarks, $p \sim m_{Qv}$, are noticeably higher than the momentum $k \sim \Lambda_{\text{QCD}}$ of a heavy quark in a hadron with a single heavy quark, since $m_{Qv} \gg \Lambda_{\text{QCD}}$;

the nonspectator contributions due to Pauli interference between the decay products of a heavy quark and the valence quarks in the initial state may amount to 30–50% of the total width, while a feature peculiar to the antisymmetric baryon color function consists in the possibility of the common sign of the interference term being either positive or negative²;

weak rescattering of quarks in the initial state (together with Pauli interference) arises in the operator expansion as an operator of highest dimensionality, which is enhanced by two-particle phase space as compared with other operators of the same dimension, for which three-particle phase space in the final state is suppressed, if it is expressed in terms of the heavy quark mass; this contribution yields up to 30% of the total width for baryons with a charmed quark.

Thus, it has been shown how the total lifetimes of baryons with two heavy quarks become ordered, the parametric dependence of estimates of total and inclusive widths upon the physical parameters of the hadron system has been revealed: the heavy diquark compactness determines its wave function — a factor in the estimation of nonspectator decays, the heavy quark masses in the operator expansion are essentially correlated for hadrons with different quark compositions, so that the known experimental data on semileptonic, nonleptonic and total widths reduce the uncertainty in the estimates. At the same time, experimental data on the inclusive widths and lifetimes of doubly heavy baryons may significantly improve the qualitative and quantitative knowledge about heavy quark dynamics, especially in the case of a consistent analysis of the data on heavy hadrons with one and with two heavy quarks.

The investigation of exclusive semileptonic and nonleptonic decays under the assumption of factorization within the framework of three-particle NRQCD sum rules permits one to obtain relations for transition form factors (i.e. hadron matrix elements) from the spin symmetry of the effective Lagrangian, to analyze the uncertainties in calculations and to draw a comparison with the predictions of potential models for exclusive decays.

In Conclusions we sum up our results in the physics of baryons with two heavy quarks. The predictions made not only pave the way for purposeful searches for such baryons, but also lay down the basis for a more detailed theoretical analysis of physical effects of unquestionable interest for hadron systems with two heavy quarks, and especially for obtaining reliable predictions concerning total and exclusive decay widths. We also analyze the feasibility of experimental observation of doubly heavy baryons.

2. Spectroscopy of doubly heavy baryons: the potential approach

In this section we analyze the principal spectroscopic characteristics of the families of doubly heavy baryons $\Xi_{QQ'} = QQ'q$ (where $q = u, d$) and $\Omega_{QQ'} = QQ's$.

Usually, two hypotheses are discussed in the description of baryons within the quark potential model: the hypothesis of pair interquark potentials, and the hypothesis of a string-like picture involving a string node. The latter is motivated by consideration of a Wilson loop, while the model with pair interactions is rather based on phenomenological approach (by analogy with the simple idea of a linear superposition of two-quark forces). From our point of view, such a linearization in the conditions of nonperturbative quark confinement that manifestly exhibit a nonlinear character represents an extremely rough approximation, including the case when two of the constituent quarks in the baryon are heavy. Nevertheless, without going into details of models with pair interactions in baryons, we note that (upon completion of a program providing detailed quantitative spectroscopic predictions) both the hypotheses permit object comparison which will become important after the first experimental data are obtained, since in a number of positions the theoretical constructions lead to clearly identifiable differences.

The general approach to calculating the masses of baryons with two heavy quarks within potential models is presented in papers [12], where the hypothesis of pair interaction between the quarks composing the baryon underlies the physical basis of the consideration and then is applied within the framework of the three-body problem. Proceeding from the above, clear consequences were derived for the mass spectra of doubly heavy baryons. Thus, the factorization approximation for the motion of the doubly heavy diquark and the light quark turned out to be quite rough in the case of charmed and beauty quark masses being preset: it resulted in the mass of the ground state and the masses of excited levels differing significantly from the estimates obtained by the calculating method for the respective three-body problem. For example, it easy to show that conventional introduction of Jacobi variables in the oscillator potential of pair interactions leads to a change in the vibration frequency $\omega \rightarrow \sqrt{3}/2 \omega$ as compared to naive expectations based on diquark factorization.

There exists another opinion about the problem of three-quark bound states in QCD: the concept of a quark–gluon string. In the stringlike picture of a doubly heavy baryon, presented in Fig. 2, the aforementioned conclusions based on

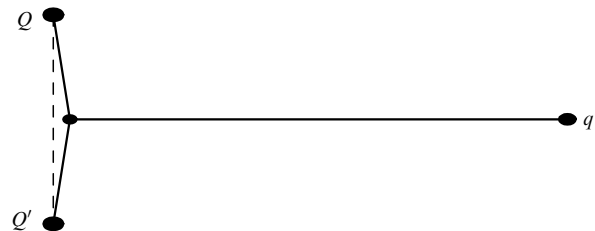


Figure 2. Representation of a $QQ'q$ baryon containing two heavy quarks with color fields forming strings between the heavy and light quarks, so that the picture of pair interactions is violated and an additional center-of-mass point is introduced, which is close to the center of mass of the two heavy quarks.

² The common sign is determined by the factors of the antisymmetric permutation of fermions and of the color factor.

the hypothesis of pair interactions and concerning the structure of the mass spectra of such baryons are essentially modified. Indeed, if one considers the center of the string (Fig. 2), which is very close to the center of mass of the doubly heavy diquark, it is seen that the light quark interacts with the heavy diquark as a whole, i.e. when the string tension coincides in magnitude with the tension in the heavy–light meson $Q\bar{q}$. Consequently, the two hypotheses about the nature of strong interactions in baryons containing two heavy quarks (pair interactions or the stringlike picture) lead to clearly distinguishable predictions for the mass spectra of doubly heavy baryons both in the ground state and at excited levels. The only criterion for testing these assumptions consists in experimental observation and measurements.

In the present review we follow the approximation of a doubly heavy diquark, which is quite reasonable, as we already explained above. To be more convincing we note that considering the masses of baryons with two heavy quarks within the QCD and NRQCD sum rules (see Section 3) results in the ground states exhibiting masses that are in good agreement with the values obtained within the potential approach with factorization of the heavy diquark.

The qualitative picture of the production of bound states in the $QQ'q$ system is clarified through the existence of two scales of distance that are given by the size r_{diq} of the doubly heavy QQ' diquark subsystem in the antitriplet state with respect to color and by the confinement scale A_{QCD} of the light quark q :

$$r_{\text{diq}} A_{\text{QCD}} \ll 1, \quad A_{\text{QCD}} \ll m_Q.$$

In these conditions a compact diquark QQ' is perceived by the light quark as a static source of the colored QCD field in the diquark local field approximation. From this standpoint, one can make use of a number of reliable results obtained with the models of heavy mesons with a single heavy quark [i.e. local static source in the antitriplet representation of the $SU(3)_c$ group]: with potential models [8] and with heavy quark effective theory (HQET) [9] in the expansion in the inverse heavy quark mass.

We apply here the nonrelativistic quark model with the Buchmüller–Tye potential [13]. In this case one can *theoretically* speak of a rough approximation for the light ($m_q^{\text{QCD}} \ll A_{\text{QCD}}$) and therefore relativistic quark in the system with a finite number of degrees of freedom and instantaneous interaction $V(\mathbf{r})$. This is due to confinement implying the light quark being dressed in a sea (infinite number of gluons and quark pairs), and nonperturbative effects with correlation times $\tau_{\text{QCD}} \sim 1/A_{\text{QCD}}$ remain beyond the framework of the potential approach. *Phenomenologically*, however, the introduction of a constituent mass $m_q^{\text{NP}} \sim A_{\text{QCD}}$ as the principal parameter determining the interaction with the QCD condensates permits one to successfully adjust the nonrelativistic potential model with a high precision ($\delta M \approx 30\text{--}40$ MeV) on the basis of available experimental data, which makes such an approach quite a reliable instrument for the prediction of masses of hadrons containing heavy and light quarks.

As to the diquark QQ' , with the exception of two very significant peculiarities it is quite similar to heavy quarkonium $\bar{Q}Q'$: $QQ'[\bar{3}_c]$ is a system with open color, and in the case of quarks of the same flavor ($Q = Q'$) it is necessary to take the Pauli exclusion principle into account for identical fermions. While the second peculiarity can be readily seen to lead to the total quark spin ($S = 0$) being forbidden for

symmetrical P -even space wave functions $\Psi_{\text{diq}}(\mathbf{r})$ of the diquark (angular momentum $L_{\text{diq}} = 2n$, where $n = 0, 1, 2, \dots$ is a nonnegative integer) and to $S = 1$ for antisymmetric P -odd functions $\Psi_{\text{diq}}(\mathbf{r})$, i.e. $L_{\text{diq}} = 2n + 1$, the nonzero color charge of the system raises two problems.

First, the confinement hypothesis in the form of a confining potential (infinite growth of energy with the size of the system) is, generally speaking, not applicable to interactions inside such an object. It is physically difficult, however, to imagine a situation when a large colored object of dimension $r > 1/A_{\text{QCD}}$ possesses limited self-action energy and at the same time, while interacting with another color source inside a white [singlet in $SU(3)_c$] state, is closed inside a hadron of dimension $r \sim 1/A_{\text{QCD}}$. Moreover, within the framework of the hadron string picture for baryons that proved to be successful, the string tension in the diquark with an external end is only two times smaller than in the quark–antiquark pair $\bar{q}q'$ of a meson, while the diquark energy also grows linearly with its size, so an effect occurs that is similar to quark confinement.

Moreover, in potential models one can consider quark coupling to be realized by the effective one-particle exchange of a colored object in the octet representation $SU(3)_c$ (usually, the sum of scalar and vector exchanges is taken), so that once again the potentials in the singlet ($\bar{q}q'$) and antitriplet (qq') states only differ by the color factor $1/2$, which points to the presence of a confining potential (a linearly growing term) in the QCD-motivated model for the heavy diquark $QQ'[\bar{3}_c]$. In this section we shall apply the nonrelativistic model with the Buchmüller–Tye potential for the diquark, too.

Second, in the singlet state $\bar{Q}Q'$ with respect to the color the total quark spin S and angular momentum L are conserved separately, since the contributions from the QCD transition operators between the levels determined by these quantum numbers are suppressed. Indeed, within the framework of the QCD multipole expansion [14], the amplitudes of chromomagnetic and chromoelectric dipole transitions are suppressed by the inverse heavy quark mass but, moreover, a decisive role is played by white radiation of the object, i.e. at least the emission of two gluons (an excess order in $1/m_Q$), and by taking into account the real phase space in the physical spectrum of massive hadrons, unlike the case of the massless gluon. On the other hand, the probability of an admixture of a hybrid state, for instance, with the octet subsystem $\bar{Q}Q'$ and an additional gluon, i.e. of a Fock state $|\bar{Q}Q'[\bar{8}_c]g\rangle$, is suppressed owing to the small size of the system and the nonrelativistic motion of the quarks (for details see Ref. [10]).

In the antitriplet state QQ' with respect to color the emission of a soft nonperturbative gluon is not forbidden in the transitions between the levels determined by spin S_{diq} and angular momentum L_{diq} in the diquark, if no other forbidings or small suppression parameters exist. If the quarks composing the diquark have identical flavors, then the Pauli exclusion principle results in transitions being possible only between the levels that either differ simultaneously in spin ($\Delta S_{\text{diq}} = 1$) and in angular momentum ($\Delta L_{\text{diq}} = 2n + 1$) or occupy the same row of radial excitations or those with $\Delta L_{\text{diq}} = 2n$.

While in the second case the transition amplitudes are suppressed by the small recoil momentum of the diquark as compared to its mass, in the first case the transition operator changing the diquark spin and its angular momentum has an excess order of smallness either due to the additional factor $1/m_Q$ or to the small size of the diquark, which results in the

presence of quasi-stationary states with quantum numbers S_{diq} and L_{diq} . In the diquark bc with quarks of differing flavors the operators of QCD dipole transitions with the emission of a sole soft gluon are not forbidden, so the excited-state lifetimes can be comparable to the formation times of a bound state or to the inverse separations between the levels themselves, and in this case one cannot definitely claim that there exists a set of diquark excitations with definite spin and angular momentum quantum numbers³.

Thus, in this review the existence of two physical scales is used in the form of factorization of the wave function in the problem with a heavy diquark and a light constituent quark within the framework of the nonrelativistic quark model, so that the problem of computation of the mass spectrum and the characteristics of a bound state in the system of a doubly heavy baryon reduces to two standard problems of searching for stationary energy levels in a two-body system. The relativistic corrections dependent on the quark spin are then taken into account for each of the two subsystems studied.

The threshold decay energy into a heavy baryon and a heavy meson can be considered a natural boundary for the domain of existence of stable states in doubly heavy baryons. As is shown in Ref. [15], the presence of such a threshold in various systems may be accompanied in QCD by the existence of a universal confinement characteristic — a limiting distance between quarks, the enhancement of which results in the quark–gluon field losing its stability, i.e. in the generation of valent quark–antiquark pairs from the sea.

In other words, a hadron string longer than the critical length has a high probability (close to unity) of decaying into shorter strings. This effect can be taken into account within the potential approach by the investigation of excited diquark levels being restricted to the region where the size of the diquark is smaller than the critical distance: $r_{\text{diq}} < r_c \approx 1.4–1.5$ fm. Besides, the model of pair interactions with the diquark structure singled out can be considered reliable only when the size of the diquark is smaller than the distance to the light quark: $r_{\text{diq}} < r_l$.

A peculiarity of the quark–diquark picture of the doubly heavy baryon consists in the possible mixing of diquark higher excited states with differing quantum numbers owing to their interaction with the light quark, so in this case it is difficult to speak of definite quantum numbers of the excitations. Below, we shall discuss this mechanism and the character of such an effect in detail.

Section 2.1 presents the general procedure for computing the mass spectrum of doubly heavy baryons within the framework of the above-formulated assumptions taking into account the quark spin-dependent corrections to the QCD-motivated potential, in Section 2.2 the results of numerical estimations are given, and our conclusions are briefly outlined in Section 2.3.

2.1 The nonrelativistic potential model

As noted in the Introduction, the problem of computing the mass spectrum of baryons containing two heavy quarks reduces to sequential calculation of the diquark energy levels and, then, of the energy levels of a point diquark with the obtained parameters and of the light constituent quark interacting with it. At each step of such calculations we

identify two stages in the agreement with the model expansion of QCD interaction in the effective theory in the inverse quark mass, so that in the first approximation a nonrelativistic Schrödinger equation is solved with a QCD-motivated model potential, while the part of perturbation is played by corrections dependent on the quark spin that are suppressed by the quark masses.

2.1.1 The potential. The potential of static heavy quarks incorporates the most important features of QCD dynamics: asymptotic freedom and confinement. The static heavy quark potential in the leading order of QCD perturbation theory at small distances and with a linear confining interaction in the infrared region was considered within the Cornell model [16], in which a simple superposition of both asymptotic limits (of the effective Coulomb and stringlike interactions) is introduced. The observed heavy quarkonia are situated in the region of intermediate distances, where the contributions from both potential terms are important for determining the mass spectra (Fig. 3). For this reason, phenomenological approximations of the potential (logarithmic and power laws [17, 18]), taking into account regularities in the mass spectra [8], could be applied with success.

The quantities most sensitive to the global properties of the potential are the quarkonium wave functions at zero point that are related to the lepton constants and the normalizations of the quarkonium production cross sections. Potentials consistent both with asymptotic freedom in one and two loops and with linear confinement were proposed by Richardson [19] and by Buchmüller and Tye [13].

In QCD, the static potential is defined in an explicitly gauge-invariant form via the vacuum expectation of the Wilson loop [20]:

$$V(r) = - \lim_{T \rightarrow \infty} \frac{1}{iT} \ln \langle \mathcal{W}_T \rangle, \\ \mathcal{W}_T = \text{Tr} \mathcal{P} \exp \left(ig \oint_{\Gamma} dx_{\mu} A^{\mu} \right). \quad (2.1)$$

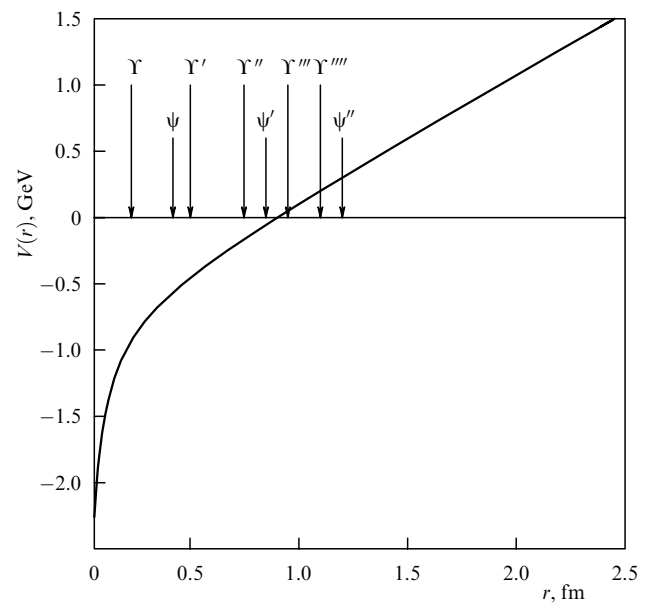


Figure 3. Cornell model of the static potential and the dimensions of a number of observable heavy quarkonia with charmed (ψ family) and beauty (Y family) quarks.

³ In other words, the existence of the gluon sea in the Ξ_{bc} baryon leads to transitions between the states with different diquark excitations such as $|bc\rangle \rightarrow |bcg\rangle$ with $\Delta S_{\text{diq}} = 1$ or $\Delta L_{\text{diq}} = 1$ that are not suppressed.

Here, Γ is a rectangular contour with sides T in time and r in space. The gauge fields A_μ are ordered along the path (symbol \mathcal{P}), while the color trace is normalized in accordance with $\widehat{\text{Tr}}(\dots) = \text{Tr}(\dots)/\text{Tr} \hat{1}$.

We note that definition (2.1) corresponds to computation of the effective action for two external sources fixed at a distance r for an indefinitely long period of time T , so in this case ordering in time coincides with ordering along the path (symbol \mathcal{P}). Moreover, the contribution to the effective action from the path lengths where the charges were separated in a finite time to a finite distance can be neglected as compared to the infinite contribution from $V(r)T$. It must be especially stressed that the static potential defined is, by construction, a renorm-invariant quantity, since the action does not depend on the normalization point by definition.

Usually, the V -scheme is considered for the effective coupling constant, while the static quark potential is defined in momentum space, making use of standard notation for the structure constants of the SU(N) group, by the formula

$$V(\mathbf{q}^2) = -C_F \frac{4\pi\alpha_V(\mathbf{q}^2)}{\mathbf{q}^2}, \quad (2.2)$$

so the newly introduced quantity α_V can be calculated both in the case of large virtualities in QCD perturbation theory and at small transferred momenta, assuming the quark confining potential to be linear in the confinement mode.

In this section we discuss two modes for QCD forces acting between static heavy quarks: asymptotic freedom and confinement. Then, following the Buchmüller–Tye method, we formulate how these modes can be represented in a unique β -function for the quantity α_V , satisfying both limits in the case of small and large QCD coupling constants.

2.1.2 Perturbative results at small distances. Technically, in a given regularization scheme (say, $\overline{\text{MS}}$) one must compute the perturbative expansion of the static quark potential, so this potential can be written as a Coulomb potential with a running constant in the so-called V -scheme. As a result, computations within perturbation theory yield matching conditions for the $\overline{\text{MS}}$ - and V -schemes. Calculations with the running constant $\alpha_s^{\overline{\text{MS}}}$ in n loops require matching with α_V in $n-1$ loops. We note that the two initial coefficients of the respective β -functions are the quantities independent of the regularization scheme and of the computation gauge, while all the remaining coefficients, generally speaking, depend on the computation procedure. The V -scheme is defined for the observable quantity, so its β -function is gauge invariant.

In QCD perturbation theory the quantity α_V can be matched with $\alpha_s^{\overline{\text{MS}}}$:

$$\begin{aligned} \alpha_V(\mathbf{q}^2) &= \alpha_s^{\overline{\text{MS}}}(\mu^2) \sum_{n=0}^{\infty} \tilde{a}_n \left(\frac{\mu^2}{\mathbf{q}^2} \right) \left(\frac{\alpha_s^{\overline{\text{MS}}}(\mu^2)}{4\pi} \right)^n \\ &= \alpha_s^{\overline{\text{MS}}}(\mathbf{q}^2) \sum_{n=0}^{\infty} a_n \left(\frac{\alpha_s^{\overline{\text{MS}}}(\mathbf{q}^2)}{4\pi} \right)^n. \end{aligned} \quad (2.3)$$

By the time the results obtained by Buchmüller and Tye were published, only the two-loop β -function and the conditions for one-loop matching with the potential were known. Noticeable progress has been made recently in computations: the two-loop conditions for matching the V - and $\overline{\text{MS}}$ -schemes [21, 22] can be combined with the three-loop running constant $\alpha_s^{\overline{\text{MS}}}$. In expansion (2.3), the coefficients a_0

of the tree approximation, of the one-loop contribution a_1 and the new results for the two-loop contribution a_2 are known (see Refs [21, 22]).

Upon introduction of $\hat{\alpha} = \alpha/(4\pi)$, the function β is defined as a derivative in the form

$$\frac{d\hat{\alpha}(\mu^2)}{d \ln \mu^2} = \beta(\hat{\alpha}) = - \sum_{n=0}^{\infty} \beta_n \hat{\alpha}^{n+2}(\mu^2), \quad (2.4)$$

so that

$$\beta_{0,1}^V = \beta_{0,1}^{\overline{\text{MS}}}, \quad \beta_2^V = \beta_2^{\overline{\text{MS}}} - a_1 \beta_1^{\overline{\text{MS}}} + (a_2 - a_1^2) \beta_0^{\overline{\text{MS}}}.$$

Fourier transformation leads to the following expression for the potential in coordinate space [21]:

$$\begin{aligned} V(r) = -C_F \frac{\alpha_s^{\overline{\text{MS}}}(\mu^2)}{r} \left\{ 1 + \frac{\alpha_s^{\overline{\text{MS}}}(\mu^2)}{4\pi} (2\beta_0 \ln(\mu r') + a_1) \right. \\ \left. + \left(\frac{\alpha_s^{\overline{\text{MS}}}(\mu^2)}{4\pi} \right)^2 \left[\beta_0^2 \left(4 \ln^2(\mu r') + \frac{\pi^2}{3} \right) \right. \right. \\ \left. \left. + 2(\beta_1 + 2\beta_0 a_1) \ln(\mu r') + a_2 \right] \right\}, \end{aligned} \quad (2.5)$$

where $r' \equiv r \exp \gamma_E$.

Upon determination of the new running constant depending on the distance:

$$V(r) = -C_F \frac{\bar{\alpha}_V(1/r^2)}{r}, \quad (2.6)$$

one can calculate its β -function from expression (2.5) [21]:

$$\bar{\beta}_2^V = \beta_2^V + \frac{\pi^2}{3} \beta_0^3, \quad (2.7)$$

and the next-to-leading coefficients $\bar{\beta}_{0,1}^V$ are equal to known values that are independent of the scheme.

We note that by construction the perturbative potential (2.5) does not depend on the normalization point, i.e. it is a renorm invariant quantity. However, in the case dealt with, breaking off a QCD perturbation theory series with coefficients that do not decrease⁴ leads to a strong residual dependence on the normalization point. Thus, when the normalization point μ is chosen to be in the region of the charmed quark mass, the two-loop potential with the three-loop running constant $\alpha_s^{\overline{\text{MS}}}$ exhibits an unremovable μ -dependent additive shift that varies within broad limits. This is a manifestation of the presence of an infrared singularity in the QCD coupling constant, so that the μ -dependent shift in the potential has the form of a pole at the point A_{QCD} .

For the static potential to be unambiguous in QCD it is necessary to deal with infrared-stable quantities. The motivation of Buchmüller and Tye consisted in writing such a β -function for α_V that would be consistent with known asymptotic modes at small and large distances. They proposed to write the function in such a manner so as to provide for an infrared-stable effective charge depending only on two parameters: a perturbative parameter — the scale in

⁴ Moreover, according to studies of the renormalon the coefficients of perturbation theory series in the perturbative potential grow factorially, so the expansion actually has an asymptotic sense.

the running constant for large virtualities, and a nonperturbative parameter — the quark – gluon string tension.

For complete definiteness it is also necessary to specify the coefficients of the β -function. The parameters of the Buchmüller–Tye potential were fixed when the mass spectra of charmonium and bottomonium were fitted in the potential model [23]. Thus, for example, the scale $\Lambda_{\overline{MS}}^{MS} \approx 510$ MeV, giving the asymptotic behavior of the QCD coupling constant in the case of large virtualities, was determined within this phenomenological approach. This value contradicts present-day data on the QCD coupling constant α_s [23]. Moreover, it is easily seen that the three-loop coefficient β_2^V of the β -function, assumed in the Buchmüller–Tye model, differs in sign and absolute value from the corresponding one calculated recently in Refs [21, 22].

Thus, it is extremely interesting to undertake modification of the Buchmüller–Tye (BT) potential for static quarks in accordance with the present-day status of perturbative calculations.

For normalization of the coupling constants in the deep-perturbative region we shall further make use of relation (2.3) for $\mathbf{q}^2 = m_Z^2$.

2.1.3 Quark confinement. The nonperturbative behavior of QCD forces between static heavy quarks at large distances r is usually represented in the form of the linear potential (see discussion in papers [25])

$$V^{\text{conf}}(r) = kr \quad (2.8)$$

that is consistent with the law of areas for the Wilson loop.

The potential (2.8) can be expressed in terms of a *constant chromoelectric field* between the sources present in the fundamental $SU(N_c)$ representation. Thus, in the Fock–Schwinger fixed point gauge $x_\mu A^\mu(x) = 0$, the gluon field can be expressed via the field strength tensor $A_\mu(x) \approx (1/2)x^\nu G_{\mu\nu}(0)$. For static quarks separated by a distance \mathbf{r} , the field is oriented in the direction from a quark to an antiquark, while for the nonperturbative quark–quark–gluon vertex we have a unique representation consistent with the requirement of the gluon field alignment and with the color structure of the quark field: $\bar{Q}_i(0) G_{m0}^a(0) Q_j(0) = (\mathbf{r}_m/r) E T_{ij}^a$, where the heavy quark fields are normalized to unity. Then the potential confining the quarks is written as

$$V^{\text{conf}}(r) = \frac{1}{2} g C_F E r.$$

Assuming that the same absolute value of the field strength corresponds to formation of the gluon condensate⁵ (Fig. 4) and introducing stochastic color sources n_i that must

⁵ We intend the model of vacuum in which the chromoelectric field equals the average field E in absolute value, while its orientation is arbitrary and varies chaotically, so when static sources are introduced at a comparatively large distance from each other, i.e. when the Coulomb contribution is small compared to the nonperturbative one, the chromoelectric field is aligned in a certain direction from the quark to the antiquark. The string configuration differs from vacuum by changing from the disoriented vacuum phase of the gluon field state to alignment in the presence of an external source, which is reminiscent of the situation with the magnetization of metals with the only difference that when the external action is removed, the disoriented phase is restored. The correlation lengths and times of vacuum fields are determined by the energy scale of confinement and are insignificant for static sources, although they may lead to power corrections in the characteristics of rapidly moving heavy quarks.

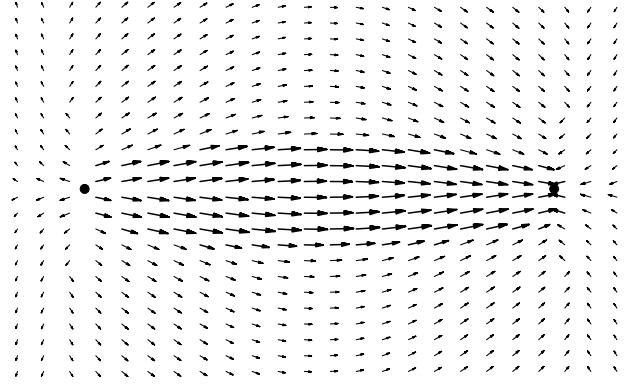


Figure 4. Vacuum chromoelectric field which in the presence of charged sources ‘is aligned’ along the line connecting the heavy quarks.

be averaged over the vacuum, we readily find

$$\langle G_{\mu\nu}^2 \rangle = 4C_F E^2 \langle \bar{n}n \rangle.$$

From Ref. [24] the following relation follows for the linear term of the potential:

$$k = \frac{\pi}{2\sqrt{N_c}} C_F \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \right\rangle^{1/2}. \quad (2.9)$$

The quantity k is usually expressed via the parameter α'_R :

$$k = \frac{1}{2\pi\alpha'_R}.$$

Following Buchmüller and Tye we assume $\alpha'_R = 1.04 \text{ GeV}^{-2}$. This value of the string tension related to the inclination of the Regge trajectories may be compared with the estimate following from expression (2.9). When $\langle (\alpha_s/\pi) G_{\mu\nu}^2 \rangle = (1.6 \pm 0.1) \times 10^{-2} \text{ GeV}^4$ [7], we have

$$\alpha'_R = 1.04 \pm 0.03 \text{ GeV}^{-2},$$

which is in good agreement with the known inclination of the Regge trajectories.

Potential form (2.8) corresponds to the limit when at small virtualities ($\mathbf{q}^2 \rightarrow 0$) the coupling constant

$$\alpha_V(\mathbf{q}^2) \rightarrow \frac{K}{\mathbf{q}^2},$$

and

$$\frac{d\alpha_V(\mathbf{q}^2)}{d \ln \mathbf{q}^2} \rightarrow -\alpha_V(\mathbf{q}^2), \quad (2.10)$$

which gives the confinement asymptotics for the function β^V .

2.1.4 Unique β -function and potential. Buchmüller and Tye proposed a constructive procedure for restoring the β -function in the entire region of charge variation from known limits in the case of asymptotic freedom in a given order of perturbation theory in α_s and in the confinement mode. Generalization of their method leads to the function β_{pert} , found within the framework of asymptotic perturbation theory in three loops, transforming into the β -function of

effective charge:

$$\begin{aligned} \frac{1}{\beta_{\text{pert}}(\hat{\alpha})} &= -\frac{1}{\beta_0 \hat{\alpha}^2} + \frac{\beta_1 + (\beta_2^V - \beta_1^2/\beta_0) \hat{\alpha}}{\beta_0^2 \hat{\alpha}} \\ &\rightarrow \frac{1}{\beta(\hat{\alpha})} = -\frac{1}{\beta_0 \hat{\alpha}^2} \left[1 - \exp\left(-\frac{1}{\beta_0 \hat{\alpha}}\right) \right]^{-1} \\ &\quad + \frac{\beta_1 + (\beta_2^V - \beta_1^2/\beta_0) \hat{\alpha}}{\beta_0^2 \hat{\alpha}} \exp\left(-\frac{l^2 \hat{\alpha}^2}{2}\right), \end{aligned} \quad (2.11)$$

where the exponential factor in the second term only determines the contribution in the order next in accuracy to the order of the three-loop case for $\hat{\alpha} \rightarrow 0$.

Function (2.11) exhibits a significant singularity as $\hat{\alpha} \rightarrow 0$, so that the expansion yields an asymptotic series in $\hat{\alpha}$. As $\hat{\alpha} \rightarrow \infty$, the function β tends toward the limit of the confinement mode, as presented in Eqn (2.10). We recall that one- and two-loop static potentials matched with the linear confinement term result, when the mass spectra of heavy quarkonia are fitted, in a contradiction with the QCD coupling constant on the Z-boson mass scale. We shall show that the static potential in the three-loop approximation is in agreement with variations in the QCD coupling constant for large virtualities.

In the perturbative limit, the usual solution for the running coupling constant

$$\begin{aligned} \hat{\alpha}(\mu^2) &= \frac{1}{\beta_0 \ln(\mu^2/A^2)} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{1}{\ln(\mu^2/A^2)} \ln \ln \frac{\mu^2}{A^2} \right. \\ &\quad \left. + \frac{\beta_1^2}{\beta_0^4} \frac{1}{\ln^2(\mu^2/A^2)} \left(\ln^2 \ln \frac{\mu^2}{A^2} - \ln \ln \frac{\mu^2}{A^2} - 1 + \frac{\beta_2^V \beta_0}{\beta_1^2} \right) \right] \end{aligned} \quad (2.12)$$

still holds valid. Making use of the asymptotic limit (2.12) one can find the equation

$$\begin{aligned} \ln \frac{\mu^2}{A^2} &= \frac{1}{\beta_0 \hat{\alpha}(\mu^2)} + \frac{\beta_1}{\beta_0^2} \ln(\beta_0 \hat{\alpha}(\mu^2)) \\ &\quad + \int_0^{\hat{\alpha}(\mu^2)} dx \left(\frac{1}{\beta_0 x^2} - \frac{\beta_1}{\beta_0^2 x} + \frac{1}{\beta(x)} \right), \end{aligned} \quad (2.13)$$

which is readily integrated, and obtain an implicit solution for the dependence of charge on the scale. The implicit equation can be inverted by the iteration method, so the approximate solution has the form

$$\hat{\alpha}(\mu^2) = \frac{1}{\beta_0 \ln(1 + \eta(\mu^2) \mu^2/A^2)}, \quad (2.14)$$

where $\eta(\mu^2)$ is expressed in terms of the coefficients of the perturbative β -function. The parameter l , in turn, is related to the inclination of Regge trajectories and the scale A (constant of integration) by the relation

$$\begin{aligned} \ln(4\pi^2 C_F \alpha'_R A^2) &= \ln \beta_0 + \frac{\beta_1}{2\beta_0^2} \left(\gamma_E + \frac{l^2}{2\beta_0^2} \right) \\ &\quad - \frac{\beta_2^V \beta_0 - \beta_1^2}{\beta_0^3} \frac{\sqrt{\pi/2}}{l}, \end{aligned} \quad (2.15)$$

which fully fixes the parameters in the β -function and the charge in terms of the scale A and inclination α'_R .

2.1.5 Choice of scales. Like above, we assume the inclination of Regge trajectories, which determines the linear part of the potential, to be $\alpha'_R = 1.04 \text{ GeV}^{-2}$. Making use of the measured QCD coupling constant [23] and assuming

$$\alpha_s^{\overline{\text{MS}}}(m_Z^2) = 0.123$$

as the main parameter of the potential, we obtain the value $\alpha_V(m_Z^2) \approx 0.1306$ which can be considered the normalization condition for $\hat{\alpha}(m_Z^2) = \alpha_V(m_Z^2)/(4\pi)$. Next we calculate the parameter A for the effective charge, depending on the number of active flavors [24].

Upon determining the momentum dependence of charge, with the aid of a Fourier transformation we find

$$V(r) = kr - \frac{8C_F}{r} u(r) \quad (2.16)$$

with the function

$$u(r) = \int_0^\infty dq \frac{1}{q} \left(\hat{\alpha}(q^2) - \frac{K}{q^2} \right) \sin(qr),$$

which is computed numerically at $r > 0.01 \text{ fm}$. At small distances, the behavior of the potential is purely perturbative, so that at $r < 0.01 \text{ fm}$ in accordance with formula (2.6) we have

$$V(r) = -C_F \frac{\bar{\alpha}_V(1/r^2)}{r}.$$

Here, the running constant $\bar{\alpha}_V(1/r^2)$ is determined by equation (2.12) with the respective value of β_2^V at $n_f = 5$ and is normalized by the matching condition with potential (2.16) at $r_s = 0.01 \text{ fm}$. As a result we obtain the value $\bar{\alpha}_V(1/r_s^2) = 0.22213$, which yields the value $A_{n_f=5}^{\overline{V}} = 617.42 \text{ MeV}$.

Thus, we have fully defined the model of the potential of static heavy quarks in QCD with a three-loop running constant. In constructing the unique β -function and the potential, the confinement mode was introduced in a phenomenological manner, and it is, naturally, not a result of evolution which, nevertheless, at virtualities $q < 3 \text{ GeV}$ starts to be sensitive to the corrections introduced owing to confinement. As a result we obtain a physically motivated and phenomenologically admissible parametrization of the potential both at small and large distances.

Figure 5 shows the potential versus the distance between the quarks. From the figure one can see that the computed potential is close in shape to the potential of the Cornell model determined phenomenologically by fitting the mass spectra of heavy quarkonia. As a result, the conclusion can be made that normalization of the QCD coupling constant at the virtuality $q^2 = m_Z^2$ by three-loop evolution of the effective charge, taking account of the linear quark confinement, yields a static potential that is consistent with phenomenological models, i.e. with calculations of mass spectra of heavy quarkonia in the nonrelativistic approximation.

Matching the potential with the QCD parameters became possible owing to two-loop consideration of the Coulomb potential in the static limit, leading to significant corrections to the β -function of the effective charge, so $\Delta\beta/\beta \sim 10\%$. Such a correction is important for determination of the critical charge determining the transition region between the perturbation theory mode and the nonperturbative limit.

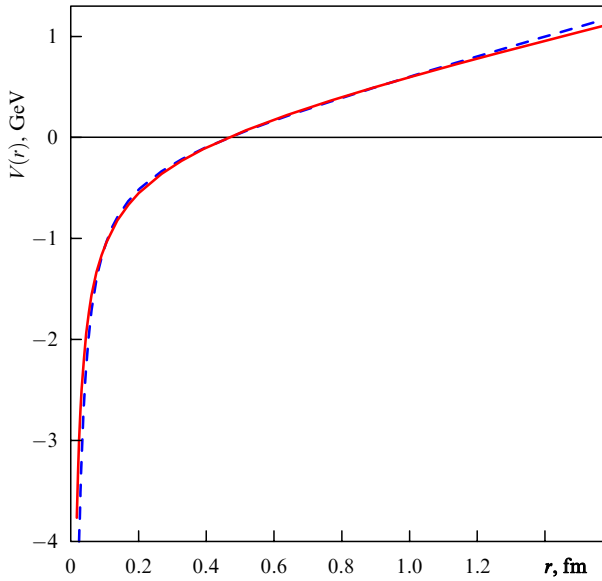


Figure 5. Potential of static heavy quarks in QCD (solid line) compared to the Cornell model (dashed line) with an accuracy up to an additive shift along the energy scale.

Moreover, the two-loop matching condition and the three-loop evolution of the running coupling constant normalized in accordance with data on m_Z at high energies fixes the range of energy scales where the change of these modes occurs. This scale is strongly correlated with data on the mass spectra of heavy quarkonia. Indeed, it is connected to the splitting between levels $1S$ and $2S$. We stress that the two-loop modification leads to correct normalization of the effective Coulomb charge at interquark distances characteristic of the average-sized heavy quarkonia, and it determines the evolution at small distances ($r < 0.08$ fm), which is essential for calculating lepton constants related to the wave functions at zero point.

A detailed analysis of the static quark potential in computations of the mass spectra of heavy quarkonia and the lepton constants of vector states, as well as a determination of heavy quark masses within the potential approach, are presented in Ref. [24]. We note that the Buchmüller–Tye potential was obtained by fitting experimental mass spectra of heavy quarkonia and is numerically very close to the static potential obtained in the present review and made consistent with the normalization of the QCD coupling constant at large virtualities. Thus, the Buchmüller–Tye potential retains its phenomenological applicability for calculating the mass levels of hadrons containing c and b quarks with an error of up to 40–70 MeV, which represents the typical uncertainty of the potential approach.

2.1.6 Level system. Following Ref. [26] we use the Buchmüller–Tye potential as the model potential in which account is taken of Coulomb-like corrections in the region of small distances with a running QCD constant in the two-loop approximation, while at large distances a linear increase of the interaction energy occurs that leads to confinement. Both modes therewith represent limiting cases for the effective Gell-Mann–Low model β -function that is given explicitly. In the antitriplet state it is necessary to take into account the numerical factor $1/2$ due to the color structure of the bound quark–quark state. The corresponding factor in the interac-

tion of the diquark with the light constituent quark is unity. We note that, as is shown in Ref. [27], the nonperturbative constituent addition to the nonrelativistic quark mass is precisely identical to the additive constant subtracted from the Coulomb-like potential.

Thus, by fitting the model to the real spectrum of charmonium and bottomonium one can reveal the heavy quark masses:

$$m_c = 1.486 \text{ GeV}, \quad m_b = 4.88 \text{ GeV}. \quad (2.17)$$

The mass of the heavy quarkonium level (for example, of charmonium) is calculated in accordance with the relation $M(c\bar{c}) = 2m_c + E$, where E is the energy of the static solution to the Schrödinger equation with the model potential V . The mass of the meson with one heavy quark is given by the expression $M(Q\bar{q}) = m_Q + m_q + E$, where already $E = \langle T \rangle + \langle V - \delta V \rangle$, and the additive supplement to the potential is introduced, because the constituent mass is determined as a part of the interaction field energy: $\delta V = m_q$, where in accordance with the fitting for the heavy meson masses $m_q = 0.385$ GeV.

The calculated results for the energy eigenvalues in the Schrödinger equation with the Buchmüller–Tye potential for various systems are presented in Table 1, while the characteristics of the respective wave functions are in Table 2. Here, the *binding energy and the wave function of the light quark are with a good precision practically independent of the heavy quark flavors*, since the large diquark mass gives a small contribution to the reduced mass of the system and results in insignificant corrections to the Schrödinger equation.

As a result, the energy levels of the light constituent quark for states beneath the decay threshold of the doubly heavy baryon into a heavy baryon and a heavy meson are as

Table 1. Spectra of bb , bc and cc diquark levels without spin splitting taken into account.

Diquark level	M , GeV	$\langle r^2 \rangle^{1/2}$, fm	Diquark level	M , GeV	$\langle r^2 \rangle^{1/2}$, fm
bb diquark					
$1S$	9.74	0.33	$2P$	9.95	0.54
$2S$	10.02	0.69	$3P$	10.15	0.86
$3S$	10.22	1.06	$4P$	10.31	1.14
$4S$	10.37	1.26	$5P$	10.45	1.39
$5S$	10.50	1.50	$6P$	10.58	1.61
$3D$	10.08	0.72	$4D$	10.25	1.01
$5D$	10.39	1.28	$6D$	10.53	1.51
$4F$	10.19	0.87	$5F$	10.34	1.15
$6F$	10.47	1.40	$5G$	10.28	1.01
$6G$	10.42	1.28	$6H$	10.37	1.15
bc diquark					
$1S$	6.48	0.48	$3P$	6.93	1.16
$2S$	6.79	0.95	$4P$	7.13	1.51
$3S$	7.01	1.33	$3D$	6.85	0.96
$2P$	6.69	0.74	$4D$	7.05	1.35
$4F$	6.97	1.16	$5F$	7.16	1.52
$5G$	7.09	1.34	$6H$	7.19	1.50
cc diquark					
$1S$	3.16	0.58	$3P$	3.66	1.36
$2S$	3.50	1.12	$4P$	3.90	1.86
$3S$	3.76	1.58	$3D$	3.56	1.13
$2P$	3.39	0.88	$4D$	3.80	1.59

Table 2. Characteristics of radial wave functions of bb, bc and cc diquarks.

nL	$R(0), \text{GeV}^{3/2}$	nL	$R'(0), \text{GeV}^{5/2}$
bb diquark			
1S	1.346	2P	0.479
2S	1.027	3P	0.539
3S	0.782	4P	0.585
4S	0.681	5P	0.343
bc diquark			
1S	0.726	2P	0.202
2S	0.601	3P	0.240
cc diquark			
1S	0.530	2P	0.128
2S	0.452	3P	0.158

follows

$$E(1s) = 0.38 \text{ GeV}, \quad E(2s) = 1.09 \text{ GeV}, \quad E(2p) = 0.83 \text{ GeV},$$

where the energy of a level was determined as the sum of the constituent mass and the eigenenergy of the stationary Schrödinger equation. In the HQET theory one introduces $\bar{A} = E(1s)$. Hence it is possible to conclude that our estimate is in good agreement with the calculations done within other approaches. This once again testifies to the reliability of such a phenomenological prediction. For the respective radial wave functions and their derivatives at zero point we find

$$R_{1s}(0) = 0.527 \text{ GeV}^{3/2},$$

$$R_{2s}(0) = 0.278 \text{ GeV}^{3/2},$$

$$R'_{2p}(0) = 0.127 \text{ GeV}^{5/2}.$$

The analogous characteristics for the bound state of the system involving the c quark and bb diquark have the values $E(1s) = 1.42 \text{ GeV}$, $E(2s) = 1.99 \text{ GeV}$, $E(2p) = 1.84 \text{ GeV}$ with the appropriate wave functions

$$R_{1s}(0) = 1.41 \text{ GeV}^{3/2},$$

$$R_{2p}(0) = 1.07 \text{ GeV}^{3/2},$$

$$R'_{2d}(0) = 0.511 \text{ GeV}^{5/2}.$$

For the constituent strange quark binding energy we add the mass from the current, $m_s \approx 100 - 150 \text{ MeV}$.

2.1.7 Spin-dependent corrections. In accordance with works [28] we introduce two types of the spin-dependent corrections that provide for the splitting of nL -levels in the diquark and in the system involving the light constituent quark and the diquark ($n = n_r + L + 1$ is the principal quantum number, n_r is the radial excitation number, and L is the angular momentum).

For a heavy diquark with identical quarks one finds

$$\begin{aligned}
V_{sd}^{\text{diq}}(\mathbf{r}) = & \frac{1}{2} \frac{\mathbf{L}_{\text{diq}} \mathbf{S}_{\text{diq}}}{2m_Q^2} \left(-\frac{1}{r} \frac{dV(r)}{dr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right) \\
& + \frac{2}{3} \alpha_s \frac{1}{m_Q^2} \frac{1}{r^3} \mathbf{L}_{\text{diq}} \mathbf{S}_{\text{diq}} + \frac{4}{3} \alpha_s \frac{1}{3m_Q^2} \mathbf{S}_{Q1} \mathbf{S}_{Q2} [4\pi \delta(\mathbf{r})] \\
& - \frac{1}{3} \alpha_s \frac{1}{m_Q^2} \frac{1}{4\mathbf{L}_{\text{diq}}^2 - 3} \frac{1}{r^3} [6(\mathbf{L}_{\text{diq}} \mathbf{S}_{\text{diq}})^2 \\
& + 3(\mathbf{L}_{\text{diq}} \mathbf{S}_{\text{diq}}) - 2\mathbf{L}_{\text{diq}}^2 \mathbf{S}_{\text{diq}}^2], \quad (2.18)
\end{aligned}$$

where \mathbf{L}_{diq} and \mathbf{S}_{diq} are the angular momentum in the diquark system and the total spin of the quarks composing the diquark, respectively.

Taking into account the interaction with the light constituent quark yields ($\mathbf{S} = \mathbf{S}_{\text{diq}} + \mathbf{S}_l$)

$$\begin{aligned}
V_{sd}^1(\mathbf{r}) = & \frac{1}{4} \left(\frac{\mathbf{L} \mathbf{S}_{\text{diq}}}{2m_Q^2} + \frac{2\mathbf{L} \mathbf{S}_l}{2m_l^2} \right) \left(-\frac{1}{r} \frac{dV(r)}{dr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right) \\
& + \frac{1}{3} \alpha_s \frac{1}{m_Q m_l} \frac{1}{r^3} (\mathbf{L} \mathbf{S}_{\text{diq}} + 2\mathbf{L} \mathbf{S}_l) \\
& + \frac{4}{3} \alpha_s \frac{1}{3m_Q m_l} (\mathbf{S}_{\text{diq}} + \mathbf{L}_{\text{diq}}) \mathbf{S}_l [4\pi \delta(\mathbf{r})] \\
& - \frac{1}{3} \alpha_s \frac{1}{m_Q m_l} \frac{1}{4\mathbf{L}^2 - 3} \frac{1}{r^3} [6(\mathbf{L} \mathbf{S})^2 + 3(\mathbf{L} \mathbf{S}) - 2\mathbf{L}^2 \mathbf{S}^2 \\
& - 6(\mathbf{L} \mathbf{S}_{\text{diq}})^2 - 3(\mathbf{L} \mathbf{S}_{\text{diq}}) + 2\mathbf{L}^2 \mathbf{S}_{\text{diq}}^2]. \quad (2.19)
\end{aligned}$$

The first term in the last expression corresponds to the relativistic correction to the effective scalar exchange, while the subsequent terms are due to the corrections to the effective one-gluon exchange with the constant α_s .

We define the effective parameter α_s in the following way. The splitting in the S -wave heavy quarkonium $\bar{Q}_1 Q_2$ is given by the expression

$$\Delta M[nS] = \frac{8}{9} \alpha_s \frac{1}{m_1 m_2} |R_{nS}(0)|^2. \quad (2.20)$$

Here $R_{nS}(0)$ is the radial wave function of quarkonium at zero point. From experimental data in the $c\bar{c}$ system one obtains

$$\Delta M[1S] = 117 \pm 2 \text{ MeV} \quad (2.21)$$

and using $R_{1S}(0)$ calculated in the model we find $\alpha_s(\Psi)$.

Further, we take into consideration the dependence of the parameter α_s on the reduced mass μ_{red} of the system within the framework of the one-loop approximation for the running QCD constant, when

$$\alpha_s(p^2) = \frac{4\pi}{\beta_0 \ln(p^2/\Lambda_{\text{QCD}}^2)}, \quad (2.22)$$

with $\beta_0 = 11 - 2n_f/3$ and $n_f = 3$ for $p^2 < m_c^2$. From the phenomenology of potential models it is known that the mean kinetic energy of the motion of bound-state quarks is practically independent of the quark flavors and takes the values

$$\langle T_{\text{diq}} \rangle \approx 0.2 \text{ GeV}, \quad (2.23)$$

$$\langle T_l \rangle \approx 0.4 \text{ GeV} \quad (2.24)$$

for the antitriplet and singlet coupling, respectively. Substituting the definition of kinetic energy

$$\langle T \rangle = \frac{\langle p^2 \rangle}{2\mu_{\text{red}}} \quad (2.25)$$

into formula (2.22) we obtain

$$\alpha_s(p^2) = \frac{4\pi}{\beta_0 \ln(2\langle T \rangle \mu_{\text{red}}/\Lambda_{\text{QCD}}^2)}, \quad (2.26)$$

where according to Eqn (2.21) the numerical estimate gives $\Lambda_{\text{QCD}} \approx 113 \text{ MeV}$.

For identical quarks in the diquark we apply the computational scheme (well-known from heavy quarkonium) for finding corrections due to LS -coupling, while for interaction with the light quark we apply the jj -coupling scheme. In this case \mathbf{LS}_l is diagonal for given $\mathbf{J}_l = \mathbf{L} + \mathbf{S}_l$, $\mathbf{J} = \mathbf{J}_l + \mathbf{\bar{J}}$, where \mathbf{J} is the total baryon spin, and $\mathbf{\bar{J}} = \mathbf{S}_{\text{diq}} + \mathbf{L}_{\text{diq}}$ is the total momentum of the diquark.

Then, for estimating various contributions and mixing of states one can avail oneself of the basis transformations ($\mathbf{S} = \mathbf{S}_l + \mathbf{\bar{J}}$, $\mathbf{J}_{\text{diq}} = \mathbf{S}_{\text{diq}} + \mathbf{L}$):

$$|J; J_l\rangle = \sum_S (-1)^{\bar{J}+S_l+L+J} \sqrt{(2S+1)(2J_l+1)} \times \left\{ \begin{matrix} \bar{J} & S_l & S \\ L & J & J_l \end{matrix} \right\} |J; S\rangle, \quad (2.27)$$

$$|J; J_l\rangle = \sum_{J_{\text{diq}}} (-1)^{\bar{J}+S_l+L+J} \sqrt{(2J_{\text{diq}}+1)(2J_l+1)} \times \left\{ \begin{matrix} \bar{J} & L & J_{\text{diq}} \\ S_l & J & J_l \end{matrix} \right\} |J; J_{\text{diq}}\rangle. \quad (2.28)$$

Thus, we have examined in detail the procedure for calculating the mass spectra of doubly heavy baryons.

2.2 Numerical results

In this section the calculated results are presented for mass spectra with due account of level splitting dependent on the quark spin. As we explained above, doubly heavy baryons with identical heavy quarks permit a quite reliable interpretation in terms of excited-state quantum numbers of the diquark (total spin and angular momentum). For a baryon with a bc diquark, a calculation was performed of the spin splitting of the ground $1S$ state, since for higher excitations of this quark the allowable emission of a soft gluon violates the simple picture of level classification. It is clear that the quark–diquark model of bound states of doubly heavy baryons leads to the most reliable results in a system with a larger quark mass, i.e. in the Ξ_{bb} system.

When classifying the quantum numbers of energy levels we make use of the notation $n_{\text{diq}} L_{\text{diq}} n_l l$, i.e. we indicate the values of the principal quantum number in the diquark, angular momentum in the diquark, the principal quantum number of the excited light quark and its angular momentum. Level splitting of the Ξ_{bb} baryon was considered in detail in papers [26]. The states with total spin $J = 3/2$ (or $1/2$) may have different values of J_l and, thus, acquire nonzero mixing in calculations by perturbation theory based on states with a certain total momentum of the light constituent quark. For $J = 3/2$, the mixing matrix can be considered with high precision to be diagonal, and for $J = 1/2$ the mixing of states with differing total momenta of the light quark is strong.

The comparative analysis for levels $1S2p$ and $2S2p$ in the Ξ_{bb} system, carried out by Gershtein [26], revealed the difference in the wave functions due to the disparity between the masses of the diquark subsystem being actually insignificant. Splitting $\Delta^{(J_{\text{diq}})}$ of the diquark D - and G -levels is suppressed [$\Delta^{(J_{\text{diq}})} < 11 \text{ MeV}$], so that within the accuracy of the method employed such corrections for excitations of a diquark with dimensions inferior to the distance from the light quark (i.e. with a small principal quantum number) are negligible ($\delta M \approx 30 - 40 \text{ MeV}$).

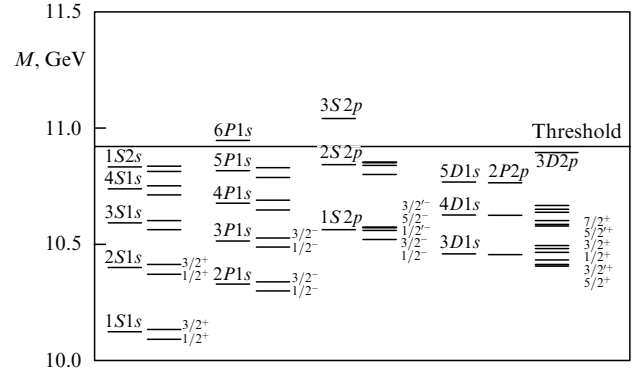


Figure 6. Mass spectrum for Ξ_{bb}^- and Ξ_{bb}^0 baryons with due account of the splitting of low-lying excitations, dependent upon the quark spins.

The hyperfine spin–spin splitting in the quark–diquark system is given by

$$\Delta_{\text{hf}}^1 = \frac{2}{9} \left[J(J+1) - \bar{J}(\bar{J}+1) - \frac{3}{4} \right] \alpha_s (2\mu_{\text{red}} T_l) \frac{1}{m_b m_l} |R_l(0)|^2, \quad (2.29)$$

where $R_l(0)$ is the radial wave function of the light constituent quark at zero point, and the diquark level shift equals

$$\Delta_{\text{hf}}^{\text{diq}} = \frac{1}{9} \alpha_s (2\mu_{\text{red}} T_{\text{diq}}) \frac{1}{m_b^2} |R_{\text{diq}}(0)|^2. \quad (2.30)$$

The mass spectrum of baryons Ξ_{bb}^- and Ξ_{bb}^0 is shown in Fig. 6, where we have restricted ourselves to presenting the S -, P - and D -wave levels, while a table containing the numerical values of the Ξ_{bb} -baryon masses is presented in Ref. [26]. From the figure one can see that the baryon masses $1S1s$ ($J^P = 3/2^+, 1/2^+$), $2P1s$ ($J^P = 3/2^-, 1/2^-$) and $3D1s$ ($J^P = 7/2^+, \dots, 1/2^+$) can be considered the most reliable predictions. Notice that the $2P1s$ level is metastable, since transition to the ground state requires the angular momentum and total spin of the heavy quarks in the diquark to change simultaneously.

The transition between ortho- and parahydrogen states in the H_2 molecule that takes place in the inhomogeneous external field created by the magnetic moments of other molecules can be considered an analog of such a process. For $2P1s \rightarrow 1S1s$, the part of the external field is assumed by the inhomogeneous chromomagnetic field of the light quark. The corresponding perturbation has the form

$$\begin{aligned} \delta V &\sim \frac{1}{m_Q} [\mathbf{S}_1 \mathbf{H}_1 + \mathbf{S}_2 \mathbf{H}_2 - (\mathbf{S}_1 + \mathbf{S}_2) \langle \mathbf{H} \rangle] \\ &= \frac{1}{2m_Q} (\mathbf{V} \mathbf{r}_{\text{diq}}) (\mathbf{S}_1 - \mathbf{S}_2) \mathbf{H} \sim \frac{1}{m_Q} \frac{\mathbf{r}_l \mathbf{r}_{\text{diq}}}{m_q r_l^5} (\mathbf{S}_1 - \mathbf{S}_2) \mathbf{J}_l f(r_l), \end{aligned}$$

where $f(r_l)$ is a dimensionless nonperturbative function depending on the coordinates of the light quark with respect to the diquark. Obviously, a perturbation δV will alter the angular momentum of the light quark and lead to mixing of states with identical J^P values. If the splitting is not small (for example, $2P1s - 1S2p$, where $\Delta E \sim \Lambda_{\text{QCD}}$), then mixing is suppressed:

$$\frac{\delta V}{\Delta E} \sim \frac{1}{m_Q m_q} \frac{r_{\text{diq}}}{r_l^4} \frac{1}{\Delta E} \ll 1.$$

Since the $1S2p$ admixture in the $2P1s$ state is small, the $2P1s$ levels are quasi-stationary, i.e. their hadron transitions (with the emission of π -mesons) to the ground state are suppressed, by the way, additionally owing to the small phase-space volume. Therefore, one should expect anomalously narrow resonances to arise in the spectra of $\Xi_{bb}\pi$ pairs owing to the decays of quasi-stationary states with $J^P = 3/2^-, 1/2^-$. The experimental detection of such energy levels would directly signify the existence of diquark excitations and would provide information on the character of the $f(r_1)$ dependence, i.e. on the origin of the inhomogeneous chromomagnetic field in the nonperturbative region.

Evidently, the $3D1s$ ($J^P = 7/2^+, 5/2^+$) states that undergo transition in the multipole QCD expansion to the ground state owing to quadrupole emission of a gluon ($E2$ transition involving hadronization $gq \rightarrow q'\pi$) are also quasi-stationary. As to the higher excitations, the $3P1s$ states, for example, are close to the $1S2p$ ($J^P = 3/2^-, 1/2^-$) levels, so even the contributions (suppressed by the inverse heavy quark mass and the small diquark size) of the operators altering the diquark angular momentum and spin can lead to significant mixing with an amplitude $\delta V_{nn'}/\Delta E_{nn'} \sim 1$. However, in our opinion it will only insignificantly shift the masses of the states. It is more important that a large $1S2p$ admixture in the $3P1s$ state renders the latter unstable with respect to transitions to the ground $1S1s$ state with emission of a gluon ($E1$ transition) which leads to decays, for instance, with the emission of π -mesons⁶ in the physical hadron spectrum. The level $1S2p$ ($J^P = 5/2^-$) possesses definite diquark and light quark quantum numbers, since there are no nearby levels with the same J^P values, but its transition width to the ground state with the emission of a π -meson is not suppressed in any manner and, most probably, is large ($\Gamma \approx 100$ MeV).

We also note that radiative processes of hadron transitions lead to various wave states depending on the P-parity of the baryons and on their spin:

$$\begin{aligned} 3/2^- &\rightarrow 3/2^+ (\pi \text{ in the } S\text{-wave}), \\ 3/2^- &\rightarrow 1/2^+ (\pi \text{ in the } D\text{-wave}), \\ 1/2^- &\rightarrow 3/2^+ (\pi \text{ in the } D\text{-wave}), \\ 1/2^- &\rightarrow 1/2^+ (\pi \text{ in the } S\text{-wave}). \end{aligned}$$

The D -wave transitions in the process are suppressed by the low baryon recoil momenta as compared to their masses. The width of the lowest-lying $J^P = 3/2^+$ state is fully determined by the radiative electromagnetic $M1$ transition to the ground state $J^P = 1/2^+$.

The calculation procedure described above yields the results presented in Table 3 for doubly charmed baryons.

As has already been noted, a heavy quark composed of quarks of different flavors is, most likely, unstable with respect to the emission of soft gluons, so that in the Fock state of a doubly heavy baryon there exists a significant nonperturbative admixture of configurations including gluons and a diquark with various values of its spin S_{diq} and angular momentum L_{diq} :

$$\begin{aligned} |B_{\text{bcq}}\rangle &= O_B |bc[\bar{3}_c, S_{\text{diq}}, L_{\text{diq}}], q\rangle \\ &+ H_1 |bc[\bar{3}_c, S_{\text{diq}} \pm 1, L_{\text{diq}}], g, q\rangle \\ &+ H_2 |bc[\bar{3}_c, S_{\text{diq}}, L_{\text{diq}} \pm 1], g, q\rangle + \dots, \end{aligned}$$

⁶ We recall that $\Xi_{QQ'}$ baryons are isodoublets.

Table 3. Mass spectra for Ξ_{cc}^{++} , Ξ_{cc}^+ , and Ω_{bbc}^0 baryons.

$nL\eta_1 h (J^P)$	$M, \text{ GeV}$	$nL\eta_1 h (J^P)$	$M, \text{ GeV}$
Ξ_{cc}^{++} and Ξ_{cc}^+ baryons			
$1S1s (1/2^+)$	3.478	$3P1s (1/2^-)$	3.972
$1S1s (3/2^+)$	3.61	$3D1s (3/2'^+)$	4.007
$2P1s (1/2^-)$	3.702	$1S2p (3/2'^-)$	4.034
$3D1s (5/2^+)$	3.781	$1S2p (3/2^-)$	4.039
$2S1s (1/2^+)$	3.812	$1S2p (5/2^-)$	4.047
$3D1s (3/2^+)$	3.83	$3D1s (5/2'^+)$	4.05
$2P1s (3/2^-)$	3.834	$1S2p (1/2'^-)$	4.052
$3D1s (1/2^+)$	3.875	$3S1s (1/2^+)$	4.072
$1S2p (1/2^-)$	3.927	$3D1s (7/2^+)$	4.089
$2S1s (3/2^+)$	3.944	$3P1s (3/2^-)$	4.104
Ω_{bbc}^0 baryon			
$1S1s (1/2^+)$	11.12	$3D1s (3/2'^+)$	11.52
$1S1s (3/2^+)$	11.18	$3D1s (5/2'^+)$	11.54
$2P1s (1/2^-)$	11.33	$1S2p (1/2^-)$	11.55
$2P1s (3/2^-)$	11.39	$3D1s (7/2^+)$	11.56
$2S1s (1/2^+)$	11.40	$1S2p (3/2'^-)$	11.58
$3D1s (5/2^+)$	11.42	$1S2p (3/2^-)$	11.58
$3D1s (3/2^+)$	11.44	$1S2p (1/2'^-)$	11.59
$3D1s (1/2^+)$	11.46	$1S2p (5/2^-)$	11.59
$2S1s (3/2^+)$	11.46	$3P1s (3/2^-)$	11.59
$3P1s (1/2^-)$	11.52	$3S1s (1/2^+)$	11.62

and the amplitudes H_1 and H_2 are not small in reference to O_B . In heavy quarkonium, the contributions from similar operators of states (octet-like in the color) are suppressed by the probability of nonrelativistic quarks emitting in the small volume determined by the size of the singlet quark – antiquark system. But here the soft gluon is only restricted by the ordinary confinement scale and no suppression exists.

In such a situation we think it is not quite justified to perform calculations of excited Ξ_{bc} -baryon masses by the above-described procedure. Therefore, we only present the result for the ground state $J^P = 1/2^+$:

$$M[\Xi'_{bc}] = 6.85 \text{ GeV}, \quad M[\Xi_{bc}] = 6.82 \text{ GeV},$$

where in the vector diquark the splitting dependent on spin is, in the case of interaction with the light constituent quark, determined by the standard contact interaction between the magnetic moments of two point systems. The diagram of baryon energy levels without the spin-dependent perturbation suppressed by the mass of the heavy quark is presented in papers [26].

2.2.1 Doubly heavy baryons $\Omega_{QQ'}$ with strangeness. In the leading approximation, the wave functions and excitation energies of a strange quark in the diquark field repeat with a good precision the characteristics of similar baryons with u and d quarks. Therefore, with a precision up to an additive upward mass shift equal to the mass due to the strange quark current, namely

$$m_s \approx M[D_s] - M[D] \approx M[B_s] - M[B] \approx 0.1 \text{ GeV},$$

the set of $\Omega_{QQ'}$ -baryon levels without account of the splitting dependent on the quark spins coincides with the set of levels of the $\Xi_{QQ'}$ baryons.

Further, the spin – spin splitting of the low-lying states of $\Omega_{QQ'}$ baryons for the levels $n_{\text{diq}} S_{\text{diq}} s$, $2P1s$ and $3D1s$ is smaller by 20 – 30% than in $\Xi_{QQ'}$ (the $m_{u,d}/m_s$ factor). Concerning the $1S2p$ level, one can repeat the calculation procedure

described above. Thus, in the Ω_{bb} baryon the mixing matrix of states with various values of the total momentum of the light constituent quark is practically diagonal. This means that the perturbation term

$$\frac{1}{4} \frac{2\mathbf{L}\mathbf{S}_1}{2m_1^2} \left(-\frac{1}{r} \frac{dV(r)}{dr} + \frac{8}{3} \alpha_s \frac{1}{r^3} \right)$$

dominates, so the splitting of the $1S2p$ level is determined by the factor $m_{u,d}^2/m_c^2$, i.e. is 40% smaller than in Ξ_{bb} , meaning very small. In the Ω_{cc} baryon, the m_s/m_c factor is not small, so that for the $1S2p$ level the mixing matrix of states with various values of the total light constituent quark momentum is non-diagonal, and the order of arrangement of the $1S2p$ spin states in Ω_{cc} may differ somewhat from that in Ξ_{cc} .

It is interesting to note the following peculiarity of $\Omega_{QQ'}$. The lowest-lying S - and P -excitations of the diquark, even with due account of the mixing of levels with various spins and angular momenta of the subsystems, are quasi-stationary with respect to decays via strong interaction, since the gluon emission is accompanied by its hadronization into K -mesons (transitions $\Omega_{QQ'} \rightarrow \Xi_{QQ'} + K$), while solitary emission of π -mesons is forbidden by isospin and strangeness conservation laws. The corresponding hadron transitions with kaons do not occur owing to the insufficient mass splitting between the $\Omega_{QQ'}$ and $\Xi_{QQ'}$ levels, and decays with the emission of pion pairs in the isosinglet state are either suppressed by the small phase-space volume or forbidden. Thus, radiative electromagnetic transitions to the ground state are the dominant decay modes of the low-lying $\Omega_{QQ'}$ excitations.

2.2.2 Ω_{bbc} baryons. Within the framework of the quark–diquark picture it is possible to construct a model of baryons with three heavy quarks: bbc . However, as revealed by calculations, the diquark dimensions are comparable to the rms distance to the charmed quark, so that the model assumption concerning the compact heavy diquark may, in this case, turn out to be insufficiently accurate for computing mass levels. As to the spin-dependent splitting, it is negligible for interactions inside the quark, as pointed out above. The spin–spin splitting of the vector diquark with a charmed quark is $\Delta(1s) = 33$ MeV, $\Delta(2s) = 18$ MeV, while the level shifts for the $1S2p$ splitting are small, so only for one of the $J^P = 1/2^-$ levels must one take into account the correction -33 MeV. The splitting in the $3D1s$ state is determined by the spin–spin interaction. The characteristics of charmed quark excitations in the model with the Buchmüller–Tye potential were presented above. As a result we obtain the energy level diagram of the Ω_{bbc} baryon, which is shown in Table 3.

Further, it must be noted that in a number of cases, owing to the small splitting between the levels, excitations of the Ω_{bbc} -baryon ground state may mix quite strongly with large amplitudes, but with small mass shifts: $3P1s-1S2p$ ($J^P = 1/2^-, 3/2^-$), $2S1s-3D1s$ ($J^P = 1/2^+, 3/2^+$). We consider quite reliable the predictions for the states $1S1s$ ($J^P = 1/2^+, 3/2^+$), $1S2p$ ($J^P = 5/2^-$), and $3D1s$ ($J^P = 5/2^+, 7/2^+$). It is for these excitations that one can perform quite precise calculations of the radiation widths of electromagnetic transitions to the ground state in the multipole expansion.

The transition widths with the participation of mixed states are to a significant degree determined by the amplitudes of admixtures, which may exhibit essential model dependence. In this connection, experimental investigation of electromagnetic transitions in the Ω_{bbc} -baryon family

could provide valuable information on the mixing mechanism of various levels in baryon systems. Notice that electromagnetic transitions together with the emission of pion pairs (if the latter processes are not forbidden by the phase space) make up the total widths of the Ω_{bbc}^0 excited levels. Characteristic total widths may, most likely, be considered to reside at the level of $\Gamma \approx 10-100$ keV. Thus, the Ω_{bbc}^0 system can be defined with a large number of narrow quasi-stationary states.

2.3 Discussion

We have performed a detailed calculation of the spectroscopic characteristics of baryons with two heavy quarks in the model of quark–diquark factorization of wave functions within the nonrelativistic model of constituent quarks with the Buchmüller–Tye potential and outlined the field of application of such approximations. We have taken into account relativistic quark spin-dependent corrections to the potential in the diquark and light quark–diquark subsystems: below the threshold of hadron decay into a heavy baryon and a heavy meson with one heavy quark one may observe a set of excited bound states that are quasi-stationary with respect to hadron transitions to the ground state. We have dealt in detail with the physical principles for quasi-stationarity occurring for baryons with two identical quarks, since together with the Pauli exclusion principle the contributions from operators responsible for hadron decays and level mixing are suppressed by the inverse heavy quark mass and by the small size of the diquark. This suppression is due to the necessity for the spin and angular momentum of the compact diquark to change simultaneously. In baryon systems with two heavy quarks and a strange quark, the quasi-stationary character of lower diquark excitations is provided for by the absence of transitions both with emission of a single kaon owing to the small level splitting and with emission of a single pion owing to isospin and strangeness conservation. The characteristics of the wave functions can be used in calculating doubly heavy baryon production cross sections in the quark–diquark approximation.

Notice that quark–diquark factorization for calculating the masses of the ground-state levels of a baryon system with two heavy quarks has also been considered in Ref. [29] within the potential approach [30]. There exists a purely numerical difference in the choice of heavy quark masses that results in the mass of the doubly heavy diquark, obtained in Ref. [29], being approximately 100 MeV higher than in the calculations presented above. This difference is decisive in the divergence of mass estimates for the ground states in the present review and in Ref. [29]. In our opinion this is due to the employment of the Cornell potential with a constant value for the effective Coulomb exchange constant contrary to our study with a running constant.

Moreover, in the potential approach the masses of heavy quarks depend on a possible additive shift in the potential, which in phenomenological models is chosen, for example, by comparing the lepton constants of quarkonia in the model with known experimental values. In the QCD-motivated potential no such arbitrariness in the additive shift in the potential exists, so that the estimates of heavy quark masses contain fewer uncertainties.

It should also be noted that in the Cornell model the lepton constants were calculated by taking into account the one-loop correction due to hard gluons that is quite significant, especially for charmed quarks, and therefore

two-loop corrections turn out to be important in dealing with lepton constants in the potential approach [24]. In Ref. [29], the constituent mass of the light quark and the potential shift were considered uncorrelated, while we assumed the constituent mass to be part of the nonperturbative energy in the potential. This may lead to an additional discrepancy of the order of 50 MeV in the estimates of baryon masses.

With due account of the above comments concerning the methodical differences one can assert that the mass estimates for the ground states of baryons with two heavy quarks, presented in Ref. [29] and in our approach, are in fairly good agreement (Table 4).

Table 4. Masses (in GeV) of ground states of baryons with two heavy quarks in various approaches.

Baryon	*	[31]	[29]	[32]	[33]	[34]	[35]
Ξ_{cc}	3.48	3.74	3.66	3.66	3.61	3.65	3.71
Ξ_{cc}^*	3.61	3.86	3.81	3.74	3.68	3.73	3.79
Ω_{cc}	3.59	3.76	3.76	3.74	3.71	3.75	3.89
Ω_{cc}^*	3.69	3.90	3.89	3.82	3.76	3.83	3.91
Ξ_{bb}	10.09	10.30	10.23	10.34	—	—	10.43
Ξ_{bb}^*	10.13	10.34	10.28	10.37	—	—	10.48
Ω_{bb}	10.18	10.34	10.32	10.37	—	—	10.59
Ω_{bb}^*	10.20	10.38	10.36	10.40	—	—	10.62
Ξ_{cb}	6.82	7.01	6.95	7.04	—	—	7.08
Ξ_{cb}'	6.85	7.07	7.00	6.99	—	—	7.10
Ξ_{cb}^*	6.90	7.10	7.02	7.06	—	—	7.13
Ω_{cb}	6.91	7.05	7.05	7.09	—	—	7.23
Ω_{cb}'	6.93	7.11	7.09	7.06	—	—	7.24
Ω_{cb}^*	6.99	7.13	7.11	7.12	—	—	7.27

The asterisk indicates results obtained by the authors of the present review. The uncertainty in the predictions, when the parameters of models are varied, amounts to 30–50 MeV; the uncertainty due to the methodical aspect is discussed in the text.

In Ref. [31], by complete analogy with Ref. [29], an analysis of relativistic spin-dependent corrections has been made within the framework of the quasi-potential approach, in which the exaggerated, in our opinion, estimate of the heavy diquark mass from Ref. [29] was used. Regretfully, the description of the calculations contains a clear error: the parameter setting the relative contribution of the scalar and vector potentials, and the anomalous chromomagnetic moment of the heavy quark were denoted by the same symbol, which leads to confusion and to numerical errors, whereas in paper [29] these characteristics are shown to possess different values. This introduces additional uncertainty in the estimates [31] at the level of 100 MeV, so that the results by Tong et al. [31] can be considered not to contradict our study (see Table 4).

Estimates based on the pair interaction hypothesis were given in Ref. [32]. In the light of the discussion at the beginning of this section it is not surprising that a difference of 200–300 MeV may be mainly due to the different character of interquark forces in a doubly heavy baryon, although the uncertainty in the heavy quark masses is also important. In Ref. [33], simple arguments were applied basing on HQET with a heavy diquark: the estimate depends on the mass of the diquark (composed of two heavy quarks) assumed in the model. Here, if one neglects the binding energy in the diquark, which is clearly related to the heavy quark masses, the mass estimates for the ground states, presented in Table 4, are obtained.

Finally, in Ref. [35] the analysis made in Ref. [36] is modified on the basis of interpolation formulae for the mass

of a bound state with due account of the dependence of spin forces on the wave functions and effective coupling constant, which are varied in accordance with the quark composition of the hadron. Here, the fitting shape contains a parameter of the additive energy shift, which changes significantly when the transition takes place from mesons to baryons: $\delta_\mu \approx 80 \text{ MeV} \rightarrow \delta_B \approx 210 \text{ MeV}$. The shift in energy provides for excellent agreement of the fitting with the heavy meson and baryon masses obtained experimentally. However, if one considers that in the character of its strong interactions a doubly heavy baryon is rather similar to a meson with a local diquark source, then a shift in the binding energy should have been used in the heavy mesons and not in the heavy baryons, where the existence of a system of two light quarks obviously leads to a significant distinction in the calculated masses of bound states, namely, to a shift in the binding energy differing from the meson case. Such a substitution of the fitting parameters would lead to a much better agreement between the results by Kaur and Khanna [35] and our findings (see Table 4).

Summing up, it can be said that the uncertainty in the heavy quark masses is decisive in calculations of the masses of doubly heavy baryons within the framework of the potential approach. The analysis of a potential with a running coupling constant at small distances and a linear nonperturbative term confining quarks at large distances, made by the authors in the QCD-motivated model and adjusted on systems with heavy quarks, yields the most reliable predictions.

Significant interest is presented by the new field of studies — radiative (electromagnetic, as well as hadronic) transitions between quasi-stationary states in the families of baryons with two heavy quarks. The first step in the investigation of this problem has been made by Dai et al. in paper [37], where preliminary qualitative results were obtained on the electromagnetic transitions between levels of the Ξ_{bc} baryon.

3. Nonrelativistic QCD sum rules: two-point correlators

Within the framework of potential models, a description was presented in Section 2 of the families of baryons with two heavy quarks, containing a set of narrow excited states (in addition to the ground states): their mass spectrum is similar to the set of levels of heavy quarkonium. With the aid of the method of QCD sum rules [7] for two-point correlators of baryon currents, calculations were performed in Ref. [38] of the masses and structure constants of baryons with two heavy quarks. The analysis presented in Ref. [38], however, exhibits a number of faults related to the instability of sum rules in the region of parameters determining baryon currents, which leads to quite large uncertainties in the calculated results.

In this section we examine the NRQCD sum rules for two-point current correlators corresponding to baryons with two heavy quarks. The major physical argument for such a study consists in the nonrelativistic motion of heavy quarks in a small-sized diquark interacting with the light quark. This leads to quite definite expressions for the structure of baryon currents written in terms of nonrelativistic heavy quarks. In the leading order in the inverse heavy quark mass and relative velocity of the heavy quarks inside the diquark it is necessary within the NRQCD sum rule to take into account hard gluon corrections for deriving the relation between nonrelativistic correlators of heavy quarks and the correlators in complete QCD. The corresponding anomalous dimensionalities of

baryon currents in the two-loop approximation were calculated in Ref. [39].

The structure of NRQCD currents correspond to a fixed choice of parameters in the expressions of complete QCD, the values of which fall in the instability region revealed in the analysis made previously [38]. We have found a simple physical reason for the loss of stability in this case: depending on the parameters of the sum rules (the Borel variable or the number of the spectral density moment) the behavior of the quantities is determined by the presence of a doubly heavy diquark inside the baryon and, as a consequence, by the difference in masses between the baryon and the diquark. The mass difference plays a dominant part, if one does not take into consideration the corrections related to the nonperturbative interaction of the doubly heavy diquark and the light quark inside the baryon.

The involvement in the study of such an interaction in the NRQCD sum rules is related to nonperturbative condensates due to operators of higher dimensions. Stability of the sum rules can be achieved by taking into account the product of quark and gluon condensates in addition to the quark, gluon and mixed condensates. This product was dropped from the analysis within the framework of complete QCD. Moreover, it is necessary to accurately take into consideration Coulomb α_s/v -corrections inside the heavy diquark, which enhance the relative contribution from the perturbative part with respect to the condensates in the correlators calculated. Then we perform comparative analysis of the sum rules for baryons with a strange and a light massless quark.

Currents are determined and spectral densities within NRQCD sum rules are computed with due account of various operators in Section 3.1. Section 3.2 deals with numerical estimates. The masses of the ground states are obtained, the values of which are close to those calculated in potential models. The results obtained are briefly summed up in Section 3.3.

3.1 Sum rules for baryons with two heavy quarks

3.1.1 Baryon currents. Currents of Ξ_{cc}^\diamond , Ξ_{bb}^\diamond , and Ξ_{bc}^\diamond baryons with two heavy quarks, where the symbol ‘ \diamond ’ denotes the baryon electric charge dependent on the light quark flavor, correspond to the quantum numbers of spin and parity $J_{\text{diq}}^P = 1^+$ and $J_{\text{diq}}^P = 0^+$ for the heavy diquark system with a flavor matrix of symmetric and antisymmetric structure, respectively. Adding a light quark to the system of heavy quarks yields $J^P = 1/2^+$ for the Ξ_{bc}^\diamond baryons as well as a pair of degenerate states $J^P = 1/2^+$ and $J^P = 3/2^+$ for the Ξ_{cc}^\diamond , Ξ_{bb}^\diamond , and $\Xi_{cc}^{*\diamond}$, $\Xi_{bb}^{*\diamond}$, $\Xi_{bc}^{*\diamond}$, $\Xi_{bb}^{*\diamond}$ baryons.

Usually, the current structure of baryons with two heavy quarks is written in the form

$$J[\Xi_{QQ}] = [Q^{iT} \hat{C} \tau Q^j] \Gamma' q^k \varepsilon_{ijk}. \quad (3.1)$$

Here, T denotes transposition, \hat{C} is the charge conjugation matrix with the properties $\hat{C} \gamma_\mu^T \hat{C}^{-1} = -\gamma_\mu$ and $\hat{C} \gamma_5^T \hat{C}^{-1} = \gamma_5$, τ is a matrix in flavor space, and i, j, k are color indices. The effective static field of the heavy quark is denoted by the symbol Q . In the leading order in both the relative velocity of heavy quarks and in their inverse masses, the field Q contains only a ‘large’ component of the Dirac spinor in the hadron rest frame.

Unlike the case of baryons with a heavy quark [40], there exists a single independent current component J for the

ground state of each baryon current:

$$\begin{aligned} J[\Xi_{QQ'}^\diamond] &= [Q^{iT} \hat{C} \tau \gamma_5 Q^{j'}] q^k \varepsilon_{ijk}, \\ J[\Xi_{QQ}^\diamond] &= [Q^{iT} \hat{C} \tau \gamma^m Q^j] \gamma_m \gamma_5 q^k \varepsilon_{ijk}, \\ J^n[\Xi_{QQ}^{*\diamond}] &= [Q^{iT} \hat{C} \tau \gamma^n Q^j] q^k \varepsilon_{ijk} + \frac{1}{3} \gamma^n [Q^{iT} \hat{C} \gamma^m Q^j] \gamma_m q^k \varepsilon_{ijk}, \end{aligned} \quad (3.2)$$

where $J^n[\Xi_{QQ}^{*\diamond}]$ satisfies the equation for a particle of spin 3/2: $\gamma_n J^n[\Xi_{QQ}^{*\diamond}] = 0$. The flavor matrix τ is antisymmetric for Ξ_{bc}^\diamond , and symmetric for Ξ_{QQ}^\diamond , $\Xi_{QQ}^{*\diamond}$. The currents in equations (3.2) are written out in the hadron rest frame. The respective expressions in an arbitrary reference frame moving with a 4-velocity v^μ can be obtained by the substitution $\gamma^m \rightarrow \gamma^\mu = \gamma^\mu - \not{v} v^\mu$. Similar expressions can be written for doubly heavy baryons with a strange quark.

For comparison with the analysis in complete QCD we present the expression for the current $J[\Xi_{bc}^\diamond]$, obtained in Ref. [38]:

$$\begin{aligned} J[\Xi_{bc}^\diamond] &= \{r_1 [u^{iT} \hat{C} \gamma_5 c^j] b^k + r_2 [u^{iT} \hat{C} c^j] \gamma_5 b^k \\ &\quad + r_3 [u^{iT} \hat{C} \gamma_5 \gamma_\mu c^j] \gamma^\mu b^k\} \varepsilon_{ijk}, \end{aligned}$$

so that the current structure in NRQCD can be obtained by the following choice of parameters:

$$r_1 = r_2 = 1, \quad r_3 = 0$$

and antisymmetric permutation of c and b flavors. As has already been pointed out, the authors of Ref. [38] noted the ‘bad’ convergence of the operator expansion in the region of NRQCD parameters. This instability leads to large uncertainties in the results. To find the reason and to remove this defect, we further perform a detailed analysis of the NRQCD sum rules.

3.1.2 Description of the method. Let us define the procedure for calculating two-point correlators in the NRQCD approximation and their relation with the physical characteristics of baryons involving two heavy quarks. We shall start with the T-ordered correlator of two baryon currents⁷ of spin 1/2:

$$\begin{aligned} \Pi(w) &= i \int d^4x \langle 0 | T \{ J(x) \bar{J}(0) \} | 0 \rangle \exp(ipx) \\ &= \not{v} F_1(w) + F_2(w). \end{aligned} \quad (3.3)$$

Here w is determined by the relationships

$$p^2 = (\mathcal{M} + w)^2, \quad \mathcal{M} = m_Q + m_{Q'} + m_s,$$

where $m_{Q, Q'}$ are the heavy quark masses, and m_s is the mass of the strange quark. It is obvious that the correlators for the currents of baryons containing a light quark, instead of a strange quark, can be obtained if one sets $m_s \rightarrow m_{u,d} \approx 0$ in the expressions presented below. The spectral densities in the case of spin-3/2 baryons assume the form

$$\begin{aligned} \Pi_{\mu\nu}(w) &= i \int d^4x \langle 0 | T \{ J_\mu(x) \bar{J}_\nu(0) \} | 0 \rangle \exp(ipx) \\ &= -g_{\mu\nu} [\not{v} \tilde{F}_1(w) + \tilde{F}_2(w)] + \dots \end{aligned} \quad (3.4)$$

⁷ Below, to reduce the notation we do not explicitly indicate the quantum numbers of currents corresponding to the baryon composition.

Further we shall not examine the contribution from other Lorentzian structures for baryons of spin 3/2, since an analysis of the scalar correlation functions $F_{1,2}$ leads (under the conditions formulated below) to consistent results for the masses of bound states and for the coupling constants of currents with hadrons. Calculations for other scalar two-point correlation functions in the case of Lorentzian structures other than those presented above can, in our opinion, only lead to a repetition of the quite reliable results already obtained by us, although, truly, this could only enhance the reliability of estimates for the masses and constants.

The scalar correlators F can be calculated in the deep-Euclidean region by applying (within the framework of NRQCD) an operator expansion for the chronological product of baryon currents in equations (3.3) and (3.4). For example, one finds

$$F_{1,2}(w) = \sum_d C_d^{(1,2)}(w) O_d, \quad (3.5)$$

where O_d stands for the local operator of dimension d :

$$O_0 = \hat{1}, \quad O_3 = \langle \bar{q}q \rangle, \quad O_4 = \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \quad \dots,$$

and the functions $C_d(w)$ are the Wilson expansion coefficients of the corresponding operators.

In this review nonperturbative contributions are taken into account that are related to the quark, gluon and mixed condensates. The operator expansion for the correlator of two quark fields [41] was used in calculating the contribution from the quark condensate:

$$\begin{aligned} \langle 0 | T q_i^a(x) \bar{q}_j^b(0) | 0 \rangle \\ = -\frac{1}{12} \delta^{ab} \delta_{ij} \langle \bar{q}q \rangle \left[1 + \frac{m_0^2 x^2}{16} + \frac{\pi^2 x^4}{288} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + \dots \right], \end{aligned} \quad (3.6)$$

where the mixed condensate is parametrized by the variable m_0^2 , the numerical value of which is approximately 0.8 GeV^2 .

With due account of the nonzero strange quark mass it is possible to obtain the following expression with a precision up to terms of the fourth order in x within the framework of the operator expansion for the quark condensate [42]:

$$\begin{aligned} \langle 0 | T s_i^a(x) \bar{s}_j^b(0) | 0 \rangle &= -\frac{1}{12} \delta^{ab} \delta_{ij} \langle \bar{s}s \rangle \left\{ 1 + \frac{x^2}{16} (m_0^2 - 2m_s^2) \right. \\ &\quad \left. + \frac{x^4}{288} \left[\pi^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle - \frac{3}{2} m_s^2 (m_0^2 - m_s^2) \right] \right\} \\ &\quad + i m_s \delta^{ab} x_\mu \gamma_{ij}^\mu \langle \bar{s}s \rangle \left[\frac{1}{48} + \frac{x^2}{24^2} \left(\frac{3}{4} m_0^2 - m_s^2 \right) \right] \\ &= -\delta^{ab} \langle \bar{s}s \rangle (\mathcal{P}_0 \delta_{ij} + \mathcal{P}_1 x_\alpha \gamma_{ij}^\alpha + \mathcal{P}_2 \delta_{ij} x^2 \\ &\quad + \mathcal{P}_3 x_\alpha \gamma_{ij}^\alpha x^2 + \mathcal{P}_4 \delta_{ij} x^4). \end{aligned} \quad (3.7)$$

We note that for $m_s \neq 0$ expansion (3.7) gives a contribution to both correlators unlike the sum rules for doubly heavy baryons $\Xi_{QQ'}$ with a light quark [43], in which in the limit $m_s = 0$ and restricting the contribution from the local quark condensate one can obtain factorization of the diquark correlator into F_2 and of the total baryon correlator into F_1 . Such factorization led methodically to instability of estimates in the sum rules.

For calculating the Wilson coefficients of single and quark–gluon operators we apply the dispersion relations in w :

$$C_d(w) = \frac{1}{\pi} \int_0^\infty d\omega \frac{\rho_d}{\omega - w}, \quad (3.8)$$

where ρ_d stands for the imaginary part of the respective Wilson coefficient in the NRQCD physical region. Thus, the problem of calculating the Wilson coefficients of the operators we are dealing with reduces to the problem of computing the respective spectral densities.

For relating the NRQCD correlators and characteristics of the hadrons we apply the dispersion representation of the two-point function for the physical spectral density in the form of a sum of the resonance part and the continuous spectrum. The baryon structure constants are determined by the expressions

$$\begin{aligned} \langle 0 | J(x) | \Xi(p) \rangle &= i Z_\Xi u(v, M) \exp(ipx), \\ \langle 0 | J^m(x) | \Xi(p, \lambda) \rangle &= i Z_\Xi u^m(v, M) \exp(ipx), \end{aligned}$$

while the spinor field with the 4-velocity v and mass M satisfy the equation

$$\psi u(v, M) = u(v, M),$$

and $u^m(v, M)$ denotes the transverse spinor, so that

$$(\gamma^m - v^m \psi) u^m(v, M) = 0.$$

We assume the density of the continuous spectrum starting from the threshold ω_{cont} to coincide with the perturbative density calculated within the framework of NRQCD. Equating the expressions for correlators calculated within NRQCD and with the aid of physical spectral densities we cancel out the contributions due to integration from ω_{cont} to infinity in both parts of the equation. Actually, such a model of the continuum cannot be accurate, and the physical density of the continuum is not described by the theoretical approximation. This leads to a dependence of the masses and structure constants calculated above upon ω_{cont} .

It is not difficult to derive nonrelativistic expressions for the physical spectral functions

$$\rho_{1,2}^{\text{phys}} = \frac{M}{2\mathcal{M}} |Z|^2 \delta(\bar{A} - \omega), \quad (3.9)$$

which are obtained by the substitution

$$\delta(p^2 - M^2) \rightarrow \frac{1}{2\mathcal{M}} \delta(\bar{A} - w),$$

where \bar{A} stands for the binding energy in the baryon, and $M = \mathcal{M} + \bar{A}$. The nonrelativistic dispersion relation for the hadronic part of the sum rules has the form

$$\int dw \frac{\rho_{1,2}^{\text{phys}}}{\omega - w} = \frac{1}{2\mathcal{M}} \frac{|Z|^2}{\bar{A} - w}. \quad (3.10)$$

Let us write down the correlators in a region significantly lower than the threshold $w = -\mathcal{M} + t$ as $t \rightarrow 0$, which corresponds to the limit $p^2 \rightarrow 0$. In the approximation of a single bound state the sum rules bring to an expression that can be expanded in a power series of t . Thus, the sum rules

result in the coefficients of identical powers t being equal:

$$\frac{1}{\pi} \int_0^{\omega_{\text{cont}}} d\omega \frac{\rho_{1,2}}{(\omega + \mathcal{M})^n} = \frac{M}{2\mathcal{M}} \frac{|Z|^2}{M^n}. \quad (3.11)$$

Here ρ_j contains the contributions from various operators for the corresponding scalar correlators F_j .

Introducing for the n th moment of the two-point correlation function the notation

$$\mathfrak{M}_n = \frac{1}{\pi} \int_0^{\omega_{\text{cont}}} d\omega \frac{\rho}{(\omega + \mathcal{M})^{n+1}}, \quad (3.12)$$

we obtain for the baryon masses and the structure constant:

$$M(n) = \frac{\mathfrak{M}_n}{\mathfrak{M}_{n+1}}, \quad (3.13)$$

$$|Z(n)|^2 = \frac{2\mathcal{M}}{M} \mathfrak{M}_n M^{n+1}, \quad (3.14)$$

where the explicit dependence of the results derived from the sum rules on the parameters of the scheme (the number n of the moment) is shown. Therefore, it is necessary to find the region of parameters in which, first, the results are stable with respect to variation of n and, second, both the correlation functions F_1 and F_2 yield the same values for physical quantities: the masses and structure constants. The problem of the analysis made within the complete QCD was the presence of a significant difference between the baryon masses and constants calculated from different F .

3.1.3 Calculation of spectral densities. We now represent the analytical expressions for perturbative spectral functions in the NRQCD approximation. When calculating spectral densities we make use of the Cutkosky rules [44] with the modifications required by NRQCD. The jump of the two-point function is calculated with the aid of the following substitutions for the propagators of heavy and light quarks, respectively:

$$\frac{1}{p_0 - (m + \mathbf{p}^2/2m)} \rightarrow 2\pi i \delta(p_0 - (m + \mathbf{p}^2/2m)),$$

$$\frac{1}{p^2 - m^2} \rightarrow 2\pi i \delta(p^2 - m^2).$$

In the leading order of the theory covering the effective heavy quarks their spin interaction splits away, which owing to spin symmetry leads to the relationships⁸

$$\rho_1[\Omega(\Xi)] = 3\rho_1[\Omega'(\Xi')] = 3\rho_1[\Omega(\Xi)^*], \quad (3.15)$$

$$\rho_2[\Omega(\Xi)] = 3\rho_2[\Omega'(\Xi')] = 3\rho_2[\Omega(\Xi)^*], \quad (3.16)$$

and for the baryon coupling constants in NRQCD one finds:

$$|Z[\Omega(\Xi)]|^2 = 3|Z[\Omega'(\Xi)]|^2 = 3|Z[\Omega(\Xi)^*]|^2. \quad (3.17)$$

For the perturbative spectral densities ρ_1 and ρ_2 in front of the unity operator in F_1 and F_2 , respectively, we will make use of the smallness of the current mass of the strange quark as compared to the heavy quark masses. Expanding in powers of m_s we obtain the following expressions for a baryon with a

scalar diquark composed of heavy quarks of differing flavors:

$$\rho_1 = \frac{\sqrt{2}}{15015\pi^3} \frac{(\mu_{\text{red}}\omega)^{3/2}}{(\mathcal{M}_{\text{diq}} + \omega)^3} (\eta_{1,0} + m_s \eta_{1,1} + m_s^2 \eta_{1,2}), \quad (3.18)$$

where $\mu_{\text{red}} = m_Q m_{Q'}/(m_Q + m_{Q'})$ is the reduced mass of the diquark, $\mathcal{M}_{\text{diq}} = m_Q + m_{Q'}$, and the coefficients η of the spectral densities are given in Appendix 7.1⁹.

The first term of the expansion reproduces the results of Ref. [43] for zero mass of the light quark. For strange baryons with a scalar diquark, the perturbative density ρ_2 is proportional to m_s and is not zero:

$$\rho_2 = \frac{2\sqrt{2}}{105\pi^3} \frac{\omega(\mu_{\text{red}}\omega)^{3/2} m_s}{(\mathcal{M}_{\text{diq}} + \omega)^2} (\eta_{2,0} + m_s \eta_{2,1} + m_s^2 \eta_{2,2}). \quad (3.19)$$

In the leading approximation of perturbative NRQCD the correlators F_2 equal zero for the massless light quark. This is due to the absence of interaction between the light quark and the heavy diquark in the given order, so no mass term is present in the correlator.

The Coulomb interaction inside the diquark can be taken into account by introducing the Sommerfeld factor C for the diquark spectral density before integrating over the invariant diquark mass in obtaining the baryon spectral densities:

$$\rho_{\text{diq}}^C = \rho_{\text{diq}}^{\text{bare}} C, \quad (3.20)$$

and

$$C = \frac{2\pi\alpha_s}{3v} \left[1 - \exp\left(-\frac{2\pi\alpha_s}{3v}\right) \right]^{-1}. \quad (3.21)$$

Here, the antitriplet color structure of the diquark has been taken into account, and v denotes the relative velocity of the heavy quarks in the diquark:

$$v = \left[1 - \frac{4m_Q m_{Q'}}{Q^2 - (m_Q - m_{Q'})^2} \right]^{1/2}, \quad (3.22)$$

where Q^2 is the heavy diquark 4-momentum squared.

In NRQCD, a passage to the limit of small velocities is performed, so the square of the diquark invariant mass $Q^2 = (\mathcal{M}_{\text{diq}} + \epsilon)^2$ and

$$C = \frac{2\pi\alpha_s}{3v}, \quad v^2 = \frac{\epsilon}{2\mu_{\text{red}}} \quad \text{for } \epsilon \ll \mu_{\text{red}}.$$

The modified spectral densities are written down as

$$\rho_1^C = \frac{\mu_{\text{red}}^2 \alpha_s \omega (2\mathcal{M}_{\text{diq}} + \omega)}{6\pi^2 (\mathcal{M}_{\text{diq}} + \omega)^3} (\eta_{1,0}^C + m_s \eta_{1,1}^C + m_s^2 \eta_{1,2}^C). \quad (3.23)$$

The leading approximation yields the results for zero light quark mass [43]. For ρ_2^C we have

$$\rho_2^C = \frac{m_s \mu_{\text{red}}^2 \alpha_s \omega (2\mathcal{M}_{\text{diq}} + \omega)}{2\pi (\mathcal{M}_{\text{diq}} + \omega)^2} (\eta_{2,0}^C + m_s \eta_{2,1}^C + m_s^2 \eta_{2,2}^C). \quad (3.24)$$

Application of the expansion in the light quark mass leads to an insignificant deviation (about 0.5%) from the precise

⁸ In the case of heavy quarks of one flavor the spectral densities for baryons with a scalar diquark become zero.

⁹ The coefficients of spectral densities not written explicitly in the subsequent text are also given in Appendix 7.1.

integral representation, so one is quite justified in using the approximate expression with the first three terms of the expansion given in their explicit analytical form.

In a similar manner it is possible to obtain the spectral functions related to the condensates of light quarks and gluons. The moments of the coefficients of quark condensates are expressed in the form

$$\begin{aligned}\mathfrak{M}_{qq}^{(1)}(n) &= -\frac{(n+1)!}{n!} \mathcal{P}_1 \mathfrak{M}_{\text{diq}}(n+1) \\ &\quad + \frac{(n+3)!}{n!} \mathcal{P}_3 \mathfrak{M}_{\text{diq}}(n+3), \\ \mathfrak{M}_{qq}^{(2)}(n) &= \mathcal{P}_0 \mathfrak{M}_{\text{diq}}(n) - \frac{(n+2)!}{n!} \mathcal{P}_2 \mathfrak{M}_{\text{diq}}(n+2) \\ &\quad + \frac{(n+4)!}{n!} \mathcal{P}_4 \mathfrak{M}_{\text{diq}}(n+4).\end{aligned}\quad (3.25)$$

Here \mathcal{P}_i are the expansion coefficients determined in Eqn (3.7), and the n th moment of the diquark two-point correlation function $\mathfrak{M}_{\text{diq}}(n)$ is obtained by integration of the spectral density

$$\rho_{\text{diq}} = \frac{12\sqrt{2} \mu_{\text{red}}^{3/2} \sqrt{\omega}}{\pi}, \quad (3.26)$$

which has to be multiplied by the Sommerfeld factor C , where ω is substituted for ϵ , since in this case there is no integration over the invariant diquark mass. The modified density

$$\rho_{\text{diq}}^C = \frac{48\pi\alpha_s \mu_{\text{red}}^2}{3} \quad (3.27)$$

is independent of ω .

It is interesting to note that within the NRQCD approximation the condensate of light massless quarks contributes only to the correlators F_2 . This fact has a simple physical explanation: in the leading order the light quark operator can be factored out in the expression for the baryon current correlator. Indeed, for the contribution of the condensate one can write the expression

$$\begin{aligned}\langle 0 | T \{ J(x) \bar{J}(0) \} | 0 \rangle \\ \approx \frac{1}{12} \langle 0 | T q_i^a(x) \bar{q}_i^a(0) | 0 \rangle \langle 0 | T \{ J_{\text{diq}}^j(x) \bar{J}_{\text{diq}}^j(0) \} | 0 \rangle + \dots,\end{aligned}$$

where $J_{\text{diq}}^j(x)$ is the current of the diquark with the color index j , defined in Eqn (3.2).

Thus, taking into account only the first x -independent term in the expansion of the quark correlator in (3.6) leads to an independent contribution of the quark correlator to the baryon correlator. Since the diquark correlator in F_2 is isolated from the baryon form factor F_1 , the NRQCD sum rules in this approximation determine the diquark mass and structure constant from F_2 and estimate the baryon mass and structure constant from F_1 . These masses and structure constants differ from each other.

A positive point in this situation consists in the possibility of calculating the binding energy for baryons with two heavy quarks: $\bar{\Lambda} = M - \mathcal{M}_{\text{diq}}$, and a disadvantage is the instability of the results of NRQCD sum rules at this stage, since various form factors or correlators yield different results. In the complete QCD sum rules, variation of the parameters in the definitions of baryon currents brings to an admixture of the

diquark correlator in various form factors, so the results of estimation exhibit large uncertainties. For example, the characteristic uncertainty in calculations of the baryon mass in complete QCD is of the order of 300 MeV, i.e. the value is close to the expected value of $\bar{\Lambda}$. This result is not unexpected within the framework of NRQCD analysis. Moreover, it is evident that the introduction of an interaction between the light quark and the heavy diquark violates the factorization of the diquark correlator. Indeed, owing to the higher-order terms in expansion (3.6) the diquark factorization is violated explicitly, which provides for convergence of the estimates of masses and constants obtained from F_1 and F_2 . This fact is numerically demonstrated below.

The contribution to the spectral density moments, determined by the condensate of light quarks together with the mixed condensate and the product of quark and gluon condensates, can be calculated with the aid of the operator expansion (3.6):

$$\begin{aligned}\mathfrak{M}_n^{qq} &= \mathfrak{M}_n^{\langle \bar{q}q \rangle} - \frac{(n+2)!}{n!} \frac{m_0^2}{16} \mathfrak{M}_{n+2}^{\langle \bar{q}q \rangle} \\ &\quad + \frac{(n+4)!}{n!} \frac{\pi^2}{288} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \mathfrak{M}_{n+4}^{\langle \bar{q}q \rangle}.\end{aligned}\quad (3.28)$$

The corrections related to the gluon condensate have the following form for the operator $O_4 = \langle (\alpha_s/\pi) G^2 \rangle$:

$$\begin{aligned}\rho_1^{G^2} &= \frac{(m_Q^2 + m_{Q'}^2 + 11m_Q m_{Q'}) \mu_{\text{red}}^{5/2} \sqrt{\omega}}{21 \times 2^{10} \sqrt{2} \pi m_Q^2 m_{Q'}^2 (\mathcal{M}_{\text{diq}} + \omega)^2} \\ &\quad \times (\eta_{1,0}^{G^2} + m_s \eta_{1,1}^{G^2} + m_s^2 \eta_{1,2}^{G^2}).\end{aligned}\quad (3.29)$$

For the nonzero light quark mass one obtains a nonzero density $\rho_2^{G^2}$ proportional to m_s :

$$\rho_2^{G^2} = \frac{m_s(m_Q^2 + m_{Q'}^2 + 11m_Q m_{Q'}) \mu_{\text{red}}^{5/2} \sqrt{\omega}}{3 \times 2^9 \sqrt{2} \pi m_Q^2 m_{Q'}^2 (\mathcal{M}_{\text{diq}} + \omega)} (\eta_{2,0}^{G^2} + m_s \eta_{2,1}^{G^2}), \quad (3.30)$$

$$\eta_{2,0}^{G^2} = -(9\mathcal{M}_{\text{diq}} + \omega), \quad \eta_{2,1}^{G^2} = \frac{9\mathcal{M}_{\text{diq}} + \omega}{\mathcal{M}_{\text{diq}} + \omega}. \quad (3.31)$$

For the condensate product

$$\langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle,$$

where the gluon fields, unlike the light quark, are related to the heavy quarks, it is possible to calculate the two-point correlation functions directly:

$$F_2^{\bar{q}q G^2}(\omega) = -\frac{\mu_{\text{red}}^{5/2} (m_Q^2 + m_{Q'}^2 + 11m_Q m_{Q'})}{2^9 \sqrt{2} \pi m_Q m_{Q'} (-\omega)^{5/2}}, \quad F_1^{\bar{q}q G^2}(\omega) = 0, \quad (3.32)$$

since we restricted ourselves to examining operators of dimension not higher than 7, while the nonzero term in F_1 arises in the fifth order of expansion (3.7). This result is presented in a form permitting analytic continuation in $\omega = -\mathcal{M} + w$.

Thus, we have formulated the NRQCD sum rules in which account is taken of perturbative terms and vacuum averages of quark–gluon operators that include the contributions from condensates of light quarks, gluons, their

product and mixed condensate. We note that the condensate product plays an important part for baryons with two heavy quarks, and we have presented the complete expression within NRQCD for this term that includes the interaction of nonperturbative gluons both with light and with heavy quarks. We have correctly taken into account the Coulomb interaction for the perturbative spectral densities of the heavy diquark that is important for the interaction of nonrelativistic heavy quarks, and, finally, we have obtained the relationships of spin symmetry for the baryon structure constants in NRQCD:

$$|Z[\Xi]|^2 = 3|Z[\Xi']|^2 = 3|Z[\Xi^*]|^2.$$

3.1.4 Anomalous dimensions of baryon currents. For relating the correlators of NRQCD sum rules with the quantities in complete QCD it is necessary to take into consideration the anomalous dimensions of effective baryon currents with nonrelativistic quarks. Indeed, the following relation holds valid in the leading order of NRQCD:

$$J^{\text{QCD}} = \mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}}) J^{\text{NRQCD}},$$

where the coefficient $\mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}})$ depends on the scale of the normalization of μ_{soft} and satisfies the normalization condition at the point $\mu_{\text{hard}} = \mathcal{M}_{\text{diq}}$.

The anomalous dimensions of NRQCD currents in the leading order in the inverse heavy quark mass are independent of the diquark spin structure, so one obtains [39]

$$\begin{aligned} \gamma &= \frac{d \ln C_J(\alpha_s, \mu)}{d \ln(\mu)} = \sum_{m=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^m \gamma^{(m)}, \\ \gamma^{(1)} &= -2C_B(3a-3) + 3C_F(a-2), \\ \gamma^{(2)} &= \frac{1}{6} \{ -48(-2+6\zeta(2))C_B^2 \\ &\quad + C_A[(104-240\zeta(2))C_B - 101C_F] \\ &\quad - 64C_B n_f T_F + C_F(-9C_F + 52n_f T_F) \}, \end{aligned} \quad (3.33)$$

with

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad C_A = N_c, \quad C_B = \frac{N_c + 1}{2N_c},$$

$T_F = 1/2$ for $N_c = 3$, and n_f is the number of light quarks. In equations (3.33) the one-loop result involving the arbitrary gauge parameter a is presented, while the two-loop anomalous dimension is written in the Feynman gauge: $a = 1$. Numerically, for $n_f = 3$ and $a = 1$ the values desired are

$$\gamma^{(1)} = -4, \quad \gamma^{(2)} \approx -188.24. \quad (3.34)$$

In the leading logarithmic approximation with a one-loop precision the coefficient \mathcal{K}_J is given by the expression

$$\mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}}) = \left(\frac{\alpha_s(\mu_{\text{hard}})}{\alpha_s(\mu_{\text{soft}})} \right)^{\gamma^{(1)}/2\beta_0}, \quad (3.35)$$

and the coefficient of the β -function is expressed as

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f = 9.$$

For calculation of the two-loop contribution to \mathcal{K}_J , knowledge is required of the corrections in the next order in α_s in addition to the anomalous dimensions. At present, these corrections have not been calculated, so we will restrict ourselves to dealing with the one-loop precision.

Further, it is necessary to determine the normalization scale of μ_{soft} for NRQCD estimates, which is given by the average transfer moment in the doubly heavy diquark:

$$\mu_{\text{soft}}^2 = 2\mu_{\text{red}} T_{\text{diq}},$$

where T_{diq} is the kinetic energy in the system of two heavy quarks, which is known to be phenomenologically independent of the quark flavors and approximately equal to 0.2 GeV. Then, one finds

$$\mathcal{K}[\Omega(\Xi)_{\text{cc}}] \approx 1.95, \quad \mathcal{K}[\Omega(\Xi)_{\text{bc}}] \approx 1.52, \quad \mathcal{K}[\Omega(\Xi)_{\text{bb}}] \approx 1.30 \quad (3.36)$$

with a characteristic uncertainty of about 10%, depending on the variation of the initial and final points $\mu_{\text{hard, soft}}$.

Finally, we note that the values of \mathcal{K}_J do not affect the estimates of baryon masses, obtained within the NRQCD sum rules, although they are essential for calculating baryon structure constants that acquire these multiplicative factors.

3.2 Numerical estimates

In the present review the spectral functions of baryon current correlators are examined within the moment scheme for NRQCD sum rules. We note that the dominant uncertainty in this scheme is related to the variation of heavy quark masses. For the analysis, the following range of masses was chosen:

$$m_b = 4.6-4.7 \text{ GeV}, \quad m_c = 1.35-1.40 \text{ GeV}, \quad (3.37)$$

which is usually applied to estimating sum rules for heavy quarkonia. An important parameter is the QCD coupling constant determining the Coulomb interaction inside the heavy diquark. Indeed, it enters linearly into the diquark perturbative functions. Thus, introduction of α_s/v -corrections is important both for the baryon structure constants and for the relative contributions from the perturbative part and from the condensates to the baryon masses.

To reduce the uncertainty, we shall apply the same approach to heavy quarkonia, where it proved to work well, and determine the characteristic values of α_s for doubly heavy systems from comparison of the calculated results with data on the lepton constants of heavy quarkonia, known experimentally for $\bar{c}c$ and $\bar{b}b$ or calculated within various approaches for $\bar{b}c$. Calculations yield the following coupling constants for Coulomb interactions:

$$\alpha_s[\bar{b}b] = 0.37, \quad \alpha_s[\bar{c}b] = 0.45, \quad \alpha_s[\bar{c}c] = 0.60. \quad (3.38)$$

Since the square of the diquark size is twice that of heavy quarkonium composed of heavy quarks of the same flavor (see the dependence of mean relative momentum squared of the heavy quarks on kinetic energy), the effective Coulomb exchange constant should be recalculated in accordance with the QCD evolution equation. In the one-loop approximation one arrives at

$$\alpha_s[QQ'] = \frac{\alpha_s[\bar{Q}Q']}{1 - (9/4\pi)\alpha_s[\bar{Q}Q'] \ln 2},$$

so that

$$\alpha_s[bb] = 0.45, \quad \alpha_s[bc] = 0.58, \quad \alpha_s[cc] = 0.85. \quad (3.39)$$

As to the dependence of the results on the quark masses, it is necessary to stress that the pole masses in QCD perturbation theory are not consistently determined owing to infrared problems usually related to renormalon arbitrariness [45]. As a result, it is important to fix the definition of the heavy quark mass [46–48].

In the given order in α_s within the NRQCD sum rules we make use of the leading approximation of the quark loop with due account of Coulomb exchange between the heavy quarks. At this stage the heavy quark masses and the Coulomb exchange constant are strictly fixed by data on the lepton constants and on the charmonium and bottomonium masses in the sum rules with the same precision. The stability or convergence of the sum rules method applied to heavy quarkonia¹⁰ leads to the following quark masses:

$$m_c = 1.40 \pm 0.03 \text{ GeV}, \quad m_b = 4.60 \pm 0.02 \text{ GeV}$$

that are in good agreement with the heavy quark masses calculated within the scheme involving subtraction of infrared contributions: the potential subtracted mass $m_b^{\text{PS}} = 4.60 \pm 0.11 \text{ GeV}$, and the kinetic mass $m_b^{\text{kin}} = 4.56 \pm 0.06 \text{ GeV}$ were obtained within the QCD sum rules for bottomonium with a two-loop precision [46, 48]. The similar 1S-mass determined in Ref. [47] has a slightly higher value.

In the leading order in α_s , the kinetic and potential masses mentioned above set the threshold for the quark contribution and can be adopted as appropriately determined heavy quark masses in calculations of the characteristics of baryons with two heavy quarks. The mass values depend on the normalization point which was chosen to be in the 1–2 GeV region. However, the interval of admissible variations in the heavy quark masses was somewhat increased in calculations.

The sum rules for bottomonium and charmonium fix the Coulomb constants, since stability in the course of variation of the moment number fixes the mass of the heavy quark, while at the same time the lepton constant depends linearly on the respective constant α_s (Fig. 7). We note that the dependence of the Coulomb constant upon the quark composition of quarkonium is consistent with the renormgroup dependence on the dimensions of the system composed of two heavy

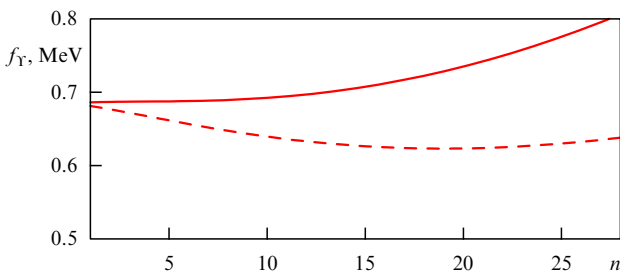


Figure 7. Lepton constant Υ in the two-point sum rules within the spectral density moment scheme for $m_b = 4.63 \text{ GeV}$ (dashed line) and $m_b = 4.59 \text{ GeV}$ (solid line).

¹⁰ We have required the ratio of the initial moments of spectral densities, calculated from data and within sum rules, to be stable.

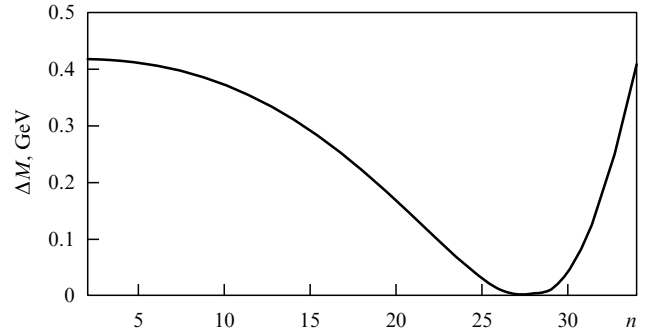


Figure 8. Mass difference for Ξ_{bc} baryons, obtained within the NRQCD sum rules for form factors F_1 and F_2 in the spectral density moment scheme.

quarks. The deviation of further estimates from conventional values of the Coulomb constants is at the level of 5%. As we already mentioned, the uncertainty due to the coefficients of matching NRQCD and complete QCD amounts to about 10% in the baryon coupling constants.

The dependence of estimates on the threshold for the continuous spectrum is not so strong as when the heavy quark masses are varied. We varied ω_{cont} within the interval

$$\omega_{\text{cont}} = 1.3 - 1.4 \text{ GeV}. \quad (3.40)$$

The range of values of quark and gluon condensates is restricted:

$$\begin{aligned} \langle \bar{q}q \rangle &= -(250 - 270)^3 \text{ MeV}^3, \\ m_0^2 &= 0.75 - 0.85 \text{ GeV}^2, \\ \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle &= (1.5 - 2) \times 10^{-2} \text{ GeV}^4. \end{aligned} \quad (3.41)$$

The main source of uncertainty in the ratio of baryon coupling constants is the ratio of quark condensates for strange and light quarks. We assume $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.2$, which corresponds to the admissible uncertainty obtained in the QCD sum rules for $m_u + m_d = 12 - 14 \text{ MeV}$ at a virtuality scale of 1 GeV [49]. The strange quark mass $m_s = 150 \pm 30 \text{ MeV}$, which is the conventional estimate consistent with the sum rules and quark current algebra, where only the quark current mass is involved. Thus, we have described the choice of parameters.

Figure 8 presents the calculated mass difference obtained with the aid of correlators¹¹ F_1 and F_2 for Ξ_{bc} baryons (we do not present the plots for Ξ_{cc} and Ξ_{bb} , since they qualitatively and quantitatively repeat the picture clearly seen in Fig. 8). From the figure one can distinctly see that at small moment numbers for the spectral densities the difference in masses between the baryon and the diquark is

$$\bar{\Lambda} = 0.40 \pm 0.03 \text{ GeV}, \quad (3.42)$$

which represents a result that is in good accordance with estimates for the case of heavy–light mesons.

¹¹ The gluon condensate value $\langle (\alpha_s/\pi) G^2 \rangle = 1.7 \times 10^{-2} \text{ GeV}^4$ was fixed in calculations, while the value of m_0^2 was chosen to be within the region given above, so as to provide a mass difference of zero, although it must be noted that variation of the parameters leads to the uncertainties indicated below.

In the stability region for the mass difference it is possible to fix the moment number of the spectral density (say, set $n = 27 \pm 1$ for Ξ_{bc}) and to calculate the respective baryon masses:

$$\begin{aligned} M[\Xi_{cc}] &= 3.47 \pm 0.05 \text{ GeV}, \\ M[\Xi_{bc}] &= 6.80 \pm 0.05 \text{ GeV}, \\ M[\Xi_{bb}] &= 10.07 \pm 0.09 \text{ GeV}. \end{aligned} \quad (3.43)$$

Here, the spin-dependent splitting given by the α_s -corrections to the interaction of the heavy diquark with the light quark was not taken into account, because the said corrections have not been calculated yet. The uncertainties in the mass values are mostly due to variation of the heavy quark masses. Thus, the stability of NRQCD sum rules permits one to improve the precision of estimates as compared to the analysis made within the complete QCD sum rules [38]. The values obtained are consistent with the results of calculations done within the framework of potential models (see Section 2).

In two-point sum rules for the Ω_{bc} mass (the derivations for other doubly heavy baryons are similar) one can see the stability of estimates with respect to the number of the spectral density moment for *both* the correlators F_1 and F_2 . This may be linked to the violation of factorization for the diquark and baryon correlators, mentioned above in the case of a massless light quark, already within the framework of the perturbative approximation unlike the case of the Ξ_{bc} baryon. The stability regions for F_1 and F_2 do not coincide, since the contributions from operators of higher dimensions become significant for various moment numbers. However, the quantity

$$\frac{1}{2} (M_1[n] + M_2[n])$$

exhibits a larger stability interval, and we shall make use of this fact in order to determine the masses of the $\Omega_{QQ'}$ and $\Xi_{QQ'}$ baryons (Fig. 9).

Thus, in the present review two stability criteria for baryon masses are examined: the first one deals with the difference between masses obtained from correlators F_1 and F_2 , while the second has to do with the half-sum of these masses. The second criterion is especially reliable for baryons with strangeness, since the stability regions of both correla-

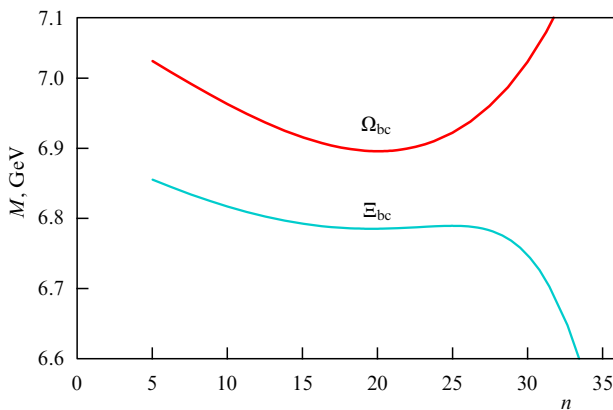


Figure 9. Masses of the Ξ_{bc} and Ω_{bc} baryons, obtained in the NRQCD sum rules when the results for the two correlators are averaged arithmetically.

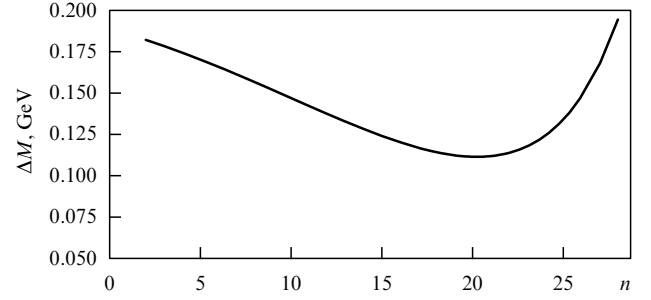


Figure 10. Mass difference $\Delta M = M_{\Omega_{bc}} - M_{\Xi_{bc}}$ obtained from the results presented in Fig. 9.

tors correspond to different moment numbers. Here, the mass difference for the two correlators within the stability regions determines the precision of estimate within the framework of QCD sum rules.

The following baryon masses were obtained in the second computing method:

$$\begin{aligned} M[\Omega_{cc}] &= 3.65 \pm 0.05 \text{ GeV}, & M[\Xi_{cc}] &= 3.55 \pm 0.06 \text{ GeV}, \\ M[\Omega_{bc}] &= 6.89 \pm 0.05 \text{ GeV}, & M[\Xi_{bc}] &= 6.79 \pm 0.06 \text{ GeV}, \\ M[\Omega_{bb}] &= 10.09 \pm 0.05 \text{ GeV}, & M[\Xi_{bb}] &= 10.00 \pm 0.06 \text{ GeV}. \end{aligned} \quad (3.44)$$

The estimates for the $\Xi_{QQ'}$ masses within methods (3.44) and (3.43) are in good agreement with each other.

We shall further examine the difference between the masses of doubly heavy baryons with and without strangeness (Fig. 10):

$$\frac{1}{2} [(M_1[\Omega_{bc}] + M_2[\Omega_{bc}]) - (M_1[\Xi_{bc}] + M_2[\Xi_{bc}])].$$

In such a scheme for determining baryon masses this quantity has the meaning of the average mass difference for which a broad stability interval is observed, which signifies that this method yields a good accuracy of the estimate:

$$\begin{aligned} \Delta M &= M[\Omega_{bb}] - M[\Xi_{bb}] = M[\Omega_{cc}] - M[\Xi_{cc}] \\ &= M[\Omega_{bc}] - M[\Xi_{bc}] = 100 \pm 30 \text{ MeV}. \end{aligned}$$

Figures 11 and 12 show the dependences of the baryon coupling constants in the moment scheme for NRQCD sum

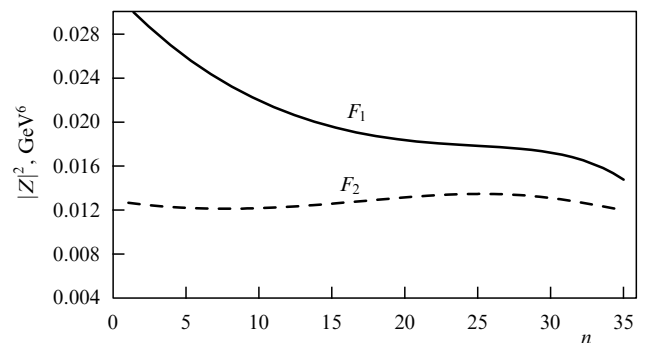


Figure 11. Coupling constants $|Z|^2$ for Ω_{bc} baryons, calculated within the NRQCD sum rules for correlators F_1 and F_2 in the spectral density moment scheme.

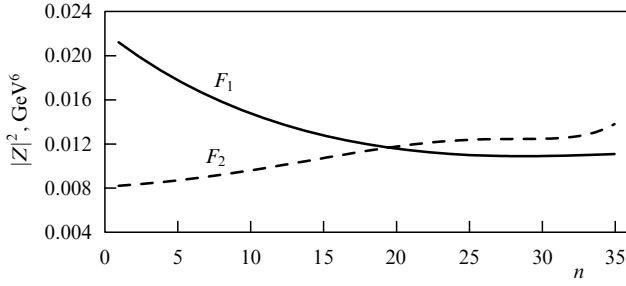


Figure 12. Coupling constants $|Z|^2$ for Ξ_{bc} baryons, calculated within the NRQCD sum rules for correlators F_1 and F_2 in the spectral density moment scheme.

rules for baryons with and without strangeness, respectively. Numerically we find

$$\begin{aligned} |Z[\Omega_{cc}]|^2 &= (10.0 \pm 1.2) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Xi_{cc}]|^2 &= (7.2 \pm 0.8) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Omega_{bc}]|^2 &= (15.6 \pm 1.6) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Xi_{bc}]|^2 &= (11.6 \pm 1.0) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Omega_{bb}]|^2 &= (6.0 \pm 0.8) \times 10^{-2} \text{ GeV}^6, \\ |Z[\Xi_{bb}]|^2 &= (4.2 \pm 0.6) \times 10^{-2} \text{ GeV}^6. \end{aligned} \quad (3.45)$$

In Fig. 13, the results derived from sum rules are presented for the ratio $|Z[\Omega_{bc}]|^2/|Z[\Xi_{bc}]|^2$ of baryon constants, so that

$$\frac{|Z[\Omega_{bc}]|^2}{|Z[\Xi_{bc}]|^2} = \frac{|Z[\Omega_{cc}]|^2}{|Z[\Xi_{cc}]|^2} = \frac{|Z[\Omega_{bb}]|^2}{|Z[\Xi_{bb}]|^2} = 1.3 \pm 0.2.$$

As mentioned above, the uncertainty of this estimate is mainly related to variation of the ratio $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.2$. From the figures one can see that the stability region for the baryon constants coincides with the region of stability for the averaged mass.

For comparison we present the relationships between the baryon structure constants and their wave functions calculated within the framework of potential models (PM) in the approximation of quark – diquark factorization:

$$|Z^{\text{PM}}| = 2\sqrt{3} |\Psi_{\text{diq}}(0)\Psi_{1,s}(0)|, \quad (3.46)$$

where $\Psi_{\text{diq}}(0)$ and $\Psi_{1,s}(0)$ are the wave functions at zero point for the heavy diquark and for the light (strange) quark – diquark system, respectively.

In the approximation used the values of $\Psi(0)$ were calculated in the Buchmüller–Tye potential [13], thus

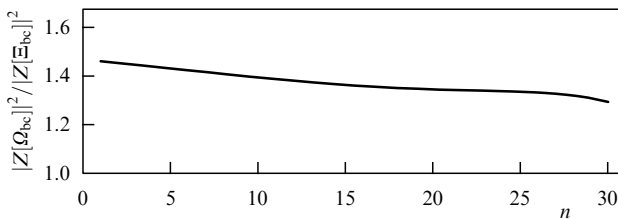


Figure 13. Ratio $|Z[\Omega_{bc}]|^2/|Z[\Xi_{bc}]|^2$ calculated within the NRQCD sum rules in the spectral density moment scheme for $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8$.

giving

$$\begin{aligned} \sqrt{4\pi} |\Psi_1(0)| &= 0.53 \text{ GeV}^{3/2}, \\ \sqrt{4\pi} |\Psi_s(0)| &= 0.64 \text{ GeV}^{3/2}, \\ \sqrt{4\pi} |\Psi_{cc}(0)| &= 0.53 \text{ GeV}^{3/2}, \\ \sqrt{4\pi} |\Psi_{bc}(0)| &= 0.73 \text{ GeV}^{3/2}, \\ \sqrt{4\pi} |\Psi_{bb}(0)| &= 1.35 \text{ GeV}^{3/2}. \end{aligned}$$

In the static limit of the potential approach we have

$$\begin{aligned} |Z^{\text{PM}}[\Omega_{cc}]|^2 &= 8.8 \times 10^{-3} \text{ GeV}^6, \\ |Z^{\text{PM}}[\Xi_{cc}]|^2 &= 6.0 \times 10^{-3} \text{ GeV}^6, \\ |Z^{\text{PM}}[\Omega_{bc}]|^2 &= 1.6 \times 10^{-2} \text{ GeV}^6, \\ |Z^{\text{PM}}[\Xi_{bc}]|^2 &= 1.1 \times 10^{-2} \text{ GeV}^6, \\ |Z^{\text{PM}}[\Omega_{bb}]|^2 &= 5.6 \times 10^{-2} \text{ GeV}^6, \\ |Z^{\text{PM}}[\Xi_{bb}]|^2 &= 3.9 \times 10^{-2} \text{ GeV}^6. \end{aligned} \quad (3.47)$$

The results of the potential model (3.47) are close to the values obtained within the NRQCD sum rules (3.45), from which it can be seen that the SU(3)-splitting of baryon constants $|Z[\Omega]|^2/|Z[\Xi]|^2$ is determined by the ratio $|\Psi_s(0)|^2/|\Psi_1(0)|^2 = 1.45$, which is consistent with the estimate from the sum rules.

The values presented must be multiplied by the Wilson coefficients obtained from expansion of the operators in complete QCD in terms of the NRQCD fields by the anomalous dimensional method. This procedure leads to the following baryon coupling constants:

$$\begin{aligned} |Z[\Omega_{cc}]|^2 &= (38 \pm 5) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Xi_{cc}]|^2 &= (27 \pm 3) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Omega_{bc}]|^2 &= (36 \pm 4) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Xi_{bc}]|^2 &= (27 \pm 3) \times 10^{-3} \text{ GeV}^6, \\ |Z[\Omega_{bb}]|^2 &= (10 \pm 1) \times 10^{-2} \text{ GeV}^6, \\ |Z[\Xi_{bb}]|^2 &= (70 \pm 8) \times 10^{-3} \text{ GeV}^6. \end{aligned} \quad (3.48)$$

Thus, reliable estimates have been obtained within the framework of the NRQCD sum rules for the masses and structure constants of baryons with two heavy quarks.

3.3 Discussion

We have considered the NRQCD sum rules for two-point correlators of baryon currents involving two heavy quarks. The nonrelativistic approximation for heavy quarks permits one to fix the structure of the currents and to take into account the Coulomb-like interactions in a doubly heavy diquark. Moreover, we have introduced operators of higher dimensionalities, responsible for quark – gluon condensates, so as to achieve convergence of the sum rules method for two scalar functions of correlators.

In the leading approximation that includes a perturbative term and the contributions from quark and gluon condensates, the correlators of the three-quark state and of the doubly heavy diquark are factorized into separate functions, when the mass of the light quark is zero. As a result, the sum rules lead to the masses and coupling constants differing in value when calculated with various functions. This means that such an approach will diverge until the contributions

from the product of the quark and gluon condensates and of the mixed condensate are taken into account. The interaction of two heavy quarks with a light quark violates factorization, which permits one to obtain reasonable estimates for the masses and coupling constants. Moreover, the calculated binding energy of the doubly heavy diquark is in good agreement with estimates made within the framework of potential models.

In doubly heavy baryons with strangeness, factorization of the diquark correlator is already violated in the quark-loop approximation in QCD perturbation theory, so that the best convergence of the sum rules method is achieved and both correlators have stability intervals when the numbers of the spectral density moment are varied. Estimates have been made of mass splittings between strange baryons and baryons with a massless light quark, $\Omega_{QQ'}$ and $\Xi_{QQ'}$, and of the ratio of their baryon coupling constants, $|Z[\Omega_{QQ'}]|^2/|Z[\Xi_{QQ'}]|^2$. Thus, the NRQCD sum rules permit the improvement of the analysis of masses and coupling constants for doubly heavy baryons and obtaining of reliable results.

4. Baryon production processes

When $\Xi_{QQ'}$ baryons with two heavy quarks are produced, the small ratio Λ/m_Q and, consequently, the small coupling constant of the QCD quark–gluon interaction [$\alpha_s \sim 1/\ln(m_Q/\Lambda) \ll 1$] permit one not only to examine the production of the two pairs of heavy quarks $\bar{Q}Q$ and $\bar{Q}'Q'$, of which $\Xi_{QQ'}$ baryons are composed, within QCD perturbation theory, but also to factor out in a certain manner the contributions due to perturbative heavy quark production and their subsequent nonperturbative binding into a heavy diquark.

For calculating the production cross sections of Ξ_{bc} S-wave states at the Z-boson peak it is sufficient to evaluate the matrix elements for the cojoint production of $b\bar{b}$ and $c\bar{c}$ pairs in the antitriplet color state of the bc pair with a definite total quark spin ($S = 0, 1$), so that the quarks move with the same velocity equal to the velocity of the diquark they compose. Then it is necessary to multiply these matrix elements by a nonperturbative factor determined from the spectroscopic characteristics of the bound state (by the wave function of the diquark, giving the probability of observing quarks at a small distance from each other in a bound state, and by the quark mass).

Such a picture is due to the characteristic virtualities of the heavy quarks in the heavy diquark being significantly smaller than their masses owing to the nonrelativistic motion of the heavy quarks in the bound state, whereas the quark virtualities at the moment of their production amount to values of the order of their masses. Therefore, when examining Ξ_{bc} production one can consider the b and c quarks in the diquark to be close to the mass surface and practically at rest with respect to each other. Thus, when the nonperturbative factor has been singled out, analysis of the heavy Ξ_{bc} baryon production is determined by the examination of matrix elements calculated within the QCD perturbation theory, if the total baryon production cross section and its differential characteristics are considered to repeat the respective quantities for the heavy diquark.

First of all, let us note that the necessity for two pairs of heavy quarks to be produced in electromagnetic and strong interaction processes for the formation of $\Xi_{QQ'}$ results in the

leading order of QCD perturbation theory having an additional factor of smallness of the order of α_s^2 compared with the leading order of perturbation theory for the production of heavy quarks of the same flavor, for instance, of the $\bar{Q}Q$ pair:

$$\frac{\sigma[\Xi_{QQ'}]}{\sigma[\bar{Q}Q]} \sim \frac{\alpha_s^2 |\Psi(0)|^2}{m_{Q'}^3}.$$

This provides for a small $\Xi_{QQ'}$ yield compared to the production of, say, heavy mesons.

Within this approach, it is important and necessary to perform analysis of the leading approximation of QCD perturbation theory for $\Xi_{QQ'}$ production, which permits one to obtain a series of analytical expressions for the $\Xi_{QQ'}$ production cross sections, among which one must especially note the fragmentation functions of heavy quarks into a heavy diquark and of the diquark into a baryon in the scaling limit ($M^2/s \rightarrow 0$). Thus, $\Xi_{QQ'}$ fragmentation production can be reliably described analytically, which opens up new possibilities in studying QCD dynamics that is essential in the complete picture of heavy quark physics.

The mechanism of baryon production with two heavy quarks in hadron collisions implies the analysis of a complete set of QCD perturbation theory fourth-order diagrams in the coupling constant, since transverse momenta, at which the fragmentation mode is not predominant, are characteristic for the total cross sections of production processes and for the dominant contributions. We shall study the role of higher twists in the transverse momentum of the doubly heavy baryon in associated production of these states in hadron interactions and quantitatively determine the applicability boundaries for the factorization mode of hard heavy quark production and their subsequent fragmentation.

4.1 The production of doubly heavy baryons in e^+e^- -annihilation

Detailed investigation of the production mechanisms of hadrons with two heavy quarks reveals that the expected yield of such hadrons compared to the production of hadrons with a single heavy quark amounts to a value of the order of $10^{-3} - 10^{-4}$. For example, the number of events with heavy quarks at the Z-boson pole is at the level of 10^6 , so the number of hadrons with two heavy quarks is of the order of $10^2 - 10^3$. Taking into account concrete decay modes of hadrons with two heavy quarks, only the registration of individual events with hadrons should be expected, which renders their observation quite problematic.

In this section we consider the production of doubly charmed $\Xi_{cc}^{(*)}$ baryons in the conditions of a B-meson factory of high luminosity ($L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$), where the $\Xi_{cc}^{(*)}$ yield is higher by two orders of magnitude than at the Z-boson pole.

4.1.1 Fragmentation mechanism. In Ref. [50], estimates have been obtained of the production cross sections for $\Xi_{cc}^{(*)}$, $\Xi_{bc}^{(*)}$, and $\Xi_{bb}^{(*)}$ baryons in the heavy quark fragmentation region at high energies. These estimates are based on an analytic computation of heavy quarkonium production in the QCD perturbation theory in the limit of a small ratio M^2/s and in the nonrelativistic potential model. The momentum spectrum of the cc diquark was considered identical to the $c\bar{c}$ heavy

quarkonium spectrum¹² up to a color factor:

$$D_{c \rightarrow cc}(z) = \frac{2}{9\pi} \frac{|R_{cc}(0)|^2}{m_c^3} \alpha_s^2 (4m_c^2) F(z), \quad (4.1)$$

where

$$F(z) = \frac{z(1-z)^2}{(2-z)^6} (16 - 32z + 72z^2 - 32z^3 + 5z^4),$$

and $R_{cc}(0)$ is the radial wave function of the bound diquark at zero point.

Notice that identical cc quarks in the antitriplet color state can only have a symmetrical spin function in the S -wave, i.e. can only be in a state with the total spin $S = 1$. Normalization of the fragmentation function $D_{c \rightarrow cc}(z)$ is determined by the model-dependent quantity $R_{cc}(0)$. In Ref. [50], quite a rough approximation of the Coulomb potential was applied to the heavy quark system. This factor introduces a noticeable uncertainty¹³ in the estimates of the $\Xi_{cc}^{(*)}$ yield. Moreover, expression (4.1) obtained in the scaling limit ($M^2/s \rightarrow 0$) is not appropriate for estimating the $\Xi_{cc}^{(*)}$ production at B -factories, where M^2/s is not small. We propose another approach to estimating the yield of hadrons containing two heavy quarks on the basis of quark – hadron duality.

4.1.2 Estimates from the quark – hadron duality. The production cross sections of B_c -mesons in S -wave states at the Z -boson pole, calculated in the fragmentation model, are in good agreement with the estimated production cross sections of $\bar{b}c$ quark pairs in the singlet color state with a small invariant mass:

$$m_b + m_c < M(\bar{b}c) < M_{th} = M_B + M_D + \Delta M, \quad (4.2)$$

so that $\Delta M = 0.5 - 1$ GeV.

Over the duality interval (4.2), the bc diquark production cross section is approximately equal to the $\bar{b}c$ pair production cross section. Isolating the antitriplet color state of bc by multiplication by the factor $2/3$, we obtain an estimate of the $\Xi_{bc}^{(*)}$ production cross section at the level

$$\frac{\sigma[\Xi_{bc}^{(*)}]}{\sigma[\bar{b}b]} \approx 6 \times 10^{-4},$$

which is six times larger than the estimate obtained by Falk et al. [50] for the production of $1S$ states. This difference stems, first, from the contribution of higher diquark excitations being taken into account in the quark – hadron duality approach and, second, from strong suppression due to the quantity $R_{bc}(0)$ which is clearly underestimated in the Coulomb potential.

Now, consider the production of $\Xi_{cc}^{(*)}$ baryons at the energy of the B -meson factory ($\sqrt{s} = 10.58$ GeV). Once more we recall that expression (4.1) is not applicable at the given energy, since the power corrections in M^2/s are significant. We have presented a detailed description of the method of numerical computation in the leading order of QCD perturbation theory in Refs [51 – 54].

In the quark – hadron duality method, the cross section of associated production of a bound quarkonium state can be evaluated by the formula

$$\begin{aligned} & \sum_{nL, J} \sigma[e^+e^- \rightarrow (nL(\bar{c}c)_J)\bar{c}c] \\ &= \int_{M_i}^{M_{th}} dM_{\bar{c}c} \frac{d}{dM_{\bar{c}c}} \sigma[e^+e^- \rightarrow (\bar{c}c)_{\text{singlet}}\bar{c}c]. \end{aligned} \quad (4.3)$$

Here, $M_i = 2m_c$ is the kinematical threshold of $\bar{c}c$ pair production, and $M_{th} = 2M_D + \Delta M$, where $\Delta M = 0.5 - 1$ GeV. Numerical calculation of QCD perturbation theory diagrams for the production of bound $1S$ and $2S$ levels of charmonium for $\sqrt{s} = 10.58$ GeV, $\alpha_s = 0.2$ and with values of the radial wave function at zero point, $R_{cc}(0)$, determined from experimental data on $\psi(nS)$ lepton decay widths [23], results in the following estimates for the cross sections required:

$$\begin{aligned} \sigma[\eta_c(1S)] &= 0.025 \text{ pb}, & \sigma[\eta_c(2S)] &= 0.003 \text{ pb}, \\ \sigma[\psi(1S)] &= 0.055 \text{ pb}, & \sigma[\psi(2S)] &= 0.010 \text{ pb}. \end{aligned}$$

The sum of the cross sections taken over the S -wave states of charmonium below the decay threshold into a $\bar{D}D$ meson pair is

$$\sigma\left[\sum \eta_c, \psi\right] = 0.093 \text{ pb}. \quad (4.4)$$

We note that the ratio between the vector state and pseudoscalar state yields at the energy considered amounts to $\omega_V/\omega_P \approx 2.2$ as compared to the ratio $\omega_V/\omega_P \approx 1$ obtained from the fragmentation mechanism [50].

Estimates of the integral in the right-hand part of Eqn (4.3) give

$$\sigma_{\bar{c}c}(\Delta M = 0.5 \text{ GeV}) = 0.093 \text{ pb}, \quad (4.5)$$

$$\sigma_{\bar{c}c}(\Delta M = 1 \text{ GeV}) = 0.110 \text{ pb}, \quad (4.6)$$

where $m_c = 1.4$ GeV was assumed. From equations (4.4) – (4.6) it follows that the quark – hadron duality relation (4.3) is satisfied well for bound states of the $\bar{c}c$ system.

As shown by calculations, the invariant mass spectra for $\bar{c}c$ and cc pairs practically repeat each other in the region of small invariant masses. Therefore, the estimates for the production cross sections of the cc diquark and of the $\bar{c}c$ pair are approximately the same in the duality interval considered [compare with formulas (4.5) and (4.6)]:

$$\sigma_{cc}(\Delta M = 0.5 \text{ GeV}) = 0.086 \text{ pb}, \quad (4.7)$$

$$\sigma_{cc}(\Delta M = 1 \text{ GeV}) = 0.115 \text{ pb}. \quad (4.8)$$

Isolating the antitriplet color state, we obtain the total production cross section of $\Xi_{cc}^{(*)}$ baryons:

$$\sigma[\Xi_{cc}^{(*)}] = (70 \pm 10) \times 10^{-3} \text{ pb}, \quad (4.9)$$

so the relative yield of doubly charmed baryons equals

$$\frac{\sigma[\Xi_{cc}^{(*)}]}{\sigma[\bar{c}c]} = 7 \times 10^{-5}. \quad (4.10)$$

The number of events involving $\Xi_{cc}^{(*)}$ production at the luminosity $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ amounts to $N(\Xi_{cc}^{(*)}) = 7 \times 10^3$

¹² Ref. [50] erroneously contains an additional factor of 2.

¹³ Calculating the diquark wave function in the model with the Martin potential with due account of the numerical factor $1/2$ for the antitriplet color quark state enhances the corresponding factor by approximately one order of magnitude.

per year, which is higher by two orders of magnitude than the $\Xi_{cc}^{(*)}$ yield at LEP. The cc diquark spectrum obtained at the antisymmetric collider KEK is presented in Ref. [55].

Thus, in this section we have calculated the production cross sections of the doubly charmed $\Xi_{cc}^{(*)}$ baryon in the leading order of QCD perturbation theory on the basis of quark–hadron duality, as well as the $\Xi_{cc}^{(*)}$ production cross sections at the energies of a B-meson factory, where the fragmentation model [50] cannot be applied.

The main theoretical uncertainty in the estimation of production cross sections of doubly heavy baryons is related to the description of the heavy cc-diquark hadronization process. A significant fraction (1/3) of the diquarks are produced in the sextet color state and can form both exotic four-quark states $cc\bar{q}\bar{q}$ and DD meson pairs. Like in work [50], we assume the antitriplet color state to undergo hadronization into the $\Xi_{cc}^{(*)}$ baryon with a 100% probability. Thus, 10^4 $\Xi_{cc}^{(*)}$ -production events per year can be expected given a luminosity $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ at a B-meson factory.

4.1.3 Exclusive diquark pair production. In the near-threshold region of doubly heavy baryon production in e^+e^- -annihilation a noticeable contribution to the cross section can be made by pair production. For estimating the yield of such events, a computation was performed in Ref. [56] of the cross sections of exclusive pair production of doubly heavy diquarks. Under the assumption of 100% diquark fragmentation into baryons, the yields of diquark pairs and of baryons can be considered equal to each other. The authors of Ref. [56] dealt with both axial-vector and scalar states of S -wave diquarks: calculations were done of amplitudes, as well as differential and total production cross sections for scalar–scalar, scalar–vector and vector–vector pairs. For details we refer the reader to the original paper [56]. For illustration, we present the expression covering the total production cross section for scalar pairs as a function of the square of the total energy s :

$$\sigma_{00} = 256\pi^3 \frac{f_{00}^2}{9s^3} |\Psi_{\text{diq}}(0)|^4 \left(1 - \frac{4M^2}{s}\right)^{3/2}. \quad (4.11)$$

Here, $\Psi_{\text{diq}}(0)$ is the diquark wave function at zero point, while the form factor has the form

$$f_{00} = \alpha_s \alpha_{em} M \left[\left(\frac{q_2}{m_1^2} + \frac{q_1}{m_2^2} \right) - \frac{2M^2}{s} \left(\frac{q_2 m_2}{m_1^3} + \frac{q_1 m_1}{m_2^3} \right) \right], \quad (4.12)$$

where $q_{1,2}$ are the heavy quark charges, $m_{1,2}$ are their masses, and $M = m_1 + m_2$.

Numerical estimates show that the production of axial-vector heavy diquark pairs predominates, and that, as compared with the yield of heavy quark pairs, the fraction of diquark pairs amounts to $(2-6) \times 10^{-6}$. This means that, say, at B-meson factories among the events with charm one can also try to search for doubly heavy baryon pair production events that amount to 10% of single particle production events.

4.2 Perturbative diquark fragmentation

In this section we examine baryon production under fragmentation of heavy vector and scalar particles that interact with the quark. From the QCD standpoint, a small-sized doubly heavy diquark represents a local triplet field, so the results obtained can be applied to calculating the

fragmentation of vector and scalar diquarks into baryons. We employ QCD perturbation theory for computing the hard fragmentation amplitude that is factorized from the soft bound-state production amplitude. Doubtless, such a method is quite precise, if the hardness is provided for by the large mass of the quark that together with the diquark forms a hadron state — a baryon, for instance, in the case of bb fragmentation into bbc. The expressions obtained, however, can also be applied for light quarks as QCD-motivated parametrizations.

Fragmentation of the scalar triplet color local field was considered in Ref. [57]. A new problem arising in the case of a vector diquark is the choice of the Lagrangian liable for the interaction of the vector colored particle with the gluon field. It is possible to add to the Lagrangian obtained by elongation of the derivatives in the free vector field Lagrangian

$$-\frac{1}{2} H_{\mu\nu} \bar{H}^{\mu\nu}, \quad H_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu,$$

where U_μ is the vector complex field, a gauge invariant term proportional to the interaction of the spin tensor $S_{\mu\nu}^{\alpha\beta}$ with the gluon field strength tensor $G^{\mu\nu}$:

$$S_{\mu\nu}^{\alpha\beta} G^{\mu\nu} U_\beta \bar{U}_\alpha, \quad S_{\mu\nu}^{\alpha\beta} = \frac{1}{2} (\delta_\mu^\alpha \delta_\nu^\beta - \delta_\nu^\alpha \delta_\mu^\beta),$$

which leads to a parameter appearing at the interaction vertex of the diquark and the gluon (the anomalous magnetic moment). Our interest is in considering the high-energy production of a spin-1/2 bound state containing a heavy vector particle in relation to the behavior of this parameter.

At high transverse momenta, the predominant production mechanism for bound heavy baryon states is diquark fragmentation that can be computed within perturbative QCD [58] upon separating the soft bound-state production factor obtained within the framework of nonrelativistic potential models [8]. The fragmentation function is universal for all high-energy processes involving direct baryon production.

In the leading order in α_s , the fragmentation function has a scaling shape which is considered the initial condition for perturbative QCD evolution resulting from hard gluon emission by the diquark before hadronization. The splitting function differs from the similar function for the heavy quark owing to the spin structure of the gluon coupling with the diquark that is a vector or scalar particle in a triplet color state.

In this review the fragmentation scaling function is calculated in the leading order of perturbation theory. The limit of an infinitely heavy diquark ($M_{\text{diq}} \rightarrow \infty$) is obtained from examining fragmentation in QCD. Computing the heavy baryon distribution function over transverse momentum with respect to the fragmentation axis within the framework of the leading order of QCD perturbation theory, we find the splitting kernel in the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) evolution, and we obtain and solve the one-loop equations of the renormgroup for moments of the fragmentation function. These equations are universal, since they are independent of whether the diquark is in a bound or free state at low virtualities, where the mode of perturbative evolution ceases. As a result, expressions are obtained for the integral probabilities of diquark fragmentation into doubly heavy baryons.

4.2.1 The fragmentation function in the leading order. The contribution from fragmentation to direct heavy baryon production takes the form

$$d\sigma[\Xi(p)] = \int_0^1 dz d\hat{\sigma} \left[\text{diq} \left(\frac{p}{z} \right), \mu \right] D_{\text{diq} \rightarrow \Xi}(z, \mu).$$

Here, $d\sigma$ is the differential production cross section of a baryon with a 4-momentum p , $d\hat{\sigma}$ is the hard production cross section of a diquark with a momentum p/z , and $D(z)$ is interpreted as the fragmentation function depending on the fraction of the momentum, z , carried away by the bound state. The quantity μ determines the scale of factorization.

In accordance with the general form of DGLAP evolution, the μ -dependent fragmentation function satisfies the equation

$$\frac{\partial D_{\text{diq} \rightarrow \Xi}(z, \mu)}{\partial \ln \mu} = \int_z^1 dy \frac{1}{y} P_{\text{diq} \rightarrow \text{diq}} \left(\frac{z}{y}, \mu \right) D_{\text{diq} \rightarrow \Xi}(y, \mu), \quad (4.13)$$

where P is the kernel due to the emission of hard gluons by the diquark before the production of a heavy quark pair. Therefore, the initial form of the fragmentation function is determined by the diagram shown in Fig. 14. Consequently, the corresponding initial factorization scale is specified by $\mu = 2m_Q$. Moreover, the fragmentation function can be calculated by expansion in a power series of $\alpha_s(2m_Q)$. The leading order contribution is computed in this section.

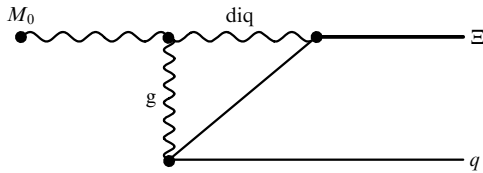


Figure 14. Fragmentation diagram of the diquark (diq) into the heavy Ξ baryon.

Let us consider the fragmentation diagram in a system where the initial diquark momentum $q = (q_0, 0, 0, q_3)$, and the momentum of the baryon is p , so that

$$q^2 = s, \quad p^2 = M^2.$$

In the static approximation for a bound state of a diquark and a heavy baryon we have the following relations for the heavy quark and diquark masses:

$$m_Q = rM, \quad \mathcal{M}_{\text{diq}} = (1-r)M = \bar{r}M.$$

The interaction vertex of a vector diquark and a gluon is written as follows

$$T_{\alpha\mu\nu}^{VVg} = -igt^a \{ g_{\mu\nu}(q + \bar{r}p)_\alpha - g_{\mu\alpha}[(1+\kappa)\bar{r}p - \kappa q]_\nu - g_{\nu\alpha}[(1+\kappa)q - \kappa\bar{r}p]_\mu \}, \quad (4.14)$$

where κ is the anomalous magnetic moment, and t^a is the QCD group generator in the fundamental representation.

The sum over the polarizations of the vector diquark with a momentum q depends on the choice of gauge of the free Lagrangian field (for example, the Stueckelberg gauge), but

the fragmentation function being calculated does not depend on the gauge parameter changing the form of the contribution from longitudinal field components. Thus, without restriction of the general character of the analysis the sum over polarizations is chosen in the form

$$P(q)_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{s}.$$

The matrix element for fragmentation into the spin-1/2 state is written as

$$\mathfrak{M} = -\frac{2\sqrt{2}\pi\alpha_s}{9\sqrt{M^3}} \frac{R(0)}{r\bar{r}(s - \mathcal{M}_{\text{diq}}^2)^2} P(q)_{\nu\delta} P(\bar{r}p)_{\mu\eta} T_{\alpha\mu\nu}^{VVg} \times \rho_{\alpha\beta} \bar{q}_\gamma^\beta (\hat{p} - M)\gamma^\eta \gamma^5 \xi \mathfrak{M}_0^\delta, \quad (4.15)$$

whereas the sum over the gluon polarizations — in the axial gauge with $n = (1, 0, 0, -1)$:

$$\rho_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{kn}, \quad k = q - (1-r)p.$$

The spinors ξ and \bar{q} in relation (4.15) correspond to the baryon and the heavy quark accompanying fragmentation, \mathfrak{M}_0 stands for the matrix element for hard diquark production at high energies, and $R(0)$ is the radial wave function at zero point.

Squaring the matrix element and summing over the helicities of the particles produced, the following expression is obtained:

$$|\overline{\mathfrak{M}}|^2 = W_{\mu\nu} \mathfrak{M}_0^\mu \mathfrak{M}_0^\nu.$$

In the limit of high energies ($qn \rightarrow \infty$), the tensor $W_{\mu\nu}$ behaves like

$$W_{\mu\nu} = -g_{\mu\nu} W + R_{\mu\nu}, \quad (4.16)$$

where $R_{\mu\nu}$ may depend on gauge parameters and on expansion in the Lorentz structures leads to scalar quantities that are small compared with W in the limit $qn \rightarrow \infty$.

We shall denote

$$z = \frac{pn}{qn}.$$

The fragmentation function is determined by the expression [59]

$$D(z) = \frac{1}{16\pi^2} \int ds \theta \left(s - \frac{M^2}{z} - \frac{m_Q^2}{1-z} \right) W$$

[W is given in Eqn (4.16)]. The integral in the expression for the fragmentation function at a constant anomalous magnetic moment differing from -1 diverges logarithmically.

We have considered two cases of anomalous magnetic moment behavior. At $\kappa = -1$, the fragmentation function coincides with that for a scalar diquark with an accuracy up to the spin factor $1/3$:

$$D(z) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{M^3 r^2 \bar{r}^2} \frac{z^2(1-z)^2}{(1-\bar{r}z)^6} \times [3 + 3r^2 - (6 - 10r + 2r^2 + 2r^3)z + (3 - 10r + 14r^2 - 10r^3 + 3r^4)z^2], \quad (4.17)$$

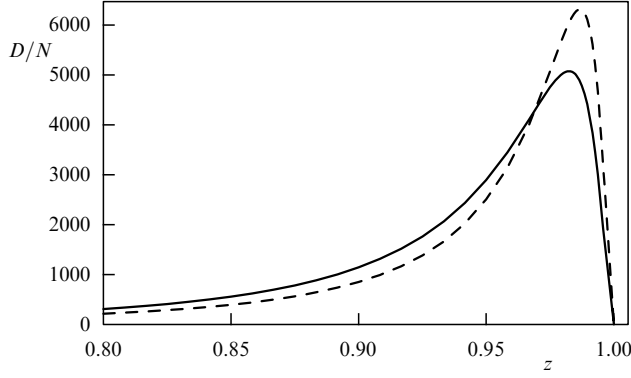


Figure 15. Fragmentation function of a diquark into a heavy baryon for $r = 0.02$; the dashed line represents $z = -1$, the solid line $1 + z = 3M^2/(s - \mathcal{M}_{\text{diq}}^2)$, and the factor $N = (8\alpha_s^2/243\pi)|R(0)|^2/[M^3r^2(1-r)^2]$.

which tends toward

$$\tilde{D}(y) = \frac{8\alpha_s^2}{243\pi y^6} \frac{|R(0)|^2}{m_Q^3} \frac{(y-1)^2}{r} (8 + 4y + 3y^2), \quad (4.18)$$

as $r \rightarrow 0$ and $y = [1 - (1-r)z]/(rz)$.

The limit $\tilde{D}(y)$ is in agreement with the general analysis of the expansion in $1/m$ for the fragmentation function [60], and here

$$\tilde{D}(y) = \frac{1}{r} a(y) + b(y).$$

We note that in this situation the dependence upon y turned out to be the same as for fragmentation of a heavy quark into quarkonium [59].

The case when $1 + z = AM^2/(s - \mathcal{M}_{\text{diq}}^2)$ has been dealt with in Ref. [61]. The perturbative fragmentation functions in the leading order in α_s are shown in Fig. 15. They represent quite hard distributions that become softer with due account of evolution (see Ref. [57]).

4.2.2 Transverse baryon momentum. In the system of a diquark undergoing fragmentation and exhibiting an infinitely large momentum, its invariant mass is expressed via the fraction of the longitudinal baryon momentum, z , and the transverse baryon momentum p_\perp with respect to the fragmentation axis:

$$s = \mathcal{M}_{\text{diq}}^2 + \frac{M^2}{z(1-z)} \{ [1 - (1-r)z]^2 + t^2 \},$$

where $t = p_\perp/M$. Calculation of the diagram depicted in Fig. 14 give the double distribution for the fragmentation probability:

$$\frac{d^2P}{ds dz} = \mathcal{D}(z, s),$$

with the function ($z = -1$)

$$\begin{aligned} \mathcal{D}(z, s) = & \frac{256\alpha_s^2}{81\pi} \frac{|R(0)|^2}{r^2 \bar{r}^2} \frac{M^3}{(1 - \bar{r}z)^2 (s - \mathcal{M}_{\text{diq}}^2)^4} \\ & \times \left\{ r\bar{r}^2 + \bar{r}[1 + r - z(1 + 4r - r^2)] \frac{s - \mathcal{M}_{\text{diq}}^2}{M^2} \right. \\ & \left. - z(1 - z) \frac{(s - \mathcal{M}_{\text{diq}}^2)^2}{M^4} \right\}. \end{aligned} \quad (4.19)$$

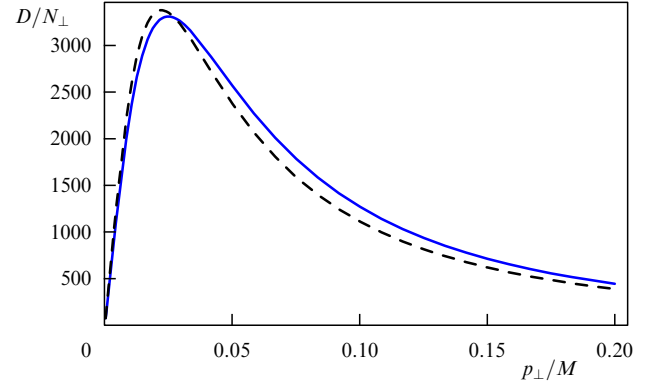


Figure 16. Distribution over transverse momentum with respect to the axis of diquark fragmentation into a heavy baryon for $r = 0.02$: dashed line — $z = -1$, and solid line — $1 + z = 3M^2/(s - \mathcal{M}_{\text{diq}}^2)$; the factor $N_\perp = (8\alpha_s^2/81\pi)|R(0)|^2/[M^4r^2(1-r)^7]$.

It is readily seen that the transverse momentum distribution can be obtained by integration over z :

$$D(t) = \int_0^1 dz \mathcal{D}(z, s) \frac{2M^2 t}{z(1-z)}.$$

As a result we obtain quite a cumbersome expression which is presented in Appendix 7.2. The typical shape of baryon transverse momentum distributions with respect to the diquark fragmentation axis is displayed in Fig. 16.

4.2.3 Hard gluon emission. The one-loop contribution of hard gluon emission can be calculated by the same method as in Section 4.2.1. The probability of the process turns out to be independent of the anomalous magnetic moment and therefore the splitting kernel for the vector diquark coincides with the splitting kernel for a scalar diquark:

$$P_{\text{diq} \rightarrow \text{diq}}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[\frac{2x}{1-x} \right]_+, \quad (4.20)$$

where the subscript ‘+’ indicates standard action

$$\int_0^1 dx f_+(x) g(x) = \int_0^1 dx f(x) [g(x) - g(1)].$$

The splitting function can be compared with the similar function for a heavy quark:

$$P_{Q \rightarrow Q}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[\frac{1+x^2}{1-x} \right]_+,$$

which has the same normalizing factor as $x \rightarrow 1$.

Further, multiplying the evolution equation by z^n and integrating over z , it is possible to obtain from Eqn (4.13) by the renormgroup method the μ -dependence of moments $a_{(n)}$ for the fragmentation function in the one-loop approximation:

$$\frac{\partial a_{(n)}}{\partial \ln \mu} = -\frac{8\alpha_s(\mu)}{3\pi} \left[\frac{1}{2} + \dots + \frac{1}{n+1} \right] a_{(n)}, \quad n \geq 1. \quad (4.21)$$

When $n = 0$, the right-hand part in Eqn (4.21) is zero. This means that the integral probability for the diquark to undergo fragmentation into a heavy baryon remains unchanged in the

course of evolution and is determined by the initial fragmentation function calculated above in QCD perturbation theory [57].

The solution of equation (4.21) assumes the form

$$a_{(n)}(\mu) = a_{(n)}(\mu_0) \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{\frac{16}{3\beta_0} \left[\frac{1}{2} + \dots + \frac{1}{n+1} \right]}, \quad (4.22)$$

where the one-loop expression is used for the QCD coupling constant:

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln(\mu/\Lambda_{\text{QCD}})},$$

with $\beta_0 = 11 - 2n_f/3$, and n_f being the number of flavors of quarks with $m_q < \mu < \mathcal{M}_{\text{diq}}$.

Relation (4.22) is universal, because it is independent of whether the diquark is free or bound with virtualities smaller than μ_0 . If, when fragmentation into a heavy baryon takes place, evolution is taken into account, then the diquark may lose about 20% of its momentum before hadronization [57].

4.2.4 Integral fragmentation probabilities. As was noted above, evolution conserves the integral fragmentation probability which can be computed analytically:

$$\int dz D(z) = \frac{8\alpha_s^2}{81\pi} \frac{|R(0)|^2}{16m_Q^3} w(r), \quad (4.23)$$

so, when $\kappa = -1$, we have

$$w(r) = \frac{16}{15(1-r)^7} [(8 + 15r - 60r^2 + 100r^3 - 60r^4 - 3r^5) + 30r(1-r+r^2+r^3) \ln r]. \quad (4.24)$$

The function $w(r)$ is plotted in Fig. 17 for two choices of κ and small r .

Thus, we have considered the dominant production mechanism for spin-1/2 bound states of the vector local color field (for example, a diquark) with a heavy antiquark in high-energy processes with large transverse momenta, where the role of the leading term is assumed by the contribution due to fragmentation. We have discussed two cases of the anomalous magnetic moment behavior. At $\kappa = -1$, the fragmentation function differs only by a numer-

ical factor from the corresponding function for fragmentation of a scalar colored particle into a bound state with a heavy antiquark. (For other values of the constant anomalous magnetic moment the integral in the expression for the fragmentation function diverges.) In the limit of an infinitely heavy diquark, the function $D(z)$ assumes a form consistent with what is expected from the general study of the $1/m$ -expansion for the fragmentation function.

The distributions over baryon transverse momentum with respect to the diquark fragmentation axis have also been computed in the leading order of QCD perturbation theory. The corrections from hard gluons that are due to vector diquark splitting can be taken into account perturbatively, which leads to the corresponding one-loop equations for the moments of the fragmentation function [see Eqns (4.21) and 4.22)].

Numerical estimates reveal that the probability of fragmentation into bound states of a heavy vector diquark with a mass between 3 and 10 GeV depends on the choice of effective mass for the quark present in the baryon. In this case, the ratio of the yields of baryons with and without strangeness amounts to $\sigma[\Omega_{QQ'}]/\sigma[\Xi_{QQ'}] \approx 0.2$. Doubtless, when light and strange quarks are also considered, the estimates made cannot be rigorously substantiated, since they assume the constituent masses to lead to an effective description of the dominant contributions from infrared dynamics. One can, however, make use of perturbative expressions as models for fragmentation into hadrons containing light and strange quarks, since in such processes the ‘fast’ valence degrees of freedom in the baryon can be, in the approximation of small dispersion, reliably approximated by the ratio between fractions of the longitudinal parton moment in the hadron, and one may not consider the contribution from the soft sea of light quarks and gluons.

Another approach to the fragmentation production mechanism of doubly heavy baryons was applied in works [62], in which the perturbative form factor of a doubly heavy diquark and of a heavy–light diquark was computed along with the fragmentation function of a heavy quark into a baryon due to pair production of vector diquarks: $Q \rightarrow \Xi_{QQ'} + (\bar{Q}'\bar{q})_{\text{diq}}$. Such estimates exhibit the character of a lower limit, since they are based on the elastic diquark form factor. The hierarchy of interaction scales, $m_Q \gg m_{Qv} \gg \Lambda_{\text{QCD}}$, also implies that upon hard production of a heavy quark (m_Q) the rapid formation of a doubly heavy diquark (m_{Qv}) first takes place and then the less rapid diquark hadronization into a baryon (Λ_{QCD}).

In Ref. [63], a comparative analysis has also been made of fragmentation into the triply charmed Ω_{ccc} baryon via the cascade process involving transformations of quark into diquark and diquark into baryon, as well as via the process of quark transformation into baryon owing to elastic production of a vector diquark. Regretfully, the main conclusion of Ref. [63] concerning the noticeable dominance of direct fragmentation over cascade one is not correct. First, using the elastic form factor of the vector diquark in the cascade process leads to the fusion factor of charmed quarks into a diquark being taken into account twice: in the quark fragmentation into a diquark, and in the elastic form factor where, also, the projection of the incoming quark state onto the bound diquark is taken. This gives a ‘superfluous’ smallness factor $\alpha_s^2 |\Psi_{cc}(0)|^2 \sim 10^{-3}$. Second, the idea of the formation of a cc diquark with subsequent hard production of a charmed quark on the diquark is certainly not correct, since

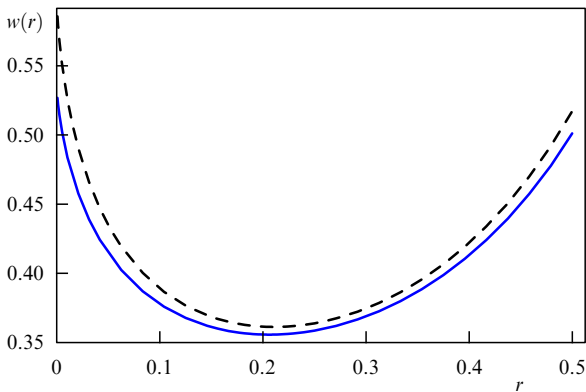


Figure 17. Function w for vector diquark fragmentation into a heavy baryon vs. the ratio $r = m_Q/M$: dashed line — $\kappa = -1$, and solid line — $1 + \kappa = 3M^2/(s - \mathcal{M}_{\text{diq}}^2)$.

the diquark formation time is significantly longer than the production time of the charmed quark.

4.3. Hadron production

Recent years have witnessed a rapid growth of the number of charmed particles registered in modern experiments. At the FNAL E831 and E781 fixed-target facilities, studies of the order of 10^6 events with charmed particles are to be expected. In experiments of the next generation this number is expected to be increased by more than two orders of magnitude. Besides the standard problems of CP-violation in the sector of charmed quarks, measurements of rare decays, etc., the investigation of processes involving the production of more than one pair of c quarks becomes urgent. The production of an additional $\bar{c}c$ pair significantly reduces the cross section of such a process, which is especially important to take into account in fixed-target experiments where quark-parton luminosities are strongly suppressed in the region of heavy mass production.

One of the interesting processes is the production of doubly charmed baryons. The $\Xi_{cc}^{(*)}$ baryon represents a totally new object as compared with ordinary hadrons composed of light quarks. The ground state of such a baryon is similar to that of the $Q\bar{q}$ meson, in which one of the quarks is heavy, and the other light. The role of the heavy quark in the ccq baryon is assumed by the heavy cc diquark [64], which is in an antitriplet color state and is small in size compared to the scale of light quark confinement. Investigation of the ccq states is interesting from the point of view of understanding their production mechanism. Production of the ccq baryon has been considered in a series of publications [50, 54, 55, 65, 66]. The main task of calculating the production cross section of this baryon reduces to evaluating the production cross section of the cc diquark in the $\bar{3}$ -plet color state. Such a diquark is assumed to transform nonperturbatively into a ccq baryon with unit probability.

Hadron production of a diquark proceeds in two stages. The first stage corresponds to hard production of two $\bar{c}c$ pairs in the processes $gg \rightarrow cc\bar{c}\bar{c}$, $q\bar{q} \rightarrow cc\bar{c}\bar{c}$ and is described by fourth-order diagrams in α_s in QCD perturbation theory (Fig. 18). The second stage corresponds to the nonperturba-

tive fusion of two c quarks with small relative momenta into the cc diquark, which is described by the diquark wave function at the origin of the reference frame, $R(0)$, in the case of S -wave states.

The main distinction in existing estimates of the yield of doubly charmed baryons consists in the differing approaches to calculating the hard production cross section of the diquark. Thus, in Ref. [67], instead of taking into account the complete set of fourth-order diagrams, only those responsible for fragmentation of a c quark into a cc diquark were used. As is shown by Berezhnuy et al. [65], such an approximation is not quite correct, since it is valid only in the case of large transverse momenta ($p_{\perp}^{\min} > 35$ GeV), where the production mechanism enters the fragmentation mode. Application of the fragmentation approximation is not justified within the remaining region of kinematic variables, and it leads to erroneous results, especially in conditions when the energy \sqrt{s} is low compared to p_{\perp}^{\min} .

But even if the complete set of diagrams is taken into account, like in Refs [65, 66], a significant uncertainty remains in the estimate of the ccq -baryon yield. The main parameters affecting this uncertainty in the cc diquark production model are the quantities α_s , m_c and $R_{cc}(0)$. Besides this, it remains unclear how valid the hypothesis of unit hadronization probability of the cc diquark into a ccq baryon is, since diquark interaction with gluons is not suppressed, like, for example, in the case of quark-antiquark $\bar{c}c$ pair production in a colorless state, when the quarkonium dissociation implies exchange with the quark-gluon sea by at least two hard gluons with greater virtualities than the inverse size of quarkonium.

It is possible to reduce the uncertainties in cross section estimates by comparing ccq baryon production with a similar process — production of the J/ψ particle accompanied by a $\bar{c}c$ pair that is described by practically the same fourth-order diagrams with the J/ψ -particle wave function known at zero point¹⁴. Thus, by reference to the $J/\psi + \bar{D}D$ production process it is possible to remove part of the uncertainties in the cc diquark production process, which are due to variations in α_s and m_c .

Further in this section combined estimates are presented for the cross sections of these processes at π^-p and pp collisions, a model for ccq baryon and $J/\psi + \bar{D}D$ productions is described, the ccq and $J/\psi + \bar{D}D$ production cross sections are calculated for the conditions of fixed-target experiments (E781, HERA-B) and at collider energies (Tevatron, FNAL, LHC), and, finally, various feasible versions are discussed of searches for $\Xi_{cc}^{(*)}$ baryons.

4.3.1 Production mechanism. As already mentioned above, diquark production proceeds via two stages. First, the production cross section is calculated of four free quarks in the subprocesses

$$gg \rightarrow cc\bar{c}\bar{c}, \quad q\bar{q} \rightarrow cc\bar{c}\bar{c}.$$

The computational technique applied in the present review is similar to the technique for calculating the hadron production of B_c -mesons [53, 68], but in this case, instead of a quark and antiquark, a bound state is formed by two quarks: Q_1 and Q_2 [54, 55, 65].

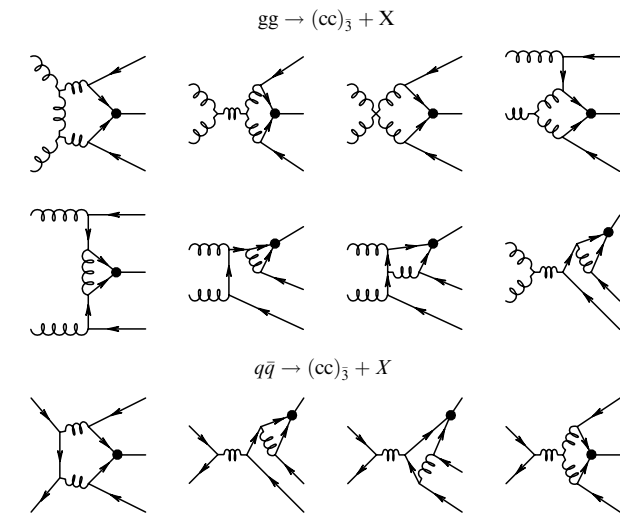


Figure 18. Diagrams of gluon – gluon and quark – antiquark production of a cc diquark. The solid lines with arrows indicate quarks, and the spiral lines indicate gluons.

¹⁴ The quantity $|R_{\psi}(0)|$ is given by the $J/\psi \rightarrow l^+l^-$ lepton decay width with due account of the hard gluon correction, so that numerically $|R_{\psi}(0)| = \sqrt{\pi M/3} \hat{f}_{\psi}$, where $\hat{f}_{\psi} = 540$ MeV.

The diquark binding energy is much smaller than the constituent quark masses, so they are on the mass surface and their 4-momenta are related to the 4-momentum P_{diq} of the diquark $Q_1 Q_2$:

$$p_1 = \frac{m_1}{\mathcal{M}_{\text{diq}}} P_{\text{diq}}, \quad p_2 = \frac{m_2}{\mathcal{M}_{\text{diq}}} P_{\text{diq}}, \quad (4.25)$$

where $\mathcal{M}_{\text{diq}} = m_1 + m_2$ is the diquark mass, m_1 and m_2 are the quark masses.

Within such an approach diquark production can be described with the aid of 36 Feynman diagrams corresponding to the production of four free quarks, by combining two quarks into an antitriplet color diquark with definite quantum numbers. This is done through the agency of projection operators:

$$\mathcal{N}(0, 0) = \left(\frac{2\mathcal{M}_{\text{diq}}}{2m_1 2m_2} \right)^{1/2} \frac{1}{\sqrt{2}} \{ \bar{u}_1(p_1, +) \bar{u}_2(p_2, -) - \bar{u}_1(p_1, -) \bar{u}_2(p_2, +) \} \quad (4.26)$$

for the scalar diquark states — the $\Xi'_{QQ'}$ ($J = 1/2$) baryon, and

$$\begin{aligned} \mathcal{N}(1, -1) &= \left(\frac{2\mathcal{M}_{\text{diq}}}{2m_1 2m_2} \right)^{1/2} \bar{u}_1(p_1, -) \bar{u}_2(p_2, -), \\ \mathcal{N}(1, 0) &= \left(\frac{2\mathcal{M}_{\text{diq}}}{2m_1 2m_2} \right)^{1/2} \frac{1}{\sqrt{2}} \{ \bar{u}_1(p_1, +) \bar{u}_2(p_2, -) + \bar{u}_1(p_1, -) \bar{u}_2(p_2, +) \}, \\ \mathcal{N}(1, +1) &= \left(\frac{2\mathcal{M}_{\text{diq}}}{2m_1 2m_2} \right)^{1/2} \bar{u}_1(p_1, +) \bar{u}_2(p_2, +) \end{aligned} \quad (4.27)$$

for the vector diquark states — the $\Xi_{QQ'}$ ($J = 1/2$) and $\Xi_{QQ'}^*$ ($J = 3/2$) baryons. For the quarks composing the diquark to be produced in the $\bar{3}_c$ -state, it is necessary for the diquark production vertex to include the color wave function $\varepsilon_{ijk}/\sqrt{2}$, where i and j are color indices of the first and second heavy quarks. For identical quarks $Q_1 = Q_2$ with equal momenta, an antisymmetrized state can exist only with spin $S = 1$.

The diquark production amplitude $A_k^{Ss_z}$ is expressed via the amplitudes $T_k^{Ss_z}(p_i)$ of free quark production in the kinematics (4.25) with substitution of projection operators for the product $\bar{u}_1 \bar{u}_2$ and under the condition that the two heavy quarks are in the color state $\bar{3}$:

$$A_k^{Ss_z} = \frac{R_{\text{diq}}(0)}{\sqrt{4\pi}} T_k^{Ss_z}(p_i), \quad (4.28)$$

where $R_{\text{diq}}(0)$ is the diquark radial wave function at zero point, k is the color state of the diquark, S and s_z are the total diquark spin and its projection, respectively.

The following parameters were adopted in calculations:

$$\alpha_s = 0.2, \quad m_c = 1.7 \text{ GeV}, \quad m_b = 4.9 \text{ GeV}, \quad (4.29)$$

$$R_{c(1S)}(0) = 0.601 \text{ GeV}^{3/2}, \quad R_{b(1S)}(0) = 0.714 \text{ GeV}^{3/2}.$$

The quantities $R_{\text{diq}}(0)$ were evaluated when solving the Schrödinger equation with the Martin potential multiplied by a factor of 1/2 that corresponds to the quark antitriplet color state. In calculating production cross sections for a

diquark consisting of two c quarks it is necessary to take into account their being identical. It is easy to understand that this circumstance leads to the scalar diquark production amplitude becoming zero, while the vector cc -diquark production amplitude can be obtained by substitution of equal masses into the production amplitude of the vector heavy diquark with quarks of different flavors and taking into account the factor 1/2 for identical quarks and antiquarks. In this approach the diquark is considered to form a baryon with unit probability, i.e. it picks up a light quark from the quark – antiquark sea at low p_\perp or undergoes fragmentation into a baryon at high p_\perp .

Typical fourth-order diagrams describing parton subprocesses are presented in Fig. 18. They can be divided into two groups: the first corresponds to the fragmentation type diagrams in which the $\bar{c}c$ pair produced subsequently emits an additional $\bar{c}c$ pair, while the second corresponds to independent gluon dissociation into $\bar{c}c$ pairs with subsequent recombination into a diquark. We shall term the second group recombination type diagrams.

The authors of some publications (see, e.g., Ref. [67]) restricted themselves to considering fragmentation type diagrams alone, reducing the expression for the cross section to the product of the $\bar{c}c$ -pair production cross section and the quark fragmentation function into a cc diquark. As is shown in Ref. [65], such an approximation is valid when two conditions are satisfied: $\mathcal{M}_{\text{diq}}^2 \ll \hat{s}$ and $p_\perp \gg \mathcal{M}_{\text{diq}}$. In the remaining region of kinematic variables the contribution of recombination diagrams is dominant. A typical transverse momentum at which the contribution from fragmentation starts to dominate in the cc diquark production amounts to $p_\perp > 35 \text{ GeV}$ (Fig. 19). Therefore, it is clear that at real p_\perp values it is necessary to take into consideration all contributions, including those due to recombination.

Such an accounting was first rendered in Ref. [65] and subsequently confirmed in Ref. [66]. Calculations were performed only for the case of gluon–gluon production,

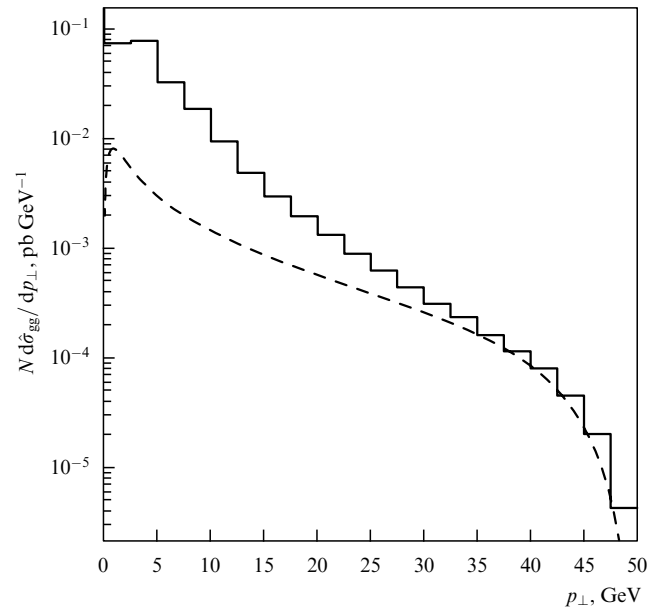


Figure 19. Differential cross section of associated cc diquark production in the gluon–gluon subprocess at 100 GeV (histogram), compared to the prediction of the fragmentation model (dashed line).

which is a good approximation for collider energies. In the case of fixed-target experiments, the total energy is drastically reduced and, consequently, the energy in the parton subprocesses, too. The role of quark–antiquark annihilation becomes essential, especially in collisions where primary valence antiquarks exist. In this review calculation is performed of the contribution of quark–antiquark annihilation to four charmed quarks, which is taken into account in estimating the doubly charmed baryon yield.

4.3.2 Doubly charmed baryon production in fixed-target experiments. In Figs 20 and 21, the calculated results are presented of total cross sections for subprocesses as functions of the subprocess energy $\sqrt{\hat{s}}$ at the above-indicated values of the parameters α_s , m_c and $R_{cc}(0)$. We shall present parametrizations for the dependence of the total cc-diquark production cross section on the subprocess energy:

$$\hat{\sigma}_{gg}^{(cc)} = 213 \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.9} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.35} \text{ pb}, \quad (4.30)$$

$$\hat{\sigma}_{q\bar{q}}^{(cc)} = 206 \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.8} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{2.9} \text{ pb}. \quad (4.31)$$

It must be noted that the numerical coefficients contain the model parameters, so that $\hat{\sigma} \sim \alpha_s^4 |R(0)|^2 / m_c^5$.

As already noted, in this review the production of J/ψ particles in subprocesses $gg \rightarrow J/\psi + c\bar{c}$ and $q\bar{q} \rightarrow J/\psi + c\bar{c}$ is computed in parallel. Parametrization of the dependence of the total J/ψ -particle production cross section upon the energy $\sqrt{\hat{s}}$ of the subprocess is given by the formulae

$$\hat{\sigma}_{gg}^{J/\psi} = 518 \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{3.0} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.45} \text{ pb}, \quad (4.32)$$

$$\hat{\sigma}_{q\bar{q}}^{J/\psi} = 699 \left(1 - \frac{4m_c}{\sqrt{\hat{s}}}\right)^{1.9} \left(\frac{4m_c}{\sqrt{\hat{s}}}\right)^{2.97} \text{ pb}. \quad (4.33)$$

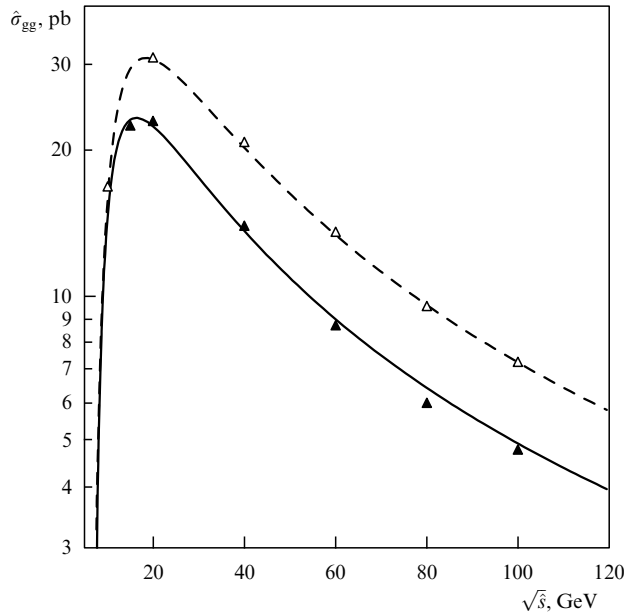


Figure 20. Total gluon–gluon production cross sections of cc diquark (dark triangles) and $J/\psi + DD$ (empty triangles), compared to approximations (4.30) and (4.32) (solid and dashed lines, respectively).

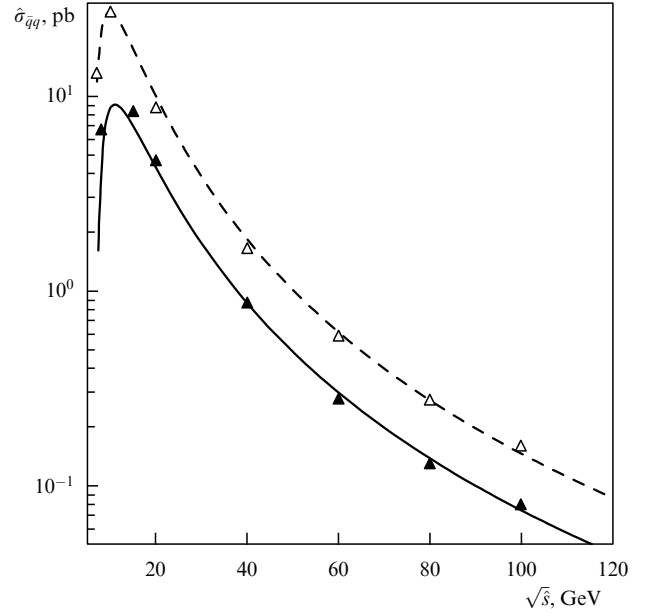


Figure 21. Total quark–antiquark production cross sections of cc diquark (dark triangles) and $J/\psi + DD$ (empty triangles), compared to approximations (4.31) and (4.33) (solid and dashed lines, respectively).

Like in the case of associated $B_c + b\bar{c}$ and $\Xi_{cc} + c\bar{c}$ productions, the following regularity is observed for the parton process of $J/\psi + c\bar{c}$ production: the fragmentation mode occurs at $p_{\perp} > 25\text{--}30$ GeV, which is clear from Fig. 22 for the differential cross section of the process $gg \rightarrow J/\psi + c\bar{c}$ at $\sqrt{\hat{s}} = 100$ GeV. Thus, for the associated $J/\psi + c\bar{c}$ and $\Xi_{cc} + c\bar{c}$ productions, fragmentation ‘works’ at $p_{\perp} \gg m_c$.

The above-presented parametrizations describe the results of accurate calculations at $\sqrt{\hat{s}} < 150$ GeV quite well

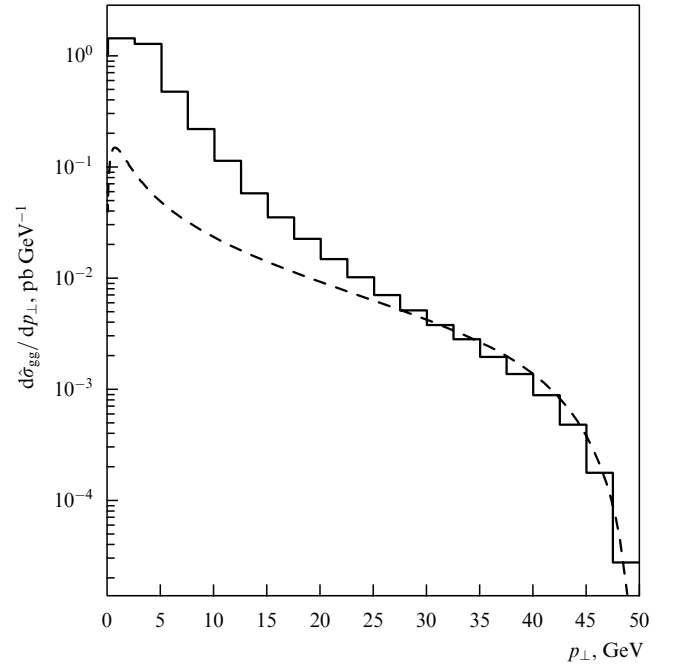


Figure 22. Differential cross section of associated $J/\psi + c\bar{c}$ production in the gluon–gluon subprocess at 100 GeV (histogram), compared with the prediction of the fragmentation model (dashed line).

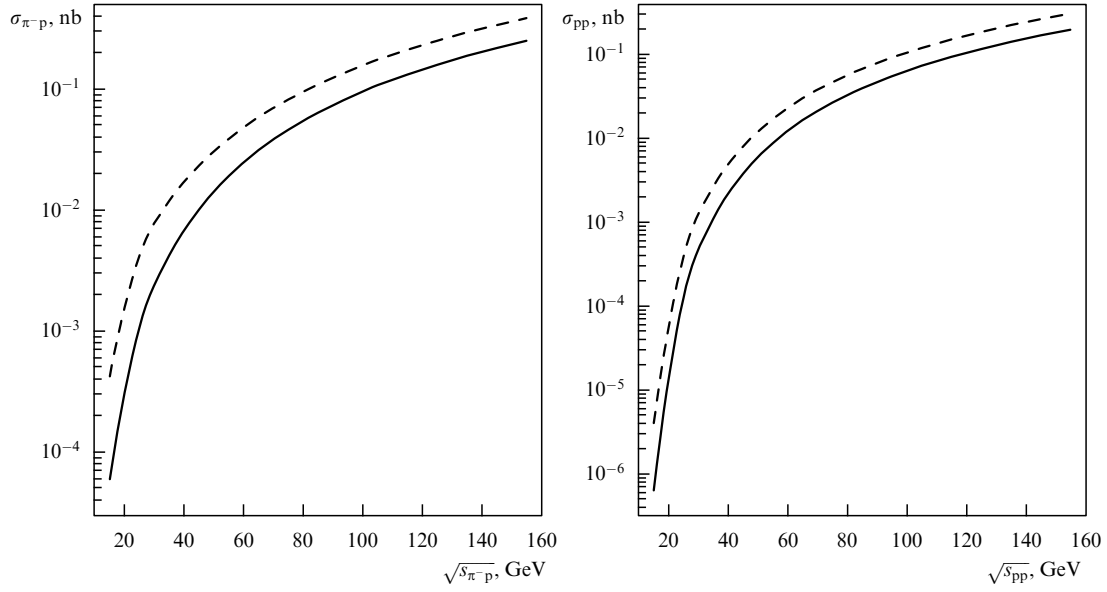


Figure 23. Total cc-diquark and $J/\psi + \bar{D}D$ production cross sections (solid and dashed lines, respectively) in π^-p - and pp -collisions.

and are entirely suitable for approximate estimates of the total cc-diquark and J/ψ -particle production cross section by convolution with parton distributions:

$$\sigma = \sum_{i,j} \int dx_1 dx_2 f_{i/A}(x_1, \mu) f_{j/B}(x_2, \mu) \hat{\sigma}, \quad (4.34)$$

where $f_{i/A}(x, \mu)$ is the distribution of the i th parton in the A hadron. The distribution function CTEQ4 [69] is adopted for the proton, and the function Hpdf [70] for the π^- -meson. In both cases the virtuality scale μ was chosen equal to 10 GeV.

Convolutions of cross sections with gluon and quark luminosities both for the cc diquark and for J/ψ -particles are presented in Fig. 23 for π^-p - and pp -collisions. As one can see from the plots, the cc-diquark and $J/\psi + \bar{c}c$ production cross sections are strongly suppressed at low energies as compared with collider energies. While in the latter case the suppression relative to the total charm production cross section is $\sigma_{cc}/\sigma_{\text{charm}} \sim 10^{-4} - 10^{-3}$, in fixed-target experiments this factor amounts to a value of the order of $10^{-6} - 10^{-5}$. The same is true for the associated production of $J/\psi + \bar{D}D$ -mesons.

Differential ccq -baryon and J/ψ -particle production cross sections are presented in paper [71] for center-of-mass energies of 35 and 40 GeV, respectively. The rapidity distribution reveals the clearly central character of ccq -baryon and J/ψ -particle production. The transverse momentum shapes of the differential cross sections are also very similar to each other for both particles (at low energies the cc diquark does not undergo fragmentation into a baryon, but simply picks up a light quark from the sea of quark–antiquark pairs). Thus, the $J/\psi + \bar{D}D$ production process can serve for normalization in estimating ccq -baryon yields, in the calculation of the production cross sections of which there exist additional uncertainties related to cc-diquark hadronization and to the unknown quantity $|R_{cc}(0)|^2$.

From the above-presented estimates it is seen that in experiments, where the expected number of events with charm should be at a level of 10^6 (for example, in the E781 experiment, when the value $\sqrt{s} = 35$ GeV) about one event is

to be expected with a $\Xi_{cc}^{(*)}$ baryon. The situation seems more favorable in the case of pp -collisions at 800 GeV (HERA-B). Here, in the conditions of an experiment aimed at observing 10^8 events with b quarks, the processes considered provide a yield of the order of $10^5 \Xi_{cc}^{(*)}$ baryons and approximately the same number of events with associated $J/\psi + \bar{D}D$ production.

4.3.3 The production of Ξ_{cc} baryons at colliders. The discussion in Section 4.3.2 revealed that observation of the $\Xi_{cc}^{(*)}$ baryon in dedicated experiments aimed at the investigation of charmed particles represents quite a complicated task. As a rule, these are fixed-target experiments, which essentially reduces the effective energy for subprocesses: the relative yield of doubly charmed baryons in these experiments with respect to the charmed particle production cross section is at the level of $10^{-6} - 10^{-5}$. The production of ccq baryons at colliders turns out to exhibit higher intensities at large p_\perp . In this case the cross section accumulates in the region of such energies of gluon–gluon and quark–antiquark subprocesses, where the threshold effect already becomes insignificant and parton luminosities at $x \sim M/\sqrt{s}$ are quite high, so that the suppression factor with respect to single $\bar{c}c$ -pair production is much smaller and amounts to $10^{-4} - 10^{-3}$. In Ref. [71], p_\perp distributions are presented for $\Xi_{cc}^{(*)}$ baryons and for J/ψ -particles, summed over rapidity at $|y| < 1$ for the Tevatron and LHC colliders.

It is easy to understand that the cross sections given here represent an upper estimate of the $\Xi_{cc}^{(*)}$ -baryon production cross section, since the heavy diquark may dissociate into a pair of D -mesons. Even if the colored object, such as the cc diquark, hadronizes with unit probability, it is necessary to introduce the fragmentation function that describes how it is dressed by a light meson at sufficiently high p_\perp . The simplest form of such a function can be chosen by analogy with fragmentation of a heavy quark:

$$D(z) \sim \frac{1}{z} \left(M^2 - \frac{\mathcal{M}_{\text{diq}}^2}{z} - \frac{m_q^2}{1-z} \right)^{-2}, \quad (4.35)$$

where M is the $\Xi_{cc}^{(*)}$ baryon mass, \mathcal{M}_{diq} is the diquark mass, and m_q is the light quark mass (we assume it equal to 300 MeV). The fragmentation function (4.35) actually repeats the form of the doubly heavy diquark fragmentation function into a baryon, computed above within the framework of QCD perturbation theory.

In Ref. [71], the distributions are presented for the hadron production of doubly charmed baryons with due regard for fragmentation in accordance with Eqn (4.35). It must be noted that the relative yield of Ξ_{cc} and Ξ_{cc}^* in the leading approximation in the inverse heavy quark mass is determined by the simple counting rule of spin states, so the $\sigma[\Xi_{cc}]/\sigma[\Xi_{cc}^*] = 1/2$. Here, the possible distinction in form of the fragmentation functions for baryons of different spins is not taken into account, unlike the case of perturbative fragmentation functions for heavy mesons and for quarkonia [51, 59].

4.3.4 Hadron production of Ξ_{bc} baryons. In Fig. 24, the energy dependence of the total cross section for the gluon production of Ξ'_{bc} and $\Xi_{bc}^{(*)}$ baryons is presented. For comparison, the predictions of the fragmentation mechanism are also presented. From the figure it is seen that the fragmentation production mechanism assuming factorization of the cross section to be possible for $M^2/s \ll 1$ by the formula

$$\frac{d}{dz} \sigma[gg \rightarrow \Xi'_{bc}(\Xi_{bc}^{(*)})\bar{b}\bar{c}] = \sigma[gg \rightarrow \bar{b}\bar{b}] D[b \rightarrow \Xi'_{bc}(\Xi_{bc}^{(*)})](z) \quad (4.36)$$

at $z = 2|\mathbf{p}|/\sqrt{s}$ does not work not only at low gluon energies, where it overestimates the cross section owing to erroneous estimation of phase-space volume (the two- instead of three-particle threshold), but also at high energies, where the predictions of the fragmentation mechanism turn out to be

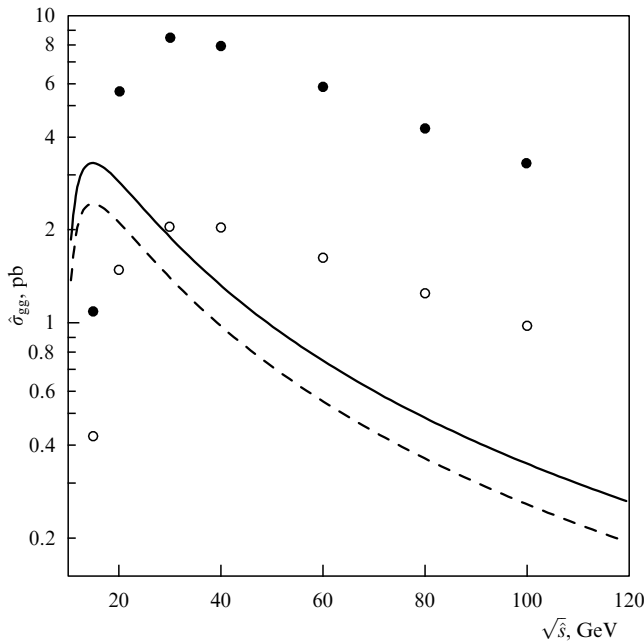


Figure 24. Total gluon–gluon production cross sections of Ξ'_{bc} (empty circles) and $\Xi_{bc}^{(*)}$ (dark circles) baryons, compared to predictions of the fragmentation mechanism for Ξ'_{bc} and $\Xi_{bc}^{(*)}$ (dashed and solid lines, respectively).

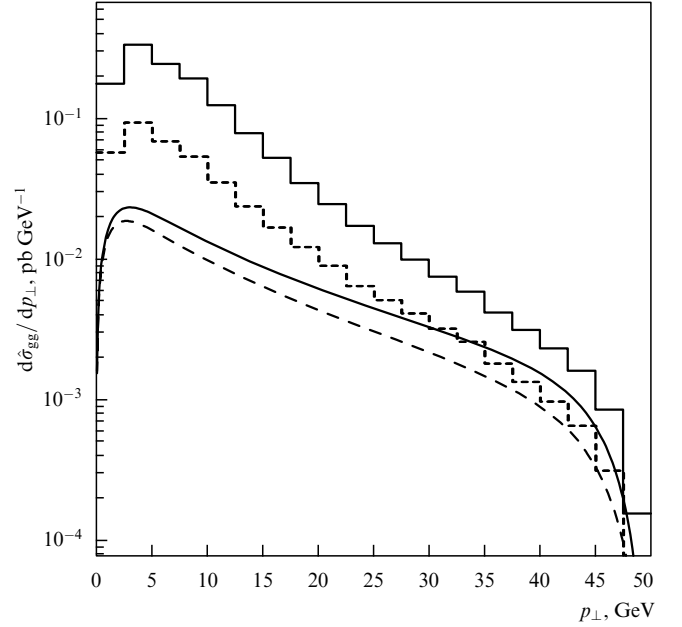


Figure 25. Transverse momentum distribution for the gluon production of Ξ'_{bc} and $\Xi_{bc}^{(*)}$ baryons (dashed and solid lines, respectively), compared to the fragmentation approximation at an interaction energy of 100 GeV. The total and fragmentation responses are represented by the histograms and the smooth curves, respectively.

significantly lower than the exact count. Thus, at $\sqrt{s} = 100$ GeV, the fragmentation mechanism underestimates the total cross section by a factor of 10 for $\Xi_{bc}^{(*)}$, and of 3 for Ξ'_{bc} . While, in accordance with fragmentation predictions, $\sigma[\Xi_{bc}^{(*)}]/\sigma[\Xi'_{bc}] \approx 1.4$, the exact count of the complete set of diagrams in the given order in the QCD coupling constant shows that $\sigma[\Xi_{bc}^{(*)}]/\sigma[\Xi'_{bc}] \approx 3.5$ even at $\sqrt{s} = 100$ GeV.

Agreement with the fragmentation mechanism at $\sqrt{s} = 100$ GeV occurs at large transverse baryon momenta, which is seen from the p_{\perp} distributions for $\Xi_{bc}^{(*)}$ and Ξ'_{bc} , presented in Fig. 25. We note that, unlike the case of baryon production, in the gluon production of B_c - and B_c^* -mesons at $p_{\perp} > 35$ GeV and energy of 100 GeV better agreement is observed of the total count and the fragmentation prediction, like for doubly charmed baryons. In the case of Ξ_{bc} baryons, divergence is observed up to the highest p_{\perp} values, although, probably, at a higher energy of the gluon–gluon subprocess the region of applicability of the fragmentation mode extends towards larger transverse momenta.

In Ref. [71], the differential cross section $d\sigma/dp_{\perp}$ is presented for Ξ'_{bc} and $\Xi_{bc}^{(*)}$ baryon production in $\bar{p}p$ -interaction at the energy $\sqrt{s} = 1.8$ TeV in comparison with the fragmentation mechanism, from which the conclusion can be made that the fragmentation approach provides a very rough estimate for the yield and momentum spectrum of Ξ_{bc} baryons. For the chosen parameters and with account of the cut-offs in transverse momentum and rapidity ($p_{\perp} > 5$ GeV, $|y| < 1$), we estimate the production cross section of bcq baryons in S -states and their anti-particles as $\sigma_{bcq} \approx 1$ nb (without cut-offs the cross section σ_{bcq} is about two times larger). By completion of Run Ib at the Tevatron with a total accumulated luminosity of 100–150 pb^{-1} this corresponds to $(1.0-1.5) \times 10^5$ events with the production of a bcq baryon.

4.3.5 Pair production of baryons with two heavy quarks. At the energies of fixed-target experiments, the luminosity of parton subprocesses with valence quarks is not inferior to the luminosity of gluon–gluon collisions in the region of large invariant masses. Here, a noticeable fraction of the total production cross section of baryons with two heavy quarks is due to the baryon pair production.

Total and differential cross sections of baryon pair production were considered in Ref. [56], where the contributions from scalar and axial-vector diquarks were taken into account. Thus, the expression for the total production cross section of scalar pairs, depending on the square of the total energy s , has the form

$$\sigma_{00} = \frac{8\pi^3}{81s^3} |\Psi_{\text{diq}}(0)|^4 \left(1 - \frac{4M^2}{s}\right)^{3/2} \times \left[\frac{16}{3} (\tilde{f}_{00}^{[1]})^2 + \frac{11}{20} \left(1 - \frac{4M^2}{s}\right) (\tilde{f}_{00}^{[2]})^2 \right]. \quad (4.37)$$

Here, $\Psi_{\text{diq}}(0)$ is the diquark wave function at zero point, while the form factors are determined by the functions

$$\tilde{f}_{00}^{[1]} = M\alpha_s^2 \left[\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} \right) - \frac{2M^2}{s} \left(\frac{m_2}{m_1^3} + \frac{m_1}{m_2^3} \right) \right], \quad (4.38)$$

$$\tilde{f}_{00}^{[2]} = \frac{M^5}{m_1^3 m_2^3} \alpha_s^2, \quad (4.39)$$

where m_1 and m_2 are the heavy quark masses, and $M = m_1 + m_2$. Numerical estimates reveal the pair production of vector diquarks to be predominant and amount to about 10% of the yield of single doubly heavy baryon production in the cross section of the parton subprocess.

4.3.6 Discussion. On the basis of perturbative calculations of the hard production of a heavy doubly charmed diquark undergoing fragmentation into a baryon we have shown that the observation of $\Xi_{cc}^{(*)}$ baryons in hadron collisions is not a simple task, since the yield of baryons relative to the production cross section of charmed particles amounts to $\sigma[\Xi_{cc}^{(*)}]/\sigma_{\text{charm}} \sim 10^{-6} - 10^{-3}$, depending on the total energy of the process. Suppression of the yield is attributed to the strong threshold effect at the energies of fixed-target experiments.

Calculations of the total cross section for the experiment at the HERA-B facility give

$$\sigma[\Xi_{cc}] \approx 2 \times 10^{-3} \text{ nb},$$

in the E781 experiment

$$\sigma[\Xi_{cc}] \approx 4.6 \times 10^{-3} \text{ nb},$$

while at the Tevatron collider

$$\sigma[\Xi_{cc}] \approx 12 \text{ nb},$$

and, finally, at LHC

$$\sigma[\Xi_{cc}] \approx 122 \text{ nb}.$$

The small production cross section of doubly charmed baryons in fixed-target experiments permits one to expect the

number of events (involving baryon production) to be of the order of 10^5 at HERA-B. With due regard for the cut-offs in transverse momentum and rapidity ($p_\perp > 5 \text{ GeV}$, $|y| < 1$) at the Tevatron for an integral luminosity of 100 pb^{-1} the yield of $\Xi_{cc}^{(*)}$ baryons is at the same level as at HERA-B. In experiments at LHC, the larger luminosity and energy permit one to expect a 10^4 -fold increase of the yield of the baryons considered. With due account of the luminosity enhancement of the hadron collider at FNAL, the experimental task of registering Ξ_{bc} and Ξ_{cc} baryons becomes feasible.

Given a sufficiently large yield of $\Xi_{cc}^{(*)}$ baryons, the question arises as to how they can be observed. First of all, it is interesting to obtain estimates for the ground state lifetimes of Ξ_{cc}^{++} and Ξ_{cc}^+ . Simple examination of quark diagrams reveals that, like in the D^+ -meson decays, in decays of the Ξ_{cc}^{++} baryon the Pauli interference effect exists for the decay products of the charmed quark with a valence quark in the initial state. In decays of the Ξ_{cc}^+ baryon there should be a manifestation of the W-boson exchange between the valence quarks, like in decays of the D^0 -meson. Therefore, the mechanisms indicated¹⁵, most likely, result in the same relation for the baryon lifetimes as for the respective D-mesons:

$$\tau[\Xi_{cc}^+] \approx 0.4 \tau[\Xi_{cc}^{++}].$$

The presence of two charmed quarks in the initial state clearly leads to the relations

$$\tau[\Xi_{cc}^{++}] \approx 0.5 \tau[D^+] \approx 0.53 \text{ ps},$$

$$\tau[\Xi_{cc}^+] \approx 0.5 \tau[D^0] \approx 0.21 \text{ ps}.$$

By analogy with the decays of a baryon with a single charmed quark it is possible to single out the following among the decay modes:

$$\begin{aligned} \text{Br} [\Xi_{cc}^{++} \rightarrow K^{0(*)} \Sigma_c^{++(*)}] &\approx \text{Br} [\Xi_{cc}^+ \rightarrow K^{0(*)} (\Sigma_c^{+(*)} + \Lambda_c^+)] \\ &\approx \text{Br} [\Lambda_c \rightarrow K^{0(*)} p] \approx 4 \times 10^{-2}. \end{aligned}$$

In such modes without regard for the decay reconstruction efficiency of the device it is possible to observe about 4×10^3 events at HERA-B and at the Tevatron. The yield to be expected at LHC amounts to 4×10^7 decays. We also note the processes $\Xi_{cc}^{++} \rightarrow \pi^+ \Xi_c^+$ and $\Xi_{cc}^+ \rightarrow \pi^+ \Xi_c^0$ with relative probabilities close to 1%. Excited Ξ_{cc}^* states decay into Ξ_{cc} owing to the emission of a photon, and the branching ratio of this transition is 100%, since, unlike decays of the D^* -meson, the emission of π -mesons is impossible owing to the small splitting between the ground and excited states.

In conclusion we shall point out one more possibility for enhancing the yield of doubly charmed baryons in fixed-target experiments. The intrinsic charm model [72] assumes that, besides the usual state $|uud\rangle$ of the proton with three light valence quarks, it also contains a nonperturbative admixture of an exotic hybrid state $|\bar{c}cuud\rangle$, the probability P_{ic} of which is suppressed at the level of 1%. The charmed valence quark from such a state can undergo recombination with the newly produced c quark in the hard parton subprocess of $\bar{c}c$ -pair production. The energy dependence of

¹⁵ Here, naive estimates of lifetimes are presented; a rigorous examination of the operator expansion for inclusive decays of doubly heavy baryons is performed in Section 5.

the production cross section of a doubly charmed baryon repeats the energy dependence of single charm production in QCD perturbation theory with an accuracy up to suppression factors of the exotic state and of fusion of charmed quark pairs into a diquark ($K \approx 0.1$).

Such a mechanism of $\Xi_{cc}^{(*)}$ -baryon production exhibits no production threshold for the four-quark state $cc\bar{c}\bar{c}$, like the one occurring in the perturbative production dealt with above of the doubly charmed diquark. Therefore, at low energies in fixed-target experiments, where threshold suppression of the perturbative mechanism is great, the intrinsic charm model could provide the dominant contribution to $\Xi_{cc}^{(*)}$ -baryon production at the level of $\sigma[\Xi_{cc}^{(*)}]/\sigma_{\text{charm}} \sim 10^{-3}$, i.e. the yield of baryons in this model is enhanced by three orders of magnitude. At high energies perturbative $\Xi_{cc}^{(*)}$ production is comparable with the contribution of intrinsic charm. We also note that the nonperturbative exotic seven-quark state $|cc\bar{c}c\bar{c}uud\rangle$ suppressed at the level of 3×10^{-4} could also enhance the yield of doubly charmed baryons in hadron-hadron collisions at low energies.

Thus, observation of $\Xi_{cc}^{(*)}$ baryons in hadron interactions seems quite a feasible task that opens up new possibilities in examining the interactions of heavy quarks. Investigation of $\Xi_{cc}^{(*)}$ -baryon production at the energies of fixed-target experiments [73] would permit one to study the production mechanism and to reveal the role of various contributions, including perturbative production and intrinsic charm that essentially increase the baryon production cross section.

5. Lifetimes and decays of $\Xi_{QQ'}$ baryons

Within the framework of the operator expansion in terms of the inverse heavy quark mass, new quite definite schemes have been developed for taking QCD effects into account consistently in calculating various characteristics of hadrons containing heavy quarks [9, 10, 74]. This permits one, on the basis of predictions of such a method, to single out the parameters of the electroweak interaction of heavy quarks against the background of the dynamics of strong interactions of quarks and gluons composing the hadrons observed in experiments. The accuracy of the QCD description in the sector of heavy quarks is extremely important for revealing subtle effects such as the violation of CP-invariance, deviations from predictions of the Standard Model, and clarification of the mechanisms of influencing the virtual corrections of the ‘new’ physics with characteristic scales exceeding teraelectron-volt energies. Therefore, studying the properties of operator expansion in the inverse heavy quark mass seems to be quite an informative problem that deserves all possible attention. Of particular interest is the complex investigation of various systems with heavy quarks, making use of comparative analysis of various characteristics: convergence of the expansion in the inverse mass and in the QCD coupling constant, the relative and absolute values of various contributions and their dependences on the system composition, qualitative inferences about the influence of various mechanisms [75] and the uncertainties of quantitative estimates.

The efficiency of this approach has been convincingly demonstrated in the description of weak decays of hadrons with one heavy quark within the framework of HQET [9], in the annihilation and radiative decays of heavy quarkonia $\bar{Q}Q$ containing heavy quarks of the same flavor within the framework of NRQCD [10], and in weak decays of the long-

lived heavy B_c^+ quarkonium of mixed flavor¹⁶ [74]. Here it must be noted that experimental data on the weak decays of hadrons with two heavy quarks are capable of introducing a significant quantitative certainty into the parameters describing systems with heavy quarks. This is due to the presence in NRQCD (as compared with HQET) of an additional parameter of smallness: the relative velocity of motion of the heavy quarks, v . Besides, there exists a significant variety of characteristics of bound states of heavy quarks in various hadrons, which permits one to study the dependence of the operator expansion on nonperturbative parameters that can be simulated, for instance, within potential quark approaches.

Fresh opportunities for the description of systems with heavy quarks have been opened up due to baryons with two heavy quarks, $QQ'q$, for which it is possible to apply a combined approach based on HQET and NRQCD [9, 10, 74], if the quark-diquark picture of such a bound state is adopted. Then, expansion in the inverse mass of the heavy quark serves for the heavy diquark QQ' as a direct generalization of the NRQCD technique [10, 74] from singlet to antitriplet color systems, taking into account the interaction of the diquark with the light quark, for which the HQET methods are assumed to work reliably.

In this section a consistent procedure is presented for calculating the lifetimes of the doubly heavy baryons Ξ_{bc}^+ , Ξ_{bc}^0 , Ξ_{cc}^+ , and Ξ_{cc}^{++} . Here, in developing the computing method we follow Refs [6, 74] with due regard for the necessary generalizations to the case of hadrons with two heavy quarks and with the introduction of a series of corrections. Such calculations are based on the optical theorem for the inclusive decay width, on the operator expansion and passage to nonrelativistic quark fields in the hadron matrix elements. First, in the operator expansion, owing to the presence of heavy quarks in the initial state, the energy released in each of their decays is, generally speaking, large compared to the energy of the bound state, so it is possible to apply expansion with respect to the scale ratios. Technically, this repeats the procedure for the inclusive decays of heavy-light mesons, a review of which can be found in Ref. [76]. Second, the nonrelativistic QCD approximation [10] permits one to reduce the calculation of the matrix elements of operators, responsible for the interaction of heavy quarks in the diquark, to a series expansion in powers of p_c/m_c , where $p_c = m_c v_c \sim 1$ GeV represents the typical momentum of the heavy quark in the baryon. The same procedure for operator matrix elements responsible for the interaction of heavy quarks with the light one leads to an expansion in powers of Λ_{QCD}/m_c .

It must be noted that in the leading approximation of the operator expansion the inclusive widths are determined by the mechanism of spectator decays of free quarks, for which corrections are taken into account by proceeding on QCD perturbation theory. Inclusion of subsequent terms of the series expansion in the inverse mass of the heavy quark¹⁷ enables one to examine the contributions due to quark confinement inside the hadron. Here, the following nonperturbative characteristics of concrete quark systems start to

¹⁶ The first experimental observation of the B_c -meson was announced in the work of the CDF collaboration [5], and a review of the theoretical status of the B_c -meson is to be found in Ref. [4].

¹⁷ As is shown in Ref. [76], there is no term linear in $1/m_Q$, and the corrections start from the second order.

play an essential role: the motion of the heavy quark inside the hadron and the ensuing time dilation in the hadron rest frame, as compared to the quark proper time, and the influence of the chromomagnetic interaction of quarks. An essential role for such corrections in baryons with two heavy quarks is played by the presence of the compact heavy diquark that determines the largest contributions to the correcting terms.

One more peculiarity consists in the quantitative influence of the hadron composition on the total lifetimes of baryons with two heavy quarks, since in the third order in the inverse mass of the heavy quark two-quark correlations in the total width are enhanced in the effective Lagrangian owing to the two-particle phase volume of the intermediate state (see discussion in Ref. [6]). In this case it is necessary to take into account the effects of Pauli interference between the decay products of heavy quarks and quarks in the initial state and of weak exchanges between the quarks composing the hadron. Here, corrections related to the masses of light and strange quarks are dealt with in estimating nonperturbative parameters in nonrelativistic models with constituent quarks. Passage to nonrelativistic heavy quarks is realized with the use of the effective weak Lagrangian with due account of the evolution of Wilson coefficients from a scale of the order of magnitude of the heavy quark masses to energies characteristic for the quark coupling in the hadron.

In Section 5.1, a general scheme is presented for constructing the operator expansion for total widths of baryons with two heavy quarks with due regard for corrections to the spectator widths. In Section 5.2, the procedure is considered for estimation of nonperturbative matrix elements for the operators of nonrelativistic heavy quarks, taken over the doubly heavy baryon states. Section 5.3 is devoted to numerical estimation of the lifetimes of baryons with two heavy quarks and to the discussion of variously typed contributions and existing uncertainties. In Section 5.4, calculation is performed of exclusive decay widths within the framework of NRQCD sum rules. The results are summarized in Section 5.5.

5.1 Operator expansion for heavy baryons

We shall now proceed to describe the method for calculating the total lifetime for the example of doubly charmed baryons. Within the framework of integral quark–hadron duality, the optical theorem permits one to relate the total decay width Γ of a particle with the imaginary part of the forward scattering amplitude. For Ξ_{cc}° baryons, where the symbol ‘ \circ ’ stands for the electric charge of the system, it is possible to write down the following relation

$$\Gamma[\Xi_{cc}^\circ] = \frac{1}{2M} \langle \Xi_{cc}^\circ | \mathcal{T} | \Xi_{cc}^\circ \rangle. \quad (5.1)$$

The state of Ξ_{cc}° with mass M in the last relation exhibits the usual relativistic normalization:

$$\langle \Xi_{cc}^\circ | \Xi_{cc}^\circ \rangle = 2EV,$$

and the transition operator \mathcal{T} is determined by the formula

$$\mathcal{T} = \text{Im} \int d^4x \, T \{ H_{\text{eff}}(x) H_{\text{eff}}(0) \}. \quad (5.2)$$

The standard effective Hamiltonian H_{eff} in Eqn (5.2), describing interactions of the initial quarks with the decay products, for example, for transitions of a c quark into a u quark and

quarks of flavors $q_{1,2}$ and charge $-1/3$, has the following form

$$H_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{uq_1} V_{cq_1}^* [C_+(\mu) O_+ + C_-(\mu) O_-] + \text{h.c.}, \quad (5.3)$$

where V is the mixing matrix of charged quark currents, and

$$O_\pm = [\bar{q}_{1\alpha} \gamma_\nu (1 - \gamma_5) c_\beta] [\bar{u}_\gamma \gamma^\nu (1 - \gamma_5) q_{2\delta}] (\delta_{\alpha\beta} \delta_{\gamma\delta} \pm \delta_{\alpha\delta} \delta_{\gamma\beta}),$$

the subscripts α and β denote quark color states, the coefficients

$$C_+ = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{6/(33-2n_f)}, \quad C_- = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{-12/(33-2n_f)},$$

and n_f is the number of flavors.

The released energy is significant in decays of heavy quarks, and it is possible to realize operator expansion of the transition Hamiltonian (5.3). As a result, there arises a series of local operators of increasing dimensions, whose contribution to the decay width Γ is suppressed by the inverse powers of the heavy quark masses. This formalism has been applied earlier in calculating total lifetimes of hadrons containing one heavy quark (see monograph [76], as well as papers [6, 77]). We stress that for doubly heavy baryons the expansion is performed simultaneously in the inverse mass of the heavy quark and in the relative velocity of motion of the heavy quarks in the baryon. This is a manifestation of the difference from the case of heavy–light mesons (expansion in Λ_{QCD}/m_c) and heavy–heavy mesons [74] (expansion in the relative velocity of motion of the heavy quark in the hadron, where the nonrelativistic QCD scaling rules [10] are applied).

In the present review a combined approach is developed in the case of baryons containing two heavy quarks. Expansion in the inverse mass leads to the relation

$$\mathcal{T} = C_1(\mu) \bar{c}c + \frac{1}{m_c^2} C_2(\mu) \bar{c}g\sigma_{\mu\nu} G^{\mu\nu}c + O\left(\frac{1}{m_c^3}\right). \quad (5.4)$$

The leading contribution is determined by the operator $\bar{c}c$ corresponding to spectator decay of c quarks. Application of the equations of motion for heavy quarks provides for the absence in the expansion of 4-dimension operators. There exists only one operator of dimension 5: $Q_{GQ} = \bar{Q}g\sigma_{\mu\nu} G^{\mu\nu}Q$. As is shown below, significant corrections arise owing to operators of dimension 6: $Q_{2Q2q} = \bar{Q}\Gamma q\bar{q}\Gamma'Q$, responsible for Pauli interference and for electroweak scattering in the case of Ξ_{cc}^{++} and Ξ_{cc}^+ , respectively, and reinforced by the factor of two-particle phase volume. The contributions of additional operators of dimension 6: $Q_{61Q} = \bar{Q}\sigma_{\mu\nu}\gamma_l D^\mu G^{\nu l}Q$, $Q_{62Q} = \bar{Q}D_\mu G^{\mu\nu}\Gamma_\nu Q$, are not taken into account, since they are suppressed by the aforementioned smallness of the three-particle phase volume, and the expansion can be considered complete only with an accuracy up to $1/m^2$.

Thus, various terms in the operator expansion can be represented in the form

$$\mathcal{T}[\Xi_{cc}^{++}] = \mathcal{T}_{35c} + \mathcal{T}_{6,\text{PI}}, \quad \mathcal{T}[\Xi_{cc}^+] = \mathcal{T}_{35c} + \mathcal{T}_{6,\text{WS}}.$$

The first terms in the expressions presented correspond to taking into account operators of dimensions 3 and 5: O_{3Q} and O_{GQ} , respectively, the second terms correspond to effects of Pauli interference and of electroweak scattering. In accor-

dance with Refs [76, 78, 79], the explicit expressions are given by the formulae

$$\mathcal{T}_{35c} = 2 \left\{ \Gamma_{c, \text{spec}} \bar{c}c - \frac{\Gamma_{0c}}{m_c^2} [(2 + K_{0c})P_1 + K_{2c}P_2] O_{Gc} \right\}, \quad (5.5)$$

$$\Gamma_{0c} = \frac{G_F^2 m_c^2}{192\pi^3}, \quad K_{0c} = C_-^2 + 2C_+^2, \quad K_{2c} = 2(C_+^2 - C_-^2).$$

The factors related to integration over phase space P_i are determined by expressions [76, 80]

$$P_1 = (1 - y)^4, \quad P_2 = (1 - y)^3,$$

where $y = m_s^2/m_c^2$. The symbol $\Gamma_{c, \text{spec}}$ in formulae (5.5) represents the contribution to the total width from the decay of one of the free c quarks, the explicit expression for which is given in paper [81]. The quite cumbersome expressions for the contributions to the inclusive widths of effects due to Pauli interference and to electroweak scattering are presented in Appendix 7.3.

Threshold effects related to the b -quark mass and, also, to the c -quark mass (in the case of Pauli interference and electroweak scattering) are taken into account in numerical estimates of the coefficients C_+ and C_- . In the expressions for C_+ and C_- , the scale μ is approximately equal to m_c . In the case of effects related to Pauli interference and electroweak scattering the scale in the factor k (see Appendix 7.3) is chosen so as to achieve agreement between the experimental difference in the lifetimes of Λ_c , Ξ_c^+ , Ξ_c^0 baryons and the theoretical predictions based on taking into account the aforementioned effects. This issue is dealt with in detail below. Naturally, in any case the choice of scales admits certain variations, and complete clarification of this issue requires calculations in the next order of perturbation theory.

The contribution from the leading operator $\bar{c}c$ corresponds to the imaginary part of the diagram depicted in Fig. 26, and it enters into expression (5.5). The coefficient of $\bar{c}c$ can be obtained in a standard way through the projection of the contribution from the diagram onto the operator $\bar{c}c$. This coefficient gives the expression for the decay probability of a free quark in the order next to the leading order in QCD perturbation theory [82–86] and includes effects related to the mass of the s quark in the final state [86]. To take into consideration logarithmic effects, it is necessary to know the Wilson coefficients in the effective Lagrangian with a given accuracy and the one-gluon corrections to the diagram shown in Fig. 26. In numerical estimations the expression used for Γ_{spec} includes corrections of the order of magnitude of α_s^2 in QCD perturbation theory and a correction due to the nonzero mass of the s quark in the final state. Cabibbo-suppressed channels in the decays of c quarks are not considered, since they are negligible. The cumbersome expression for the spectator decay of the c quark is presented in the Appendix to paper [81].

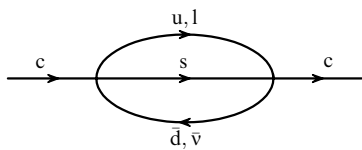


Figure 26. Spectator contribution to the total decay width of doubly charmed baryons.

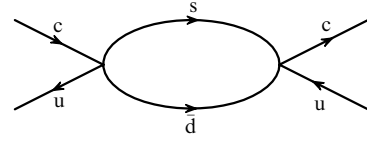


Figure 27. Contribution of Pauli interference of the c -quark decay products with the valence quark in the initial state in the case of Ξ_{cc}^{++} baryons.

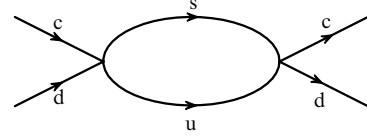


Figure 28. Contribution of the electroweak scattering of valence quarks in the initial state in the case of Ξ_{cc}^+ baryons.

Similarly, the contribution of the operator O_{GQ} is obtained when the external gluon line is attached in all possible ways to the internal quark lines in the diagram presented in Fig. 26. The coefficient functions for this operator are known in the leading logarithmic approximation. Diagrams for 6-dimension operators are shown in Figs 27 and 28. Their contributions correspond to effects of Pauli interference and electroweak scattering. Expressions for them are known with due regard for the s -quark mass and logarithmic renormalization of the effective electroweak Lagrangian in the low-energy region, the form of which is given in Appendix 7.3.

The following formulae [79] (see, also, Ref. [86]) are used for calculating the contribution of semilepton modes to the total decay width of the baryons considered (the electron and muon channels):

$$\begin{aligned} \Gamma_{sl} = & 4\Gamma_c [1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln \rho \\ & + E_c(5 - 24\rho + 24\rho^2 - 8\rho^3 + 3\rho^4 - 12\rho^2 \ln \rho) \\ & + K_c(-6 + 32\rho - 24\rho^2 - 2\rho^4 + 24\rho^2 \ln \rho) \\ & + G_c(-2 + 16\rho - 16\rho^3 + 2\rho^4 + 24\rho^2 \ln \rho)], \end{aligned} \quad (5.6)$$

where

$$\begin{aligned} \Gamma_c = & |V_{cs}|^2 G_F^2 \frac{m_c^5}{192\pi^3}, \quad \rho = \frac{m_s^2}{m_c^2}, \quad E_c = K_c + G_c, \\ K_c = & -\frac{1}{2m_c^2} \langle \Xi_{cc}^\circ(v) | \bar{c}_v(i\mathbf{D})^2 c_v | \Xi_{cc}^\circ(v) \rangle, \\ G_c = & \frac{1}{4m_c^2} \langle \Xi_{cc}^\circ(v) | \bar{c}_v g G_{\alpha\beta} \sigma^{\alpha\beta} c_v | \Xi_{cc}^\circ(v) \rangle, \end{aligned} \quad (5.7)$$

and here the spinor c_v represents the standard field in the effective theory of heavy quarks:

$$c(x) = \exp(-im_c v x) \left[1 + \frac{i D_\mu \gamma^\mu}{2m_c} \right] c_v(x), \quad (5.8)$$

while the symbol D_μ denotes derivatives with respect to the coordinates.

A similar scheme for calculating the widths Γ of the Ξ_{bc}° baryon was developed in Ref. [87]. As a result, the following

expressions are obtained for the total widths:

$$\begin{aligned}\mathcal{T}[\Xi_{bc}^+] &= \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{6,PI}^{(1)} + \mathcal{T}_{6,WS}^{(1)}, \\ \mathcal{T}[\Xi_{bc}^0] &= \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{6,PI}^{(2)} + \mathcal{T}_{6,WS}^{(2)}.\end{aligned}$$

Here, the first two terms stand for contributions to the Q -quark decay from the operators of dimensions 3 and 5, while the subsequent terms are due to the effects of interference and rescattering of the constituents (see the explicit form in Ref. [87]).

Computation of the Pauli interference of the decay products of heavy quarks with quarks in the initial state and of weak rescattering of the quarks composing the hadron yields a sum over various decay channels:

$$\begin{aligned}\mathcal{T}_{6,PI}^{(1)} &= \mathcal{T}_{PI,u\bar{d}}^c + \mathcal{T}_{PI,s\bar{c}}^b + \mathcal{T}_{PI,d\bar{u}}^b + \sum_l \mathcal{T}_{PI,l\bar{v}_l}^b, \\ \mathcal{T}_{6,PI}^{(2)} &= \mathcal{T}_{PI,s\bar{c}}^b + \mathcal{T}_{PI,d\bar{u}}^b + \mathcal{T}_{PI,d\bar{u}}^{b'} + \sum_l \mathcal{T}_{PI,l\bar{v}_l}^b, \\ \mathcal{T}_{6,WS}^{(1)} &= \mathcal{T}_{WS,bu} + \mathcal{T}_{WS,bc}, \\ \mathcal{T}_{6,WS}^{(2)} &= \mathcal{T}_{WS,cd} + \mathcal{T}_{WS,bc},\end{aligned}\quad (5.9)$$

where

$$\begin{aligned}\mathcal{T}_{PI,d\bar{u}}^b &= \mathcal{T}_{PI,s\bar{c}}^b(z_- \rightarrow 0), \\ \mathcal{T}_{PI,e\bar{\nu}_e}^b &= \mathcal{T}_{PI,\mu\bar{\nu}_\mu}^b = \mathcal{T}_{PI,\tau\bar{\nu}_\tau}^b(z_\tau \rightarrow 0).\end{aligned}\quad (5.10)$$

Now, α_s -corrections to semilepton quark widths are also considered. Thus, the calculation of the total lifetime of baryons containing two heavy quarks is reduced to the task of estimating the matrix elements of operators, which is the subject of Section 5.2.

5.2 Hadron matrix elements

In accordance with the equations of motion, the matrix element of the operator $\bar{Q}Q$ can be expanded in a power series of $1/m_Q$:

$$\begin{aligned}\frac{1}{2M} \langle \Xi_{\bar{Q}Q'}^\circ | \bar{Q}Q | \Xi_{\bar{Q}Q'}^\circ \rangle &= 1 - \frac{1}{4Mm_Q^2} \langle \Xi_{\bar{Q}Q'}^\circ | \bar{Q}(i\mathbf{D})^2 Q | \Xi_{\bar{Q}Q'}^\circ \rangle \\ &+ \frac{i}{8Mm_Q^2} \langle \Xi_{\bar{Q}Q'}^\circ | \bar{Q}\sigma G Q | \Xi_{\bar{Q}Q'}^\circ \rangle + O\left(\frac{1}{Mm_Q^3}\right).\end{aligned}\quad (5.11)$$

Thus, it is necessary to submit estimates of numerical values for the following set of operators:

$$\begin{aligned}\bar{Q}(i\mathbf{D})^2 Q, \\ \frac{i}{2} \bar{Q}\sigma G Q, \\ \bar{Q}\gamma_\alpha(1-\gamma_5)Q\bar{q}\gamma^\alpha(1-\gamma_5)q, \\ \bar{Q}\gamma_\alpha\gamma_5 Q\bar{q}\gamma^\alpha(1-\gamma_5)q, \\ \bar{Q}\gamma_\alpha\gamma_5 Q\bar{Q}\gamma^\alpha(1-\gamma_5)Q, \\ \bar{Q}\gamma_\alpha(1-\gamma_5)Q\bar{Q}\gamma^\alpha(1-\gamma_5)Q.\end{aligned}\quad (5.12)$$

The first of them corresponds to quark motion within the hadron and leads to an effect (suppressed by the square of mass) due to the time dilation in the rest frame of the hadron as compared with the proper quark time. The second operator corresponds to switching on the chromomagnetic interaction

of quarks. The subsequent operators depend on four quark fields and are related to Pauli interference and weak rescattering.

Further, following the general methods of effective theories, we introduce the effective field Ψ_Q which in this case represents the nonrelativistic spinor of the heavy quark; we take also into account, within the framework of QCD perturbation theory, contributions with virtualities μ for which $m_Q > \mu > m_Q v_Q$, and, finally, we express nonperturbative effects in the matrix elements via effective nonrelativistic fields. As a result we obtain

$$\begin{aligned}\bar{Q}Q &= \Psi_Q^\dagger \Psi_Q - \frac{1}{2m_Q^2} \Psi_Q^\dagger (i\mathbf{D})^2 \Psi_Q + \frac{3}{8m_Q^4} \Psi_Q^\dagger (i\mathbf{D})^4 \Psi_Q \\ &- \frac{1}{2m_Q^2} \Psi_Q^\dagger g\sigma\mathbf{B} \Psi_Q - \frac{1}{4m_Q^3} \Psi_Q^\dagger g\mathbf{D}\mathbf{E} \Psi_Q + \dots,\end{aligned}\quad (5.13)$$

$$\bar{Q}g\sigma_{\mu\nu}G^{\mu\nu}Q = -2\Psi_Q^\dagger g\sigma\mathbf{B}\Psi_Q - \frac{1}{m_Q} \Psi_Q^\dagger g\mathbf{D}\mathbf{E}\Psi_Q + \dots\quad (5.14)$$

Here we have dropped the term $\Psi_Q^\dagger g\sigma[\mathbf{E} \times \mathbf{D}]\Psi_Q$ corresponding to spin-orbit interactions which are equated to zero for the ground state of the baryons considered. Assuming the normalization

$$\int d^3x \Psi_Q^\dagger \Psi_Q = \int d^3x Q^\dagger Q,\quad (5.15)$$

for

$$Q \equiv \exp(-imt) \begin{pmatrix} \phi \\ \chi \end{pmatrix},\quad (5.16)$$

we arrive at

$$\Psi_Q = \left(1 + \frac{(i\mathbf{D})^2}{8m_Q^2}\right)\phi.\quad (5.17)$$

Let us stress the distinction between describing the interactions of a heavy quark with a light quark and the interactions of a heavy quark with a heavy quark. In the heavy subsystem there exists an additional parameter — the relative velocity of motion of the quarks, which sets the energy scale $m_Q v$. Therefore, for example, the Darwin term \mathbf{DE} in the heavy subsystem turns out to be of the same order of smallness in the inverse heavy quark mass as compared with the chromomagnetic term $\sigma\mathbf{B}$ (of the same degree of smallness in the velocity v). This becomes especially clear if one makes use of the scaling relationships in nonrelativistic QCD [10]:

$$\begin{aligned}\Psi_Q &\sim (m_Q v_Q)^{3/2}, \quad |\mathbf{D}| \sim m_Q v_Q, \quad g|\mathbf{E}| \sim m_Q^2 v_Q^3, \\ g|\mathbf{B}| &\sim m_Q^2 v_Q^4, \quad g \sim v_Q^{1/2}.\end{aligned}$$

There exists no small parameter of relative velocity for the interaction of the heavy quark with the light one, so that the Darwin term is suppressed by an additional factor $k/m_Q \sim \Lambda_{\text{QCD}}/m_Q$.

5.2.1 Ξ_{cc}^+ and Ξ_{cc}^{++} baryons. We shall now proceed to calculate matrix elements within potential models for a bound state. We note that the kinetic energy of the heavy quark in the baryon is composed of two summands: due to the quark motion inside the diquark, and to the diquark itself. In

accordance with the phenomenology of potential meson models in the region of intermediate interquark distances ($0.1 \text{ fm} < r < 1 \text{ fm}$), the mean quark kinetic energy is a constant quantity independent of the quark composition of the meson and the quantum numbers of the excitation from the ground state. Bearing this in mind we denote the mean kinetic energy of the diquark and the light quark by $T = \mathcal{M}_{\text{diq}} v_{\text{diq}}^2/2 + m_1 v_1^2/2$, and the kinetic energy of motion of the heavy quarks in the diquark by $T/2 = m_c v_1^2/2 + m_c v_2^2/2$ (the coefficient $1/2$ takes into account the fact that the diquark under consideration is in the antisymmetric color state). Then we find

$$\frac{1}{2Mm_c^2} \langle \Xi_{cc}^\circ | \Psi_c^\dagger (\mathbf{iD})^2 \Psi_c | \Xi_{cc}^\circ \rangle \approx v_c^2 \approx \frac{m_1 T}{2m_c^2 + m_c m_1} + \frac{T}{2}. \quad (5.18)$$

Making use of the value $T \approx 0.4 \text{ GeV}$, we obtain the estimate $v_c^2 = 0.146$, where the contribution of the quark motion in the diquark is dominant.

We now proceed to calculate the matrix element of the operator corresponding to the interaction of heavy quarks with the chromomagnetic field of the light quark. To this end we introduce the following operators

$$O_{\text{mag}} = \sum_{i=1}^2 \frac{g_s}{4m_c} \bar{c}^i \sigma_{\mu\nu} G^{\mu\nu} c^i,$$

$$O_{\text{mag}} \propto \lambda [J(J+1) - S_{\text{diq}}(S_{\text{diq}}+1) - S_l(S_l+1)].$$

Here, S_{diq} is the diquark spin (as was noted, only the vector state of the cc diquark is not forbidden in the ground state), S_l is the spin of the light quark, and J is the total baryon spin. Since the contributions of both c quarks enter additively into the total decay width of the baryons under consideration, we apply the concept of the diquark and substitute the diquark spin for the sum of the c -quark spins. In the last analysis this leads to the parametrization adopted above for O_{mag} and permits one to relate the matrix element of the operator O_{mag} to the difference between the masses of the excited and ground states:

$$O_{\text{mag}} = -\frac{2}{3} (M[\Xi_{cc}^{*\circ}] - M). \quad (5.19)$$

The interactions of heavy quarks inside the diquark determine the chromomagnetic and ‘Darwin’ terms:

$$\frac{1}{2M} \langle \Xi_{cc}^\circ | \Psi_c^\dagger g \mathbf{\sigma B} \Psi_c | \Xi_{cc}^\circ \rangle = \frac{2g^2}{9m_c} |\Psi_{\text{diq}}(0)|^2, \quad (5.20)$$

$$\frac{1}{2M} \langle \Xi_{cc}^\circ | \Psi_c^\dagger g \mathbf{DE} \Psi_c | \Xi_{cc}^\circ \rangle = \frac{2g^2}{3} |\Psi_{\text{diq}}(0)|^2, \quad (5.21)$$

where $\Psi_{\text{diq}}(0)$ is the diquark wave function at zero point. Collecting the results together, we obtain for the matrix element of the dominant spectator decay operator the following relations:

$$\begin{aligned} \frac{1}{2M} \langle \Xi_{cc}^\circ | \bar{c} c | \Xi_{cc}^\circ \rangle &= 1 - \frac{1}{2} v_c^2 - \frac{1}{3} \frac{M[\Xi_{cc}^{*\circ}] - M}{m_c} \\ &\quad - \frac{g^2}{9m_c^3} |\Psi_{\text{diq}}(0)|^2 - \frac{g^2}{6m_c^3} |\Psi_{\text{diq}}(0)|^2 + \dots \\ &\approx 1 - 0.074 - 0.004 - 0.003 - 0.005 + \dots \end{aligned} \quad (5.22)$$

From the last relation it is seen that the largest contribution to reduction of the decay width is due to the time dilation related to the motion of a heavy quark in the baryon. For the matrix element of the operator $\bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c$ we write

$$\begin{aligned} &\frac{1}{2Mm_c^2} \langle \Xi_{cc}^\circ | \bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c | \Xi_{cc}^\circ \rangle \\ &= -\frac{4}{3} \frac{M[\Xi_{cc}^{*\circ}] - M}{m_c} - \frac{4g^2}{9m_c^3} |\Psi_{\text{diq}}(0)|^2 - \frac{g^2}{3m_c^3} |\Psi_{\text{diq}}(0)|^2. \end{aligned} \quad (5.23)$$

We now proceed to calculate the matrix elements of the four-quark operators responsible for effects of the Pauli interference and electroweak scattering. Calculations within the framework of nonrelativistic QCD give

$$\langle \bar{c} \gamma_\mu (1 - \gamma_5) c \bar{q} \gamma^\mu (1 - \gamma_5) q \rangle = 2m_c V^{-1} (1 - 4S_c S_q), \quad (5.24)$$

$$\langle \bar{c} \gamma_\mu \gamma_5 c \bar{q} \gamma^\mu (1 - \gamma_5) q \rangle = -4S_c S_q 2m_c V^{-1}. \quad (5.25)$$

Here we have introduced $V^{-1} = |\Psi_1(0)|^2$, where $\Psi_1(0)$ is the light quark wave function at zero point in the rest frame of one of the c quarks. For estimating $|\Psi_1(0)|$ we take the value characteristic for D -mesons:

$$|\Psi_1(0)|^2 \approx \frac{f_D^2 m_D^2}{12m_c}. \quad (5.26)$$

We note that numerical value of $|\Psi_1(0)|$, obtained from the lepton constant $f_D \approx 200 \text{ MeV}$ of the charmed meson, is about two times smaller than the value of the light quark wave function, calculated in Section 2 within the approximation of quark–diquark factorization. This is due to the lepton constants of charmed hadrons acquiring greater corrections both of the logarithmic type and of power character in the inverse charmed quark mass. For example, the lepton constant of the D -meson in potential models is about twice the value obtained in QCD sum rules taking into account the aforementioned corrections. Therefore, approximation (5.26) for the wave function of the light quark in the baryon can be considered quite justified.

Furthermore, taking into account the fact that the contributions of both c quarks enter additively into the total decay width and making use of the diquark concept, in estimating the matrix elements of operators we substitute the diquark spin S_{diq} for the sum $S_1 + S_2$. As a result we find

$$\langle \Xi_{cc}^\circ | [\bar{c} \gamma_\mu (1 - \gamma_5) c] [\bar{q} \gamma^\mu (1 - \gamma_5) q] | \Xi_{cc}^\circ \rangle = 12m_c |\Psi_1(0)|^2, \quad (5.27)$$

$$\langle \Xi_{cc}^\circ | (\bar{c} \gamma_\mu \gamma_5 c) [\bar{q} \gamma^\mu (1 - \gamma_5) q] | \Xi_{cc}^\circ \rangle = 8m_c |\Psi_1(0)|^2. \quad (5.28)$$

The color antisymmetry of the baryon wave function relates the baryon matrix elements of operators with various color summations:

$$\begin{aligned} &\langle \Xi_{cc}^\circ | (\bar{c}_i T_\mu c_k) [\bar{q} \gamma^\mu (1 - \gamma_5) q_i] | \Xi_{cc}^\circ \rangle \\ &= -\langle \Xi_{cc}^\circ | (\bar{c} T_\mu c) [\bar{q} \gamma^\mu (1 - \gamma_5) q] | \Xi_{cc}^\circ \rangle, \end{aligned}$$

where T_μ is an arbitrary spinor structure. Thus, we have formally constructed the procedure for estimating matrix elements resulting from the operator expansion \mathcal{T} for a baryon with two identical heavy quarks.

5.2.2 Ξ_{bc}^+ and Ξ_{bc}^0 baryons. In considering baryons with two heavy quarks of differing flavors we will underline which modifications must be made for estimation of hadron matrix elements of the quark operators, determining the widths of inclusive decays. In accordance with the quark–diquark factorization for kinetic terms we have

$$\frac{1}{2Mm_c^2} \langle \Xi_{bc}^\circ | \Psi_c^\dagger (\mathbf{iD})^2 \Psi_c | \Xi_{bc}^\circ \rangle \approx v_c^2$$

$$\approx \frac{2m_l T}{(m_l + m_b + m_c)(m_b + m_c)} + \frac{m_b T}{m_c(m_c + m_b)}, \quad (5.29)$$

$$\frac{1}{2Mm_b^2} \langle \Xi_{bc}^\circ | \Psi_b^\dagger (\mathbf{iD})^2 \Psi_b | \Xi_{bc}^\circ \rangle \approx v_b^2$$

$$\approx \frac{2m_l T}{(m_l + m_b + m_c)(m_b + m_c)} + \frac{m_c T}{m_b(m_c + m_b)}. \quad (5.30)$$

Numerically $T \approx 0.4$ GeV, which results in the values of $v_c^2 = 0.195$ and $v_b^2 = 0.024$, where the decisive contribution is due to motion inside the diquark.

We define the following operators

$$O_{\text{mag}} = \frac{g_s}{4m_c} \bar{c} \sigma_{\mu\nu} G^{\mu\nu} c + \frac{g_s}{4m_b} \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b, \quad (5.31)$$

$$O_{\text{mag}} = \frac{\lambda}{m_c} [S_{cl}(S_{cl} + 1) - S_c(S_c + 1) - S_l(S_l + 1)]$$

$$+ \frac{\lambda}{m_b} [S_{bl}(S_{bl} + 1) - S_b(S_b + 1) - S_l(S_l + 1)], \quad (5.32)$$

with $S_{bl} = S_b + S_l$, $S_{cl} = S_c + S_l$, S_b and S_c are the b- and c-quark spins, while S_l is the light quark spin. The operator studied is related to hyperfine splitting in the baryon system with a vector diquark $S_{bc} = 1$:

$$\langle S = 3/2 | O_{\text{mag}} | S = 3/2 \rangle - \langle S = 1/2 | O_{\text{mag}} | S = 1/2 \rangle$$

$$= \langle S = 3/2 | V_{\text{hf}} | S = 3/2 \rangle - \langle S = 1/2 | V_{\text{hf}} | S = 1/2 \rangle, \quad (5.33)$$

where S is the total spin of the system.

The spin-dependent perturbation is given by

$$V_{\text{hf}} = \frac{8}{9} \frac{\alpha_s}{m_l m_c} S_l S_c |R_l(0)|^2 + \frac{8}{9} \frac{\alpha_s}{m_l m_b} S_l S_b |R_l(0)|^2. \quad (5.34)$$

Here $R_l(0)$ is the radial wave function of the quark–diquark system at zero point. Unlike the diquark system with identical quarks, this operator is off-diagonal in the basis of S and S_{bc} . For calculations we take advantage of the substitutions

$$|S; S_{bc}\rangle = \sum_{S_{bl}} (-1)^{S+S_l+S_c+S_b} [(2S_{bl} + 1)(2S_{bc} + 1)]^{1/2}$$

$$\times \begin{Bmatrix} \bar{S}_l & S_b & S_{bl} \\ S_c & S & S_{bc} \end{Bmatrix} |S; S_{bl}\rangle, \quad (5.35)$$

$$|S; S_{bc}\rangle = \sum_{S_{cl}} (-1)^{S+S_l+S_c+S_b} [(2S_{cl} + 1)(2S_{bc} + 1)]^{1/2}$$

$$\times \begin{Bmatrix} \bar{S}_l & S_c & S_{cl} \\ S_b & S & S_{bc} \end{Bmatrix} |S; S_{cl}\rangle. \quad (5.36)$$

The result of substitutions gives

$$\lambda = \frac{4}{9} \frac{\alpha_s}{m_l} |R_l(0)|^2, \quad (5.37)$$

however, for the state with zero heavy diquark spin, considered below, one obtains

$$\frac{1}{2M} \langle \Xi_{bc}^\circ | O_{\text{mag}} | \Xi_{bc}^\circ \rangle = 0. \quad (5.38)$$

Taking into account the chromomagnetic and Darwin interactions inside the diquark leads to the formulae

$$\frac{1}{2M} \langle \Xi_{bc}^\circ | \Psi_c^\dagger g \boldsymbol{\sigma} \mathbf{B} \Psi_c | \Xi_{bc}^\circ \rangle = -\frac{2}{3m_b} g^2 |\Psi_{\text{diq}}(0)|^2, \quad (5.39)$$

$$\frac{1}{2M} \langle \Xi_{bc}^\circ | \Psi_c^\dagger g \mathbf{D} \mathbf{E} \Psi_c | \Xi_{bc}^\circ \rangle = \frac{2}{3} g^2 |\Psi_{\text{diq}}(0)|^2. \quad (5.40)$$

Similar matrix elements¹⁸ for operators with beauty quarks can be obtained from those presented above by substitution of the heavy quark masses.

Combining the results we arrive at

$$\frac{1}{2M} \langle \Xi_{bc}^\circ | \bar{c} c | \Xi_{bc}^\circ \rangle = 1 - \frac{1}{2} v_c^2 + \frac{g^2}{3m_b m_c^2} |\Psi_{\text{diq}}(0)|^2$$

$$- \frac{g^2}{6m_c^3} |\Psi_{\text{diq}}(0)|^2 + \dots$$

$$\approx 1 - 0.097 + 0.004 - 0.007 + \dots \quad (5.41)$$

The dominant part in the corrections is assumed by the term related to the time dilation due to the quark motion inside the diquark. Furthermore, for the operator $cg\sigma_{\mu\nu}G^{\mu\nu}c$ we have

$$\frac{1}{2Mm_c^2} \langle \Xi_{bc}^\circ | \bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c | \Xi_{bc}^\circ \rangle$$

$$= \frac{4g^2}{3m_b m_c^2} |\Psi_{\text{diq}}(0)|^2 - \frac{g^2}{3m_c^3} |\Psi_{\text{diq}}(0)|^2 \approx 0.002. \quad (5.42)$$

Permutations of the quark masses lead to similar expressions for the operators $\bar{b}b$ and $\bar{b}g\sigma_{\mu\nu}G^{\mu\nu}b$.

Making use of relations (5.35) and (5.36), we write down for the baryon ground states:

$$\langle \Xi_{bc}^\circ | [\bar{b}\gamma_\mu(1 - \gamma_5)b] [\bar{c}\gamma^\mu(1 - \gamma_5)c] | \Xi_{bc}^\circ \rangle$$

$$= 8(m_b + m_c) |\Psi_{\text{diq}}(0)|^2, \quad (5.43)$$

$$\langle \Xi_{bc}^\circ | (\bar{b}\gamma_\mu\gamma_5b) [\bar{c}\gamma^\mu(1 - \gamma_5)c] | \Xi_{bc}^\circ \rangle$$

$$= 6(m_b + m_c) |\Psi_{\text{diq}}(0)|^2, \quad (5.44)$$

$$\langle \Xi_{bc}^\circ | [\bar{b}\gamma_\mu(1 - \gamma_5)b] [\bar{q}\gamma^\mu(1 - \gamma_5)q] | \Xi_{bc}^\circ \rangle$$

$$= 2(m_b + m_l) |\Psi_1(0)|^2, \quad (5.45)$$

$$\langle \Xi_{bc}^\circ | (\bar{b}\gamma_\mu\gamma_5b) [\bar{q}\gamma^\mu(1 - \gamma_5)q] | \Xi_{bc}^\circ \rangle = 0, \quad (5.46)$$

$$\langle \Xi_{bc}^\circ | [\bar{c}\gamma_\mu(1 - \gamma_5)c] [\bar{q}\gamma^\mu(1 - \gamma_5)q] | \Xi_{bc}^\circ \rangle$$

$$= 2(m_c + m_l) |\Psi_1(0)|^2, \quad (5.47)$$

$$\langle \Xi_{bc}^\circ | (\bar{c}\gamma_\mu\gamma_5c) [\bar{q}\gamma^\mu(1 - \gamma_5)q] | \Xi_{bc}^\circ \rangle = 0. \quad (5.48)$$

Now we can proceed to deal with numerical estimates of inclusive widths.

5.3. Numerical estimates

Summing up the various contributions described above we will estimate the lifetimes of Ξ_{cc}^{++} and Ξ_{cc}^+ baryons. First of all, we present the values of the parameters and comment on their

¹⁸ The expressions obtained differ from the results for the B_c -meson [74] by the factor 1/2 accounting for the color structure of the state.

choice:

$$\begin{aligned} m_c &= 1.6 \text{ GeV}, & m_s &= 0.45 \text{ GeV}, \\ m_l &= 0.30 \text{ GeV}, & |V_{cs}| &= 0.9745, \\ M[\Xi_{cc}^{++}] &= 3.56 \text{ GeV}, & M[\Xi_{cc}^+] &= 3.56 \text{ GeV}, \\ M[\Xi_{cc}^{*\circ}] - M[\Xi_{cc}^\circ] &= 0.1 \text{ GeV}, \\ T &= 0.4 \text{ GeV}, & |\Psi_{\text{diq}}(0)| &= 0.17 \text{ GeV}^{3/2}. \end{aligned} \quad (5.49)$$

For masses $M[\Xi_{cc}^{++}]$, $M[\Xi_{cc}^+]$, and $M[\Xi_{cc}^{*\circ}] - M[\Xi_{cc}^\circ]$, the mean values available in the scientific literature are given; their calculation was performed within the potential model for doubly charmed baryons with a Buchmüller–Tye potential [26] and, also, in Refs [29, 32, 33, 38]. For f_D , the value given in Refs [6, 76] was used, while the value of T was set in accordance with Ref. [88]. The parameter m_c corresponded to the pole mass of the c quark. To determine it, the lifetime and width of the semilepton D^0 -meson channel were fitted. Such a choice of the c-quark mass effectively takes into account the unknown contributions from higher orders of QCD perturbation theory to the total decay width of the baryons studied.

The renormalization scale is $\mu_1 = m_c$ in the case of c-quark decays, and $\mu_2 = 1.2 \text{ GeV}$ in the case of Pauli interference and electroweak scattering effects. The renormalization scale was obtained by comparison of theoretical predictions for the differences between the lifetimes of Λ_c , Ξ_c^+ , Ξ_c^0 baryons and their experimental findings. We note that the formulae in Ref. [89] only take into account the effect of logarithmic renormalization, while the mass corrections related to the s quark in the final state are omitted. The dependence of baryon widths on their quark composition is given by the formulae

$$\begin{aligned} \Delta\Gamma_{\text{nl}}[\Lambda_c] &= c_d \langle O_d[\Lambda_c] \rangle + c_u \langle O_u[\Lambda_c] \rangle, \\ \Delta\Gamma_{\text{nl}}[\Xi_c^+] &= c_s \langle O_s[\Xi_c^+] \rangle + c_u \langle O_u[\Xi_c^+] \rangle, \\ \Delta\Gamma_{\text{nl}}[\Xi_c^0] &= c_d \langle O_d[\Xi_c^0] \rangle + c_s \langle O_s[\Xi_c^0] \rangle. \end{aligned} \quad (5.50)$$

Here the notation was used:

$$\langle O_q[X_c] \rangle = \langle X_c | O_q | X_c \rangle, \quad O_q = (\bar{c}\gamma_\mu c)(\bar{q}\gamma^\mu q)$$

with $q = u, d$ or s , and

$$\begin{aligned} c_d &= \frac{G_F^2 m_c^2}{4\pi} \left[C_+^2 + C_-^2 + \frac{1}{3}(4k^{1/2} - 1)(C_-^2 - C_+^2) \right], \\ c_u &= -\frac{G_F^2 m_c^2}{16\pi} \left[(C_+ + C_-)^2 \right. \\ &\quad \left. + \frac{1}{3}(1 - 4k^{1/2})(5C_+^2 + C_-^2 - 6C_+C_-) \right], \\ c_s &= -\frac{G_F^2 m_c^2}{16\pi} \left[(C_+ - C_-)^2 \right. \\ &\quad \left. + \frac{1}{3}(1 - 4k^{1/2})(5C_+^2 + C_-^2 + 6C_+C_-) \right]. \end{aligned} \quad (5.51)$$

The mean D-meson mass can be expressed in terms of the light quark mass $m_l = \bar{A}$:

$$m_D = m_c + \bar{A} + \frac{\mu_\pi^2}{2m_c} = m_c + m_l + \frac{m_l}{m_c + m_l} T = 1.98 \text{ GeV}. \quad (5.52)$$

The mass of the s quark is related to m_l :

$$m_s = m_l + 0.15 \text{ GeV}. \quad (5.53)$$

As mentioned above, the spectator decay width of the c quark, $\Gamma_{c, \text{spec}}$, is known in the next order after the leading order of QCD perturbation theory [82–86]. The most complete computation, including mass effects related to the s quark in the final state, has been performed in Ref. [86]. We make use of it in our estimation. In the semilepton decay width one can neglect the masses of the electron and muon in the final state, which in the given context seems reasonable. Further, we do not take into consideration the τ -lepton mode, which is significantly suppressed owing to the small phase-space volume.

Let us analyze the contributions of various baryon decay channels to the total decay width. From Table 5 it is seen what a significant role in the decays of doubly charmed baryons is played by Pauli interference and electroweak scattering effects. Pauli interference contributes a correction of the order of 63% in the case of Ξ_{cc}^{++} baryons, and an electroweak scattering of about 61% in the case of Ξ_{cc}^+ . As was already stressed, these effects occur in various baryons, and they thus enhance the difference between the lifetimes of the hadrons studied.

Table 5. Contribution of various modes to the total decay width of doubly charmed baryons.

Mode or decay mechanism	Width, ps ⁻¹	Branching ratio, % (Ξ_{cc}^{++})	Branching ratio, % (Ξ_{cc}^+)
$c \rightarrow sdu$	2.648	127	31
$c \rightarrow se^+v$	0.380	18	4.2
PI	-1.317	-63	—
WS	5.254	—	60.6
$\Xi_{cc}^{++} \rightarrow X$	2.089	100	—
$\Xi_{cc}^+ \rightarrow X$	8.660	—	100

We recall that the difference between the lifetimes of D^{+-} and D^0 -mesons is mainly explained by the effect of Pauli interference between the c-quark decay products and the antiquark in the initial state, whereas in the case of doubly charmed baryons the clear predominance of electroweak scattering is seen. This is not surprising, since the formula for the Pauli interference operator in the case of D-mesons reproduces the expression for electroweak scattering in the event of baryons containing a c quark. As a result, for the total lifetimes of doubly charmed baryons we have

$$\tau[\Xi_{cc}^{++}] = 0.48 \text{ ps}, \quad \tau[\Xi_{cc}^+] = 0.12 \text{ ps}.$$

We note that fitting the data on the semilepton decay widths of D-mesons, on the difference in decay widths for baryons with charmed quarks, and using the spectroscopic characteristics (as described in detail above) enables one to achieve a significant reduction in the variation of model parameters: the quark masses, the normalization scale for Wilson coefficients, and the light quark wave function in the nonrelativistic model. Here, it turns out to be possible to essentially reduce the uncertainty of theoretical estimates. Variations of the c-quark mass within the limits 1.6–1.65 GeV and of the mass difference between the s quark and the light quark within the limits 0.15–0.2 GeV leads to the following uncertainties in the lifetimes of the baryons

studied:

$$\delta\tau[\Xi_{cc}^{++}] = 0.1 \text{ ps}, \quad \delta\tau[\Xi_{cc}^+] = 0.01 \text{ ps},$$

and, as is readily seen, the uncertainties in the widths amount to

$$\delta\Gamma[\Xi_{cc}^{++}] = 0.4 \text{ ps}^{-1}, \quad \delta\Gamma[\Xi_{cc}^+] = 0.9 \text{ ps}^{-1}.$$

Since the width of the Ξ_{cc}^+ baryon is significantly enhanced by the contribution of electroweak exchange between the constituent quarks, the relative uncertainty in the estimate of the baryon lifetime is noticeably lower: 10% as compared with 20% for Ξ_{cc}^{++} .

In calculations of inclusive decay widths of Ξ_{bc}^+ and Ξ_{bc}^0 baryons it is necessary to supplement the set of parameters with the b-quark mass:

$$m_b = m_c + 3.5 \text{ GeV}. \quad (5.54)$$

The mass of the baryons was taken to be 7 GeV. For the wave function in the diquark subsystem, the results were used of calculations in the nonrelativistic model with the Buchmüller–Tye potential [13]:

$$\Psi_{\text{diq}}(0) = 0.193 \text{ GeV}^{3/2}.$$

Since the estimates for the spectator decay widths of free heavy quarks are independent of the system in which they are, it is possible to make use of the results of calculations performed in Ref. [74], because the values of quark masses given in that work and in the present review coincide. This leads to the results presented in Table 6.

Table 6. Inclusive spectator decay widths of b and c quarks (in ps^{-1}).

Mode	$b \rightarrow c\bar{u}d$	$b \rightarrow c\bar{c}s$	$b \rightarrow c\bar{e}^-\bar{\nu}$	$b \rightarrow c\tau^-\bar{\nu}$	$c \rightarrow s\bar{d}u$	$c \rightarrow s\bar{e}^+\nu$
Γ	0.310	0.137	0.075	0.018	0.905	0.162

In accordance with the procedure described above, for the total lifetimes of Ξ_{bc}^+ and Ξ_{bc}^0 baryons we have

$$\tau[\Xi_{bc}^+] = 0.33 \text{ ps}, \quad \tau[\Xi_{bc}^0] = 0.28 \text{ ps}, \quad (5.55)$$

so the difference in lifetimes due to decay processes taking into account Pauli interference and weak rescattering amounts to about 20%. The relative contributions of various terms to the total widths of the baryons examined are presented in greater detail in Table 7. The contributions depending on the baryon composition are quite significant: 40–50%. The corrections due to quark–gluon operators of dimension 5 are numerically very small. Essentially more important are the corrections to the 3-dimension operator, where the role of the heavy quark time dilation in the rest frame of the hadron is noticeable.

For the semilepton decay modes, the branching ratios of which are presented in Table 7, the largest arising corrections are to b-quark decays and are due to Pauli interference: the respective widths are practically doubled. This leads to the semilepton widths of b and c quarks in the electron mode

Table 7. Relative contributions of various inclusive decay modes of Ξ_{bc}^+ and Ξ_{bc}^0 baryons (in %).

Mode	$b \rightarrow X$	$c \rightarrow X$	PI	WS	$c \rightarrow e\nu X$	$b \rightarrow e\nu X$	$b \rightarrow \tau\nu X$
Ξ_{bc}^+	20	37	23	20	5.0	4.9	2.3
Ξ_{bc}^0	17	31	21	31	4.2	4.1	1.9

becoming comparable in value, while for the spectator decays the width of the charmed quark is twice that of the beauty quark. As to the sign of terms due to Pauli interference, it is determined mainly by the sign of the leading contribution from interference of the charmed quark residing the initial state with the charmed quark from the b-quark decay. Here, the antisymmetric color structure of the baryon wave function leads to the Pauli interference being positive.

We note that the uncertainties in the estimates obtained are related to the predictions:

(1) of the spectator width of the charmed quark, where the uncertainty reaches about 50%, since the agreement of theoretical evaluations with the lifetimes of the charmed hadrons is only qualitative (for the baryons under consideration this contribution introduces an uncertainty $\delta\Gamma/\Gamma \approx 10\%$);

(2) of Pauli interference effects in the beauty quark decays and in its weak rescattering with the charmed quark residing the initial state, where the nonrelativistic model-dependent diquark wave function is used, which results in an uncertainty in these contributions at the 30% level (the uncertainty in the total width is of the order of $\delta\Gamma/\Gamma \approx 15\%$).

Thus, the uncertainty in the predictions of the total Ξ_{bc}^+ and Ξ_{bc}^0 baryon widths can be considered to fall within 20% limits.

5.3.1 Dependence of results on the parameters. In spite of the expected precision in predictions of inclusive widths and lifetimes of baryons with two heavy quarks presented above, we shall deal with this issue in greater detail owing to its importance.

First of all, we shall examine the dependence of widths on the masses of heavy quarks present in the baryons. The spectator decay widths of heavy quarks are determined by the fifth power of the masses, and the dominant corrections due to Pauli interference and weak rescattering of the constituents — by the third power of the heavy quark masses. Here, the issue arises in a natural manner of the applicability of quark–hadron duality with expansion of quark operators, a consequence of which is the zero contribution linear in the inverse heavy quark mass. Such setting to zero is derived from the Ademollo–Gatto theorem, according to which the introduction of a term with the constant λ that violates symmetry of the Lagrangian results in corrections to the conserved observables arising only in the second order in λ . Therefore, when interactions suppressed by the heavy quark mass are introduced, the decay widths of the heavy hadrons contain no contributions linear in $1/m_Q$, if quark–hadron duality holds valid.

In this connection, extreme importance must be attributed to the problem of the lifetime of the Λ_b baryon, the experimental total width of which is 20% greater than the B-meson widths, which contradicts the predictions of heavy quark theory [90]. The authors of Ref. [91] put forward the hypothesis of strong violation of quark–hadron duality, i.e. of the possible significant contribution from terms linear in $1/m_Q$ to the inclusive heavy hadron widths. This assumption actually signifies that the effective heavy quark mass determining the contribution of the leading term varies depending on the hadron mass and composition. Therefore, the looser system of the Λ_b baryon, where a light diquark is present (in which the string tension is two times smaller than that in a meson), implies the introduction of a larger effective heavy quark mass, since it is determined on a lower energy

scale (the cloud of virtual gluons and quarks has larger dimensions). As a result, the total Λ_b width increases.

Such an approach is not admissible in the operator expansion with quark–hadron duality, in which the heavy quark mass is the same for all types of hadrons, since otherwise corrections linear in $1/m_Q$ arise in the widths. However, the hypothesis about strong violation of quark–hadron duality has been practically discarded by the experimental examination of the B_c -meson lifetime. In accordance with the ideology of Ref. [91], the lifetime $\tau[B_c] \approx 1.3–1.5$ ps was predicted in Ref. [92], since the quarks in heavy quarkonium are strongly bound and their effective masses (and available phase volume in the final state) decrease, which leads to significant suppression of the \bar{b} - and c -quark decay widths in the B_c -meson. Experimental investigation yields the value of $\tau[B_c] = 0.48 \pm 19$ ps, which is in excellent agreement with estimates submitted in the operator expansion [74, 93], with QCD sum rules [94] and potential models [4]. Thus, at the moment the operator expansion with quark–hadron duality is a correct instrument for calculating the inclusive widths and lifetimes of hadrons with heavy quarks.

As has already been mentioned, the estimates presented at the beginning of Section 5.3 were obtained under the assumption that the chosen value of the charmed quark mass provides for quite an accurate theoretical description of the semilepton D-meson widths. After the publication of paper [81], similar calculations in Ref. [95], in which a significantly smaller mass is adopted for the charmed quark ($m_c = 1.35$ GeV), revealed that the same method does not permit the description of the semilepton widths of charmed mesons. Such a preference is, most likely, due to, first, the low value of the c -quark current mass being obtained within the QCD sum rules for charmonium and, second, the description of the inclusive D-meson widths exhibiting rather a qualitative than quantitative character, because the charmed quark mass is not too large and convergence of the power expansion in $1/m_Q$ may turn out to be slow (once again, data on the B_c -meson are not taken into account). Such premises only yield qualitative predictions for the lifetimes of baryons with two charmed quarks in Ref. [95], where the results of calculations differ by two-three times from the estimates presented above. Indeed, the significant reduction of the leading contribution due to variation of the charmed quark mass leads to negative Pauli interference strongly reducing the total Ξ_{cc}^{++} baryon width, and the expected lifetime increases significantly.

Another important uncertainty factor in the estimates of charmed quark decay widths is the strange quark mass. Since the charmed quark has a mass of about 1.5 GeV, the decay phase space mainly depends on the strange quark mass (either the current mass of 150–200 MeV or the constituent mass close to the K-meson mass). Suppression of the actual phase-space volume is obviously determined by the constituent mass. The issue of the dependence of inclusive widths on the heavy and strange quark masses has been investigated in

greater detail in Ref. [96] in connection with an examination of the lifetime of the B_c -meson, for which the uncertainty due to simulation of the wave function is small (heavy quarkonium is well described owing to the abundance of data on charmonium and bottomonium) and the contribution from weak annihilation operators (second-order corrections in $1/m_Q$) is small: about 10%.

Estimates have been obtained in Ref. [96] under condition (5.54) (which is dictated by the analysis of data on B-meson decays) that are presented in Table 8. From the table it is seen that, first, the small value for the charmed quark mass chosen in Ref. [95] yields a clearly overestimated B_c -meson lifetime. Second, the choice of the current mass of the strange quark seems somewhat preferable, since it results in a smaller value for the B_c -meson lifetime, which is in better agreement with the central value of the experimental interval, although the uncertainty in the data does, also, permit a description with the constituent mass of the strange quark. We note that this analysis is in agreement with the experimental values for the semilepton D-meson widths.

Together with the quark masses that to a large extent can now be considered quantities with not so large uncertainties, variation of the light quark wave function plays an essential part in calculations of the inclusive widths of baryons with two heavy quarks. As indicated above, this quantity was fixed under the assumption of similarity between the wave functions of D-mesons and baryons, i.e. the similarity of corrections to estimates within the framework of the potential model.

In the analysis made in works [95, 96], the following relation was considered to hold valid for the wave function of the light quark in the doubly charmed baryon:

$$|\psi_1(0)|^2 = \frac{2}{3} \frac{f_D^2 M_D k^{-4/9}}{12}, \quad (5.56)$$

where $f_D = 170$ MeV, and the factor $k^{-4/9}$ is due to ‘hybrid’ logarithms for the nonrelativistic heavy quarks. Expression (5.56) is obtained if scaling is assumed of the hyperfine spin–spin splitting in charmed mesons and baryons, and, also, if account is taken of spin factors and of doubling of the mass of the diquark composed of two heavy quarks. The assumption itself of the independence of such splitting on the meson or baryon hadron state seems quite illusory. Nevertheless, if one digresses from physical motivations, the numerical effect reduces to a decrease in the factor of the light quark wave function by two-three times. On the other hand, calculations within the potential model lead to this factor being approximately doubled. Thus, utilization of the numerical value adopted at the beginning of Section 5.3 yields the central value for widths upon variation of the light quark wave function.

Comparison of the estimates [96] involving the underestimated value of the light quark wave function (Table 9) with the results presented in Table 5 gives an idea of the degree

Table 8. Lifetimes of the B_c -meson and contributions of spectator widths and corrections due to Pauli interference (PI) and weak annihilation (WA) for different values of the quark masses.

Parameters, GeV	$\Gamma[\bar{b} \rightarrow \bar{c}], \text{ps}^{-1}$	$\Gamma[c \rightarrow s], \text{ps}^{-1}$	$\Gamma_{\text{PI}}, \text{ps}^{-1}$	$\Gamma_{\text{WA}}, \text{ps}^{-1}$	$\tau[B_c], \text{ps}$
$m_b = 5.0, m_c = 1.5, m_s = 0.20$	0.694	1.148	–0.115	0.193	0.54
$m_b = 4.8, m_c = 1.35, m_s = 0.15$	0.576	0.725	–0.132	0.168	0.75
$m_b = 5.1, m_c = 1.6, m_s = 0.45$	0.635	1.033	–0.101	0.210	0.55
$m_b = 5.1, m_c = 1.6, m_s = 0.20$	0.626	1.605	–0.101	0.210	0.43
$m_b = 5.05, m_c = 1.55, m_s = 0.20$	0.623	1.323	–0.107	0.201	0.48
$m_b = 5.0, m_c = 1.5, m_s = 0.15$	0.620	1.204	–0.114	0.193	0.53

Table 9. Lifetimes and inclusive widths for Ξ_{cc}^{++} , Ξ_{cc}^+ , and Ω_{cc}^+ baryons.

Parameters, GeV	$c \rightarrow sX$, ps ⁻¹	$\Gamma_{PI} + \Gamma_{WS}$, ps ⁻¹	τ , ps
Ξ_{cc}^{++} baryon			
$m_c = 1.35$, $m_s = 0.15$	1.638	– 0.616	0.99
$m_c = 1.6$, $m_s = 0.45$	2.397	– 0.560	0.56
$m_c = 1.55$, $m_s = 0.2$	3.104	– 0.874	0.45
Ξ_{cc}^+ baryon			
$m_c = 1.35$, $m_s = 0.15$	1.638	1.297	0.34
$m_c = 1.6$, $m_s = 0.45$	2.397	2.563	0.20
$m_c = 1.55$, $m_s = 0.2$	3.104	1.776	0.20
Ω_{cc}^+ baryon			
$m_c = 1.35$, $m_s = 0.15$	1.638	1.780	0.30
$m_c = 1.6$, $m_s = 0.45$	2.397	0.506	0.34
$m_c = 1.55$, $m_s = 0.2$	3.104	1.077	0.24

of variation of theoretical predictions for the inclusive widths of doubly charmed baryons. We recall that calculations with a small mass of the charmed quark are only illustrative and cannot be adopted owing to contradictions with data on the B_c -meson lifetime. Examination of the uncertainties due to the quark masses and to variation in the wave functions of the light quark in the baryon leads to the most realistic estimates (Table 10).

Table 10. Lifetimes of doubly heavy baryons.

Baryon	τ , ps	Baryon	τ , ps	Baryon	τ , ps
Ξ_{cc}^{++}	0.46 ± 0.05	Ξ_{bc}^+	0.30 ± 0.04	Ξ_{bb}^0	0.79 ± 0.02
Ξ_{cc}^+	0.16 ± 0.05	Ξ_{bc}^0	0.27 ± 0.03	Ξ_{bb}^-	0.80 ± 0.02
Ω_{cc}^+	0.27 ± 0.06	Ω_{bc}^0	0.22 ± 0.04	Ω_{bb}^-	0.80 ± 0.02

In Ref. [97], a comparative analysis has been made of the structure of operator expansion for heavy hadrons, based on the symmetry properties of hadron matrix elements determining the contributions of Pauli interference and weak rescattering of the constituents¹⁹. With a precision up to corrections in the inverse heavy quark mass and to logarithmic terms given by the anomalous dimensionalities of respective operators, the scaling relations have the form

$$\frac{\Gamma[B^-] - \Gamma[B^0]}{\Gamma[D^+] - \Gamma[D^0]} = \frac{\Gamma[\Xi_b^-] - \Gamma[\Xi_b^0]}{\Gamma[\Xi_c^+] - \Gamma[\Xi_c^0]} = \frac{\Gamma[\Xi_{bb}^-] - \Gamma[\Xi_{bb}^0]}{\Gamma[\Xi_{cc}^{++}] - \Gamma[\Xi_{cc}^+]} = \frac{m_b^2}{m_c^2} \frac{|V_{cb}|^2}{|V_{cs}|^2}. \quad (5.57)$$

The precision of these relations is to be estimated at the 50% level, since, for example, in accordance with the examination of heavy meson lepton constants the hadron matrix elements of quark currents with charmed and light quarks are subject to large corrections (about 50–90%) owing to $1/m_Q$ -terms and logarithmic renormalization. Since the considered cc diquark is two times heavier than the charmed quark, it is possible to consider in the first approximation that in the case of doubly charmed baryons the aforementioned corrections may be two times smaller.

¹⁹ Actually, the issue concerns taking advantage of the wave function of the light quark in the gluon field of an infinitely heavy source being independent of heavy quark flavor.

Making use of the data from Table 10 for testing the last equality in relations (5.57) we see that the accuracy of theoretical estimates does not permit one to make convincing quantitative conclusions concerning the difference between the lifetimes of baryons with two b quarks. On the other hand, if one only deals with the arithmetic of central values, then the examined part of equations (5.57) is indeed satisfied with an uncertainty of 50%, which points to the qualitative applicability of these relations, while their quantitative precision is depressingly low.

5.4 Exclusive decays in NRQCD sum rules

In this section the calculation is presented for exclusive semilepton cascade decays of doubly heavy baryons and, also, of two-particle hadron decays in the approximation of factorization of the weak quark transitional current [98].

Within the framework of NRQCD sum rules in Ref. [98], the baryon current

$$J[\Xi_{QQ}] = \varepsilon^{\alpha\beta\gamma} : (Q_\alpha^T C \gamma_5 q_\beta) Q_\gamma' : \quad (5.58)$$

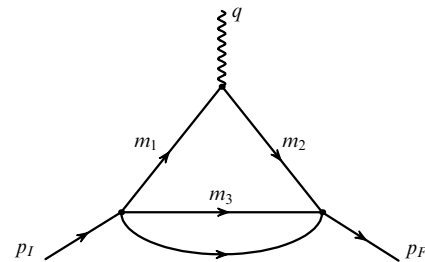
was considered that leads to the necessity of antisymmetrization at a diagram level, since the baryon can contain two identical heavy quarks. In relation (5.58) there exists a component giving a nonphysical contribution, but it becomes equal to zero when the matrix element is taken over the baryon and over vacuum. Within this approach, baryon coupling constants are, generally speaking, different from those calculated in Section 3, so it is necessary to make additional analysis of two-point correlation functions, like in Ref. [98]. However, such a choice of the current has a formal advantage in dealing with three-point correlators determining the form factors of, say, semilepton decays (Fig. 29), which will be clearly seen below in the course of investigation of the sum rules.

Let us consider the correlator

$$\Pi_\mu = i^2 \int d^4x d^4y \langle 0 | T \{ J_F(x) J_\mu(0) \bar{J}_I \} | 0 \rangle \exp(i p_F x - i p_I y), \quad (5.59)$$

where J_μ is the weak decay current of the heavy quark, and indices I and F are the baryon quantum numbers in the initial and final states, respectively. In accordance with the dispersion relations, the theoretical part of the sum rules admits the representation

$$\Pi_\mu^{\text{theor}}(s_1^0, s_2^0, q^2) = \frac{1}{(2\pi)^2} \int_{m_I^2}^{\infty} ds_1 \int_{m_F^2}^{\infty} ds_2 \frac{\rho_\mu(s_1, s_2, q^2)}{(s_1 - s_1^0)(s_2 - s_2^0)} + \dots \quad (5.60)$$

**Figure 29.** Quark loop for three-point correlator in the baryon decay.

Here, the suspension points stand for possible subtractions providing for convergence of the integrals. The spectral densities ρ_μ were calculated in Ref. [98], where the limit of spin symmetry in the effective Lagrangian of heavy non-relativistic quarks was considered within the quark-loop approximation with due account of the condensate of light quarks. In this case it is sufficient to determine only one scalar correlator.

Indeed, the hadron part of the sum rules is given by the expression

$$\Pi_\mu^{\text{phen}}(s_1^0, s_2^0, q^2) = \sum_{\text{spins}} \frac{\langle 0 | J_F | \Xi_F(p_F) \rangle}{s_2^0 - M_F^2} \times \langle \Xi_F(p_F) | J_\mu | \Xi_I(p_I) \rangle \frac{\langle \Xi_I(p_I) | \bar{J}_I | 0 \rangle}{s_1^0 - M_I^2}, \quad (5.61)$$

and the form factor for the decay of a baryon with spin 1/2 into a baryon with spin 1/2 can be written in the general form as follows

$$\langle \Xi_F(p_F) | J_\mu | \Xi_I(p_I) \rangle = \bar{u}(p_F) \{ \gamma_\mu G_1^V + v_\mu^I G_2^V + v_\mu^F G_3^V + \gamma_5 (\gamma_\mu G_1^A + v_\mu^I G_2^A + v_\mu^F G_3^A) \} u(p_I). \quad (5.62)$$

All six form factors in Eqn (5.62) are independent. However, the NRQCD Lagrangian in the leading approximation possesses spin symmetry, so for a small recoil momentum that restricts the virtualities of gluon exchanges in the hadron state it is possible to obtain relations connecting form factors yielding nonzero contributions.

At small recoils, when the baryon 4-velocities v_I and v_F in the initial and final states differ insignificantly from each other, and their scalar product is close to unity [$w = (v_I v_F) \rightarrow 1$], the correlation function for the decay of a heavy quark into a heavy quark has the form

$$\Pi_\mu^{\text{theor}} \propto \xi^{\text{IW}}(w) (1 + \tilde{\psi}_F) \gamma_\mu (1 - \gamma_5) (1 + \tilde{\psi}_I), \quad (5.63)$$

where

$$\tilde{v}_I = v_I + \frac{m_3}{2m_1} (v_I - v_F), \quad (5.64)$$

$$\tilde{v}_F = v_F + \frac{m_3}{2m_2} (v_F - v_I). \quad (5.65)$$

From Eqn (5.63) it is seen that at the minimum recoil momentum the correlation function is determined by a sole form factor ξ^{IW} which, however, is not universal, since it depends on the quark composition of the baryon.

For the decay of a heavy quark into a light quark, the correlation function is written as

$$\Pi_\mu^{\text{theor}} \propto \{ \xi_1(w) \tilde{\psi}_I + \xi_2(w) \tilde{\psi}_F + \xi_3(w) \} \gamma_\mu (1 - \gamma_5) (1 + \tilde{\psi}_I). \quad (5.66)$$

Hence it is possible to obtain spin symmetry relations:

$$G_1^V + G_2^V + G_3^V = \xi^{\text{IW}}(w), \quad (5.67)$$

$$G_1^A = \xi^{\text{IW}}(w), \quad (5.68)$$

and the connection between functions $\xi_i(w)$:

$$\xi_1(w) + \xi_2(w) = \xi_3(w) = \xi^{\text{IW}}(w).$$

Owing to heavy hadrons existing in the initial and final states, we arrive at the conclusion that only two form factors are not suppressed by the heavy quark mass. These form factors are related by the formula $G_1^V = G_1^A = \xi(w)$. In the case of zero recoil, the reduced matrix element is determined from the equation

$$\xi^{\text{IW}}(w=1) = \frac{1}{(2\pi)^2} \frac{1}{8M_I M_F Z_I Z_F} \times \int_{(m_1+m_3)^2}^{s_I^{\text{theor}}} \int_{(m_1+m_2)^2}^{s_F^{\text{theor}}} \rho(s_I, s_F, q^2) \times \exp \left(-\frac{s_I - M_I^2}{B_I^2} - \frac{s_F - M_F^2}{B_F^2} \right) ds_I ds_F, \quad (5.69)$$

Here, B_I and B_F are parameters of the Borel transformation with respect to invariant masses squared in the initial and final decay states, while Z_I and Z_F are the coupling constants of baryons with quark currents.

Numerical estimates. From the sum rules (5.69) estimates are obtained for the analog of the Isgur–Wise function for the form factors of semilepton decays of doubly heavy baryons into baryon states of spin 1/2, which are presented in Table 11 and compared with the values calculated in the potential model. The difference between the values of $\xi(1)$ in these models do not result in contradictions, since the systematic uncertainty amounts to about 10%. Figure 30 shows the result for normalization of the Isgur–Wise form factor in the $\Xi_{bb} \rightarrow \Xi_{bc}$ decay in the Borel scheme of NRQCD sum rules.

For calculating exclusive widths it is necessary to define the dependence of the form factor upon the transferred momentum. With the sufficient accuracy one can adopt the

Table 11. Normalization of the Isgur–Wise form factor $\xi(1)$ in the case of zero recoil momentum in sum rules and in the potential model.

Mode	Sum rules	Potential model
$\Xi_{bb} \rightarrow \Xi_{bc}$	0.85	0.91
$\Xi_{bc} \rightarrow \Xi_{cc}$	0.91	0.99
$\Xi_{bc} \rightarrow \Xi_{bs}$	0.9	0.99
$\Xi_{cc} \rightarrow \Xi_{cs}$	0.99	1.0

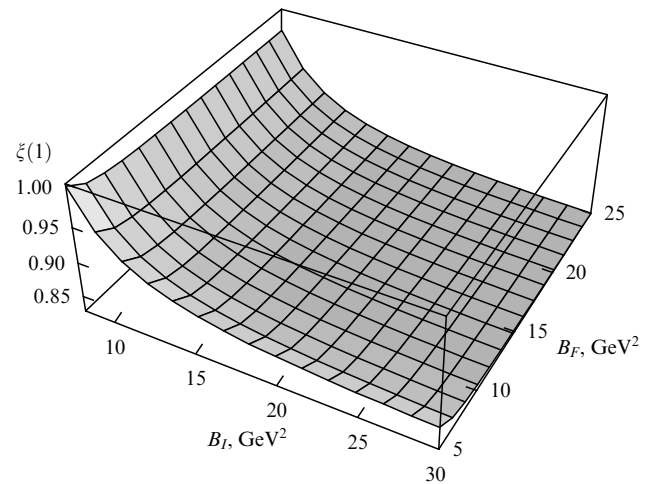


Figure 30. Form factor $\xi(1)$ for the transition $\Xi_{bb}^\circ \rightarrow \Xi_{bc}^\circ$ in the Borel scheme of NRQCD sum rules.

pole model:

$$\xi^{IW}(w) = \xi_0 \frac{1}{1 - q^2/m_{\text{pole}}^2}, \quad (5.70)$$

with

$$m_{\text{pole}}[b \rightarrow c] = 6.3 \text{ GeV}, \quad m_{\text{pole}}[c \rightarrow s] = 1.85 \text{ GeV}.$$

The results of calculations of the branching ratios for the exclusive widths of doubly heavy baryons within the NRQCD sum rules are presented in Table 12, where the total widths were assumed equal to the mean values given in Table 10. The estimates also include the contribution from cascade decays into baryons of spin 3/2. To this end the results of Ref. [99] were used, in which the semilepton modes $\Xi_{bc} \rightarrow \Xi_{cc} + l\bar{\nu}$ were considered. In Ξ_{bb}° and Ξ_{cc}° decays antisymmetrization was taken into account for identical quarks, and the correction factor 0.62 due to negative Pauli interference was introduced for transitions $\Xi_{cc}^{++} \rightarrow \Xi_c^+ X$. We note that the authors of Ref. [98] claimed that the values obtained in the sum rules are consistent both with the estimates in the potential approach [99] and with the estimates for inclusive decay widths, if summation over the exclusive channels (presented in Table 12) is performed.

Table 12. Branching ratios (Br) for exclusive decays of baryons with two heavy quarks.

Mode	Br, %	Mode	Br, %
$\Xi_{bb}^\circ \rightarrow \Xi_{bc}^\circ \bar{l}\nu_l$	14.9	$\Xi_{bc}^+ \rightarrow \Xi_{cc}^{++} \bar{l}\nu_l$	4.9
$\Xi_{bc}^\circ \rightarrow \Xi_{cc}^+ \bar{l}\nu_l$	4.6	$\Xi_{bc}^+ \rightarrow \Xi_b^0 \bar{l}\nu_l$	4.4
$\Xi_{bc}^\circ \rightarrow \Xi_b^- \bar{l}\nu_l$	4.1	$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \bar{l}\nu_l$	16.8
$\Xi_{cc}^+ \rightarrow \Xi_c^0 \bar{l}\nu_l$	7.5	$\Xi_{bb}^\circ \rightarrow \Xi_{bc}^\circ \pi^-$	2.2
$\Xi_{bb}^\circ \rightarrow \Xi_{bc}^\circ \rho^-$	5.7	$\Xi_{bc}^+ \rightarrow \Xi_{cc}^{++} \pi^-$	0.7
$\Xi_{bc}^\circ \rightarrow \Xi_{cc}^+ \pi^-$	0.7	$\Xi_{bc}^+ \rightarrow \Xi_{cc}^{++} \rho^-$	1.9
$\Xi_{bc}^\circ \rightarrow \Xi_{cc}^+ \rho^-$	1.7	$\Xi_{bc}^+ \rightarrow \Xi_b^0 \pi^+$	7.7
$\Xi_{bc}^\circ \rightarrow \Xi_b^- \pi^+$	7.1	$\Xi_{bc}^+ \rightarrow \Xi_b^0 \rho^+$	21.7
$\Xi_{bc}^\circ \rightarrow \Xi_b^- \rho^+$	20.1	$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$	15.7
$\Xi_{cc}^+ \rightarrow \Xi_c^0 \pi^+$	11.2	$\Xi_{cc}^{++} \rightarrow \Xi_c^+ \rho^+$	46.8
$\Xi_{cc}^+ \rightarrow \Xi_c^0 \rho^+$	33.6		

5.5 Discussion

Above, within the framework of a consistent study of operator expansion in the inverse heavy quark mass, calculations have been done of the total lifetimes of baryons with two heavy quarks. The leading contribution in the expansion is determined by the spectator widths of the inclusive decays of heavy quarks, and significant corrections arise when the effects of Pauli interference and electroweak scattering are taken into account that turn out to be at the 20–30% level for Ξ_{cc} and Ξ_{bc} baryons.

Measurement of the lifetimes of doubly heavy baryons will permit one to perform comparative analysis of the decay mechanisms for hadrons with heavy quarks, which is especially important in the light of studies of subtle effects involving combined CP-parity violation in the sector of heavy quarks, since the quark interaction characteristics enter measurable quantities with factors formed by the hadron matrix elements due to quark currents.

To a great extent, the reliable knowledge of the properties of these matrix elements can be enhanced by

studying the decays and lifetimes of baryons with two heavy quarks. Such studies will permit one to analyze quantitatively the effects of possible violation of the quark–hadron duality (which, as noted above, are, most likely, small). It is topical to consider the dependence of heavy quark widths upon the hadron composition, which may be essential in clarification of the reasons for the large deviation from unity of the ratio between the lifetime of the Λ_b baryon and that of B-mesons. Measurement of the lifetimes of doubly heavy baryons will also make it possible to study the confinement characteristics of heavy quarks in various systems.

A field open to activities is represented by calculations of the exclusive decay widths of baryons with two heavy quarks. The estimates obtained within the framework of NRQCD sum rules and potential models are, doubtless, preliminary, since the question still remains concerning the role of corrections in the inverse heavy quark mass, which may be quite significant for hadrons with a charmed quark.

Moreover, studies of exclusive hadron decays of doubly heavy baryons are of significant practical interest. Among the hadron decays those channels are to be especially singled out that do not involve cascade decays of two heavy quarks (since such modes require reconstruction of three secondary vertices, like, for example, $\Xi_{cc} \rightarrow \Omega_c + X \rightarrow \Xi + X$), but are processes with weak rescattering of the constituent heavy quarks, resulting in the presence of only a single heavy quark in the final state.

Figure 31 illustrates such a decay channel for the Ξ_{bc} baryon — only one heavy hadron has to be detected. The contribution of weak rescattering to the total width is quite large (about 20%). Simple estimates of the suppression factors show that the decay branching ratio should be at the level of some thousandths.

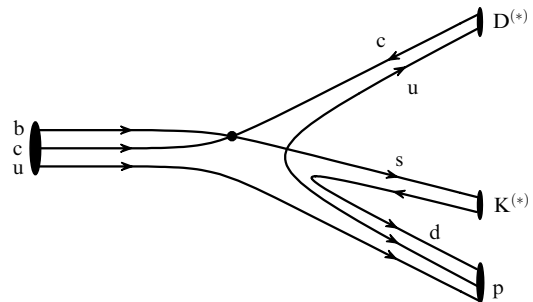


Figure 31. Baryon decay $\Xi_{bc} \rightarrow D^{(*)} K^{(*)} p$ in a process with weak rescattering of the constituent b and c quarks.

We believe experimental studies of doubly heavy baryons to be quite a feasible task, first of all, at hadron colliders. Measurements of their decay characteristics are capable of essentially enriching our knowledge about heavy quark decay mechanisms.

6. Conclusions

The main physical characteristics of baryons containing two heavy quarks have been presented in this review. The description of such hadrons is based on the hierarchy of energy scales which determine the times and distances that are typical for strong interactions in baryon-forming subsystems. Thus, owing to the nonrelativistic motion of heavy quarks

with respect to each other, the formation time of a system of two heavy quarks is longer than the time of hard heavy-quark formation and than the time required for hard gluons to 'dress' the quarks.

On the other hand, the formation time of a heavy diquark is much shorter than the characteristic interaction time in the case of light quark confinement (low-frequency strongly interacting fields). Owing to the hierarchy of strong interactions in the quark systems dealt with it turns out to be possible to develop methods of effective heavy quark theory for baryons with two heavy quarks, in which one can formally single out the leading approximation and construct a systematic method for calculating corrections to it.

The following special methods for dealing with two-quark baryons have been developed within the description of hadron systems with heavy quarks, based on general approaches:

- the static quark potential and a potential model for doubly heavy baryons, separation of the motion of heavy quarks in the diquark and of the light quark in the diquark field;

- formulation of two-point NRQCD sum rules for quark currents corresponding to baryons with two heavy quarks, calculation of the masses of the ground baryon states and of their coupling constants with currents within the approach taking into account corrections to the local condensate of light and strange quarks;

- evaluation of the anomalous dimensions of baryon currents involving two heavy quarks;

- scaling functions for diquark fragmentation;

- numerical calculations of the complete set of fourth-order QCD diagrams, analysis of higher twists in the transverse momentum;

- generalization of the method of operator expansion for calculating inclusive widths of baryons with two heavy and light quarks;

- formulation of three-point NRQCD sum rules for exclusive semileptonic decays and hadron decays in the approximation of factorization of the transitional current.

The most impressive physical effects in baryons containing two heavy quarks are the following:

- the existence of a system of quasi-stationary excited levels in baryons with identical heavy quarks due to suppression of the transition operators to low-lying states, which is caused by the necessity of altering the diquark quantum numbers, since for a number of states (in accordance with the Pauli exclusion principle) the operators, not suppressed by the heavy quark mass and the small size of the diquark, must become zero;

- cascade processes of fragmentation at high transverse momenta, for which it is possible to obtain analytical results for the universal fragmentation functions in perturbative QCD (of a heavy quark into a heavy diquark or of a heavy diquark into a doubly heavy baryon) and to describe their evolution due to the emission of hard gluons within the framework of the QCD renormalization group;

- separation of the fragmentation and recombination modes in hadron processes by taking into account higher twists in transverse momentum, which can be described within the framework of QCD perturbation theory when calculating the complete set of diagrams for the given order in the coupling constant;

- large (20–50%) contributions from nonspectator Pauli interference and weak rescattering effects, dependent on the

baryon composition, to the total lifetimes of doubly heavy baryons, in particular, in the presence of a charmed quark, which leads to strong splitting of the baryon lifetimes:

$$\tau[\Xi_{cc}^{++}] > \tau[\Omega_{cc}^{+}] > \tau[\Xi_{cc}^{+}],$$

$$\tau[\Xi_{bc}^{+}] > \tau[\Xi_{bc}^{0}] > \tau[\Omega_{bc}^{0}],$$

$$\tau[\Xi_{bb}^{-}] \approx \tau[\Omega_{bb}^{-}] > \tau[\Xi_{bb}^{0}];$$

cascade decay mechanisms of baryons with two heavy quarks, and the existence of special decay modes due to weak rescattering, which have a large branching ratio.

Doubtless, direct mass measurements of the ground and excited states will permit one to essentially specify the formation dynamics of bound states with heavy quarks. Detailed theoretical investigation is required of radiative transition processes (both electromagnetic and hadronic) between quasi-stationary levels of doubly heavy baryons, for which the chiral Lagrangian method can be developed in the case of soft emission of Goldstone mesons, for instance, of pions.

In the present review quite a full picture has been presented for the production mechanisms of doubly heavy baryons. Searches for such baryons at hadron colliders with high luminosity seem to present the best prospects. Transverse momentum distributions of baryons, which contain information on the production modes, could become rich in content and clarify the reasons for the disagreement between theory and experimental data on the yield of b-hadrons.

The experimental information that, probably, can be considered the most interesting information consists in the data on the lifetimes of doubly heavy baryons, since they are closely related to the entire system for describing inclusive decays of heavy hadrons. Here, it is important to know the branching ratios for semileptonic widths, which reveal the contribution from gluon corrections to the nonleptonic Lagrangian of weak charged quark currents. The lifetimes will essentially enrich the knowledge of heavy quark masses and of the relative role of various decay mechanisms, to which they are extremely sensitive. The description of exclusive hadron decays of baryons with two heavy quarks will, most likely, require significant theoretical effort.

On the whole, the physics of baryons containing two heavy quarks is extremely rich in content and informative. It is justified in occupying a worthy place in experimental studies in the light of the optimistic prospects put forward by modern theory in this field.

The authors express their gratitude to coauthors S S Gershtein, A V Berezhnoi, A É Koval'skiĭ and M V Shevlyagin, together with whom a number of results discussed in this review were obtained, and, in particular, the significant contribution by A I Onishchenko to the investigations presented should be noted. We are also grateful to V A Rubakov, A L Kataev, A Ali, W Buchmüller, K A Ter-Martirosian and V A Saleev for fruitful discussions of the studies that formed the basis of the material for the review.

This work has been partially supported by grants of the Russian Foundation for Basic Research (01-02-99315, 01-02-16585, 00-15-96645) and by the Ministry of Education of the Russian Federation (00-3.3-62).

7. Appendices

7.1 Spectral density coefficients

The spectral densities in equation (3.18) have the coefficients

$$\begin{aligned}\eta_{1,0} &= 16\omega^2 (429\mathcal{M}_{\text{diq}}^3 + 715\mathcal{M}_{\text{diq}}^2\omega + 403\mathcal{M}_{\text{diq}}\omega^2 + 77\omega^3), \\ \eta_{1,1} &= 104\omega (231\mathcal{M}_{\text{diq}}^3 + 297\mathcal{M}_{\text{diq}}^2\omega + 121\mathcal{M}_{\text{diq}}\omega^2 + 15\omega^3), \\ \eta_{1,2} &= \frac{10}{(\mathcal{M}_{\text{diq}} + \omega)^2} (3003\mathcal{M}_{\text{diq}}^5 + 9009\mathcal{M}_{\text{diq}}^4\omega \\ &\quad + 9438\mathcal{M}_{\text{diq}}^3\omega^2 + 4290\mathcal{M}_{\text{diq}}^2\omega^3 + 871\mathcal{M}_{\text{diq}}\omega^4 + 77\omega^5). \end{aligned} \quad (7.1)$$

The coefficients of the spectral densities in equation (3.19) are written in the form

$$\begin{aligned}\eta_{2,0} &= 42\omega (\mathcal{M}_{\text{diq}}^2 + 48\mathcal{M}_{\text{diq}}\omega + 14\omega^2), \\ \eta_{2,1} &= 3 (35\mathcal{M}_{\text{diq}}^2 + 28\mathcal{M}_{\text{diq}}\omega + 5\omega^2), \\ \eta_{2,2} &= \frac{1}{(\mathcal{M}_{\text{diq}} + \omega)^2} (105\mathcal{M}_{\text{diq}}^3 + 315\mathcal{M}_{\text{diq}}^2\omega \\ &\quad + 279\mathcal{M}_{\text{diq}}\omega^2 + 77\omega^3). \end{aligned} \quad (7.2)$$

The spectral densities with due account of Coulomb corrections in equation (3.23) are determined by the coefficients

$$\begin{aligned}\eta_{1,0}^{\text{C}} &= (2\mathcal{M}_{\text{diq}} + \omega)^2\omega^2, \\ \eta_{1,1}^{\text{C}} &= \frac{3(2\mathcal{M}_{\text{diq}} + \omega)\omega}{\mathcal{M}_{\text{diq}} + \omega} (4\mathcal{M}_{\text{diq}}^3 + 6\mathcal{M}_{\text{diq}}^2\omega + 4\mathcal{M}_{\text{diq}}\omega^2 + \omega^3), \\ \eta_{1,2}^{\text{C}} &= \frac{1}{(\mathcal{M}_{\text{diq}} + \omega)^2} (12\mathcal{M}_{\text{diq}}^4 + 24\mathcal{M}_{\text{diq}}^3\omega + 32\mathcal{M}_{\text{diq}}^2\omega^2 \\ &\quad + 20\mathcal{M}_{\text{diq}}\omega^3 + 5\omega^4). \end{aligned} \quad (7.3)$$

With allowance for Coulomb corrections the coefficients of spectral densities in equation (3.24) take the form

$$\begin{aligned}\eta_{2,0}^{\text{C}} &= (2\mathcal{M}_{\text{diq}} + \omega)\omega, \\ \eta_{2,1}^{\text{C}} &= \frac{2}{\mathcal{M}_{\text{diq}} + \omega} (2\mathcal{M}_{\text{diq}}^2 + 2\mathcal{M}_{\text{diq}}\omega + \omega^2), \\ \eta_{2,2}^{\text{C}} &= \frac{2}{(\mathcal{M}_{\text{diq}} + \omega)^2} (2\mathcal{M}_{\text{diq}}^2 + 2\mathcal{M}_{\text{diq}}\omega + \omega^2). \end{aligned} \quad (7.4)$$

For the spectral densities with the gluon condensate in equation (3.29) we find

$$\begin{aligned}\eta_{1,0}^{G^2} &= 84\mathcal{M}_{\text{diq}}^3 + 196\mathcal{M}_{\text{diq}}^2\omega + 133\mathcal{M}_{\text{diq}}\omega^2 + 11\omega^3, \\ \eta_{1,1}^{G^2} &= -\frac{2}{\mathcal{M}_{\text{diq}} + \omega} (210\mathcal{M}_{\text{diq}}^3 + 70\mathcal{M}_{\text{diq}}^2\omega \\ &\quad + 21\mathcal{M}_{\text{diq}}\omega^2 + 3\omega^3), \\ \eta_{1,2}^{G^2} &= \frac{2}{(\mathcal{M}_{\text{diq}} + \omega)^2} (210\mathcal{M}_{\text{diq}}^3 + 70\mathcal{M}_{\text{diq}}^2\omega \\ &\quad + 21\mathcal{M}_{\text{diq}}\omega^2 + 3\omega^3). \end{aligned} \quad (7.5)$$

7.2 Transverse momentum distribution

In the case of fragmentation of a vector diquark, in the scaling limit the distribution over $t = p_{\perp}/M$ for the baryon state

about the fragmentation axis is given by the function

$$\begin{aligned}D(t) &= \frac{64\alpha_s^2}{81\pi} \frac{|R(0)|^2}{3(1-r)^5 M^3} \frac{1}{t^6} \\ &\times \left(t(30r^3 - 30r^4 - 61t^2r + 45r^2t^2 + 33r^3t^2 \right. \\ &\quad - 17r^4t^2 + 3t^4 - 9rt^4 + 15r^2t^4 - 9r^3t^4) \\ &\quad - (30r^4 - 99r^2t^2 - 54r^3t^2 + 27r^4t^2 + 9t^4 + 18rt^4 - 6r^2t^4 \\ &\quad + 18r^3t^4 + 3r^4t^4 + 3t^6 - 6rt^6 + 9r^2t^6) \arctan \frac{(1-r)t}{r+t^2} \\ &\quad \left. + 24(2r^3t + rt^3 + r^2t^3) \ln \frac{r^2(1+t^2)}{r^2+t^2} \right). \end{aligned} \quad (7.6)$$

7.3 Spectator effects in baryon decays

The operators of Pauli interference and of electroweak scattering in Ξ_{cc} baryons have the form

$$\begin{aligned}\mathcal{T}_{\text{PI}} &= -\frac{2G_{\text{F}}^2}{4\pi} m_{\text{c}}^2 \left(1 - \frac{m_{\text{c}}}{m_{\text{u}}} \right)^2 \\ &\times \left\{ \left[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4} \right) \right. \right. \\ &\quad \times (\bar{c}_i \gamma_{\alpha} (1-\gamma_5) c_i) (\bar{q}_j \gamma^{\alpha} (1-\gamma_5) q_j) \\ &\quad + \left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3} \right) (\bar{c}_i \gamma_{\alpha} \gamma_5 c_i) (\bar{q}_j \gamma^{\alpha} (1-\gamma_5) q_j) \Big] \\ &\quad \times \left[(C_+ + C_-)^2 + \frac{1}{3} (1-k^{1/2}) (5C_+^2 + C_-^2 - 6C_-C_+) \right] \\ &\quad + \left[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4} \right) \right. \\ &\quad \times (\bar{c}_i \gamma_{\alpha} (1-\gamma_5) c_j) (\bar{q}_j \gamma^{\alpha} (1-\gamma_5) q_i) \\ &\quad + \left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3} \right) (\bar{c}_i \gamma_{\alpha} \gamma_5 c_j) (\bar{q}_j \gamma^{\alpha} (1-\gamma_5) q_i) \Big] \\ &\quad \left. \times k^{1/2} (5C_+^2 + C_-^2 - 6C_-C_+) \right\}, \end{aligned} \quad (7.7)$$

$$\begin{aligned}\mathcal{T}_{\text{WS}} &= \frac{2G_{\text{F}}^2}{4\pi} p_+^2 (1-z_+)^2 \\ &\times \left[\left(C_+^2 + C_-^2 + \frac{1}{3} (1-k^{1/2}) (C_+^2 - C_-^2) \right) \right. \\ &\quad \times (\bar{c}_i \gamma_{\alpha} (1-\gamma_5) c_i) (\bar{q}_j \gamma^{\alpha} (1-\gamma_5) q_j) \\ &\quad + k^{1/2} (C_+^2 - C_-^2) (\bar{c}_i \gamma_{\alpha} (1-\gamma_5) c_j) (\bar{q}_j \gamma^{\alpha} (1-\gamma_5) q_i) \Big], \end{aligned} \quad (7.8)$$

with $p_+ = p_{\text{c}} + p_{\text{q}}$, $p_- = p_{\text{c}} - p_{\text{q}}$, and $z_{\pm} = m_{\text{c}}^2/p_{\pm}^2$. In the expressions for p_+ and p_- we use their threshold values

$$p_+ = p_{\text{c}} \left(1 + \frac{m_{\text{q}}}{m_{\text{c}}} \right), \quad p_- = p_{\text{c}} \left(1 - \frac{m_{\text{q}}}{m_{\text{c}}} \right).$$

Formulae (7.7) and (7.8) were derived by taking into account the low-energy renormalization of the nonleptonic weak interaction Lagrangian for nonrelativistic heavy

quarks [6, 89]

$$\begin{aligned}
L_{\text{eff, log}} = & \frac{G_F^2 m_c^2}{2\pi} \left[\frac{1}{2} \left(C_+^2 + C_-^2 + \frac{1}{3} (1 - k^{1/2}) (C_+^2 - C_-^2) \right) \right. \\
& \times (\bar{c} \Gamma_\mu c) (\bar{d} \Gamma^\mu d) + \frac{1}{2} (C_+^2 - C_-^2) k^{1/2} (\bar{c} \Gamma_\mu d) (\bar{d} \Gamma^\mu c) \\
& + \frac{1}{3} (C_+^2 - C_-^2) k^{1/2} (k^{-2/9} - 1) (\bar{c} \Gamma_\mu t^a c) j_\mu^a \\
& - \frac{1}{8} \left((C_+ + C_-)^2 + \frac{1}{3} (1 - k^{1/2}) (5C_+^2 + C_-^2 - 6C_+ C_-) \right) \\
& \times \left(\bar{c} \Gamma_\mu c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c \right) (\bar{u} \Gamma^\mu u) \\
& - \frac{1}{8} k^{1/2} (5C_+^2 + C_-^2 - 6C_+ C_-) \\
& \times \left(\bar{c}_i \Gamma_\mu c_k + \frac{2}{3} \bar{c}_i \gamma_\mu \gamma_5 c_k \right) (\bar{u}_k \Gamma^\mu u_i) \\
& - \frac{1}{8} \left((C_+ - C_-)^2 + \frac{1}{3} (1 - k^{1/2}) (5C_+^2 + C_-^2 + 6C_+ C_-) \right) \\
& \times \left(\bar{c} \Gamma_\mu c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 c \right) (\bar{s} \Gamma^\mu s) \\
& - \frac{1}{8} k^{1/2} (5C_+^2 + C_-^2 + 6C_+ C_-) \\
& \times \left(\bar{c}_i \Gamma_\mu c_k + \frac{2}{3} \bar{c}_i \gamma_\mu \gamma_5 c_k \right) (\bar{s}_k \Gamma^\mu s_i) \\
& - \frac{1}{6} k^{1/2} (k^{-2/9} - 1) (5C_+^2 + C_-^2) \left(\bar{c} \Gamma_\mu t^a c + \frac{2}{3} \bar{c} \gamma_\mu \gamma_5 t^a c \right) j^{a\mu} \Big], \\
\end{aligned} \tag{7.9}$$

where

$$\Gamma_\mu = \gamma_\mu (1 - \gamma_5), \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_c)}, \quad \text{and}$$

$$j_\mu^a = \bar{u} \gamma_\mu t^a u + \bar{d} \gamma_\mu t^a d + \bar{s} \gamma_\mu t^a s$$

is the color current of light quarks ($t^a = \lambda^a/2$ are the color generators).

Here, it is necessary to make a comment relevant to the terms of the effective Lagrangian containing the color current of light quarks. We have neglected these terms, because they are present in the Lagrangian with the factor $k^{-2/9} - 1$, the numerical value of which is of the order of 0.054.

References

- Weinberg S *Phys. Rev. Lett.* **19** 1264 (1967); Salam A, in *Elementary Particle Theory. Relativistic Groups and Analyticity. Proc. of the 8th Nobel Symp., May, 1968, Aspenäs garden, Lerum* (Ed. N Svartholm) (Stockholm: Almqvist & Wiksell, 1968) p. 367; Glashow S L, Iliopoulos J, Maiani L *Phys. Rev. D* **2** 1285 (1970)
- Stone S *Nucl. Instrum. Methods A* **462** 323 (2001); hep-ph/0012162; Rosner J L “CP violation in B decays”, hep-ph/0011355; *Lecture Notes of TASI-2000* (Ed. J L Rosner) (Singapore: World Scientific, 2001); Faccioli P *Nucl. Instrum. Methods A* **462** 313 (2001); hep-ph/0011269
- Chetyrkin K, Narison S *Phys. Lett. B* **485** 145 (2000); Gershtein S S, Likhoded A K, Prokoshkin Yu D *Z. Phys. C* **24** 305 (1984)
- Gershtein S S et al., Talk given at 4th Intern. Workshop “Progress in Heavy Quark Physics”, Rostock, Germany, 20–22 September 1997; Gershtein S S et al. *Usp. Fiz. Nauk* **165** 3 (1995) [*Phys. Usp.* **38** 1 (1995)]
- Abe F et al. (CDF Collab.) *Phys. Rev. Lett.* **81** 2432 (1998); *Phys. Rev. D* **58** 112004 (1998)
- Voloshin M B, Shifman M A *Yad. Fiz.* **41** 187 (1985) [*Sov. J. Nucl. Phys.* **41** 120 (1985)]; *Zh. Eksp. Teor. Fiz.* **91** 1180 (1986) [*Sov. Phys. JETP* **64** 698 (1986)]
- Shifman M A, Vainshtein A I, Zakharov V I *Nucl. Phys. B* **147** 385 (1979); Reinders L J, Rubinstein H R, Yazaki S *Phys. Rep.* **127** 1 (1985); Novikov V A et al. *Phys. Rep.* **41** 1 (1978)
- Quigg C, Rosner J L *Phys. Rep.* **56** 167 (1979)
- Neubert M *Phys. Rep.* **245** 259 (1994)
- Bodwin G T, Braaten E, Lepage G P *Phys. Rev. D* **51** 1125 (1995); **55** 5853 (1997); Mannel T, Schuler G A *Z. Phys. C* **67** 159 (1995)
- Brambilla N et al. *Nucl. Phys. B* **566** 275 (2000); *Phys. Rev. D* **60** 091502 (1999)
- Savage M J, Wise M B *Phys. Lett. B* **248** 117 (1990); Savage M J, Springer R P *Int. J. Mod. Phys. A* **6** 1701 (1991); Fleck S, Richard J M *Part. World* **1** 760 (1989); *Prog. Theor. Phys.* **82** 760 (1989); Lichtenberg D B, Roncaglia R, Predazzi E *Phys. Rev. D* **53** 6678 (1996); Stong M L “Spectra of baryons containing two heavy quarks”, hep-ph/9505217; Richard J M *Phys. Rep.* **212** 1 (1992)
- Buchmüller W, Tye S-H H *Phys. Rev. D* **24** 132 (1981)
- Gottfried K *Phys. Rev. Lett.* **40** 598 (1978); Voloshin M *Nucl. Phys. B* **154** 365 (1979); Peskin M *Nucl. Phys. B* **156** 365 (1979)
- Rosner J L *Phys. Lett. B* **385** 293 (1996)
- Eichten E et al. *Phys. Rev. D* **17** 3090 (1978); **21** 203 (1980)
- Quigg C, Rosner J L *Phys. Lett. B* **71** 153 (1977)
- Martin A *Phys. Lett. B* **93** 338 (1980)
- Richardson J L *Phys. Lett. B* **82** 272 (1979)
- Susskind L, in *Weak and Electromagnetic Interactions at High Energy* (Eds R Balian, C H Llewellyn-Smith) (Amsterdam: North-Holland, 1977) p. 207; Fischler W *Nucl. Phys. B* **129** 157 (1977); Appelquist T, Dine M, Muzinich I J *Phys. Lett. B* **69** 231 (1977); *Phys. Rev. D* **17** 2074 (1978); Billoire A *Phys. Lett. B* **92** 343 (1980); Eichten E, Feinberg F L *Phys. Rev. Lett.* **43** 1205 (1979); *Phys. Rev. D* **23** 2724 (1981)
- Peter M *Nucl. Phys. B* **501** 471 (1997); *Phys. Rev. Lett.* **78** 602 (1997)
- Schröder Y *Phys. Lett. B* **447** 321 (1999)
- Caso C et al. *Eur. Phys. J. C* **3** 1 (1998)
- Kiselev V V, Kovalsky A E, Onishchenko A I *Phys. Rev. D* **64** 054009 (2001)
- Simonov Yu A *Phys. Rep.* **320** 265 (1999); *Usp. Fiz. Nauk* **166** 337 (1996) [*Phys. Usp.* **39** 313 (1996)]; Simonov Yu A, Titard S, Ynduráin F J *Phys. Lett. B* **354** 435 (1995)
- Gershtein S S et al. *Yad. Fiz.* **63** 334 (2000) [*Phys. Atom. Nucl.* **63** 274 (2000)]; *Phys. Rev. D* **62** 054021 (2000); *Heavy Ion Phys.* **9** 133 (1999); *Mod. Phys. Lett. A* **14** 135 (1999)
- Hoang A H et al. *Phys. Rev. D* **59** 114014 (1999)
- Eichten E, Feinberg F *Phys. Rev. D* **23** 2724 (1981); Gromes D *Z. Phys. C* **26** 401 (1985)
- Ebert D et al. *Z. Phys. C* **76** 111 (1997)
- Logunov A A, Tavkhelidze A N *Nuovo Cimento* **29** 380 (1963)
- Tong S-P et al. *Phys. Rev. D* **62** 054024 (2000)
- Roncaglia R, Lichtenberg D B, Predazzi E *Phys. Rev. D* **52** 1722 (1995)
- Körner J G, Krämer M, Pirjol D *Prog. Part. Nucl. Phys.* **33** 787 (1994)
- Itoh Ch et al. *Phys. Rev. D* **61** 057502 (2000)
- Kaur H, Khanna M P “Masses of heavy hadrons”, hep-ph/0005077
- Singh C P, Khanna M P *Lett. Nuovo Cimento* **30** 276 (1981)
- Dai W-Sh et al. *Phys. Rev. D* **62** 114026 (2000)
- Bagan E, Chabab M, Narison S *Phys. Lett. B* **306** 350 (1993); Bagan E et al. *Z. Phys. C* **64** 57 (1994)
- Kiselev V V, Onishchenko A I “Two-loop anomalous dimensions for currents of baryons with two heavy quarks in NRQCD”, hep-ph/9810283
- Groote S, Körner J G, Yakovlev O I *Phys. Rev. D* **54** 3447 (1996)
- Smilga A *Yad. Fiz.* **35** 473 (1982) [*Sov. J. Nucl. Phys.* **35** 271 (1982)]
- Kiselev V V, Kovalsky A E *Phys. Rev. D* **64** 014002 (2001)
- Kiselev V V, Onishchenko A I *Nucl. Phys. B* **581** 432 (2000)
- Cutkosky R E *J. Math. Phys.* **1** 429 (1960)
- Beneke M *Phys. Rep.* **317** 1 (1999)
- Beneke M, Signer A *Phys. Lett. B* **471** 233 (1999); hep-ph/9906475
- Hoang A H *Phys. Rev. D* **61** 034005 (2000)
- Melnikov K, Yelkhovsky A *Phys. Rev. D* **59** 114009 (1999)
- Ovchinnikov A I, Pivovarov A A *Phys. Lett. B* **163** 231 (1985)

50. Falk A F et al. *Phys. Rev. D* **49** 555 (1994)
51. Kiselev V V, Likhoded A K, Shevlyagin M V *Z. Phys. C* **63** 77 (1994)
52. Kiselev V V, Likhoded A K, Shevlyagin M V *Yad. Fiz.* **57** 733 (1994) [*Phys. Atom. Nucl.* **57** 689 (1994)]
53. Berezhnoy A V, Kiselev V V, Likhoded A K *Z. Phys. A* **356** 79 (1996)
54. Berezhnoy A V, Kiselev V V, Likhoded A K *Z. Phys. A* **356** 89 (1996)
55. Kiselev V V, Likhoded A K, Shevlyagin M V *Phys. Lett. B* **332** 411 (1994); *Yad. Fiz.* **58** 1092 (1995) [*Phys. Atom. Nucl.* **58** 1018 (1995)]
56. Braguta V V, Chalov A E “Pair production of doubly heavy baryons”, hep-ph/0005149
57. Kiselev V V *Phys. Rev. D* **58** 054008 (1998); *Yad. Fiz.* **62** 335 (1999) [*Phys. Atom. Nucl.* **62** 300 (1999)]
58. Braaten E, Fleming S, Yuan T C *Ann. Rev. Nucl. Part. Sci.* **46** 197 (1997)
59. Braaten E, Yuan T C *Phys. Rev. Lett.* **71** 1673 (1993); Braaten E, Cheung K, Yuan T C *Phys. Rev. D* **48** 4230, 5049 (1993)
60. Jaffe R L, Randall L *Nucl. Phys. B* **412** 79 (1994)
61. Kiselev V V, Koval’sky A É *Yad. Fiz.* **63** 1728 (2000) [*Phys. Atom. Nucl.* **63** 1640 (2000)]
62. Martynenko A P, Saleev V A *Yad. Fiz.* **60** 517 (1997) [*Phys. Atom. Nucl.* **60** 443 (1997)]; *Phys. Lett. B* **385** 297 (1996)
63. Saleev V A *Yad. Fiz.* **63** 1716 (2000) [*Phys. Atom. Nucl.* **63** 1629 (2000)]
64. Georgi H, Wise M B *Phys. Lett. B* **243** 279 (1990); Carone C D *Phys. Lett. B* **253** 408 (1991); Savage M J, Wise M B *Phys. Lett. B* **248** 177 (1990)
65. Berezhnoy A V, Kiselev V V, Likhoded A K *Yad. Fiz.* **59** 909 (1996) [*Phys. Atom. Nucl.* **59** 870 (1996)]
66. Baranov S P *Phys. Rev. D* **54** 3228 (1996)
67. Doncheski M A, Steegborn J, Stong M L *Phys. Rev. D* **53** 1247 (1996)
68. Berezhnoy A V, Likhoded A K, Shevlyagin M V *Yad. Fiz.* **58** 730 (1995) [*Phys. Atom. Nucl.* **58** 672 (1995)]; Berezhnoy A V, Likhoded A K, Yushchenko O P *Yad. Fiz.* **59** 742 (1996) [*Phys. Atom. Nucl.* **59** 709 (1996)]
69. Lai H L et al. *Phys. Rev. D* **55** 1280 (1997)
70. Owens J F *Phys. Rev. D* **30** 943 (1984)
71. Berezhnoy A V, Kiselev V V, Likhoded A K, Onishchenko A I *Phys. Rev. D* **57** 4385 (1998)
72. Vogt R, Brodsky S J *Nucl. Phys. B* **478** 311 (1996)
73. Moenster M A Z. *Phys. A* **355** 349 (1996); Kaplan D M, Kwan S (Eds) *The Future of High Sensitivity Charm Experiments: Proc. of the CHARM 2000 Workshop, Fermilab, June 7–9, 1994*, Preprint Fermilab-Conf-94/190 (Batavia, IL: Fermilab, 1994)
74. Beneke M, Buchalla G *Phys. Rev. D* **53** 4991 (1996)
75. Okun’ L B *Leptony i Kvarki* (Leptons and Quarks) (Moscow: Nauka, 1981) [Translated into English (Amsterdam: North-Holland, 1984)]
76. Bigi I et al. *B Decays* (Ed. S Stone) 2nd ed. (Singapore: World Scientific, 1994)
77. Guberina B, Rückl R, Trampetic J Z. *Phys. C* **33** 297 (1986)
78. Bigi I, Uraltsev N, Vainshtein A *Phys. Lett. B* **293** 430 (1992); **297** 477 (1993); Blok B, Shifman M *Nucl. Phys. B* **399** 441 (1993); Bigi I et al. *Phys. Lett. B* **323** 408 (1994)
79. Manohar A V, Wise M B *Phys. Rev. D* **49** 1310 (1994)
80. Falk A F et al. *Phys. Lett. B* **326** 145 (1994); Koyrakh L *Phys. Rev. D* **49** 3379 (1994)
81. Kiselev V V, Likhoded A K, Onishchenko A I *Phys. Rev. D* **60** 014007 (1999)
82. Altarelli G et al. *Nucl. Phys. B* **187** 461 (1981)
83. Buras A J, Weisz P H *Nucl. Phys. B* **333** 66 (1990)
84. Buchalla G *Nucl. Phys. B* **391** 501 (1993)
85. Hokim Q, Pham X Y *Phys. Lett. B* **122** 297 (1983); *Ann. Phys. (New York)* **155** 202 (1984)
86. Bagan E et al. *Nucl. Phys. B* **432** 3 (1994); *Phys. Lett. B* **342** 362 (1995); **351** 546 (1995)
87. Kiselev V V, Likhoded A K, Onishchenko A I *Eur. Phys. J. C* **16** 461 (2000)
88. Gershtein S S et al. *Phys. Rev. D* **51** 3613 (1995)
89. Voloshin M B *Phys. Lett. B* **385** 369 (1996)
90. Voloshin M B “Inclusive weak decay rates of heavy hadrons”, hep-ph/0004257
91. Altarelli G et al. *Phys. Lett. B* **382** 409 (1996)
92. Quigg C, in *Proc. of the Workshop on B Physics at Hadron Accelerators, Snowmass, Colorado, 1993*, Preprint Fermilab-Conf-93/267 (Eds P McBride, C S Mishra) (Batavia, IL: Fermilab, 1993) p. 439
93. Bigi I *Phys. Lett. B* **371** 105 (1996)
94. Kiselev V V, Likhoded A K, Onishchenko A I *Nucl. Phys. B* **569** 473 (2000); Kiselev V V, Koval’sky A E, Likhoded A K *Nucl. Phys. B* **585** 353 (2000)
95. Guberina B, Melic B, Stefancic H *Eur. Phys. J. C* **9** 213 (1999)
96. Onishchenko A I “Doubly heavy systems: decays and OPE”, hep-ph/9912424; Likhoded A K, Onishchenko A I “Lifetimes of doubly heavy baryons”, hep-ph/9912425
97. Guberina B, Melic B, Stefancic H *Phys. Lett. B* **484** 43 (2000)
98. Onishchenko A I “Exclusive decays of $\Xi_{QQ'}$ baryons in NRQCD sum rules”, hep-ph/0006271; “Inclusive and exclusive decays of doubly heavy baryons”, hep-ph/0006295; talk given at the 5th Intern. Workshop “Heavy Quark Physics”, Dubna, Russia, 2000
99. Sanchis-Lozano M A *Nucl. Phys. B* **440** 251 (1995)