Physics – Uspekhi **44** (3) 291 – 305 (2001)

PHYSICS OF OUR DAYS

PACS numbers: **04.70.–s**, 04.80.Nn, 97.60.Lf, 98.62.Js

Black holes in the Universe

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DOI: 10.1070/PU2001v044n03ABEH000773

Contents

Abstract. Diverse physical and astrophysical aspects of black holes are reviewed. We start by describing a membrane paradigm approach in which a black hole is treated as a physical body with very special properties. In particular, a black hole behaves as a conducting sphere with a universal finite electrical resistivity, so that when rotating in an external magnetic field it becomes a unipolar inductor capable of producing a huge potential difference. Astrophysical applications of this mechanism are described and the properties of spacetime inside a black hole are briefly considered. In the bulk of the review, possible sources of observational evidence for the existence of black holes are discussed. Prospects for the detection of gravitational waves from black holes in future by gravitational wave observatories are also examined. The review is concluded with a discussion of the universality phenomenon discovered recently in a study of critical gravitational collapse.

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Received 19 April 2000
Uspekhi Fizicheskikh Nauk **171** (3) 307 – 324 (2001)
Translated by A F Zakharov; edited by A Radzig

1. Introduction

Some 30 years ago very few scientists thought that black holes could really exist. Attention focussed on the black hole hypothesis after neutron stars had been discovered. It was rather surprising that astrophysicists immediately 'welcomed' black holes. They found their place not only in the remnants of supernova explosions but also in the nuclei of galaxies and quasars.

A black hole is perhaps the most fantastic of all conceptions of the human mind. Black holes are neither bodies nor radiation. They are clots of gravity. The study of black hole physics extends our knowledge of the fundamental properties of space and time. Quantum processes occur in the neighborhood of black holes, so that the most intricate structure of the physical vacuum is revealed. Even more powerful (catastrophically powerful) quantum processes follow inside black holes (in the vicinity of the singularity). One may say that black holes are a door to a new, very broad field of investigation of the physical world.

In this paper we will give a brief review of some problems centering around the physics and astrophysics of black holes. For a systematic discussion of the problems see the books by Thorne et al. [1], Novikov and Frolov [2], the section 'Black Holes' in the book by Kawaler et al. [3], and Frolov and Novikov [4].

2. Physics outside a black hole

Let us start with physics. By definition, a *black hole* is the region in spacetime from which no information-carrying signal can escape to an external observer. A black hole's boundary is the so-called *event horizon*. After the gravitational collapse of a celestial body and the formation of a black hole, its external gravitational field asymptotically approaches a standard equilibrium configuration known as the Kerr – Newman field and which is characterized by only three parameters: mass, angular momentum and charge.

Spacetime in the vicinity of black holes is highly curved. If a black hole has a nonzero angular momentum, then anything near a black hole will be dragged along into rotation by the vortex gravitational field. In this section we consider a black hole without an electric charge (a Kerr black hole). The horizon's surface area can be written in terms of its mass M and angular momentum $J = aM$, where a is the angular momentum per unit mass ($c = 1$, $G = 1$):

$$A = 4\pi(r_{\mathrm{H}}^2 + a^2)\,, \tag{1}$$

$$r_{\mathrm{H}} = M + \sqrt{M^2 - a^2}\,. \tag{2}$$

The rotational energy or corresponding mass M_{rot} of a Kerr black hole is the following

$$M_{\mathrm{rot}} = M - \left[\frac{1}{2}\,M\Big(M + \sqrt{M^2 - a^2}\Big)\right]^{1/2}. \tag{3}$$

This rotational energy (energy of the vortex gravitational field) can be extracted (in principle) from a black hole.

The black hole is a clot of gravity; there is no real matter on the horizon. In spite of this fact the horizon for an external observer (outside the black hole) looks and behaves as a physical membrane which is made from a two-dimensional viscous fluid with definite mechanical, electrical and thermodynamic properties. This remarkable viewpoint is known as the membrane paradigm (see Thorne et al. [1] for a review). According to this paradigm, the interaction of the horizon with the external universe is described in terms of familiar laws for the horizon fluid, for example, the Navier – Stokes equation, Maxwell equations, a tidal force equation, and the equations of thermodynamics. It is very important to emphasize that the membrane paradigm is not an approximation method or some analogy. It is an exact formalism which gives exactly the same results as the standard formalism of general relativity. Because the laws governing the horizon's behavior have familiar forms, they are powerful for understanding intuitively and computing quantitatively the interaction of black holes with complex environments.

In subsequent parts of this section we consider some manifestations of the physical properties of the black hole's membrane that resides in three-dimensional space.

2.1. Mechanical properties of the horizon membrane

According to the membrane formalism, from the point of view of an external observer the black hole's membrane has a definite surface mass density, surface pressure and viscosity. The mass density is defined by the following expressions

$$\sigma = -\frac{1}{8\pi}\,\theta^{\mathrm{H}}\,, \qquad \theta^{\mathrm{H}} \equiv \frac{\mathrm{d}(\Delta A)}{\Delta A\,\mathrm{d}t}\,, \tag{4}$$

where θ^{H} is the fractional change of area of a surface element per unit time for an observer at infinity. The value of θ^{H} is always nonnegative for classical processes, consequently σ is always nonpositive. One can see that for the case of a black hole in equilibrium [for example, a nonrotating (Schwarzschild) or a Kerr black hole in empty space] $\sigma = 0$.

There is surface pressure p^{H} in the membrane. For a Schwarzschild black hole it is

$$p^{\mathrm{H}} = \frac{1}{32\pi M} \approx \frac{M_{\odot}}{M} \times 10^{42}\ \mathrm{dyn\,cm^{-2}}\,, \tag{5}$$

where $M_{\odot} \approx 2 \times 10^{33}$ g is the mass of the Sun. From the point of view of the membrane formalism, the gravity of a black hole in equilibrium is produced by p^{H}.

The horizon's shear viscosity η^{H} and the horizon's bulk viscosity ζ^{H} are correspondingly

$$\eta^{\mathrm{H}} = \frac{1}{16\pi} \approx 10^{37}\ \mathrm{g\,s^{-1}}\,, \tag{6}$$

$$\zeta^{\mathrm{H}} = -\frac{1}{16\pi} \approx -10^{37}\ \mathrm{g\,s^{-1}}\,. \tag{7}$$

Because the membrane paradigm regards a black hole as a two-dimensional membrane with familiar mechanical properties, it is quite easy to understand intuitively and compute quantitatively what happens with a black hole under some definite conditions. Let us consider a few examples.

If a black hole is created in the gravitational collapse of an asymmetric nonrotating celestial body, then a nonspherical hole arises at first. The black hole's membrane is deformed and there is no balance between the surface pressure of the membrane and its gravity. So the membrane vibrates and radiates gravitational waves. The waves carry away the energy of the membrane deformation. This effect together with the membrane viscosity makes the horizon settle down into an absolutely spherical equilibrium shape.

Another example is the shape of the membrane for a rotating black hole. Centrifugal forces make the hole's membrane bulge out in the equatorial plane. The balance between the surface pressure, gravity and centrifugal forces determines the shape of the horizon membrane.

Let us consider one very unusual property of the horizon membrane. We emphasized above that the differential equations which describe the interaction of the horizon with the external universe are familiar physical laws (e.g., the Navier – Stokes equation and so on). But the solutions of the equations are also determined by the boundary conditions. In the case of conventional physics, the boundary conditions must be imposed at some initial moment or in the infinite past. This is not so for the black hole's horizon! The point is that the horizon is the boundary between the light-speed signals that can and those that cannot ever escape to spatial infinity. But this fact depends on processes in the future, not in the past.

Whether a signal can escape depends on the region of spacetime in the future of the signal's source. This means that the motion of the horizon at any instant of time depends not on what has happened to the horizon in the past but what will happen to the horizon in the future.

This property can be illustrated by the problem of free fall of a thin spherical shell of a matter of mass ΔM into a Schwarzschild hole with the mass M. The spacetime geometry is Schwarzschildian both inside and outside the shell. Inside the shell, the Schwarzschild mass is M, while outside the mass is $M + \Delta M$. Now the light-speed signals with world lines at $r = 2M$ cannot be the boundary of the nonescape region because these signals and outgoing signals just outside the region $r = 2M$ will get caught and pulled into the hole by the added gravity of the shell, when in the future the shell will pass through them. The real boundary (that is the event horizon) is generated by null world lines propagating just outside the surface $r = 2M$. In the past, long before the shell crosses the horizon, this null surface practically coincides with $r = 2M$. Then, the null surface starts to expand. This occurs because the world lines of its generators go farther and farther from $r = 2M$. This is their property in the Schwarzschild spacetime, and it does not depend on the approaching shell. When the shell finally passes through it, the added shell's gravity starts to influence the motions of the generators of the surface: the horizon suddenly stops expanding and freezes at $r = 2(M + \Delta M)$. This behavior of the horizon is dictated by the properties of propagation of the light-speed signals forming the horizon and the condition that the horizon is at $r = 2(M + \Delta M)$ *after* being crossed by the shell. Thus, the position of the horizon and its expansion *before* the shell crosses it depend on the events in the future (the collapse of the massive shell).

One might refer to this dependence on future events as the 'theological' nature of the horizon (see Thorne et al. [1]). We would like to emphasize that it looks as if the hole's membrane lives in time which flows in the *opposite direction*: from the future into the past. Indeed, in this case the change of the size of the horizon looks very natural and causal. If we accept this point of view, we should consider the extraction of the shell from the hole, and just after this extraction of the shell from the membrane at $r = 2(M + \Delta M)$, the horizon starts contracting and settles down at $r = 2M$. We shall see in Section 3 that this unusual property, namely, 'receiving' information from the infinite future of the external observer, is a characteristic property not only of the horizon but also of the interior of a black hole.

2.2 Black-hole electrodynamics

In the presence of an external electromagnetic field, a black hole's horizon behaves as an electrically conducting surface. To understand this, let us ask what could be the external manifestation of the electric conductivity of a body in a flat spacetime? The simplest manifestation is the following. If one brings a positive electric charge close to a metal sphere, then free electrons on the sphere's metal surface will be displaced with respect to the ions by the Coulomb electric forces. This polarizes the sphere. As a result, the electric field lines form a characteristic configuration in the space around the sphere. Now if one moves the charge parallel to the surface of the sphere from one position to another one, the characteristic configuration of the electric field lines comes to a new place with some delay. This delay is determined by the resistivity of the sphere's metal surface. It turns out that if one brings a charge close to a nonspinning black hole, there is a similarity between the picture of the field lines in the vicinity of the black hole and the analogous pattern in the vicinity of a metal sphere in a flat spacetime. Now the curvature of spacetime distorts the field lines rather than the displacement of real charges on the horizon. Nevertheless, it looks like the electric field of the charge polarizes the horizon.

If one moves the charge parallel to the hole's horizon to another position, then the configuration of the electric field lines will settle down at the new place with some delay. Now this effect is determined by the finite propagation time of electromagnetic signals. Nevertheless, one can interpret it as an effective resistance of the horizon.

In general one can say that a horizon membrane behaves as a conducting sphere with a surface resistance equal to $R_{\mathrm{H}} = 4\pi \approx 377\ \Omega$.

The membrane paradigm gives insight into the possible behavior of rotating black holes interacting with a magnetized plasma. We shall draw an analogy with a dynamo. The motion of the wire coils of a dynamo rotor in a magnetic field produces an electromotive force compelling the charges to flow through the conductor. A black hole is also a special dynamo of a giant size. If a spinning black hole is immersed in an external magnetic field, a strong electric field is generated in its vicinity. The magnetic field is created by the interstellar gas flowing into a black hole. The magnetic field lines will tend to rotate along with the spinning black hole. The motion of any magnetic field generates an electric field. In the case of a rapidly rotating, magnetized black hole, the electric field generated near its edge can produce an enormous potential difference between the poles of the hole and its equatorial region:

$$\Delta V \approx \frac{a}{M}\,\frac{M}{10^9 M_\odot}\,\frac{B}{10^4\ \mathrm{G}}\ 10^{20}\ \mathrm{V}\,, \tag{8}$$

where B is the magnetic field induction in the vicinity of the black hole. It is as though the spinning black hole were a huge battery. The electric field is responsible for accelerating the charged particles of the plasma and causing them to move along the magnetic lines of force. The total power output is

$$P \approx \left(\frac{a}{M}\right)^2 \left(\frac{M}{10^9 M_\odot}\right)^2 \left(\frac{B}{10^4\ \mathrm{G}}\right)^2\ 10^{45}\ \mathrm{erg\ s^{-1}}\,. \tag{9}$$

Probably, this mechanism is the main 'engine' of active galactic nuclei (see Section 4.4).

2.3 Thermodynamics of black holes

Of many aspects of black hole thermodynamics, we discuss here only two problems: the black hole's thermal quantum radiation and the thermal atmosphere of a black hole.

Hawking [5] discovered that a black hole should emit thermal radiation with a temperature

$$T_{\mathrm{H}} = \frac{\hbar}{8\pi k_{\mathrm{B}}}\,M^{-1} \approx \frac{M_\odot}{M}\ 10^{-7}\ \mathrm{K}\,. \tag{10}$$

How, in simple physical terms, could one understand that a black hole behaves like an ordinary body with the temperature T_{H}? A key insight into the thermal emission from a black hole comes from theoretical discoveries in the mid-1970s (see Ref. [6]). The crucial point is the existence of the event horizon for some definite classes of observers. For example, a uniformly accelerated observer in a flat spacetime has a horizon. This observer cannot receive information from the region over the horizon. The virtual particles' vacuum fluctuation waves are not confined solely to the region above the horizon: part of each fluctuation wave is over the horizon and part is within the region which the observer can see. According to quantum mechanics, this principle lack of information about vacuum fluctuation waves leads to the conclusion (for an accelerated observer) that they are real waves. As a result, this observer is plunged in a perfect thermal radiation thermostat with temperature $T = \hbar a/(2\pi k)$, where a is the observer's acceleration. Since a static observer just above a Schwarzschild horizon can be viewed as analogous to an accelerated observer in flat spacetime with acceleration $a = c^2/z$, where z is the proper distance from the horizon, such an observer should feel himself bathed in thermal radiation with a local temperature $T = \hbar/(2\pi k z)$. This thermal radiation forms the thermal atmosphere of the black hole. The radiation breaking through the hole's gravitational field would be redshifted by a factor $(1 - 2M/r)^{1/2}$. It will emerge with temperature T_{H}. Most of the photons and other particles fly outward the hole a short distance and are then trapped again by the hole's enormous gravitational field. A few of the particles manage to escape the hole's gravitational grip and evaporate into space. These particles form the Hawking radiation.

Note that a free falling observer does not feel this thermal atmosphere. He 'sees' only the 'usual' zero-point vacuum fluctuations.

The process of Hawking quantum evaporation is very slow. The total lifetime is proportional to the cube of the black hole mass. For a 20-solar-mass black hole it is 10^{70} years. In principle, for special processes the interaction of a black hole with the external universe can essentially change the efficiency of release of the thermal energy from a black hole atmosphere (Unruh and Wald [7]).

3. Physics inside a black hole

3.1 A black hole's interior

What can one say about the interior of a black hole? This problem has been the subject of very active investigation in recent decades. There has been a great progress in this research. We knew some important properties of a realistic black hole's interior, but some details and crucial problems are still the subject of much debate.

A very important point for understanding the problem of the black hole's interior is the fact that the path into the gravitational abyss of the interior of a black hole offers a progression in time. We recall that inside a spherical hole, for example, the radial coordinate is time-like. It means that the problem of the black hole's interior is an *evolutionary problem*. In this sense it is completely different from the problem of the internal structure of other celestial bodies, stars for example.

In principle, if we know the conditions on the border of a black hole (on the event horizon), we can integrate the Einstein equations in time and learn the structure of the progressively deeper layers inside the black hole. Conceptually it looks simple, but there are two principal difficulties which prevent realizing this idea consistently.

The first difficulty is the following. The internal structure of a generic rotating black hole even soon after its formation depends crucially on the conditions on the event horizon in very distant future of the external observer (formally in the infinite future). This happens because a light-like signal can come from the very distant future to those regions inside a black hole which are deep enough in the hole. The limiting light-like signals which propagate from (formally) infinite future of the external observer form a border inside a black hole, which is called a *Cauchy horizon*.

Thus, the structure of the regions inside a black hole crucially depends on the fate of the black hole in the infinite future of an external observer. For example, it depends on the final state of the black hole evaporation, on possible collisions of the black hole with other black holes, and on the fate of the universe itself. It is clear that theoreticians feel themselves uncomfortable under such circumstances.

The second serious problem is related to the existence of a singularity inside a black hole. Close to the singularity, where the curvature of the spacetime approaches the Planck value, classical general relativity is not applicable. We have no final version of the quantum theory of gravity yet, thus any extension of the discussion about the physics in this region would be highly speculative. Fortunately, as we shall see, these singular regions are deep enough in the black hole's interior and are *in the future* with respect to overlying and *preceding* layers of the black hole, where curvatures are not so high and which can be described by well-established theory.

The first attempts to investigate the interior of a Schwarzschild black hole were made in the late 70s. It was demonstrated that in the absence of external perturbations, those regions of the black hole's interior which are located long after the black hole formation are virtually free of perturbations. This happens because the gravitational radiation from aspherical initial excitations becomes infinitely diluted as it reaches these regions. But this result is not valid in the general case when the angular momentum or the electric charge does not vanish. The reason for this is related to the fact that the topology of the interior of a rotating or charged black hole differs drastically from that of a Schwarzschild one. The key point is that the interior of this black hole possess a Cauchy horizon. This is a surface of infinite blueshift. Infalling gravitational radiation propagates inside the black hole along paths approaching the generators of the Cauchy horizon, and the energy density of this radiation will suffer an infinite blueshift as it approaches the Cauchy horizon.

In general, the evolution with time into the depths of a black hole looks like the following. There is a weak flux of gravitational radiation into a black hole through the horizon because of small perturbations outside it. When this radiation approaches the Cauchy horizon it suffers an infinite blueshift. The infinitely blueshifted radiation together with the radiation scattered by the curvature of spacetime inside the black hole gives rise to a tremendous growth of the black hole's internal mass parameter ('mass inflation', after Poisson and Israel [8]) and finally leads to the formation of the curvature singularity of the spacetime along the Cauchy horizon. Infinite tidal gravitational forces arise here. This result was confirmed by considering different models of the ingoing and outgoing fluxes in the interior of charged and rotating black holes. It was shown that the singularity on the Cauchy horizon is quite weak. In particular, the integral of the tidal force in the freely falling frame of reference over the proper time remains finite.

A detailed discussion can be found in the following works [8 – 11] and references cited therein.

3.2 Quantum effects

In the previous discussion we emphasized that the internal structure of black holes is a problem of evolution in time starting from boundary conditions on the event horizon for all moments of time up to the infinite future of the external observer.

It is very important to know the boundary conditions up to infinity because we observed that the essential events — mass inflation and singularity formation — happened along the Cauchy horizon which brought information from the infinite future of the external spacetime. However, even an isolated black hole in an asymptotically flat spacetime cannot exist forever. It will evaporate by emitting Hawking quantum radiation. So far we discussed the problem without taking into account this ultimate fate of black holes. Even without going into details it is clear that quantum evaporation of the black holes is crucial for the whole problem.

What can we say about the general picture of the black hole's interior accounting for quantum evaporation? To account for the latter process we have to change the boundary conditions on the event horizon as compared to the boundary conditions discussed above. Now they should include the flux of negative energy across the horizon, which is related to the quantum evaporation. The last stage of quantum evaporation, when the mass of the black hole becomes comparable to the Planck mass $m_{\rm Pl} = (\hbar c/G)^{1/2} \approx 2.2 \times 10^{-5}$ g, is unknown. At this stage the spacetime curvature near the horizon reaches $l_{\rm Pl}^{-2}$, where $l_{\rm Pl}$ is the Planck length:

$$l_{\rm Pl} = \left(\frac{G\hbar}{c^3}\right)^{1/2} \approx 1.6 \times 10^{-33}\ \text{cm}\,.$$

This means that from the point of view of semiclassical physics a singularity arises here. Probably at this stage the black hole has the characteristics of an extreme black hole, when the external event horizon and internal Cauchy horizon coincide.

As for the processes inside a true singularity in the black hole's interior, they can be treated only in the framework of an unified quantum theory incorporating gravitation, which is unknown.

4. Astrophysics of black holes

Do black holes exist in the universe or are they only an abstract concept of the human mind? In principle, a black hole could be built artificially. However, this meets such grandiose technical difficulties that it seems impossible to master them, at least in the immediate future. In fact, the artificial building of a black hole looks even more problematic than the artificial creation of a star. Thus we have to conclude that the physics of black holes, as the physics of stars, concerns the celestial bodies. Stars definitely exist, but what may one say about the existence of astrophysical black holes?

Modern astrophysics deals with two types of black holes in the universe:

(1) *Stellar black holes*, i.e. black holes of stellar masses that were born when massive stars died.

(2) *Supermassive black holes* with masses up to $10^9 M_\odot$ and greater at the centers of galaxies.

These two types of black holes have been discovered. The third possible type of astrophysical black hole — a primordial black hole — will be discussed in Section 4.5. Our main attention in Section 4 is focused on the possible observational manifestation of black holes.

4.1 The origin of stellar black holes

"When all the thermo-nuclear sources of energy are exhausted, a sufficiently heavy star will collapse" — this is the first phrase of the abstract of a remarkable paper by Oppenheimer and Snyder (1939) [12]. Every statement of this paper accords with ideas that remain valid today. The authors conclude the abstract with the following sentence: "... an external observer sees the star shrinking to its gravitational radius". This is the modern prediction of the formation of black holes, when massive stars die.

How heavy should a star be to turn into a black hole? The answer is not simple. A star that is not massive enough ends up either as a white dwarf or a neutron star. There are upper limits on the masses of both these types of celestial bodies. For white dwarfs, this is the *Chandrasekhar limit* which is about $(1.2-1.4) \times M_\odot$. For neutron stars, it is the *Oppenheimer–Volkoff limit*. The exact value of this limit depends on the equation of state at a matter density higher than the density of nuclear matter, $\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} . The modern theory gives for the maximum mass of a nonrotating neutron star the estimate $(2-3) \times M_\odot$. Rotation can increase the maximum mass of a nonrotating neutron star only slightly — up to 25%. Thus one can believe that the upper mass limit for neutron stars should not be greater than $M_0 \approx 3M_\odot$. If a star at the very end of its evolution has mass greater than M_0, it must turn into a black hole. However, this does not mean that all normal stars (on the 'main sequence' of the Herzsprung–Russell diagram) with masses $M > M_0$ are black hole progenitors. The point is that the final stages of evolution of massive stars are poorly understood. Steady mass loss, catastrophic mass ejection and even disruption in supernovae explosions are possible. These processes can considerably reduce the mass of a star at the end of its evolution. Thus the initial mass of black hole progenitors could be essentially greater than M_0.

There are different estimates for the minimum mass M_* of a progenitor star that still forms a black hole. The uncertainty reaches $M_* \approx (10-40)M_\odot$ and even more. Numerical simulations show that besides the prompt direct gravitational collapse of a progenitor, black holes can also be formed in supernova explosions. In the latter case, the fallback of a part of the matter after the explosion drives the compact object in the remnant core beyond the maximum neutron star mass, causing it to collapse into a black hole. There are indications that more massive progenitors (of mass more than $40M_\odot$) can form black holes directly, while progenitors of smaller mass create black holes in the delayed collapse owing to fallback (see e.g. Ref. [13]). Recently Israelian et al. [14] reported evidence for a supernova origin of the black hole in the binary system GRO J1655-40. By studying the optical spectrum from the subgiant companion star with mass $1.7-3.3M_\odot$, they found evidence of so-called α-elements — O, Mg, Si and S — with abundances six to ten times higher than in the Sun. These elements can be produced only in the inner cores of $25-40M_\odot$ massive stars. A proposed explanation is that the companion star got these elements during the supernova explosion which produced the black hole in the binary.

Let us emphasize that the evolution of stars in close binary systems differs from the evolution of solitary stars because of mass transfer from one star to another. The conclusions about masses of black hole progenitors in this case could be essentially different. In particular, a black hole can be produced in the binary where originally, besides a normal star, there was a neutron star. A black hole can be formed as a result of the flux of matter from the star companion onto the neutron star, which finally makes the mass of the latter greater than the neutron mass limit.

One can try to estimate how many black holes have been created by stellar collapse in our Galaxy during its existence. The estimates give a number of order 10^9.

4.2 Disk accretion onto black holes

For the purpose of finding and investigating black holes, two specific cases of accretion are of particular importance: accretion in binary systems and accretion onto the supermassive black holes that probably reside at the centers of galaxies. In both cases, the accreting gas has a large specific angular momentum. As a result the gas elements circle around the black hole in Keplerian orbits, forming a disk or a torus around it. Viscosity plays a crucial role in the accretion. It removes angular momentum from each gas element, permitting it to gradually spiral inward toward the black hole. At the same time the viscosity heats the gas, causing it to radiate. Probable sources of viscosity are turbulence in the gas disk and random magnetic fields. Unfortunately, we are not near a good physical understanding of the effective viscosity. Large-scale magnetic fields may also play an important role in the physics of accretion.

The properties of the accreting disk are determined by the rate of gas accretion. An important measure of any accretion luminosity for a black hole is provided by the Eddington critical luminosity

$$L_{\mathrm{E}} = \frac{4\pi G M_{\mathrm{h}} m_{\mathrm{p}} c}{\sigma_{\mathrm{T}}} = \frac{M_{\mathrm{h}}}{M_{\odot}}\, 1.3 \times 10^{38}\ \mathrm{erg\ s^{-1}}\,. \qquad (11)$$

Here, M_{h} is the mass of the black hole, m_{p} is the rest mass of the proton, and σ_{T} is the Thomson cross section. It is the luminosity at which the radiation pressure just balances the gravitational force of the mass M_{h} for a fully ionized plasma.

A useful measure of the accretion rate $\dot{M}$ is the so-called 'critical accretion rate'

$$\dot{M}_{\mathrm{E}} = L_{\mathrm{E}} c^{-2}\,, \qquad (12)$$

where L_{E} is given by equation (11). We shall also use the dimensionless ratio $\dot{m} \equiv \dot{M}/\dot{M}_{\mathrm{E}}$.

The first models of the disk accretion were rather simple. They focused on the case of a moderate rate of accretion, $\dot{m} < 1$. Subsequently theories for $\dot{m} \sim 1$ and $\dot{m} \gg 1$ were developed. They take into account complex processes in radiating plasma and various types of instabilities.

The source of luminosity for disk accretion is the gravitational energy that is released when gas elements in the disk spiral down. Most of the gravitational energy is released, generating most of the luminosity, from the inner parts of the disk. According to the theory for these simple models the total luminosity of the disk is equal to

$$L = q\,\frac{\dot{M}}{10^{-9} M_{\odot}\ \mathrm{yr}^{-1}}\, 3 \times 10^{36}\ \mathrm{erg\ s^{-1}}\,, \qquad (13)$$

where the coefficient q depends on the angular velocity of the black hole. It is of the order of 1 for a nonrotating black hole, and of the order of 10 for an extremely rotating one.

The accretion rate $\dot{M}$ makes up an arbitrary external parameter which is determined by the source of gas (for example, by the gas flux from the upper atmosphere of the companion star in a binary system). We normalized $\dot{M}$ to the value $\dot{M}_0 = 10^{-9} M_{\odot}\ \mathrm{yr}^{-1}$, because this is probably the typical rate at which a normal star dumps gas onto a companion black hole. In this model, the accretion gas is assumed to be relatively cool, with its temperature much less than the virial temperature corresponding to the potential energy in the gravitational field. As estimates show, a *geometrically thin disk* (with heights $h \ll r$) might be formed under these conditions. This is the so-called *standard disk model* (see Refs [15 – 17]). In this model the electron and ion temperatures are equal, and the disk is effectively optically thick. The gas temperature in the inner parts of the disk reaches $T \approx 10^7 - 10^8$ K. In this region, electron scattering opacity modifies the emitted spectrum so that it is no longer a blackbody spectrum. Instead, the total spectrum of the disk radiation is fitted by a power law $F \sim \omega^{1/3}$ with an exponential cut off at high frequencies. The innermost regions of such 'standard' disks are probably unstable.

The thin accretion disk model is unable to explain the hard spectra observed in accretion flows around black holes in many observable cases. A few types of hot accretion flow models have been proposed. Among them a model with a hot corona above a standard thin accretion disk. In another model, the ions in the inner region are hot, $T_{\mathrm{i}} \approx 10^{11}$ K, but the electrons are considerably cooler: $T_{\mathrm{e}} \approx 10^9$ K. This inner disk is thicker than that in the 'standard' model and produces most of the X-ray emission. The models with hot ions and cooler electrons are optically thin.

Further development of the theory of disk accretion led to more sophisticated models. It has been demonstrated that when the luminosity reaches a critical level (corresponding to $\dot{m} \equiv \dot{M}/\dot{M}_{\mathrm{E}}$ of the order of unity), the radiation pressure in the inner parts of the disk dominates the gas pressure and the disk is thermally and viscously unstable. For especially big values of $\dot{m} > 80$, the essential part of the plasma energy is lost by advection into the black hole's horizon because the radiation is trapped by the accretion gas and is unable to escape from the system of interest. This process stabilizes the gas flow against perturbations. Advection can also be important for smaller $\dot{m}$. For higher mass accretion rates, the height of the accretion disk becomes comparable to its radius. In modern models, the radial pressure gradients and the motion of gas elements along the radius are taken into account. In the innermost parts of the disk and down to the black hole, the flow of gas is supersonic.

Recently, a new class of optically thin hot disk solutions has been discovered. In this model, most of the viscously dissipated energy is advected with the accreting gas, with only a small fraction of the energy being radiated. It is because the gas density is so low that the radiative efficiency is very poor. These models are called *advection-dominated*. They have been applied successfully to a few concrete celestial objects.

In conclusion we note that in some models of disk accretion electron – positron pair production can be important. We believe that new models involving recent achievements of plasma physics will play a key role in the modern astrophysics of black holes.

4.3 Evidence for black holes in stellar binary systems

Probably the best evidence that black holes exist comes from studies of X-ray binaries as predicted by Novikov and Zeldovich [18]. The arguments used in proving that an X-ray binary contains a black hole are as follows:

(1) The X-ray emitting object in a binary system is very compact, and therefore cannot be an ordinary star. Thus it is either a neutron star or a black hole. This argument mainly comes from analysis of the features of emitted X-rays.

(2) Analysis of the observational data allows one to determine the orbital motion in the binary system and makes it possible to obtain an estimate of the mass of the compact object. The observed velocity of the optical companion star is of the most importance. Notice that the Newtonian theory is always sufficient for the analysis. The technique of weighing stars in binaries is well known in astronomy. If the mass of the compact component is greater than the maximum possible mass of a neutron star, $M_0 \approx 3M_{\odot}$ (see Section 4.1), then it is a black hole.

It is worth noting that this evidence is somewhat indirect because it does not confront us with the specific relativistic effects that occur near black holes and which are peculiar to

Table 1. Black-hole candidates in binary systems (after Cherepashchuk [19]).

System	Spectral type of the optical companion	Orbital period, days	Mass of the compact companion in $M_\odot$	Mass of the optical companion in $M_\odot$	X-ray luminosity, erg s^{-1}
Cyg X-1 (V 1357 Cyg)	O9.7Iab	5.6	7 – 18	20 – 30	$\sim 8 \times 10^{37}$
LMC X-3	B(3 – 6) II – III	1.7	7 – 11	3 – 6	$\sim 4 \times 10^{38}$
LMC X-1	O(7 – 9) III	4.2	4 – 10	18 – 25	$\sim 2 \times 10^{38}$
A0620-00 (V616 Mon)	K(5 – 7)V	0.3	5 – 17	~ 0.7	$\leqslant 10^{38}$
GS 2023+338 (V404 Cyg)	K0IV	6.5	10 – 15	0.5 – 1.0	$\leqslant 6 \times 10^{38}$
GRS 1121-68 (XN Mus 1991)	K(3 – 5)V	0.4	9 – 16	0.7 – 0.8	$\leqslant 10^{38}$
GS 2000+25 (QZ Vul)	K(3 – 7)V	0.3	5.3 – 8.2	~ 0.7	$\leqslant 10^{38}$
GRO J0422+32 (XN Per 1992 = V518 Per)	M(0 – 4)V	0.2	2.5 – 5.0	~ 0.4	$\leqslant 10^{38}$
GRO J1655-40 (XN Sco 1994)	F5IV	2.6	4 – 6	~ 2.3	$\leqslant 10^{38}$
XN Oph 1977	K3	0.7	5 – 7	~ 0.8	$\leqslant 10^{38}$

black holes alone. However, it is the best that modern astronomy has proposed so far. In spite of these circumstances, we believe that the logic of the arguments is fairly reliable.

According to the generally accepted interpretation, we have the necessary observational confirmation only for a few systems at the present time. For these systems, we have strong reasons to believe that the compact X-ray emitting companions are black holes. Some characteristics of these leading black-hole candidates are summarized in Table 1 (according to Cherepashchuk [19]).

The most plausible masses of compact objects in these systems are considerably larger than $M_0 \approx 3M_\odot$. The strongest candidates are those which have a *dynamical lower limit* of the mass of the compact object (or so-called *mass function* [1]) greater than $3M_\odot$. From this point of view the strongest candidates are GS 2023 + 338 with $f(M) = 6.5M_\odot$, GS 2000 + 25 with $f(M) = 5M_\odot$, and XN Oph 1977 with $f(M) = 4M_\odot$.

The total number of systems that are frequently mentioned as possible candidates for black holes of stellar mass is about 20. All seriously discussed candidates are X-ray sources in binary systems. Some of them are persistent, others are transient. Begelman and Rees [20] summarized the present status as follows: "There is also overwhelming evidence for black holes in our own galaxy, formed when ordinary massive stars die, each weighing a few times as much as the Sun". Most of experts now agree with this unambiguous conclusion.

During the more than 25 years since the discovery of the first black-hole candidate Cyg X-1 only a few new candidates have been added. This is in sharp contrast to the rapid increase of the number of identified neutron stars. At present more than a thousand neutron stars have been identified in the Galaxy. About 100 of them are in binary systems. One might conclude that black holes in binary systems are exceedingly rare objects. This is not necessarily true, however. The small number of identified black-hole candidates may well be related to the specific conditions which are necessary for their observable manifestation.

According to estimates, the evolutionary stage when a black-hole binary continuously radiates X-rays may last only 10^4 years, that is during the period when an intense gas flux from the stellar atmosphere to the black hole exists. We can thus detect it only during this short period. In effect, the population of black-hole binaries may be much larger than what we can presently observe. Such systems may be as common as neutron star binaries.

At the end of this section we mention an interesting mechanism for electron – positron outflow from stellar black holes which are created by the collapse of a neutron star [21]. If there exists a flow of matter onto a rotating magnetized neutron star, which makes its mass greater than the critical one, it will collapse producing a rapidly rotating black hole immersed in a strong magnetic field. This field can be confined to the emitted matter of the neutron star which forms an accretion disk around the black hole. In this system, the produced potential difference ΔV given by Eqn (8) can be very high. If the captured magnetic field is of the order of the critical value 4.4×10^{13} G, such a black hole can produce a radiation of power of the order of $10^{47}(M/M_\odot)^2$ erg s^{-1}. Van Putten [21] discusses the possible relation of such objects to gamma-ray bursts.

4.4 Supermassive black holes in galactic centers

Since the middle of this century astronomers have come across many violent or even catastrophic processes associated with galaxies. These processes are accompanied by a powerful release of energy and are fast not only by astronomical but also by earthly standards. They may last only a few days or even minutes. Most such processes occur in the central parts of galaxies — the galactic nuclei.

About one percent of all galactic nuclei eject radio-emitting plasma and gas clouds, and are themselves powerful sources of radiation in the radio, infrared, and especially, the 'hard' (short wavelength) ultraviolet, X-ray and gamma regions of the spectrum. The full luminosity of the nuclei in some cases reaches $L \approx 10^{47}$ erg s^{-1}. This is millions of times greater than the luminosity of the nuclei of quieter galaxies, such as ours. These objects are called active galactic nuclei (AGN) (see Ref. [22]). Practically all the energy of activity and the giant jets released by galaxies originate from the centers of their nuclei.

Quasars form a special subclass of AGN. Their characteristic property is that their total energy release is hundreds of times greater than the combined radiation of all the stars in a large galaxy. At the same time, the average linear dimensions of the radiating regions are small: a mere one-hundred-millionth of the linear size of a galaxy. Quasars are the most powerful energy sources registered in the universe to date. What processes are responsible for the extraordinary outbursts of energy from AGN and quasars?

Learning about the nature of these objects involves measuring their sizes and masses. This is not easy at all. The

[1] The mass function $f(M)$ is defined as $f(M) = M^3 \sin^3 i/(M + M_1)^2$. Here, M is the mass of the compact object, M_1 is the mass of the optical star companion, and i is the angle between the orbit axis and the direction to an observer.

central emitting regions of AGN and quasars are so small that a telescope view reveals them as just point-like sources of light. Fortunately quite soon after the discovery of the quasar 3C 273 it was shown that its luminosity changed. Sometimes it changes very rapidly, in less than a week. After this discovery, even faster variability (on a time-scale of a few hours or less) was detected in other galactic nuclei. From these variations one could estimate the dimensions of the central parts of the nuclei that are responsible for radiation. The conclusion was drawn that these regions were not more than a few light-hours in diameter. That is, they are comparable in size to the solar system.

In spite of the rather small linear dimensions of quasars and many galactic nuclei, their masses turned out to be enormous. They were first estimated by using formula (11) [23]. For quasi-static objects, the luminosity cannot be essentially greater than $L_{\rm E}$. A comparison of the observed luminosity with the expression (11) gives an estimate for the lower limit of the central mass. In some quasars this limit is $M \approx (1-10^2) \times 10^7 M_{\odot}$. These estimates are supported by data on the velocities within the galactic nuclei of stars and gas clouds accelerated in the gravitational fields of the nucleus centers. We shall discuss this issue at the end of the section.

Great mass but small linear dimensions prompt the guess that there could be a black hole. This would account for all the extraordinary properties of these objects. Now it is generally accepted that in AGN there are supermassive black holes with accretion gas (and maybe also dust) disks. One of the most important facts implied by observations, especially by means of radio telescopes, is the existence of directed jets from the nuclei of some active galaxies. For some of the objects there is evidence that radio components move away from the nucleus with ultrarelativistic velocities. The existence of an axis of ejection strongly suggests the presence of some stable compact gyroscope, probably a rotating black hole. In some cases one can observe evidence that there is also precession of this gyroscope. An essential role in the physics of processes in the centers of AGN is probably played by *black-hole electrodynamics*.

In the model of a supermassive black hole with an accretion disk as an AGN one requires sources of fuel — gas or dust. The following sources have been discussed: gas from a nearby galactic companion (as the result of interaction between the host galaxy and the companion), interstellar gas of the host galaxy, disruption of stars by high-velocity collisions in the vicinity of the black hole, disruption of stars by the tidal field of the black hole and some others [24].

Clearly, the processes taking in quasars and other galactic nuclei are still a mystery in many respects. But the suggestion that we are witnessing the work of a supermassive black hole with an accretion disk seems rather plausible. M Rees advocates a hypothesis that massive black holes are not only located in active galactic nuclei but also in the centers of 'normal' galaxies (including nearby galaxies and our own Milky Way) [25, 26]. They are quiescent because they are now starved of fuel (accretion gas). Observations show that galactic nuclei were more active in the past. Thus, 'dead quasars' (massive black holes without fuel) should be common in the present epoch [27].

How can these black holes be detected? It has been pointed out that black holes produce cusp-like gravitational potentials and hence they should produce cusp-like density distributions of stars in the central regions of galaxies. Some authors have argued that the brightness profiles of the central regions of particular galaxies imply that they contain black holes. However, the arguments based only on surface brightness profiles are inconclusive. The point is that a high central number density of stars in a core with small radius could be the consequence of dissipation, and a cusp-like profile could be the result of anisotropy of the velocity dispersion in stars. Thus these properties taken alone are not sufficient evidence for the presence of a black hole.

A reliable way to detect black holes in galactic nuclei is analogous to the case of black holes in binaries. Namely, one must prove that there is a large dark mass in a small volume, and that it can be nothing but a black hole. In order to obtain such a proof we can use arguments based on both stellar kinematics and surface photometry of the galactic nuclei.

If the distributions of the mass M and the luminosity L as functions of the radius are known, we can determine the mass-to-luminosity ratio M/L (in solar units) as a function of radius. This ratio is well known for different types of stellar populations. As a rule this ratio is between 1 and 10 for elliptical galaxies and globular clusters (old stellar populations dominate there). If for particular galaxy the ratio M/L is almost constant at rather large radii (and has a 'normal' value between 1 and 10) but rises rapidly toward values much larger than 10 as one approaches the galactic center, then this is evidence for a central dark object (probably a black hole).

As an example consider the galaxy NGC 3115 which is at a distance of 9.2 Mpc from us [28]. For this galaxy $M/L \approx 4$ and is almost constant over a large range of radii $r > 4''$ (in angular units). This value is normal for a bulge of this type of galaxies. At radii $r < 2''$, the ratio M/L rises rapidly up to $M/L \approx 40$. If this is due to a central dark mass added to a stellar distribution with constant M/L, then $M_{\rm H} = 10^{9.2\pm0.5} M_{\odot}$.

Is it possible to give another explanation of the large mass-to-light ratio in the central region of a galaxy? We cannot exclude the possibility that a galaxy contains a central compact cluster of dim stars. But this is unlikely. The central density of stars in the galaxy NGC 3115 is not peculiar. It is the same as in the centers of globular clusters. The direct observational data (spectra and colors) of this galaxy do not give any evidence against a dramatic stellar population gradient near the center. Thus, the most plausible conclusion is that there is a central massive black hole.

Unfortunately, it is difficult to detect massive black holes in giant elliptical galaxies with active nuclei, where we are almost sure black holes must exist because we observe their active manifestation [29]. The reason for this is a fundamental difference between giant elliptical galaxies (the nuclei of some of them are among the most extreme examples of AGN), dwarf elliptical galaxies and spiral galaxies. Dwarf ellipticals rotate rapidly and the stellar velocity dispersions are nearly isotropic. Giant elliptical galaxies do not rotate significantly and they have anisotropic velocity dispersions. It is not so easy to model these dispersion distributions. Furthermore, giant elliptical galaxies have large nuclei and shallow brightness profiles. Consequently, the projected spectra are dominated by light from large radii, where the black hole has no effect.

The technique described above has been used to search for black holes in galactic nuclei. Another possibility is to observe rotational velocities of gas in the vicinity of the galactic center. Information about some of the supermassive black-hole candidates is given in Table 2 (see Refs [28 – 32]).

Table 2. Estimated masses of black holes in galactic nuclei. Data from Ref. [32].

Galaxy	Mass of black hole in $M_\odot$
M31	3×10^7
M32	3×10^6
Milky Way	2.4×10^6
NGC 4594	10^9
NGC 3115	2×10^9
NGC 3377	1.4×10^8
M87	3×10^9
NGC 4258 (M106)	7×10^7
NGC 4261	9×10^8
NGC 4374	3.6×10^8
NGC 4486B	10^7

Special investigations were performed in the case of the galaxy M87 [33] (for a review of earlier works see Ref. [34]). This is a giant elliptical galaxy with an active nucleus and a jet from the center. At present there is secure stellar-dynamical evidence for a black hole with mass $M \approx 3 \times 10^9 M_\odot$ in this galaxy. The Hubble Space Telescope has revealed a rotating disk of gas orbiting the central object in the galaxy [35, 36]. The estimated mass of the central object is $M = 3 \times 10^9 M_\odot$. The presence of a black hole in M87 is especially important for our understanding of the nature of the central regions of galaxies because in this case we also observe the activity of the 'central engine'.

Radio observations of the nucleus of the galaxy NGC 4258 are of special interest [30]. Using the radio interferometry technique for observing maser lines of water molecules in gas clouds orbiting in the close vicinity of the nucleus, the observers obtained an angular resolution 100 times better than the observations by the Hubble Space Telescope. The spectral resolution was 100 times better as well. According to the interpretation of the observations, the center of NGC 4258 harbors a thin disk which was measured on scales of less than one light year. The mass of the central object is $7 \times 10^7 M_\odot$. According to Begelman and Rees [20]: "It represents truly overwhelming evidence for a black hole... NGC 4258 is the system for which it is hardest to envisage that the mass comprises anything but a single black hole".

In Table 2 we list the estimates of masses of black holes in the nuclei of some galaxies [29, 30, 32, 37, 38]. By the summer of 2000, the total number of candidates for supermassive black holes was 34 (see Ref. [39]).

Perhaps the most convincing evidence that a strong gravitational field is present in active galactic nuclei comes from the measurements on the shape of the Fe K_α fluorescence line [40–42]. The inner part of the accretion disk is illuminated by X-rays. This creates luminescence of various elements in the disk. Analysis shows that the strongest discrete spectral line is the 6.4-keV Fe K_α fluorescent line. This line is very sharp and has a width ~ 150 eV. Since the matter of the disk is moving, the frequency of the radiation arriving from different parts of the disk is Doppler-shifted. It also has a redshift because of the gravitational field. In the calculations of a line profile the relativistic effects must be taken into account. The emission from the disc is beamed in the direction of motion, which means that the blue horn appears brighter than the red one. The transverse Doppler effect and gravitational redshift skew the line profile. The result is a skewed, broad line which has a characteristic two-pronged structure. The form of the line profile is very sensitive to the inclination of the accretion disk and angular momentum of the black hole. The X-ray observations of Seyfert galaxies clearly showed broad, skewed lines in the X-ray spectra of most Seyfert 1 galaxies [40, 43–45]. The line profiles indicate that the disc emission region is at 3–30 Schwarzschild radii and therefore that a relativistic accretion disk is present. In the Seyfert 1 galaxy MCG-6-30-15, the line shift indicates that the inner part of the disk is closer than 3 Schwarzschild radii and hence the central black hole in this case must be spinning.

Progress in this field is very rapid and in the near future our knowledge about evidence for supermassive black holes in galactic nuclei will be more profound.

4.5 Primordial black holes

Modern astrophysics also considers a third possible type of black holes in the universe — primordial black holes (PBH). In the now adopted 'standard' cosmology the universe starts its evolution at some very early time with cosmological inflation. During this stage, density fluctuations are produced from initial zero-point vacuum fluctuations, which later result in the observable large-scale structure of the universe. Inflation ends, through the preheating/reheating transition, giving way to a period of radiation-dominated universe. This phase is very important at the age of 1 s and makes it possible for nucleosynthesis to proceed. Finally, at the redshift $z \approx 24000 \Omega_0 h^2$ (Ω_0 is the ratio of mass density of the universe at present epoch to the critical mass density, and h is the Hubble constant in units 100 km s^{-1} Mpc^{-1}) the radiation-dominated era gives way to the matter-dominated era during which stars and galaxies form.

Primordial black holes might be created at the very beginning of the expansion of the universe [46–48]. Black holes which might be created before or during the inflation seem to play no role since inflation rapidly dilutes the gas of such black holes. Primordial black holes created after inflation might have interesting observational consequences. The masses of primordial black holes are arbitrary, but primordial black holes with $M \leqslant 10^{15}$ g would have radiated away their masses by the Hawking quantum process in a time $t \leqslant 10^{10}$ years (the age of the universe). Only primordial black holes with a mass $M > 10^{15}$ g could exist in the contemporary universe. The value $M \sim 10^9$ g is another mass scale which is of interest for cosmology. Primordial black holes with such a mass evaporate around the time of nucleosynthesis, which is well enough understood to tolerate only modest interference from products of black hole evaporation.

Several mechanisms of primordial black hole formation were proposed. The simplest one describes the formation of black holes in the collapse of large-amplitude, short-wavelength density perturbations in the early universe. If the equation of state during the epoch of primordial black hole formation is $p = \gamma\rho$ with $0 < \gamma < 1$ ($\gamma = 1/3$ for the radiation-dominated stage), then in order to collapse against the pressure, the overdense region must be larger than the Jeans length which is $\gamma^{1/2}$ times smaller than the cosmological horizon size. This simple estimate (confirmed by numerical calculations) implies that the density fluctuation must exceed γ on the epoch horizon.

The mass fraction of the primordial black holes can be characterized by a quantity $\beta = \rho_{\mathrm{pbh}}/\rho_{\mathrm{tot}}$, where ρ_{pbh} is the mass density in the form of the black holes, and ρ_{tot} is the total mass density. The ratio β depends on time. We denote β_{i} the value of the ratio β at the point in time of primordial black hole formation. If the matter fluctuations have a Gaussian distribution and are spherically symmetric, then the fraction of regions of mass M which collapse is given by [49]

$$\beta_{\mathrm{i}} \sim \epsilon(M) \exp\left\{-\frac{\gamma^2}{2\left[\epsilon(M)\right]^2}\right\}, \tag{14}$$

where $\epsilon(M)$ is the amplitude of the fluctuations when the horizon mass is M. Two important conclusions follow directly from this result: (1) primordial black holes would form more efficiently if the equation of state were softer, $\gamma \ll 1$, for example, during a phase transition in the universe; (2) the PBH mass spectrum can be extended only if $\epsilon(M) \approx \mathrm{const}$, that is the fluctuation spectrum is scale-invariant.

The cosmological fluctuations which give origin to primordial black holes can be of diverse origin. The fluctuations can be either primordial or they can be spontaneously generated at some epoch. Density fluctuations generated during the inflation from vacuum zero-point fluctuations are one natural source of primordial black holes. The amplitudes of these fluctuations depend on the form of the inflationary potential [50 – 56]. Other mechanisms of primordial black hole creation do not depend on the existence of primordial density fluctuations. Examples are the formation of black holes in the collision of bubbles with broken symmetry during cosmological phase transitions [57 – 59] and in the collapse of cosmic strings [60 – 65].

A population of PBHs whose influence today is small may have been more important in the earlier epochs of the evolution of the universe. Radiation from PBHs could perturb the accepted picture of cosmological nucleosynthesis, distort the microwave (relic) background and produce too much entropy in relation to the matter density of the universe. Limits on the density of PBHs, now or at earlier times, can be used to provide information on the homogeneity and isotropy of the very early universe, when they were formed. For a relevant review see Refs [52, 66, 67].

The fate of primordial black holes depends on their masses. Primordial black holes with $M > 10^{15}$ g will survive until the present epoch. Limitations on the mass ratio for these black holes can be obtained from the following simple observation [46]. One can consider such black holes as a gas of *non-relativistic* particles. The energy density of this gas falls as a^{-3} as the scale a of the universe increases. At the radiation-dominated epoch the energy density of the remainder of the matter falls as a^{-4}. Thus the relative black hole mass contribution grows as a. As these black holes form so early this factor could be extremely large. In order that the matter density in the form of black holes should not now exceed the observable mass density in the universe, the fraction of the matter which collapses into black holes of masses $M > 10^{15}$ g must be extremely small.

Much stronger constraints on β_i can be obtained for black holes which were small enough to have evaporated by now. These constraints are summarized in Fig. 1.

Searches for PBHs attempted to detect a diffuse photon (or another particle) background from a distribution of PBHs or to search directly for the final emission stage of individual black holes. Using the theoretical spectra of particles and radiation emitted by evaporating black holes of different masses, one can calculate the theoretical backgrounds of photons and other particles produced by a distribution of PBHs emitting over the lifetime of the universe. The level of this background depends on the integrated density of PBHs with initial masses found in the range considered.

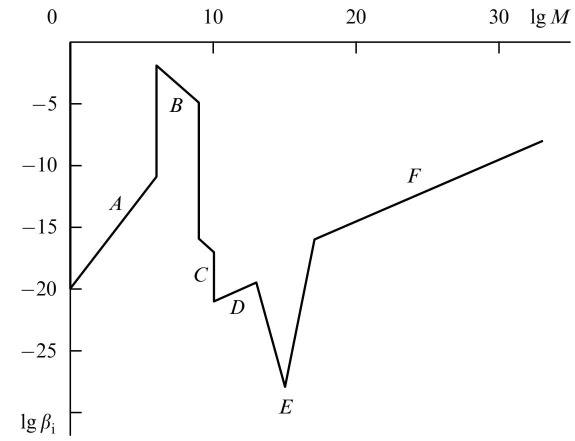

Figure 1. Constraints on $\beta_{\mathrm{i}}(M)$: relics (A), entropy (B), helium (C), deuterium (D), γ (E), density (F). Data from Ref. [67].

Black holes of mass $M \approx 10^{14.5}$ g should be evaporating in the present epoch. The constraint on such black holes can be obtained from γ-ray observations [68, 69]. A comparison of the theoretical estimates with the observable cosmic ray and γ-ray backgrounds place an upper limit on the integrated density of PBHs with initial masses discovered within this range. According to estimates of MacGibbon and Carr [69], this limit corresponds to $\approx 10^{-6}$ of the integrated mass density of the visible matter in the universe (matter in the visible galaxies). The comparison of the theory with other observational data gives weaker limits [67, 70, 71].

The search for high-energy γ-ray bursts as a direct manifestation of the final emission of the evaporating (exploding) individual PBHs has continued for more than 20 years. No positive evidence for the existence of PBHs has been reported thus far [67, 72, 73].

The evaporation of lighter black holes can affect $n\bar{n}$-production at nucleosynthesis [74], cause deuterium destruction [75] and helium-4 spallation [76]. Black holes of masses 10^9 g$< M < 10^{13}$ g contribute to the entropy per baryon [77, 78]. Constraints below 10^6 g can be imposed by assuming that a black hole leaves a stable Planck mass relics (*maximons* studied by Markov [79]) [52, 68, 80].

It should be emphasized that the black hole constraint limits matter density perturbations on scales which are much shorter than those which can be probed using information on the large-scale structure and the cosmological microwave background, though these scales are similar to those which could be probably tested by gravitational wave interferometers, such as LIGO, VIRGO and GEO (see next section) [81].

As the result of quantum evaporation the mass of a black hole decreases. Once the mass falls below 10^{14} g, a black hole begins radiating hadrons. According to quantum chromodynamics (QCD) hadrons are composite particles and at a temperature higher than the confinement scale ($T_{\mathrm{QCD}} = 250-300$ GeV) one must consider the emission of fundamental particles such as quarks and gluons. Since there exist 12 quark degrees of freedom per flavor and 16 gluon degrees of freedom, the phase space of emitted particles increase dramatically at the confinement scale. One can show that for $T_{\mathrm{BH}} > T_{\mathrm{QCD}}$ the time interval between emission of two subsequent quanta is much greater than the time of emission and is much less than the characteristic time scale of strong interactions, $T_{\rm QCD}^{-1}$. For this reason the emission of quarks and gluons resembles similar collider events and results in the generation of quark and gluon jets. The jets decay into hadrons at distances much greater than the gravitational radius, where gravity is not important any more. Using this approach, MacGibbon and Webber obtained emission spectra for a $T = 1$ GeV black hole. All particle spectra show a peak at 100 MeV due to pion decays, and at 1 MeV due to neutron decay (see also Carr and MacGibbon [67]).

The final stage of black hole evaporation is still unclear. There is a possibility that the endpoint of black hole evaporation is a stable relic (a maximon, see Ref. [79]). The possible role of such relics in cosmology was first discussed by MacGibbon [68]; for a more recent review see Ref. [80], and a new approach to the problem was made in Ref. [82].

5. Probing black holes with gravitational waves

The quantum decay of primordial black holes is a direct consequence of the existence of the event horizon and hence its observation would directly testify to the existence of small black holes. Unfortunately, we have no such evidence. Observations of stellar and massive black holes in optics and X- and γ-rays do not provide us with direct information about spacetime regions close to a black hole, since the radiation is generated in regions far from the horizon. To explore the region close to the horizon in detail may well require using a new information channel in astrophysics — gravitational waves. With the construction of new gravitational wave observatories this option becomes very important.

Among the most promising sources of gravitational waves which can be observed by the gravity wave detectors are astrophysical compact binaries. Three types of compact binaries are mainly discussed: neutron-star – neutron-star (NS/NS) binaries, neutron-star–black-hole (NS/BH) binaries, and black-hole – black-hole (BH/BH) binaries. Because of the emission of gravitational waves at some stage of their evolution, compact binaries enter the inspiral phase which ends with a coalescence. During these final stages of the binary system evolution they emit powerful gravitational waves.

An international network of ground-based gravitational wave detectors is now under construction. It includes two detectors of the American Laser Interferometer Gravitational-wave Observatory (LIGO) [83], the French/Italian 3-kilometer-long arms interferometer VIRGO near Pisa (Italy) [84], and the British/German 600-meter interferometer GEO-600 near Hannover (Germany) [85].

The LIGO detector, which is now under construction, consists of two vacuum facilities with two 4-km-long orthogonal arms. One of these detectors is in Hanford (state Washington) and the other in Livingston (state Louisiana). Their coincident operation will start in 2002. Gravitational waves coming from far-distant sources effectively change the relative length of the arms, which can be measured by the phase shift between two laser beams in the two orthogonal arms. With expected accuracies of the arm-length difference $\Delta L \sim 10^{-16}$ cm, the expected sensitivity of the detector would be $\Delta L/l \sim 10^{-21} - 10^{-22}$. This sensitivity will be achieved in LIGO within the frequency range from 40 to 120 Hz. The efficiency of LIGO is effectively reduced by photon counting statistics ('shot noise') at higher frequency and by seismic noise at lower frequency. The LIGO facilities are designed to house many successive generations of upgraded interferometers. The second generation, LIGO II, is planned to start to be designed in 2005, and to be observing before 2007. Working in the same frequency range, it will have an approximately two orders of magnitude higher sensitivity. Table 3 gives the limiting distances up to which the LIGO detectors would be able to observe different types of binaries.

Table 3. List of the sources detectable by LIGO I and LIGO II. Neutron stars are assumed to have mass 1.4 $M_\odot$, and black holes are assumed to have mass 10 $M_\odot$. Data from Ref. [83].

Systems	Distance for LIGO I	Distance for LIGO II
Inspiral NS/NS binaries	20 Mpc	450 Mpc
Inspiral NS/BH binaries	40 Mpc	1000 Mpc
Inspiral BH/BH binaries	100 Mpc	2000 Mpc

Black hole binary evolution and its emitted gravitational waveforms can be divided into the following three stages: inspiral, coalescence, and ringdown. The inspiral epoch for a BH/BH binary requires post-Newtonian expansions for its understanding and is qualitatively the same as for other compact binaries. Gravitational radiation during coalescence and ringdown epochs contains information which allows the BH/BH case to be singled out. Supercomputer simulations are required to determine the dynamics of two merging black holes and to produce templates which can be used to decode the information encoded in emitted gravitational waves. The ringdown epoch is much better understood. At this stage, two initial black holes form a new final one, which is in a very excited state. Its further evolution involves a decay of these excitations. These excitations are a nonlinear superposition of quasi-normal modes. The decay of the quasi-normal modes produces a characteristic 'ringing' in the gravitational waveforms.

Gravitational waves emitted at the stages of BH/BH coalescence and ringdown carry information about the highly nonlinear, large-amplitude dynamics of spacetime curvature, and for this reason the study of these signals tests Einstein gravitational equations in their full complexity. Table 4 gives an estimate of the amplitude signal-to-noise (S/N) ratio for coalescences at a 1000-Mpc distance for two black holes of equal mass.

Table 4. Amplitude signal-to-noise (S/N) ratio for coalescences at 1000-Mpc distance for two black holes of equal mass (data from Ref. [83]).

BH/BH coalescences	S/N for LIGO I	S/N for LIGO II
$10M_\odot/10M_\odot$	0.5	10
$25M_\odot/25M_\odot$	2	30
$100M_\odot/100M_\odot$	4	90

The time- and length-scales for double black-hole dynamics (including the gravitational radiation from such systems) are proportional to the total mass. Other parameters (such as the black hole mass ratio, black hole angular momentum and so on) enter through dimensionless combinations. The total number of cycles spent in the LIGO/VIRGO band for a BH/BH of $10M_\odot$ is about 600. These detectors will be able to detect and study gravitational waves emitted during last few minutes of their evolution for black hole binaries with a total mass of up to $10^3 M_\odot$. For larger masses, a gravitational wave detector must have a much lower frequency band. Future space-based gravitational wave interferometers will work in this band. LISA is an example of such a project.

The Laser Interferometer Space Antenna (LISA) consists of 3 spacecraft flying 5×10^6 km apart in the shape of an equilateral triangle. The center of the triangle will be at the ecliptic plane at the same distance from the Sun as the Earth and 20° behind the Earth on the orbit. The three spacecraft will act as a giant interferometer measuring distortions in space caused by gravitational waves. This project was proposed in 1993 by the United States and European scientists as a joint NASA/ESA (National Aeronautics and Space Administration/European Space Agency) mission. If approved, the project will start in 2005 with a launch planned in 2008 [86].

The frequency band of LISA covers $10^{-4} - 1$ Hz, that is 10,000 times lowers than the frequency band of LIGO/VIRGO. Its sensitivity in this frequency band will be at the level of 10^{-23}. LISA will be able to register gravitational waves emitted by BH/BH binaries for a total mass in the range $10^3 M_\odot - 10^8 M_\odot$ (massive and supermassive black holes), away from each other by a distance corresponding to redshifts of $z \sim 3000$. Since it is very unlikely that massive and supermassive black holes form so early (until they are primordial), this means that LISA will be able to observe practically *all* coalescing black hole binaries in the visible universe within this range of mass.

For a discussion of gravitational-wave radiation from colliding black holes it is very important to know how many BH/BH binaries exist in the universe. Unfortunately, this is not known. The scatter between the most optimistic and most pessimistic estimates is quite wide. However, for BH/BH binaries with a total mass of $5 - 50\ M_\odot$ that are created from main-sequence progenitors, one can expect a coalescence rate in our Galaxy of 1 per 1 – 30 million years [87 – 89]. If these estimates are correct, LIGO/VIRGO will see one coalescence per year for such binaries up to the distance of 300 – 900 Mpc. The event rate for supermassive black hole coalescences is much more uncertain — from 0.1 to 1000 per year. But even for the pessimistic rate value, LISA will be able to observe 3 BH/BH binaries with a total mass of $3{,}000 - 10^5 M_\odot$ that are 30 years away from their final coalescence [89, 90].

To summarize, there is a good chance that in the near future gravitational waves from coalescing black holes will be observed and, hence, for the first time we shall be able to probe almost directly our theoretical predictions concerning black holes.

6. Critical gravitational collapse

Now we discuss a problem of so-called *critical gravitational collapse* which has recently attracted a lot of attention. This problem can be formulated as follows. Consider an isolated initial distribution of gravitating matter and allow it to evolve. A black hole is one possible final state of such a system. It is also possible that in the collapse no black hole is created. Hence, the phase space of isolated gravitating systems is naturally divided into basins of attraction, one of which contains black holes. For any given initial conditions it is practically impossible to decide whether a black hole is formed until the nonlinear Einstein equations which determine the evolution are solved. Thus, a rather complete description of basins of attraction in general relativity is an extremely complicated problem. A remarkable recent achievement is that general ideas from dynamical systems theories can be applied to study the 'boundaries' separating different basins of attraction and provide a qualitative understanding of the dynamics of self-gravitating systems near such 'boundaries'.

The behavior of black holes at the threshold of their formation was first investigated by Choptuik [91] who established a number of interesting general relations characterizing this behavior. M Choptuik numerically solved spherically symmetric gravitational equations minimally coupled to a scalar massless field. He studied the gravitational collapse for different sets of one-parameter families of initial conditions. Suppose that for a fixed family a parameter p is chosen so that for small values of p the gravitational field during evolution is too weak to form a black hole, while for large values of p a black hole is produced. In general, between these two extremes there is a *critical* parameter value, p^*, where black hole formation first occurs. We will refer to the solutions with $p < p^*$ and $p > p^*$ as subcritical and supercritical, respectively. Choptuik presented convincing numerical evidence that there is *no mass gap* in black hole production; arbitrarily small black holes can be formed in a collapse [2]. Moreover, for $p > p^*$ the mass of sufficiently small black holes is given by

$$M_{\mathrm{BH}} \sim |p - p^*|^{\beta}\,, \tag{15}$$

where $\beta \approx 0.37$ is a universal exponent (this relation is referred to as *scaling*). The most surprising fact is that this result remains the same for all families of solutions which have been studied.

Moreover, for marginal data, both supercritical and subcritical, the evolution approaches a certain universal solution which is the same for all the families of initial data. This solution, which is unique and corresponds to the field configuration exactly at the threshold p^* of black hole formation, is called the *critical solution*, and sometimes is referred to as the *choptuon*. This solution acts as an *intermediate attractor* in the sense that the time evolution first converges onto it, but then eventually diverges from it to either form a black hole or to disperse.

The critical solution for a spherically symmetric gravitational collapse of the massless scalar field has a discrete symmetry: it is periodic in the logarithm of the spacetime scale

$$t' = \exp(-\Delta)t\,, \qquad r' = \exp(-\Delta)r\,, \tag{16}$$

$$\mathrm{d}s'^2 = \exp(-2\Delta)\,\mathrm{d}s^2\,, \qquad \phi(t', r') = \phi(t, r) \tag{17}$$

with a period $\Delta \approx \ln 30 \approx 3.4$, which is a constant belonging to the choptuon (the instant of time $t = 0$ captures the formation of the black hole). This behavior of the critical solution is referred to as *echoing*, because the solution repeats itself at ever-decreasing time- and length-scales, or discrete self-similarity (DSS).

Later more accurate numerical calculations [94, 95] demonstrated that a periodic 'wiggle' or 'fine' structure is superimposed on the straight line relating $\lg M$ to $\lg(p - p^*)$, the period of 'wiggles' also being universal and related to the critical exponent β.

[2] It should be emphasized that quantum effects could modify this conclusion. Since the curvature at the surface of a black hole of mass M is of order M^{-2}, it reaches the Planckian value for black holes of Planckian mass. In this regime quantum effects dominate. In particular, higher curvature corrections may create a mass gap [92] (see also Ref. [93]).

Calculations of the gravitational collapse of the massless scalar field using different coordinate systems and numerical algorithms [96, 97] confirmed that the effects observed by Choptuik are not numerical artifacts.

These features of the near-critical gravitational collapse first discovered for a self-gravitating scalar field appear to be quite general. Abrahams and Evans [98] found a similar phenomenon in the axisymmetric collapse of a gravitational wave with almost the same value of the critical exponent $\beta \approx 0.38$. The corresponding choptuon is also discrete self-similar, but the constant Δ appears to be different: $\Delta \approx \ln 1.8 \approx 0.6$. Hirschmann and Eardley [99, 100] obtained the spherically symmetric solutions for coupled Einstein-complex-scalar-field equations which possess Choptuik-type universal scaling and echoing behavior.

In some cases, the critical solution possesses a stronger symmetry than the discrete self-similarity described above, namely, continuous self-similarity (CSS) or homotheticity. The presence of this symmetry, allowing the elimination of one of the coordinates from the equations, is one of the reasons why it is easier to deal with continuous self-similar solutions: most analytical calculations use a continuous self-similarity ansatz. An example of critical behavior with continuous self-similarity was found by Evans and Coleman [101] in the model of the spherically symmetric collapse of a radiating fluid. The critical exponent in their case is $\beta \approx 0.36$.

In all the cases when the critical behavior was observed, the generic feature is that the spacetime is asymptotically flat; there is transportation of the energy from a collapsing system to infinity, and the matter content is 'massless'.

The first calculations in different models gave very close values for the mass scaling exponent β. These results were first interpreted as an indication that the meaning of the universality might be extended to the independence of the critical exponent from details of the system, though initially this meant its independence from initial data. Later calculations for a wider class of models did not confirm this conclusion.

For example, the exact analytical solution to the collapse of a thin shell coupled with an outgoing null fluid [102] and perturbative analysis of the collapse of a perfect fluid with the equation of state $p = \gamma\rho$ (with γ in the range $0 \leqslant \gamma \leqslant 0.88$ [103]) both have a critical exponent β strongly dependent on the parameters of the matter model.

The universality of β for a massless scalar field, gravitational waves, and a radiating fluid seems to be connected with the fact that these three are massless fields, but there is no proof of why it should be so [3]. The observed nonuniversality goes beyond varying β for different matter models: it affects more fundamental properties of critical solutions. In particular, it was shown by Hirschmann and Eardley [100] that in the spherically symmetric gravitational collapse of the massless complex scalar field the critical solution is unstable. That is, it has an instability other than the obvious black hole one, apparently an oscillatory instability toward the original real choptuon. Especially intriguing is an example [104] of gravitational collapse of a Yang–Mills field, where there exist two distinct critical solutions: one with discrete self-similarity and allowing black holes of arbitrarily small mass, and the other one with a mass gap. Some of the results of studying critical phenomena in gravitational collapse for different models are presented in Table 5. For more information see the recent review [105].

[3] Note, however, that $\beta = 0.387$ for a massless complex scalar field [100], which is slightly but nevertheless noticeably different from $\beta = 0.37$ for the above three fields.

Table 5. Critical behavior in the gravitational collapse.

Model	References	β	Symmetry
Scalar fields			
Massless scalar field	[91, 96, 97]	0.37	DSS
	[106–108]	0.374	DSS
	[109–112]	1/2	CSS
	[113–115]	1	CSS
Complex scalar field	[99, 100]	0.387	CSS
Charged scalar field	[116]	0.37	DSS
Other matter models			
Gravitational waves	[98]	0.37	DSS
Radiating fluid	[101]	0.36	CSS
	[117]	0.356	CSS
Perfect fluid	[103, 118]	varies	CSS
Thin shell	[102]	varies	–
Yang–Mills field	[104]	0.20	DSS
Other theories			
Axion-dilaton field	[119, 120]	0.264	CSS
2D dilaton gravity	[121]	0.53	–
Nonlinear σ model	[122]	varies	both
Brans–Dicke theory	[123–125]	varies	both

The results of (mostly numerical) investigation of critical collapse strongly support the following general picture (see, e.g., review [105]. For isolated systems typically three kinds of final states are possible. The matter either collapses to a black hole or forms a star or disperses, leaving empty spacetime behind. Kerr–Newman black holes form a set of stable points in the basin of the black hole attraction. The Minkowski empty space is a point of attraction for the basin of dispersing configurations. The boundary between these two basins of attraction is a *critical surface* of co-dimension one. If a system starts its evolution at this critical surface, it always remains on it. For most systems that were studied there exists a special 'critical' solution which is an attractor on the critical surface. Solutions close to the critical surface have infinite number of decaying perturbation modes tangential to the critical surface, and a *single* growing mode that is not tangential. Such solutions stay close to the critical surface, moving towards the critical solution for some period of time until the growing mode develops and brings the solution away either into the black hole basin or into the Minkowski one. During this relatively long stage when a solution is 'close' to the critical one, the information about initial conditions is lost. If a black hole is formed, this process for small masses is dominated by properties of the critical solution and does not depend on the details of the initial conditions. This explains the universality properties of the critical collapse.

This picture has an evident similarity to the critical phenomena in condensed matter physics. Namely, time evolution of near critical solutions for the gravitational collapse problem can be considered as a renormalization group flow in the phase space of solutions. For calculation of the critical exponent for this process, one can use the same methods as for calculations of the critical exponent governing the correlation length near the critical point in statistical mechanics. This method was adapted to the critical gravitational collapse by Evans and Coleman [101] and developed later by Koike, Hara and Adachi [117]. Briefly the idea of the method is as follows.

The critical solution obtained for $p = p^*$ obeys the self-similarity property, and it is usually much easier to find it than to solve the full problem. The characteristic feature of solutions with initial data close to those of the critical solutions is that they first approach the latter, but eventually run away from them; that is, they contain a factor $\exp(\sigma t)$. Evans and Coleman [101] proposed to use a linear stability analysis for studying these run-away solutions. Quite general arguments show that the mass of a black hole which is formed as a result of this instability is proportional to $(p - p^*)^{1/\sigma}$, so the critical index is $\beta = 1/\sigma$. This method allows both the calculation of β and a test of the stability of the critical solution, and it has been used in the investigation of various matter models [101, 103, 113 – 115, 117].

The discovery of universal properties of critical collapse is one of the most profound achievements in numerical relativity.

We would like to conclude this section with the following general remark. A characteristic property of black holes is the extremality of the gravitational field at their surface. This field is so strong that only very special field and matter configurations are possible in the vicinity of the horizon. Since the boundary conditions on the black hole surface are so special, black holes in their interaction with an external world behave in a most universal way. For this reason, if one considers a black hole as a physical body, the physical properties of this body are quite simple and universal. Some examples were given at the beginning of the paper, e.g., black hole viscosity, conductivity, and thermodynamic properties. The critical collapse discussed in this section also implies that the very formation of small-mass black holes possesses universality properties similar to scaling laws for critical phenomena in condensed matter physics. Namely these kinds of universal properties single out black holes from other matter, and at the same time make the physics of black holes so interesting and deep.

7. Conclusions

Black holes are absolutely unusual objects. In spite of all the progress the nature of space and time in black holes remains a mystery to a large extent. Some aspects of the problem still appear as scientific toys, interesting only to specialists.

As for the practical realization of new ideas, we would like to conclude the paper by recalling that in the middle of the 19th century even such a practical (now!) thing as electricity appeared as a scientific toy. When, in that period, the prime minister asked M Faraday about the practical worth of electricity, Faraday answered: "One day your government will tax it".

Being optimists, we believe in the enormous promise of the new field of research concerning the physics and astrophysics of black holes.

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