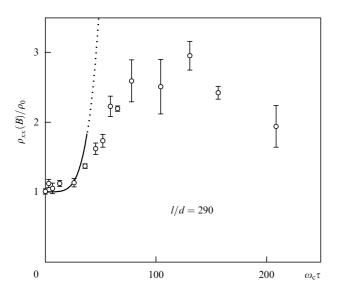


**Figure 3.** Magnetoresistivity (normalized by the Drude value) in a random magnetic field from numerical simulations for three different strengths of disorder; the full line corresponds to Eqn (10). Parameter  $\alpha = (d/2l)^{1/2}$ .



**Figure 4.** Magnetoresistivity in a random potential from computer simulations in comparison with Eqn (11).

theoretical prediction of a strong positive MR (10) crossing over to a negative one at  $\overline{B} \sim B_0 (l/d)^{1/6}$  is confirmed by the data. At moderately small d/l the positive MR still exists, but becomes weak; this is the region of d/l relevant to the CF system. The numerically calculated MR for  $d/l \sim 0.1-0.2$  agrees well [3] with the experimental data [10] around  $v_e=1/2$ . At sufficiently large d/l the region of positive MR disappears, and  $\rho_{xx}$  drops monotonously with  $\overline{B}$  [3]. The numerically found MR for the random potential case (Fig. 4) shows good agreement with the theoretical result (11) up to  $\Delta\rho_{xx}/\rho_0 \sim 1$ . At larger  $\overline{B}$ ,  $\rho_{xx}$  deviates from (11) and starts to decrease, as expected.

In conclusion, we have analyzed non-Markovian effects in quasiclassical transport of a two-dimensional electron gas subject to a long-range Gaussian disorder. Particular attention has been paid to the composite-fermion description of transport in a half-filled Landau level. We have shown that the ac conductivity has a nonanalytic correction  $\propto |\omega|$ . We have also calculated the quasiclassical magnetoresistance: the memory effects lead to a strong positive magnetoresistance, which we argue to have been observed in the composite-fermion system near half-filling.

We are grateful to DE Khmel'nitskiĭ for discussions of the role of inelastic scattering, to Y Levinson for attracting our attention to Ref. [5], and to Yu V Nazarov for pointing out a question about the role of dynamical screening. This work was supported by SFB 195 and the Schwerpunktprogramm "Quanten-Hall-Systeme" of the Deutsche Forschungsgemeinschaft, by INTAS (Grants 97-1342 and 99-1705, and by the German – Israeli Foundation.

#### References

- Wilke J et al. Phys. Rev. B 61 13774 (2000)
- 2. Mirlin A D et al. Phys. Rev. Lett. 83 2801 (1999)
- 3. Evers F et al. Phys. Rev. B 60 8951 (1999)
- 4. Halperin B I, Lee P A, Read N Phys. Rev. B 47 7312 (1993)
- Gantsevich S V, Kagan V D, Katilyus R Zh. Eksp. Teor. Fiz. 80 1827 (1981) [Sov. Phys. JETP 53 946 (1981)]
- 6. Ernst M H, Weyland A *Phys. Lett. A* **34** 39 (1971)
- 7. Fogler M M et al. Phys. Rev. B 56 6823 (1997)
- 8. Khveshchenko D V Phys. Rev. Lett. 77 1817 (1996)
- 9. Mirlin A D, Polyakov D G, Wölfle P Phys. Rev. Lett. 80 2429 (1998)
- See, e.g., Smet J H, in *Composite Fermions* (Ed. O Heinonen) (Singapore: World Scientific, 1998) p. 443

# Density of states near the Anderson transition in the $(4 - \epsilon)$ -dimensional space

I M Suslov

Abstract. The calculation of the density of states for the Schrödinger equation with a Gaussian random potential is equivalent to the problem of a second-order transition with a 'wrong' sign of the coefficient of the quartic term in the Ginzburg-Landau Hamiltonian. The special role of the dimension d = 4 for such Hamiltonian can be seen from different viewpoints but fundamentally is determined by the renormalizability of the theory. Construction of  $\epsilon$ -expansion in direct analogy with the phase transitions theory gives rise to a problem of a 'spurious' pole. To solve this problem, the proper treatment of the factorial divergency of the perturbation series is necessary. In  $(4 - \epsilon)$ -dimensional theory, the terms of the leading order in  $1/\epsilon$  should be retained for  $N \sim 1$  (N is an order of the perturbation theory) while all degrees of  $1/\epsilon$  are essential for large N in view of the fast growth of their coefficients. The latter are calculated in the leading order in N from the Callan – Symanzik equation with results of Lipatov method using as boundary conditions. The qualitative effect consists in shifting of the phase transition point to the complex plane. This results in elimination of the 'spurious' pole and in regularity of the density of states for all energies.

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#### 1. Introduction

Let us consider a usual Shrödinger equation with random potential V(x):

$$\left[\frac{\hat{p}^2}{2m} + V(x)\right]\Psi(x) = E\Psi(x). \tag{1}$$

Qualitative features of the equation are well known. In the absence of disorder the density of states v(E) has a power-law behavior at E>0 and is identically zero at E<0. When a random potential is included, the van Hove singularity is smeared out and the density of states becomes finite formally for all values of the energy, i.e., the fluctuation tail arises. Near the bare edge of the band there exists a special energy  $E_c$  (mobility edge), separating the regions of localized and extended states. If the Fermi level  $E_F$  crosses  $E_c$  when parameters of the system vary, then the metal—insulator transition referred to as the Anderson transition occurs. It is characterized by the singular behavior of conductivity  $\sigma$  and localization radius  $\xi$  of the wave functions, which are conventionally described by power dependences:

$$\sigma \sim |E_{\rm F} - E_{\rm c}|^s, \quad \xi \sim |E_{\rm c} - E_{\rm F}|^{-\nu}, \tag{2}$$

with the critical exponents s and v introduced in the manner similar to the theory of phase transitions.

As for the density of states v(E), it generally believed to have a smooth behavior at the mobility edge, i.e.,  $E_c$  is the singular point for one category of quantities but a regular point for another. Formally, it is a consequence of the fact that the density v(E) is determined by the average Green's function  $\langle G(x,x')\rangle$ , while kinetic properties are specified by correlator  $\langle G^R(x_1,x_2)G^A(x_3,x_4)\rangle$ .

Therefore, there is no problem of the critical behavior of the density of states, it is trivial. Nevertheless, there is a problem of the quantitative description of its behavior in the critical region. All conventional methods are not applicable there and the calculation of the density of states run into fundamental problems similar to those for kinetic properties.

# 2. Relationship with the theory of phase transitions

Let us specify the model and consider the Gaussian random potential with zero average and the point-like correlator:

$$\langle V(x)\rangle = 0$$
,  $\langle V(x)V(x')\rangle = W^2\delta(x - x')$ . (3)

For such a model, there exists the exact formal equivalence of the problem of Anderson transition and the fluctuational theory of phase transitions [1, 2]. The average Green's function of a disordered system is determined by the same functional integral as in the phase transitions theory:

$$\langle G(x, x') \rangle \sim \int D\varphi \, \varphi(x) \, \varphi(x') \, \exp\left[-H\{\varphi\}\right].$$
 (4)

Hamiltonian  $H\{\varphi\}$  has the usual Ginzburg – Landau form

$$H\{\varphi\} = \int d^d x \left(c|\nabla \varphi|^2 + \kappa^2 |\varphi|^2 + g_0 |\varphi|^4\right), \tag{5}$$

$$c = \frac{1}{2m}, \quad \kappa^2 = -E, \quad g_0 = -\frac{W^2}{2},$$
 (6)

but the coefficient  $g_0$  of the quartic term has 'wrong' sign. Here  $\varphi$  is the *n*-component vector  $(\varphi_1, \varphi_2, \dots, \varphi_n)$ , where *n* tends to zero at the end of calculations. The functional integrals with  $g_0 < 0$  imply analytical continuation from the positive  $g_0$  through the upper (lower) semiplane, which results in retarded (advanced) Green's function of the disordered system.

Similarly, the correlator  $\langle G^{R}G^{A}\rangle$  is determined by the functional integral

$$\langle G^{R}(x_1, x_2)G^{A}(x_3, x_4)\rangle \sim \int D\varphi$$

$$\times \int D\psi \ \varphi(x_1) \ \varphi(x_2) \ \psi(x_3) \ \psi(x_4) \ \exp\left[-H\{\varphi, \psi\}\right]$$
 (6)

with more complicated Hamiltonian

$$H\{\varphi,\psi\} = \int d^{d}x \left[ c|\nabla \varphi|^{2} + c|\nabla \psi|^{2} + \kappa_{1}^{2}|\varphi|^{2} + \kappa_{2}^{2}|\psi|^{2} + g_{0}\left(|\varphi|^{2} + |\psi|^{2}\right)^{2} \right], \tag{7}$$

$$\kappa_{1}^{2} = -E - i\delta, \quad \kappa_{2}^{2} = -E + i\delta,$$

depending on the two zero-component fields  $\varphi$  and  $\psi$ . In the phase transitions theory such Hamiltonian corresponds to the existence of two phase transitions of different nature with close critical points.

# 3. Problem of the 'spurious' pole

This brings up the question: Is it possible to develop the  $(4 - \epsilon)$ -dimensional theory for such Hamiltonians in analogy with Wilson's  $\epsilon$ -expansion for phase transitions? For a long time it was thought to be impossible, and the problem is related to the Landau pole.

In quantum electrodynamics there is a relation which links the observed charge e that enters into the Coulomb law with the 'bare' charge  $e_0$  in the initial Lagrangian:

$$e^{2} = \frac{e_{0}^{2}}{1 + \cos e_{0}^{2} \ln(\Lambda/m)},$$
 (8)

where m is the electron mass,  $\Lambda$  is the cutoff parameter which should tend to infinity. A similar formula, but with the opposite sign in denominator, takes place in quantum chromodynamics.

In the four-dimensional theory of phase transitions there is a similar relation between the renormalized charge g describing the long-range interaction and the bare value  $g_0$ :

$$g = \frac{g_0}{1 + \operatorname{const} g_0 \ln (\Lambda/\kappa)} \,. \tag{9}$$

At  $g_0 > 0$  the effective interaction tends to zero as  $\kappa \to 0$ , upon approaching the transition point. We have a 'zero-charge' situation in a literal sense, since the bare charge  $g_0$  and the cutoff parameter  $\Lambda$  correspond to the atomic scale and are observable quantities. In  $4 - \epsilon$  dimensions, the charge g tends to a finite value of order  $\epsilon$  in the limit  $\kappa \to 0$ . Therefore, it may be said that the success of the Wilson  $\epsilon$ -expansion is due to weak interaction in the  $(4 - \epsilon)$  dimensions.

In the theory of disordered systems, the relation (9) is valid with  $g_0 < 0$  and the situation appears asymptotically free; in approaching the mobility edge, the effective interaction does not decrease but rises due to existence of the 'spurious' pole in

the expression (9). The problem of strong coupling arises, which, as is thought [3], cannot be solved. Below we will obtain a complete solution of this problem for the density of states.

# 4. Special role of dimension d=4

The first question, which arises here is whether the dimension d=4 is special for the Anderson transition. The answer is positive, although most of researchers hold a different viewpoint. Special role of dimension d = 4 can be revealed in different ways, but fundamentally it is related to the renormalizability of the theory. As usually, the renormalizability can be investigated by counting up the power of momentum p in diagrams. The N-order diagram for the selfenergy  $\Sigma(p,\kappa)$  includes N integrations over the d-dimensional momentum and (2N-1) Green's functions, each of dimension  $1/p^2$ . As a result, the dimensionality with respect to momentum is  $p^r$ , where r = 2 + (d-4)N. At d > 4 the power of divergence at large momenta increases as the order of diagram enhances, hence, the theory cannot be renormalized. and the cutoff parameter  $\Lambda$  of order of inverse atomic scale should be explicitly introduced. At d < 4 we have r < 2 for all N and the theory is renormalizable by one subtraction: if we subtract from each diagram its value at  $p = \kappa = 0$ , the exponent r is reduced by 2, and the difference  $\Sigma(p,\kappa) - \Sigma(0,0)$  does not contain divergencies, which are absorbed by the quantity  $\Sigma(0,0)$  that only shifts the origin of the energy. At d=4 the logarithmic situation takes place: the difference  $\Sigma(p,\kappa) - \Sigma(0,0)$  contains logarithmic divergencies, which can be eliminated by renormalization of the charge and the Green's function.

### 5. Simplification of the theory for d > 4

The next question which arises is why the theory is simplified at d > 4? From diagrammatic analysis we can conclude that the increase in the order of diagram by unity give rise to an extra factor of  $W^2$  and two additional G functions. In rough estimates, the latter can be taken in the simplest functional form

$$G(p, E) = \left(E - \frac{p^2}{2m} + i\gamma\right)^{-1},\tag{10}$$

with the damping  $\gamma$ . For d < 4 the divergences at high momenta are eliminated due to renormalization, and integrations are determined by small values of p; therefore the Green's function should be considered as a quantity  $\sim 1/\gamma$  in the region of low energies E. In the case of weak disorder, the damping  $\gamma$  is small, being determined by parameter W, and, occuring in the denominator, may compensate the smallness of  $W^2$ . Detailed estimates show that it is indeed the case near the mobility edge, so the expansion parameter is of the order of unity at  $|E| \sim \gamma$ .

With d > 4, the situation is changed. Since the theory is nonrenormalizable, the integrals are determined by large momenta  $\sim \Lambda$ , and the Green's function (10) is of the order of 1/J, where J is the width of the band. Therefore, the expansion parameter is small at weak disorder,

$$\frac{W^2}{J^2} \leqslant 1. \tag{11}$$

The existence of the small parameter does not immediately simplify the problem, since the attempt to use finite number of terms in the perturbation expansion for  $\Sigma$ ,

$$\Sigma(p,\kappa) = A_1 g_0 + A_2 g_0^2 + A_3 g_0^3 + \dots, \tag{12}$$

results in the loss of the fluctuational tail. The reason is that the series is divergent and an essential contribution caused by high-order terms is ignored in this case. To obtain an asymptotically exact result for small disorder, we should approximate the perturbation series for  $\Sigma$  by the first term and by the sum of high-order terms, from some large  $N_0$  to infinity:

$$\Sigma(p,\kappa) = A_1 g_0 + \sum_{N=N_0}^{\infty} A_N g_0^N.$$
(13)

The asymptotic behavior of the high-order expansion coefficients has a the functional form (see below)

$$A_N = c \Gamma(N+b) a^N \tag{14}$$

and the sum in Eqn (13) can easily be calculated by representing the gamma-function as a definitive integral and finding the sum of the resulting geometric progression. The real part of the sum is small with respect to the first term in Eqn (13), while the imaginary part gives the substantial contribution

$$\operatorname{Im} \sum_{N=N_0}^{\infty} c \, \Gamma(N+b) a^N (g_0 + \mathrm{i} \, 0)^N$$

$$= \frac{\pi c}{(a|g_0|)^b} \exp\left(-\frac{1}{a|g_0|}\right), \tag{15}$$

which does not depend on  $N_0$ . This contribution results in the exponentially small fluctuational tail.

# 6. The Lipatov method

High-order terms of perturbation series can be calculated by the Lipatov method [4], which is based on the following simple idea. The expansion coefficients of the function F(g),

$$F(g) = \sum_{N=0}^{\infty} F_N g^N, \qquad (16)$$

are calculated using the relation

$$F_N = \int_C \frac{\mathrm{d}g}{2\pi \mathrm{i}} \, \frac{F(g)}{g^{N+1}} \,, \tag{17}$$

where the contour C encloses the point g=0 in the complex plane. Rewriting the denominator as  $\exp[-(N+1) \ln g]$ , we obtain an exponential with a large exponent at large N, allowing us to use the saddle point method. Let us apply Eqn (17) to the functional integral

$$F(g) = \int D\varphi \exp\left(-H_0\{\varphi\} - gH_{\rm int}\{\varphi\}\right). \tag{18}$$

Then we obtain

$$F_N = \int_C \frac{\mathrm{d}g}{2\pi i} \int D\varphi$$

$$\times \exp\left[-H_0\{\varphi\} - gH_{\text{int}}\{\varphi\} - (N+1)\ln g\right]. \quad (19)$$

The idea of the Lipatov method is that the saddle point of Eqn (19) is sought with respect to g and  $\varphi$ , simultaneously. It exists for all the interesting cases and is realized on a certain space-localized functions  $\varphi(x)$  (referred to as instantons). The conditions of applicability of the saddle-point method are fulfilled at large N, no matter how it is applied to the initial integral (18).

#### 7. The four-dimensional case

We gave the scheme of the calculation for high dimensions. For d=4, the difference  $\Sigma(p,\kappa)-\Sigma(0,0)$  diverges logarithmically. Arranging the contributions of the diagrams in the powers of the logarithms, for p=0 we get

$$\Sigma(0,\kappa) = \Sigma(0,0) + \kappa^2 \sum_{N=1}^{\infty} g_0^N \sum_{K=0}^N A_N^K \left( \ln \frac{\Lambda}{\kappa} \right)^K.$$
 (20)

The *N*-th order contribution involves all powers of the logarithms from zero to *N*. For exact solution of the problem we need all coefficients  $A_N^K$ ; our aim, however, will be to obtain only asymptotically exact results for weak disorder, that is for small  $g_0$ . Such results are obtained in the phase transitions theory  $(g_0 > 0)$  in the so-called main logarithmic approximation, by retaining in Eqn (20) only coefficients  $A_N^N$ , corresponding to 'parquet' diagrams. For the disordered systems  $(g_0 < 0)$ , the 'parquet' approximation is completely unsatisfactory, giving expressions with 'spurious' pole [3].

To obtain an adequate approximation for this case, the behavior of coefficients  $A_N^K$  should be carefully investigated. First of all, we know the parquet coefficients which are well studied and take on the functional form:

$$A_N^N = \operatorname{const} a^N N^{\beta} \,. \tag{21}$$

In principle, it is possible to construct the second, third etc. logarithmic approximations by including the coefficients  $A_N^{N-1}$ ,  $A_N^{N-2}$ ,... in Eqn (20). For finite K and  $N \to \infty$  we have

$$A_N^{N-K} = \operatorname{const} a^N N^{\beta} (N \ln N)^K.$$
 (22)

Information concerning the fastest-growing coefficients can be obtained by the Lipatov method. The total N-th order contribution to  $\Sigma(0,\kappa)$  has the form

$$\operatorname{const} g_0^N \Gamma(N+b) a^N (\ln N)^{-\gamma} \exp\left(\frac{\sigma \ln \Lambda}{\kappa}\right), \tag{23}$$

Comparing this with expansion (20) we get

$$A_N^K = \operatorname{const} \frac{\sigma^K}{K!} \Gamma(N+b) a^N (\ln N)^{-\gamma}.$$
 (24)

The Lipatov method reproduces coefficients  $A_N^K$  well only for small K: they are decreasing rapidly for large K, and the accuracy of the method becomes insufficient.

From these estimates we see that the terms with low powers of logarithms increase rapidly with N: the less a

power of the logarithm, the greater a rate of growth. Parquet coefficients are never dominant in high orders, and that is why the parquet approximation fails in the theory of disordered systems.

To obtain asymptotically exact results we should use the following approximation. For low-order terms we can keep in Eqn (20) only the parquet coefficients  $A_N^N$  that are distinguished by largest powers of logarithms. But for N exceeding certain  $N_0 \gg 1$ , all the coefficients  $A_N^K$  should be taken into account, since we cannot say beforehand, what range of N and K gives the dominant contribution. The parquet coefficients are well known and therefore only  $A_N^K$  with large N should be found. As indicated above, it cannot be done straightforwardly by the Lipatov method, since this method gives good results only for small K.

The calculation of  $A_N^K$  with  $K \sim N$  is based on the renormalizability of the theory. Let us explain the basic idea. Let F be an observable quantity. When we calculate it formally in the perturbation theory, it is a function of the bare charge  $g_0$  and the cutoff parameter  $\Lambda$  that has to be introduced for eliminating the divergences. The renormalizability of the theory means that the renormalized charge g can be defined in such a way that the variable F as a function of g does not diverge, and at  $\Lambda \to \infty$  tends to a finite limit,

$$F(g_0, \Lambda) = F_R(g). \tag{25}$$

The quantities that are not directly observable (like the Green's functions) may renormalize in a more complicated fashion; as a matter of fact, for all technical purposes we may confine ourselves to the category of quantities that are renormalizable in the multiplicative way,

$$F(g_0, \Lambda; p_i, \ldots) = Z(g_0, \Lambda) F_R(g; p_i, \ldots), \qquad (26)$$

In other words, from the quantity F (which depends on the momenta  $p_i$  and other variables) we separate out the diverging Z factor. Since  $F_R$  does not depend on  $\Lambda$ , we have

$$\frac{\mathrm{d}F_R}{\mathrm{d}\ln A} = 0. \tag{27}$$

Substituting here  $F_R$  from Eqn (26) and expressing the total derivative in terms of partial derivatives, we get the Callan – Symanzik equation

$$\left[\frac{\partial}{\partial \ln \Lambda} + W(g_0)\frac{\partial}{\partial g_0} + V(g_0)\right] F(g_0, \Lambda; p_i, \ldots) = 0.$$
 (28)

Defined formally, the functions W and V depend on  $\Lambda$ . The actual absence of such a dependence may be proved taking advantage of the fact that the Z factors for different F are not independent, and can be expressed one via another. Equation (28) is a mathematical expression of the existence of an exact renormalization group in the phase transitions theory.

In the case under consideration, multiplicative renormalizability is exhibited by the quantity

$$Y = \kappa^2 + \Sigma(0, \kappa) - \Sigma(0, 0), \qquad (29)$$

that differs from the series in Eqn (20) only by a trivial term. Its substitution into (28) yields a set of equations for coefficients  $A_N^K$ :

$$-KA_{N}^{K} = \sum_{M=1}^{N-K+1} [W_{M+1}(N-M) + V_{M}] A_{N-M}^{K-1},$$
 (30)

where  $W_M$  and  $V_M$  are expansion coefficients for  $W(g_0)$  and  $V(g_0)$  over  $g_0$ . This set of equations has a form of a recurrent relation determining  $A_N^K$  from given  $A_{K-1}^{K-1}$ ,  $A_K^{K-1}$ , ...,  $A_N^{K-1}$ . All the coefficients  $A_N^K$  can be calculated when  $A_N^0$  are known. But the latter coefficients are well reproduced by the Lipatov method and can be used as a boundary condition for set (30). Thus, all the required coefficients and the sum of the perturbation series can be found.

## 8. Transition to the $(4 - \epsilon)$ -dimensional theory

For  $d = 4 - \epsilon$ , the perturbation expansion for the self-energy has a following structure

$$\Sigma(0,\kappa) - \Sigma(0,0) = \kappa^2 \sum_{N=1}^{\infty} (g_0 \Lambda^{-\epsilon})^N \times \sum_{K=0}^{N} A_N^K(\epsilon) \left[ \frac{(\Lambda/\kappa)^{\epsilon} - 1}{\epsilon} \right]^K, \quad (31)$$

where the coefficients  $A_N^K(\epsilon)$  are finite at  $\epsilon \to 0$  and have the regular expansions in  $\epsilon$ :

$$A_N^K(\epsilon) = \sum_{L=0}^{\infty} A_N^{K,L} \epsilon^L \,. \tag{32}$$

After the limiting transition  $\Lambda \to \infty$ , the expansion (32) becomes analogous to Eqn (20) with replacement of the large logarithms by the powers of  $1/\epsilon$ .

In the standard procedure of  $\epsilon$ -expansion [1] only a few largest powers of  $1/\epsilon$  are retained in each order of perturbation theory. The first order of  $\epsilon$ -expansion is obtained by retaining only coefficients  $A_N^{N,0}$ , that coincide with the coefficients of main logarithms in Eqn (20). Analogously to the case of d=4, such approximation is insufficient at  $g_0<0$  due to the large rate of growth in N of the coefficients at small powers of  $1/\epsilon$ .

In fact, the coefficients  $A_N^K(\epsilon)$  are calculated directly without preliminary expansion over  $\epsilon$ . This calculation is entirely similar to the four-dimensional case, i.e., it is based on the Callan-Symanzik equation with the results of the Lipatov method used as a boundary condition. Thus, the transition to the  $(4 - \epsilon)$ -dimensional theory does not require essentially new ideas.

As a result, we have obtained explicit expressions for the density of states of a disordered system in the whole range of energies including the vicinity of the mobility edge. They are rather cumbersome and we do not present them here [5]. We restrict ourselves to only qualitative discussion. As is known from the phase transitions theory, there is a fluctuational shift of the critical point: the correct value of the critical temperature  $T_c$  differs from that calculated by the Landau theory. A similar shift occurs also in the theory of disordered systems, but now it is complex and its imaginary part arises due to instability of the corresponding field theory. If we could penetrate to the complex plane of the energy (according to Eqns (5), (6), the energy plays a role of temperature) and approach the critical point, we would observe the usual critical exponents of the phase transitions theory. However, we can deal only with the real values of the energy, so the

critical point is avoided and the density of states has no singularity.

The main difficulties of the theory are related with calculation of the imaginary part of the fluctuational shift, which is exponentially small with respect to  $g_0$  and has nonperturbative nature. This is the reason, why here we restrict ourselves by consideration of the quantity  $\Sigma(0,\kappa)$ , which determines the transition point. The rest of the calculations are the same as in the phase transitions theory; therefore the average Green's function of the disordered system is, roughly speaking, described by 'parquet' formulas with the complex  $T_c$ . The critical point is shifted in different directions of the complex plane for the retarded and advanced Green's functions. We may think that in the case of the correlator  $\langle G^R G^A \rangle$ , the critical point remains on the real axis, since it does not 'know' the direction of the shift.

Details of the calculations are described in Ref. [6] and systematized in review [5].

The work is supported by the INTAS (Grant 99-1070) and by the RFBR (Grant 00-02-1729).

#### References

- Ma S Modern Theory of Critical Phenomena (Reading, Mass.: Benjamin W A, 1976) [Translated into Russian (Moscow: Mir, 1980)]
- 2. Nitzan A, Freed K F, Cohen M N *Phys. Rev. B* **15** 4476 (1977)
- Sadovskiĭ M V Usp. Fiz. Nauk 133 223 (1981) [Sov. Phys. Usp. 24 96 (1981)]; Sov. Sci. Reviews Sect. A Phys. Rev. (Ed. I M Khalatnikov) 7 1 (1986)
- Lipatov L N Zh. Eksp. Teor. Fiz. 72 411 (1977) [Sov. Phys. JETP 45 216 (1977)]
- 5. Suslov I M Usp. Fiz. Nauk **168** 503 (1998) [Phys. Usp. **41** 441 (1998)]
- Suslov I M Zh. Eksp. Teor. Fiz. 102 1951 (1992) [Sov. Phys. JETP 75 1049 (1992)]; Zh. Eksp. Teor. Fiz. 106 560 (1994) [JETP 79 307 (1994)]; Zh. Eksp. Teor. Fiz. 111 220 (1997) [JETP 84 120 (1997)]; Zh. Eksp. Teor. Fiz. 111 1896 (1997) [JETP 84 1036 (1997)]