Some signatures of quantum chaos on dirty superconductors

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Abstract. The Anderson theory of dirty superconductivity was established a few years after the discovery of the BCS wave function. Disregarding the rich properties in the one-particle energy spectrum in dirty limit, the theory claimed that the ground state condensate is translationally invariant and free from Toulouse type of frustrations. This theory also set down the foundation of dirty superconductivity in the presence of external fields.

In this talk, I demonstrate the failure of the Anderson theory in amorphous superconducting films in general and its connection with Wigner–Dyson surmise. I will discuss the Chandrasekar–Glogston limit in which the nodes in one-particle wave functions are shown to result in a novel superconducting glass phase. I will also discuss why nature has tolerated the failure.

1. Introduction

Disordered superconductors have been studied long ago [1], right after the discovery of BCS wave functions. It was shown that the ground-state-condensate wave function is homogeneous and the critical temperature remains unchanged in the presence of weak nonmagnetic disorder. To derive this dirty superconductor theory, one has to assume that (i) the effective interaction constant in the Cooperon channel remains unchanged when a clean superconductor is disordered, (ii) the condensate wave function is translationally invariant, and (iii) the time reversal symmetry is preserved.

The first assumption, though is not true in the thin film limit where the Coulomb interaction in Cooperon channel can be greatly enhanced, is valid in the bulk limit [2]. We will assume its validity because it does not affect the result presented in this paper as far as the renormalized interaction constant is still negative.

The second assumption is concerned with the translation invariance of the pairing wave functions. The translation invariance is not a generic symmetry of the original Hamiltonian in the presence of disorder. In principle, one has to deal with the Gor'kov–Eliashberg equations written in terms of exact Green's functions (or exact eigenstates) in the presence of given impurity potentials. Practically, following Abrikosov and Gor'kov, an impurity average is taken in the course of studying dirty superconductivity [3]. The sample-specific quantum interference effect which is of the same origin as Wigner–Dyson statistics does not survive this averaging and is not taken into account. And the second assumption is true only after the impurity average is taken in the semiclassical limit.

Consequences of sample-specific quantum interference effects which break the translation invariance is one of the subjects of this paper. In a noninteracting electron system, the energy spectrum in a disordered mesoscopic sample was shown to exhibit Wigner–Dyson statistics, which is universal, only dependent on the symmetry of the Hamiltonian [4].

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For an open sample, the fluctuation of number of levels δN within the energy band of Thouless energy E_c is of order of unity,

$$\frac{\langle (\delta N)^2 \rangle}{\langle N \rangle^2} = \frac{\beta}{G^2} \tag{1}$$

for a 2D film, where the corresponding average number of levels $\langle N \rangle = L^2 \, d\nu_0 E_c = G$, with ν_0 being the average density of states at the Fermi surface. $E_c = D/L^2$(Thouless energy), L is the length of the sample, $D = v_F l/3$ is the diffusion constant of the film, v_F is the Fermi velocity, l is the elastic mean free path, β is a factor of order unity depending on the symmetry of the Hamiltonian,

$$G = k_F^2 \, dl \tag{2}$$

is the dimensionless conductance of the 2D normal metal in units of $e^2/\hbar$, k_F is Fermi wavelength and the brackets $\langle \, \rangle$ denote averaging over realizations of random potential.

The transport in disordered mesoscopic systems is governed by UCF (universal conductance fluctuation) theory [5, 6]. The conductance exhibits sample-specific fluctuations, with amplitude $e^2/\hbar$, independent of the average conductance of the sample [5, 6]. More generally, any physical quantity in a mesoscopic sample consists of an ensemble average part and a sample-specific part due to quantum interference. We will see this is also true for superconductors with properly chosen 'mesoscopic scales'.

The third aspect of Anderson theory for a dirty superconductor is the *absence of spontaneous time-reversal symmetry breaking*, that is, the stability of a BCS state with respect to possible frustrations caused by quantum chaos, even when l is much shorter than the coherence length. This issue will be addressed in this article, in connection with nodes in the spatial dependence of exchange interactions and the distribution function of the exchange interactions.

Dirty superconductor theory also lays down the foundation for dirty superconductivity in the presence of external magnetic fields: both orbital effects and Zeemann effects were studied in the same approximation. The last two assumptions were, to certain extent, taken for granted and generalized to some limits where they become false. While they are generally invalid in thin film limit, even in the bulk limit, they cease to be true at certain magnetic field. It is the purpose of this work to explore novel phases in dirty superconductors as signatures of quantum chaos.

Finally I want to remark that unlike in the noninteracting metal where the mesoscopic physics is relevant only in a finite sample smaller than the dephasing length, in the presence of off-diagonal long-range order, it reveals itself in the thermodynamic limit. One example is a superconductor in a magnetic field close to the upper critical field H_{c2}, where the magnetic field dependence of the superconducting critical temperature is determined by the mesoscopic fluctuations [7]. In general, the mesoscopic effects are not only relevant in a disordered superconductor but also vital to the global phase rigidity.

In this paper I consider the case, where the magnetic field is parallel to the thin superconducting film and the main contribution to the suppression of superconductivity by the magnetic field is due to Zeeman splitting of electron spin energy levels. To simplify the situation I should focus on the case when the transition is a second order one; results can be generalized to the case of first order transitions. For a detail derivation of the results presented here, I should refer to the original papers [8–10], on which these materials are based. Specifically, I will discuss signatures of dirty superconductivity along $T = 0$ axis and those along $H = 0$ axis.

2. Stochastic Ginzburg–Landau equations

The signatures of quantum chaos on dirty superconductor can be studied by introducing stochastic sources to Gor'kov equation; when it is close to a critical field, one can derive a stochastic Ginzburg–Landau equation, with the *effective temperature* of the stochastic sources given by the dimensionless conductance introduced before.

When $\Delta(H, \mathbf{r}) \ll \Delta_0$ (Δ_0 denotes the order parameter in zero field at zero temperature), we make an expansion of the Gor'kov equation in terms of $\Delta(\mathbf{r})$ and the gradient. As a result, we get

$$\xi_0^2\left(\nabla - \mathrm{i}\frac{2e}{c}\mathbf{A}\right)^2 + \frac{H_c^0 - H}{H_c^0}\Delta(\mathbf{r}) + \int \delta K^0(\mathbf{r}, \mathbf{r}', H, \mathbf{A})\Delta(\mathbf{r}')\,\mathrm{d}\mathbf{r}' = \frac{\Delta^3(\mathbf{r})}{2\Delta_0}\,, \qquad (3)$$

where $\mathbf{A}$ is the vector potential of external perpendicular magnetic field. $\xi_0 = \sqrt{D/\Delta_0}$. The kernel $\delta K^0(\mathbf{r}, \mathbf{r}')$ is of stochastic nature. The difference between Eqn 3 and the conventional Ginzburg–Landau equation is the third term in Eqn 3 which accounts for mesoscopic fluctuations of the kernel $K^0(\mathbf{r}, \mathbf{r}')$. It is precisely this term, which at high magnetic fields leads to the random sign of superfluid density.

We introduce a correlator

$$\mathcal{C}(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_2, \mathbf{r}_2') = \langle \delta K^0(\mathbf{r}_1, \mathbf{r}_1')\delta K^0(\mathbf{r}_2, \mathbf{r}_2')\rangle\,. \qquad (4)$$

The large-distance asymptotics of the correlation function in Eqn 4 takes the form:

$$\mathcal{C}(\mathbf{r}_1, \mathbf{r}_1'; \mathbf{r}_2, \mathbf{r}_2') = \frac{G^{-2}}{16}\left[\xi_0^2\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1' - \mathbf{r}_2')\delta(\mathbf{r}_1 - \mathbf{r}_1') + \delta(\mathbf{r}_1 - \mathbf{r}_2')\delta(\mathbf{r}_1' - \mathbf{r}_2)\frac{\xi_0^4}{|\mathbf{r}_1 - \mathbf{r}_1'|^4}\right]. \qquad (5)$$

There are two kinds of sources in the equation corresponding to two signatures of quantum chaos in dirty superconductivity. The short-range term stands for the fluctuations of local density of states; they are largely responsible for the nucleation of mesoscopic pairing states above T_c. The long-range term, whose origin will be discussed in great detail, leads to frustrations.

3. Two signatures of quantum chaos

3.1 Signature 1: nucleation of mesoscopic pairing states

Let me start with the first signature. Employing the perturbation theory with respect to $\delta K^0(\mathbf{r}, \mathbf{r}')$, from Eqn 3 we get an expression for the correlation function of the mesoscopic fluctuations of the superconducting order parameter $\delta\Delta(\mathbf{r}; H) = \Delta(\mathbf{r}; H) - \langle\Delta(H)\rangle$:

$$\langle\delta\Delta(\mathbf{r})\delta\Delta(\mathbf{r}')\rangle \propto \frac{\Delta_0^2}{G^2} \times \begin{cases} 1 - \dfrac{1}{2}\ln\left[\dfrac{\xi(H)}{\xi_0}\right]\left[\dfrac{|\mathbf{r} - \mathbf{r}'|}{\xi(H)}\right]^2, & |\mathbf{r} - \mathbf{r}'| \ll \xi(H)\,, \\ \exp\left[-\dfrac{|\mathbf{r} - \mathbf{r}'|}{\xi(H)}\right], & |\mathbf{r} - \mathbf{r}'| \gg \xi(H)\,. \end{cases} \qquad (6)$$

It follows from Eqn 6 that the amplitude of the fluctuations of the order parameter in the two-dimensional case is almost independent of H, but the average order parameter decreases with H. As a result, perturbation theory holds as long as $\langle\Delta(H)\rangle/\Delta_0 = \sqrt{(H_c^0 - H)/H_c^0} \gg G^{-1}$. The other feature in Eqn 6 is that the correlation length for the mesoscopic pairing fluctuations gets longer and longer as the critical point is approached.

One of the consequences of the mechanism discussed above is the *instability of the spin-polarized disordered Fermi liquid* well above the critical magnetic field. In the regions where the spin-polarization energy cost to form superconducting pairing state is much lower than the average energy cost, the normal metal with $\Delta = 0$ becomes unstable. As a result, above the critical field H_c^0, the superconducting pairing correlations are established at mesoscopic scales in the different regions in the normal metal and couple with each other via exchange interactions of random signs. This argument was presented in our paper [8].

What is the probability to find regions where the superconducting pairing states are formed at mesoscopic scales at $H > H_c^0$? At high magnetic fields the statistics of these pairing states can be studied with the help of the generalized Ginzburg–Landau equation, which is valid when $H - H_c^0$ is small compared with H_c^0 and when the spatial variation of the pairing wave function $\Delta(\mathbf{r})$ over distance ξ_0 is negligible.

Equation (3) is a *nonlinear* equation in terms of $\Delta(\mathbf{r})$, with a *nonlocal* $\delta K^0(\mathbf{r}, \mathbf{r}')$ potential originating from the oscillations of the wave functions of Cooper pairs. These are the generic features of *strongly correlated* mesoscopic systems. The nonlocal structure of the potential in Eqn (3) leads to the superconducting glass state.

At $H - H_c^0 \gg H_c^0/G^2$, the optimal configurations can be written as

$$\Delta(\mathbf{r}) = \sum_\alpha \Delta_\alpha \eta_\alpha(\mathbf{r}), \quad \int \mathrm{d}\mathbf{r}\eta_\alpha(\mathbf{r})\eta_\beta(\mathbf{r}) \propto \delta_{\alpha\beta}\,. \qquad (7)$$

Note that $\eta(\mathbf{r})$ introduced in this way is dimensionless. The total energy of such a configuration consists of cross terms corresponding to the coupling between different droplets. The coupling between the droplets decays as the distance increases. When the size of the droplets is much smaller than the distance between them, the typical magnitude of the coupling between different droplets is much smaller than that of the coupling within one droplet. We are going to neglect such terms in the estimate of the probability of the droplets in the leading order of $o(L_f^2/L_d^2)$. The main results are presented below.

Shape of droplet:

$$\eta_s(\mathbf{r}) = \eta_s(yL_f), \quad L_f = \xi_0\left(\frac{H_c^0}{H - H_c^0}\right)^{1/2}, \qquad (8)$$

η_s satisfies the dimensionless saddle point equation

$$\left(-\nabla_y^2+1\right)\eta_s(y) + \int dy_1\, dy_1'\, dy'\tilde{C}(y,y';y_1,y_1')\eta_s(y_1)\eta_s(y_1')\eta_s(y') = 0 \quad (9)$$

and at $y=\infty$, $\eta_s(y)=0$, $C(\mathbf{r},\mathbf{r}';\mathbf{r}_1,\mathbf{r}_1') = \frac{1}{G^2}\frac{\xi_0^2}{L_f^6}\tilde{C}(y,y';y_1,y_1')$.

Number density:

$$\rho = \frac{1}{V}\frac{\sum_l P_l l}{\sum_l P_l} = \frac{1}{L_f^2}\text{erfc}\left[\frac{BG}{A}\left(\frac{H-H_c^0}{H_c^0}\right)^{1/2}\right], \quad (10)$$

where B, A^2 are the dimensionless quantities of order of unity depending on the details of $\eta_s(y)$:

$$B = \int dy\eta_s(y)(-\nabla_y^2+1)\eta_s(y)\,,$$
$$A^2 = \int dy_1\, dy_1'\, dy_2\, dy_2' \times \tilde{C}(y_1,y_1';y_2,y_2')\eta_s(y_1)\eta_s(y_1')\eta_s(y_2')\eta_s(y_2)\,. \quad (11)$$

Typical amplitude of Δ:

The distribution function of the amplitude of the order parameter Δ in a droplet can be written as

$$P_c(\Delta^2) = \frac{2CG\Delta}{\Delta_0^2}\exp\left(-C^2G^2\frac{\Delta^2}{\Delta_0^2}\right). \quad (12)$$

3.2 Signature 2: random exchange and frustrations

The second signature is concerned with the exchange interaction between different droplets, which deserves special attention. Though the coupling between droplets does not affect the probability of finding one droplet, it determines the global phase rigidity. The typical coupling between α and β droplet is determined by the long-range stochastic term. In the limit $L_d \gg L_f$ we obtain the variance of the coupling

$$\left\langle\left[v_0\Delta_\alpha\Delta_\beta\int d\mathbf{r}\, d\mathbf{r}'\delta K^0(\mathbf{r},\mathbf{r}')\eta_s(\mathbf{r}-\mathbf{r}_\alpha)\eta_s(\mathbf{r}'-\mathbf{r}_\beta)\right]^2\right\rangle^{1/2} \propto \frac{\Delta_0}{G^2}\left(\frac{L_f}{L_d}\right)^2. \quad (13)$$

This suggests that the ground state of these coupled mesoscopic pairing states will exhibit glassy behavior in this limit, or superconducting glass state.

The existence of random Josephson coupling in the presence of a parallel magnetic field is a consequence of the Pauli spin polarization. Consider, for example, a granular superconductor, with superconducting grains embedded inside a *noninteracting* disordered polarized liquid. An electron with spin up has a different kinetic energy than an electron with spin down on the Fermi surface because of the Pauli spin polarization. As a result, the pairing wave function oscillates and develops nodes in its spatial dependence:

$$2\pi kT\sum_\omega \sigma_y^{\alpha-\alpha}\sigma_y^{\beta-\beta}G_\omega^{\alpha\beta}(\mathbf{r},\mathbf{r}')G_{-\omega}^{-\alpha-\beta}(\mathbf{r},\mathbf{r}') = \frac{v_0}{|\mathbf{r}-\mathbf{r}'|^2}\cos\left(\frac{|\mathbf{r}-\mathbf{r}'|\mu_B H}{v_F}-\frac{\pi}{4}\right). \quad (14)$$

This leads to the sign oscillations of the Josephson coupling with a period $v_F/\mu_B H$, which is much longer than the Fermi wavelength. The positions of these nodes in the spatial dependence of the coupling can be shifted in random directions when impurities are present. When $L^2/l \gg v_F/\mu_B H$, ϕ is much larger than unity, and the sign of the coupling becomes unpredictable for different impurity configurations. In this limit, the Josephson coupling averaged over impurity configurations is exponentially small,

$$\exp\left(-\frac{\sqrt{2}L}{\sqrt{D/\mu_B H}}\right),$$

while the typical amplitude of the coupling decays as L^{-2}. Therefore, when the magnetic field increases, only the position of the maximum of the distribution function moves towards zero, while the width of the distribution function barely changes. This results in the superconducting glass state. It is interesting to note that correlation effect in the presence of localized impurities can also lead to random sign of exchange interactions [11, 12].

4. Some aspects of transport

For a frustrated XY model, transport is not well understood. Phenomenlogically, we can take a hydrodynamical point of view developed by Andreev, and also by Halperin – Saslow. Then the dynamics in the long wavelength limit is determined by two macroscopic variables: (i) θ which characterizes the deviation of local phase of the order parameter from the ground state value θ_0 (due to frustration, the time reversal symmetry is broken at mean field level and θ_0 is a random quantity); (ii) μ which is the local chemical potential. They form a pair of conjugate variables. Without taking into account the motion between different energy minima, the system supports gapless sound-like waves, undamped at zero frequency limit. In this approximation, the conductance is infinity.

Motions between the different valleys are present at finite temperature under the general belief that the distribution function of barriers separating these different valleys is smooth around zero. Therefore the low-temperature dependence of conductance is completely determined by the energy barriers within kT. An estimate shows that

$$\frac{\sigma}{\sigma_{Dr}} = \frac{D_p}{D_0},\quad D_p = \frac{1}{3}\tau_p v_s^2,\quad \frac{1}{\tau_p} = \frac{T^2}{3E_B}\,. \quad (15)$$

Here σ_{Dr} is the Drude conductivity; D_p is the diffusion constant of phase θ and v_s is the 'sound velocity'. The T^2 dependence of τ_p is obtained using the thermal activation formula: one T is from the number of available barriers within kT, and the other T is from thermally activated transition between barriers: $1/E_B$ is the slope of the distribution function of the energy barriers around the zero energy.

This hydrodynamic approach, though far from complete, at least demonstrates the possibility of having anomalous temperature dependence of the conductance of the frustrated system under consideration. It is different from both an insulator and a superconductor. I am not going to pursue further.

For a finite sample, when gate voltages are applied, the mesoscopic fluctuations in Eqn (3) start to oscillate. Such oscillations should manifest themselves in the gate-voltage fingerprint experiment: the conductance as a function of gate voltage exhibits sample-specific fluctuations, with amplitude equal to the normal sample conductance. The conductance fluctuation due to pairing correlations can also much exceed the value of UCF, as discovered in Ref. [10].

One should recall that for noninteracting system, the amplitude of the conductance fluctuation is of order of $e^2/\hbar$ at $T = 0$ and becomes less than $e^2/\hbar$ when the temperature is higher than E_T [5, 6]. The above statement about mesoscopic fluctuations of the conductance remains true in weakly correlated electron systems, and most of strongly correlated systems encountered, which could be fractional-quantum-Hall systems.

In the presence of pairing correlations, mesoscopic fluctuations of conductance can greatly exceed $e^2/\hbar$, the scale of UCF. To proceed further, one has to express the conductivity of a given sample above T_c as

$$\sigma_{xx} = \sigma + \delta\sigma + \delta\sigma^M_{xx} \, ,$$

$\delta\sigma$ represents the contributions from Aslamazov – Larkin and Maki – Thompson corrections due to thermal fluctuations. It is divergent as the temperature approaches T_c. Correspondingly, the mesoscopic fluctuations of the conductivity $\delta\sigma^{\mathrm{M}}_{xx}$ therefore consist of two parts: one from the fluctuations of σ and the other from those of $\delta\sigma$.

A detailed derivation was discussed in Ref. [10]. We present here the main results of the calculations.

$$\frac{\langle(\delta\sigma^{\mathrm{M}}_{xx})^2\rangle}{\sigma^2} = \frac{\beta}{G^4}\left(\frac{T}{T - T_c}\right)^3 \left[\frac{\xi(T)}{L}\right]^3 , \qquad (16)$$

when $L \gg \xi(T)$ and saturates as

$$\frac{\langle(\delta\sigma^{\mathrm{M}}_{xx})^2\rangle}{\sigma^2} = \frac{\beta}{G^4}\left(\frac{T}{E_T}\right)^3 , \qquad (17)$$

when $L \ll \xi(T)$. Here $\beta \propto \max\{\ln(L/\xi(T)), 1\}$.

Equations (16), (17) are valid as far as $\delta\sigma^{\mathrm{M}}_{xx} < \delta\sigma < \sigma$. Following Eqns (16), (17), mesoscopic fluctuations of conductance can be much larger than $e^2/\hbar$. For instance, for a 2D film of the size of $\xi(T)$, at the temperature $T - T_c \sim T_c/g_2$ when $\delta\sigma/\sigma \sim 1$,

$$\sqrt{\langle(\delta\sigma^M_{xx})^2\rangle} \propto \frac{e^2}{t\hbar}\sqrt{g_2} \qquad (18)$$

is parametrically larger than UCF in normal metals.

The anomalous fluctuations can be probed in experiments where resistances are measured at different gate voltages. Let us consider a 2D film where a gate voltage is applied to the top of the film with capacitance C. The electric field induced by the gate is normal to the film and is screened over a Debye screening length $r_0 = (e^2\nu)^{-1/2}$. One finally obtains the gate voltage dependence of the mesoscopic fluctuations:

$$\left\langle(\delta\sigma^{\mathrm{M}}_{xx}(V_{g1}) - \delta\sigma^{\mathrm{M}}_{xx}(V_{g2}))^2\right\rangle = \left\langle(\delta\sigma^{\mathrm{M}}_{xx})^2\right\rangle F\left(\frac{|V_{g1} - V_{g2}|Cr_0^2}{\epsilon_0 t L^2 T}\right) , \qquad (19)$$

where $F(x) \propto x$ at $x \ll 1$ and of order of unity when $x \gg 1$. Following Eqn (19), in this case, the characteristic gate voltage V_g at which $\delta\sigma^M_{xx}(V_g)$ are correlated is $\epsilon_0 TL^2 t/er_0^2 C$. Mesoscopic fluctuations discussed here are also sensitive to external magnetic fields. The other possibility to observe the anomalous mesoscopic fluctuations of transport coefficients is to measure the conductance during different thermal cycles.

5. Discussion

We show the existence of a novel superconducting glass phase in disordered thin films in Glogston limit. The statistics of mesoscopic pairing states in the superconducting glass phase is universal and determined only by the sheet conductance. It is a direct consequence of Wigner – Dyson statistics of single-particle energy spectrum.

The mechanism discussed in this paper is distinct from the effect of inhomogeneity of impurity concentration, or classical pinning effect on vortex lattices. First of all, in the present case, the magnetic field couples only with spins and the wave functions are real (as far as the impurity-averaged condensate wave function is concerned); the time reversal symmetry is broken spontaneously. For classical pinning effects on vortex lattices, the time reversal symmetry is broken by the applied perpendicular magnetic field. Moreover, fluctuations of local quantities like mean free path can lead to inhomogeneous states but do not lead to spontaneous time-reversal symmetry breaking. The glass state discussed in this paper is due to random signs of long-range exchange interaction, which is purely of mesoscopic nature. Finally, the response of the state discussed here is determined *universally* by Thouless energy of the size of the coherence length and the response of a pinned vortex glass strongly depends on the range and strength of the classical pinning potential. For amorphous films where the impurity potential is perfectly screened and in the absence of granularities, the classical pinning effect is weak; the mesoscopic effects dominate in this limit.

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