Coupling of two superconductors through a ferromagnet. SFS π-junctions and intrinsically-frustrated superconducting networks

V V Ryazanov, V A Oboznov, A V Veretennikov, A Yu Rusanov, A A Golubov, J Aarts

Abstract. Reentrant superconducting behavior of the critical supercurrent temperature dependencies has been observed for the Nb–Cu/Ni–Nb Josephson SFS (superconductor–ferromagnet–superconductor) junctions. The $I_c(T)$ oscillations detected are associated with a crossover of the SFS junctions from '0'- to 'π'-state that is related to a special feature of superconducting pair flow through a ferromagnet (spatial oscillations of induced superconducting order parameter in presence of the exchange field). A triangular array of the junctions in the π-state shows evidence for a phase shift of π on the $I_c(H)$ dependence. We have argued advantages of the SFS π-junction use for the quantum bit implementation based on the superconducting loop with quantum Josephson junctions. While designs proposed previously are based on magnetically frustrated superconducting loops, we discuss the advantages offered by the π-junctions in obtaining naturally degenerate two-level systems.

1. Introduction

In recent years considerable attention has been given to possible realization of 'π-contacts', i.e. weakly coupled junctions formed in superconducting and superfluid systems which demonstrate π-shift of macroscopic phase difference in the ground state [1–6]. A brief review of experimental and theoretical works devoted to the study of Josephson structures with spontaneous π-shift of the phase difference is given in Ref. [7]. Among such structures one should mention bicrystals [3] and 's–d' contacts [4, 8] based on high-temperature superconductors with assumed 'non-trivial' (d-wave) symmetry of the order parameter, mesoscopic superconductor–normal metal–superconductor (SNS) junctions controlled by current along the N layer [5], and finally, superconductor–ferromagnet–superconductor (SFS) junctions [2, 9]. Experimental studies of Josephson thin-film SFS junctions have been taken up [7] with the aim to use the π-states in superconducting logic elements for quantum computing [10] and digital superconducting logics [11]. In this paper we present the experimental evidence for the π-junction behavior of single Josephson SFS junctions and triangular two-dimensional arrays of SFS junctions and give the brief theoretical consideration of the phenomena.

V V Ryazanov, V A Oboznov, A V Veretennikov, A Yu Rusanov Intitute of Solid State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Moscow Region, Russian Federation

A A Golubov Department of Applied Physics, University of Twente, P.O.Box 217, 7500 AE, Enschede, the Netherlands

J Aarts Kamerlingh-Onnes Laboratory, Leiden University, P.O.Box 9504, 2300 RA, Leiden, the Netherlands\

2. Proximity effect in the SF system

As in the case of a superconductor–normal metal (SN) interface, superconducting pairs in an SF structure can penetrate to the ferromagnetic F layer through a certain depth denoted here as ξ_{F1}, inducing a superconducting order parameter. The decay of the order parameter in the ferromagnetic layer increases as the exchange energy E_{ex} rises, since exchange interactions try to arrange electron spins in a certain direction, i.e. to destroy superconducting pairs formed by electrons with opposite spins. When the Curie temperature, T_m, of the ferromagnet greatly exceeds the critical temperature, T_c, of the superconductor, only exchange interaction can be considered as the depairing factor in the dirty ferromagnet. In this case the decay length of the superconducting order parameter in the ferromagnet can be estimated by substituting the exchange energy E_{ex} instead of $k_B T$ to the conventional expression for the pair coherence length in a dirty normal metal

$$\xi_{F1} \sim \left(\frac{\hbar D}{E_{ex}}\right)^{1/2} \sim \left(\frac{\hbar D}{k_B T_m}\right)^{1/2}, \quad (1)$$

where k_B is the Boltzmann constant, D is the diffusion coefficient for electrons in the ferromagnet. As distinct from the SN system, where the coherence length ξ_N responsible for the decay of the order parameter in the N layer is real, the coherence length ξ_F in a ferromagnet is complex. This means that in the ferromagnet close to the SF interface, oscillations of the induced superconducting order parameter related to the imaginary part ξ_{F2} of the coherence length should arise together with the decay determined by the length ξ_{F1}. The order parameter (superconducting wave function) can be expressed in these terms as follows:

$$\Psi_F(x) = \Psi_{F0} \exp\left(-\frac{x}{\xi_F}\right) = \Psi_{F0} \exp\left(-\frac{x}{\xi_{F1}}\right) \exp\left(-i\frac{x}{\xi_{F2}}\right). \quad (2)$$

Here Ψ_{F0} is the order parameter in the ferromagnet near the SF interface and x is the coordinate in the direction perpendicular to the interface. The wavelength of the order parameter oscillations is $2\pi\xi_{F2}$. In the diffusive limit for $E_{ex} \gg k_B T$, the values of ξ_{F1} and ξ_{F2} are equal and described by (1), which is valid [12] in the case of weak spin-orbit scattering in the ferromagnet. Then the complex coherence length can be written as

$$\xi_F = \left(\frac{\hbar D}{2iE_{ex}}\right)^{1/2}. \quad (3)$$

We discuss a physical origin of the oscillating and sign-reversal order parameter Ψ_F in the ferromagnetic superconductor before the consideration of the case of small exchange energies $E_{ex} \geq k_B T$. This phenomenon, predicted by Larkin and Ovchinnikov [13] and by Fulde and Ferrel [14], is related to arising of Cooper pairs with nonzero total momentum in presence of the exchange field. The bulk superconductors with the order parameter in this state (usually referred to as the LOFF state) have never been observed so far. However, as it will be shown below, the quite realisable modification of the LOFF state is the oscillating order parameter near the SF interface, which is predicted in Refs [9, 15, 16]. A simple physical picture of an arising of nonzero total pair momentum in the ferromagnet was proposed in Ref. [12]. The energy of each electron in a pair changes due to the exchange interaction as the Cooper pair transfers through the SF interface to the ferromagnet. The electron with the spin parallel to the exchange field should decrease its energy by $E_{\rm ex}$, while the other electron should increase its energy by the same value. The conservation of the total electron energy occurs due to the corresponding changes in their momenta:

$$|(\hbar k')^2/2m - (\hbar k)^2/2m| = E_{\rm ex}\,.$$

As a result, the total momentum of the pair, Q, is not zero and of the order of $2E_{\rm ex}/v_{\rm F}$ (where $v_{\rm F}$ is the electron Fermi velocity in the ferromagnet). This contribution to the order parameter is determined by the additional multiplier $\exp(\mathrm{i}Qx/\hbar)$. Within the approach used one should consider both the pairs where an electron with the spin parallel to the exchange field has momentum k (then the total pair momentum must be equal to Q) and the pairs where the electron with the same spin direction has the momentum $-k$. In the latter case the pair has the total momentum $-Q$. The combination of the two possibilities gives the following order parameter:

$$\Psi_{\rm F}(x) = \Psi_{\rm F0}\,\frac{\exp(\mathrm{i}Qx/\hbar) + \exp(-\mathrm{i}Qx/\hbar)}{2} = \Psi_{\rm F0}\cos\left(\frac{Qx}{\hbar}\right). \tag{4}$$

In the diffusive limit the order parameter oscillations are superimposed on the decay arising due to pair breaking by impurities in the presence of the exchange field.

Let us consider now the case of a weak exchange energy $E_{\rm ex} \geq k_{\rm B}T$, when the thermal and exchange energy make comparable contribution to the pair decay process. The general expression for the complex coherence length is following:

$$\xi_{\rm F} = \left[\frac{\hbar D}{2(\pi k_{\rm B}T + \mathrm{i}E_{\rm ex})}\right]^{1/2}. \tag{5}$$

Extracting the real and imaginary parts from this expression (for this purpose it is convenient to write

$$\pi k_{\rm B}T + \mathrm{i}E_{\rm ex} = \sqrt{(\pi k_{\rm B}T)^2 + E_{\rm ex}^2}\times\exp\left[\mathrm{i}\arctan\left(\frac{E_{\rm ex}}{\pi k_{\rm B}T}\right)\right],$$

we obtain

$$\xi_{\rm F1,2} = \sqrt{\frac{\hbar D}{[(\pi k_{\rm B}T)^2 + E_{\rm ex}^2]^{1/2} \pm \pi k_{\rm B}T}}\,. \tag{6}$$

This expression transforms to (1) at $E_{\rm ex} \gg k_{\rm B}T$ and to the well-known expression for the coherence length in normal metal at $E_{\rm ex} = 0$. Note that in the case of $E_{\rm ex} \geq k_{\rm B}T$ the decay length $\xi_{\rm F1}$ increases with decreasing temperature, whereas the oscillation wavelength $2\pi\xi_{\rm F2}$ decreases. This allows us to observe the crossover of the SFS junction to the π-state as temperature decreases.

3. π-state of the Josephson SFS junction

Since the oscillating order parameter alternates in the ferromagnet it is expected to have the different signs on superconducting banks of the SFS sandwich, when the thickness d_F of the ferromagnetic layer is close to the half-wave of the oscillations $\pi\xi_{\rm F2}$, i.e. the phase difference across the junction should be equal to π in the absence of any external fields or currents (that does not contradict to the stationary Josephson equation). The Josephson π-junction was first proposed in [1] and detailed for an SFS structure in [2].

To reveal possible distributions of the order parameter in a F layer of a Josephson SFS junction, we have calculated the Ginzburg – Landau (GL) free energy for the ordinary 0- and π-contacts and solved linearized Ginzburg – Landau equations for the order parameter in the F layer. When the axis x is perpendicular to the SF interface, and the origin of the coordinates is in the center of the F layer, the solution for the order parameter $\Psi_{\rm F}(x)$ takes on the form

$$\Psi_{\rm F}(x) = \Psi_{\rm F0}\left\{\frac{\alpha\exp[-(x+d_{\rm F}/2)/\xi_{\rm F1}]\cos[(x+d_{\rm F}/2)/\xi_{\rm F2}]}{\exp(-2d_{\rm F}/\xi_{\rm F1})\cos^2(d_{\rm F}/\xi_{\rm F2}) - 1} + \frac{\beta\exp[(x-d_{\rm F}/2)/\xi_{\rm F1}]\cos[(x-d_{\rm F}/2)/\xi_{\rm F2}]}{\exp(-2d_{\rm F}/\xi_{\rm F1})\cos^2(d_{\rm F}/\xi_{\rm F2}) - 1}\right\}, \tag{7}$$

$$\alpha = \exp(\mathrm{i}\varphi)\exp(-d_{\rm F}/\xi_{\rm F1})\cos(d_{\rm F}/\xi_{\rm F2}) - 1\,,$$
$$\beta = \exp(-d_{\rm F}/\xi_{\rm F1})\cos(d_{\rm F}/\xi_{\rm F2}) - \exp(\mathrm{i}\varphi)\,,$$

where φ is the phase difference across the SFS junction, $\Psi_{\rm F0}$ is the absolute value of the GL order parameter at the SF interfaces, and $\Psi_{\rm F}(-d/2) = \Psi_{\rm F0}$, $\Psi_{\rm F}(d/2) = \Psi_{\rm F0}\exp\mathrm{i}\varphi$.

There are two possible states with the phase difference $\varphi = 0$ and $\varphi = \pi$, which correspond to zero current flow through the junction. We refer them to as '0'- and 'π'-states. In conventional Josephson junctions the ground state corresponds to $\varphi = 0$, since the energy of this state is lower than that for $\varphi = \pi$. However, below we will show that due to the oscillations of the order parameter in the SFS junction, determined by (7), the minimum energy can occur at $\varphi = \pi$. In this case the sign of the order parameter changes over the junction. To determine which state is more favorable at given $d_{\rm F}$ and $\xi_{\rm F1,2}$ we should calculate the GL free energy of the junction. We carried out these calculations in the simplest case $E_{\rm ex} \gg k_{\rm B}T$, when $\xi_{\rm F1} = \xi_{\rm F2}$. But the results obtained are valid qualitatively for other ratios between $E_{\rm ex}$ and $k_{\rm B}T$. In the next Section we will return to the interesting case of $k_{\rm B}T \sim E_{\rm ex}$ to discuss the crossover from the 0- to π-state, which is induced by decreasing the temperature. In the diffusive case at $E_{\rm ex} \gg k_{\rm B}T$ the SFS junction free energy takes on the form:

$$F_{\rm GL} = \int_{-d_{\rm F}/2}^{-d_{\rm F}/2}\left[\Psi_{\rm F}{}^2 + \left(\frac{\mathrm{d}\Psi_{\rm F}}{\mathrm{d}x}\right)^2\right]\mathrm{d}x\,. \tag{8}$$

Substituting the solution (7) for $\xi_{\rm F1} = \xi_{\rm F2}$ in the above relation we found the following expression for the free energy of the 0-state:

$$F_{\rm GL}^0 = -\frac{\exp(-d)\{d\cos d + 2[2 - \exp(-d)\cos d\sin d]\}}{[1 + \exp(-d)\cos d]^2}\,, \tag{9}$$

and respectively for the π-state

$$F_{\rm GL}^{(\pi)} = \frac{\exp(-d)\{d\cos d + 2[2-\exp(-d)\cos d \sin d]\}}{[1-\exp(-d)\cos d]^2}\,, \tag{10}$$

where $d = d_{\rm F}/2\pi\xi_{\rm F2}$.

Figure 1 shows the distributions of the order parameter in the ferromagnetic layer of an SFS junction, calculated using (7) for various ratios d of the thickness $d_{\rm F}$ to the oscillation period $2\pi\xi_{\rm F2}$. The inset shows the dependencies of the GL free energies for the 0- and π-states on the ratio $d_{\rm F}/2\pi\xi_{\rm F2}$ in the SFS junction, calculated by (9) and (10). One can see that the π-state becomes more stable at $d_{\rm F} \simeq \pi\xi_{\rm F2}$, but then it is changed again by the 0-state as $d_{\rm F}$ approaches the integer period of oscillations. The dashed line in Fig.1 indicates the distribution of the order parameter for the 0-state at $d_{\rm F} = \pi\xi_{\rm F2}$, when the energy of the state is higher than that of the π-state. As is seen from the figure, the averaged square of the order parameter gradient for the 0-state is higher than that for the π-state.

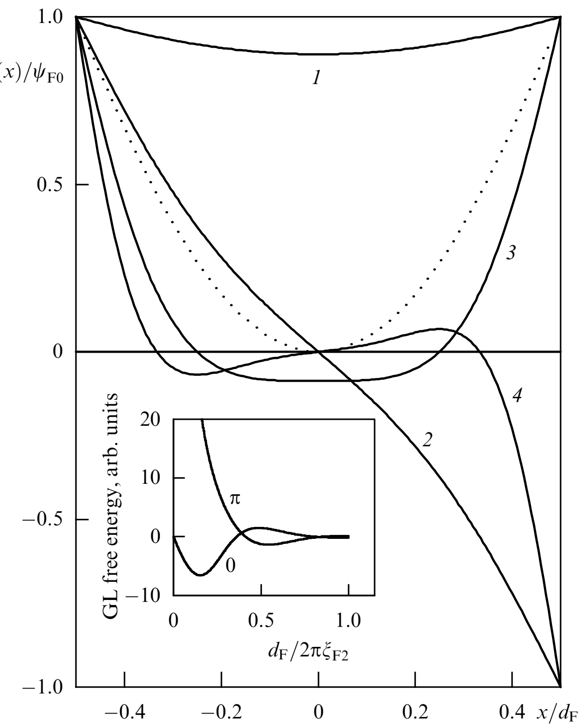

Figure 1. Spatial distribution of the superconducting order parameter in the F layer of the Josephson SFS sandwich, calculated at various ratios $d_{\rm F}/2\pi\xi_{\rm F2}$: for $d_{\rm F}/2\pi\xi_{\rm F2} = 1/2\pi$ and 1 (curves *1* and *3*) the energy minimum corresponds to 0-state, while for $d_{\rm F}/2\pi\xi_{\rm F2} = 1/2$ and 3/2 (curves *2* and *4*) the energy minimum yields π-state. For comparison the dashed line indicates the curve for 0-state at $d_{\rm F}/2\pi\xi_{\rm F2} = 1/2$, whose energy is higher than that of π-state depicted by curve *2*. The inset shows the calculations of the Ginzburg – Landau energy for 0- and π-states.

The crossover to the π-state should manifest itself as an anomalous oscillating F layer thickness dependencies of the critical temperature $T_{\rm c}$ of SF multilayers [15, 16] and the critical current of Josephson SFS sandwiches [9]. Experiments on $T_{\rm c}(d_{\rm F})$ for multilayer structures Nb/Gd [17], Nb/Fe [18], V/Fe [19], and Pb/Fe [20] are not conclusive (see discussion in Ref. [20]), that is related in particular to the fact that various samples are required to study the dependencies on the thickness. The proposed method based on the crossover to the π-state caused by temperature decrease allows us to avoid this difficulty.

4. Reentrant temperature dependence of critical current in an SFS junction: the crossover from 0-state to π-state

The experimental studies of Josephson characteristics of SFS junctions [7, 21, 22] were carried out by us on thin-film sandwiches Nb – $\rm Cu_{1-z}Ni_z$ – Nb, with z about 0.5 and the Curie temperature, $T_{\rm m}$, of the copper – nickel layer in the range 20 – 150 K. The weak ferromagnetism of the Cu/Ni alloy was important for preparing homogeneous and continuous interlayers with a thickness comparable with the decay length $\xi_{\rm F1}$ of the order of dozens nanometers. Conventional ferromagnetic metals (Co, Fe, Ni) can hardly be used for fabrication of the sufficiently uniform F layers, because the pair decay length in this case is close to 1 nm. The use of ferromagnetic alloys with low Curie temperatures allowed us to increase the pair decay length by an order, which made possible flowing of supercurrents through F layers of 20 – 30 nm in thickness, prepared with the roughness of 2 – 3 nm. Another important result of the application of the alloys with low $T_{\rm m}$ is the achievement of the $E_{\rm ex} \sim k_{\rm B}T$ limit, which allows us to observe the crossover of the Josephson SFS junction to π-state at decreased temperature.

Figure 2a shows the schematic cross-section of a thin-film Nb – $\rm Cu_{1-z}Ni_z$ – Nb sandwich. The bottom Nb electrode 100 μm wide and 110 nm thick, was fabricated by dc magnetron sputtering with subsequent photolithography and chemical etching. The deposition of copper – nickel film was carried out by rf sputtering after ion etching of the niobium surface (targets of various contents with z varying from 0.4 to 0.57 were used). Then the insulating layer with 50× 50 μm^2 'window' determining the junction area was prepared by lift-off photolithography. We used 170 nm thick SiO film as insulator, which was deposited by vacuum evaporation. The fabrication procedure was accomplished by dc magnetron sputtering of the upper niobium electrode, 80 μm wide and 240 nm thick, after preliminary ion cleaning of the surface of the copper – nickel layer. The upper electrode patterning was made by lift-off photolithography. The junction normal resistance $R_{\rm n}$ did not exceed 10^{-5} Ω, so the transport characteristics of the junctions were measured by a picovoltmeter based on a rf SQUID with sensitivity better than 10^{-11} V.

The onset of ferromagnetism for $\rm Cu_{1-z}Ni_z$ alloy is around $z = 0.44$. Upon crossing to the ferromagnetic regime of the F layer the junction critical currents dropped sharply but $I-V$ characteristics and magnetic field dependencies $I_{\rm c}(H)$ (H in the plane of the junction) were still similar [7, 22] to those for standard Josephson SNS junctions. The $I_{\rm c}(H)$ patterns for 'fresh' or well demagnetized samples were described by the Fraunhofer relation with high accuracy, which indicates the fact of high uniformity of the thickness and the magnetic properties of the F layer along the junction. The latter seems to be caused by averaging of a small-scale structure of magnetic domains in the F layer resulting in highly uniform current flow through the ferromagnetic layer with zero net magnetization. The main peak of the Fraunhofer pattern was found shifted when samples were heated above $T_{\rm c}$ (but below $T_{\rm m}$) and a small magnetic field was briefly applied leading to nonuniformity of the domain structure and

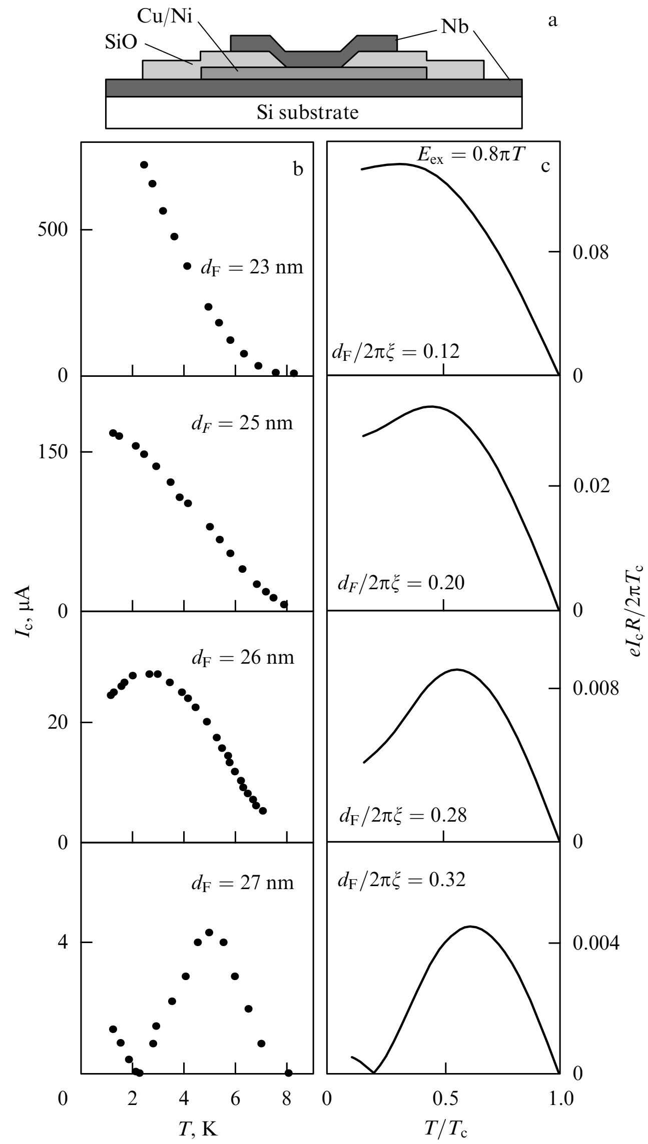

Figure 2. (a) Schematic picture of the cross-section of the SFS sandwich; (b) temperature dependencies of the critical current in Nb – $Cu_{0.48}Ni_{0.52}$ – Nb sandwiches with thickness of the F layer varied from 23 to 27 nm; (c) calculations of the temperature dependencies of the critical current in the SFS junctions at $E_{ex} = 0.8\pi T_c$ and various values of the ratio $d_F/2\pi\xi^*$, where $\xi^* = (\hbar D/2\pi k_B T_c)^{1/2}$.

residual macroscopic magnetic induction [7]. In non-magnetized samples, which will be considered below, the residual magnetic induction was not observed in the whole temperature range 1.2 – 9 K, the latter was indicated by zero field position of the main peak of the Fraunhofer pattern [21, 22].

Figure 2 shows experimentally measured (b) and calculated (c) dependencies $I_c(T)$ at various thickness d_F of the sandwich ferromagnetic interlayer from $Cu_{0.48}Ni_{0.52}$ alloys with parameters close to the crossover from the 0- to π-state. Within the narrow range of thickness variation (4 nm for the averaged thickness of the F layer) the shape of the curves changed from that of typical for SNS junction (which rises sharply with positive curvature as temperature decreases) to the reentrant oscillating dependence vanishing at $T = T_{cr} \simeq 2$ K, which had not been observed earlier in experiments with Josephson junctions. Such behavior should result from the crossover to the π-state at $T < T_{cr}$. In the crossover point, the critical current $I_c(T)$ is formally equal to zero and then should become negative. Since in real experiments we could measure only positive values of the critical current, the dependence $I_c(T)$ has a sharp cusp at $T = T_{cr}$ that is the negative branch of the curve reflected to the positive region. To investigate quantitatively this effect, we extended a theory [16], which was initially used for large thickness of F layers in SFS junctions, to the case of arbitrary thickness and small exchange energy $E_{ex} \sim k_B T_c$. Apart from it we considered barriers at SF interfaces, which are inevitably arisen due to splitting of spin subbands caused by exchange interactions. Since the thin-film SFS sandwich corresponds to the diffusive limit, we used the formalism of quasiclassical Usadel equations [23], where the Josephson junction critical current was determined by summation over Matsubura frequencies $\omega_n = \pi T(2n+1)$ of the corresponding spectrum of supercurrents $J_s(\omega_n)$. The spectrum is expressed via Usadel function $F(\pm\omega_n)$ with the typical spatial scale $\xi_{\omega_n} = [D/2(\omega_n + iE_{ex})]^{1/2}$ of changes of superconducting properties in the ferromagnet.

Usadel equation for the F layer is following [15, 16]

$$\frac{\hbar D}{2}\frac{d}{dx}\left[G(x,\tilde{\omega})\frac{d}{dx}F(x,\tilde{\omega}) - F(x,\tilde{\omega})\frac{d}{dx}G(x,\tilde{\omega})\right] = \tilde{\omega}F(x,\tilde{\omega})\,, \tag{11}$$

$$G^2(x,\tilde{\omega}) + F(x,\tilde{\omega})\tilde{F}(x,\tilde{\omega}) = 1\,, \tag{12}$$

where $\tilde{\omega} = \omega_n + iE_{ex}$ and $\tilde{F}(x,\omega_n) = F^*(x,-\omega_n)$.

The superconducting current J_s through SFS junctions is expressed as

$$J_s(\varphi) = ieN(0)D\pi T\sum_{-\infty}^{\infty}\left(F\frac{d}{dx}\tilde{F} - \tilde{F}\frac{d}{dx}F\right). \tag{13}$$

To find the solution for the Green's functions $F(x,\tilde{\omega})$, $G(x,\tilde{\omega})$ in the F layer, we took into account the potential barrier at SF interfaces, using the boundary conditions obtained in [24] for the SN case. According to [24], the interface barrier is described by the parameter

$$\Gamma_b = \frac{2}{3}\frac{l_F}{d_F}\left\langle\frac{xD_b}{1-D_b}\right\rangle^{-1},$$

where l_F is the electron mean free path in the F layer, D_b is the barrier transmission coefficient for electrons. The parameter Γ_b determines the ratio of the resistance of the interface to that of the F layer. In the practically interesting case $\Gamma_b \gg 1$, the functions $F(x,\tilde{\omega})$ are small and the Usadel equations can be linearized and solved analytically. As a result, we obtained the sinusoidal current – phase relation and the following expression for the critical current:

$$I_c = \frac{2\pi T}{eR_N}\frac{1}{\Gamma_b}\mathrm{Re}\left[\sum_{\omega_n>0}\frac{\Delta_0^2}{(\omega_n^2 + \Delta_0^2)\beta\sinh 2\beta}\right], \tag{14}$$

$$\beta = \frac{d_F}{2\xi_{\omega_n}} = d_F\sqrt{\frac{\omega_n + iE_{ex}}{2\hbar D}}\,. \tag{15}$$

Here Δ_0 is the order parameter in the bulk superconductor, R_N is the normal resistance of the SFS junction. This approximation is valid for sufficiently high temperatures when $\pi T/T_c \gg 1/\Gamma_b$. In this case the temperature dependence of I_c is not sensitive to the value of Γ_b and only the magnitude of characteristic voltage $I_c R_N$ changes as this parameter rises. Numerical calculations based on this model at various values of $d_F/2\pi\xi^*$ (where $\xi^* = (\hbar D/2\pi k_B T_c)^{1/2}$ and $\Gamma_b = 10$ are shown in Fig. 2c. They are in qualitative agreement with the experimental data and by and large confirm the simple model of the $0-\pi$ crossover induced by decreasing the temperature, which is presented in Section 2. To compare them with the experimental data we reflected negative branches of dependencies $I_c(T)$ to the positive region. Thus, the Josephson coupling energy can really change the sign as temperature decreases for the case of $E_{ex} \sim k_B T$ and $d_F \sim \pi\xi_{F2}$.

5. Triangular arrays of SFS junctions. Direct observation of the π-shift

Several experimental phase-sensitive methods for detecting the π-state crossover were suggested. The original signature of the π-state is the appearance of a spontaneous magnetic flux of half-flux quantum in a superconducting loop containing a π-contact. Josephson π-junction was first predicted in Ref. [1] and it was shown [1] that the state with a spontaneous flux could be realized in such a one-contact interferometer only if $2\pi L I_c \gg \Phi_0$, where L is the loop inductance and I_c is the contact critical current. The other limit, $2\pi L I_c \ll \Phi_0$, can be used to observe the shift of the external flux dependence of the transport critical current $I_M(\Phi)$ for a two-contact interferometer including Josephson 0- and π-junctions. The presence of the phase difference of π across the π-junction results in the circulating supercurrent close to the critical one and the extra phase shift of $\pi/2$ in both 0- and π-junctions, hence the necessary additional π phase incursion appears in the superconducting loop. So the critical transport current I_M through this interferometer (two-contact SQUID) is equal to zero in the absence of any external magnetic flux and reaches the maximum $I_M = 2I_c$ when the external magnetic flux is equal to $\Phi_0/2$. In the latter case it is the external magnetic flux that induces the additional phase shift of $2\pi\Phi/\Phi_0 = \pi$ which compensates the spontaneous π-shift over the π-junction. Thus the '$0-\pi$ interferometer' initially is in a fully frustrated state. Fabrication of 0- and π-junctions with close Josephson parameters was a rather difficult technical problem in our case. Therefore we used thin-film periodical arrays with three identical SFS junctions per cell. The two-cell array, shown in Fig. 3, is believed to be the most simple and plain structure to study the intrinsically frustrated arrays. The $Cu_{0.46}Ni_{0.54}$ alloy with T_m about 100 K was used as an F layer in the SFS contacts. The junctions with $d_F = 19$ nm showed the crossover to the π-state at $T_{cr} = 2.2$ K (see Fig. 4). Above this temperature (0-state) the $I_M(H)$ pattern was the same as the one predicted [25, 26] for a two-cell interferometer. Periodical maximal peaks were observed at external fields corresponding to an integer number of the flux quanta per cell, i.e. integer frustration parameters $f = \Phi/\Phi_0$. The small peaks at half-integer frustration parameters corresponded to the quantization on the contour of the net structure which is twice the cell square. Below the T_{cr} (in the π-state) the $I_M(H)$ pattern was found to be shifted exactly by half a period. Without considering the quantization process for the doubled cell square (that resulted in the appearance of the small peaks), the behavior of this π-junction interferometer resembled that of the $0-\pi$ SQUID described above. At $f = 0$ and other integer values of f there was a current close to the junctions critical current flowing in the outer loop of the array. The current induced the extra phase shift $2 \times \pi/2$ in each cell and compensated the odd number of the π-shifts in each cell. The maximal array transport supercurrent was close to zero, because the array was initially in the spontaneous fully frustrated state. The external magnetic flux equal to half-integer quanta per cell produced the necessary phase shift of π in each cell in the absence of any circular currents in the

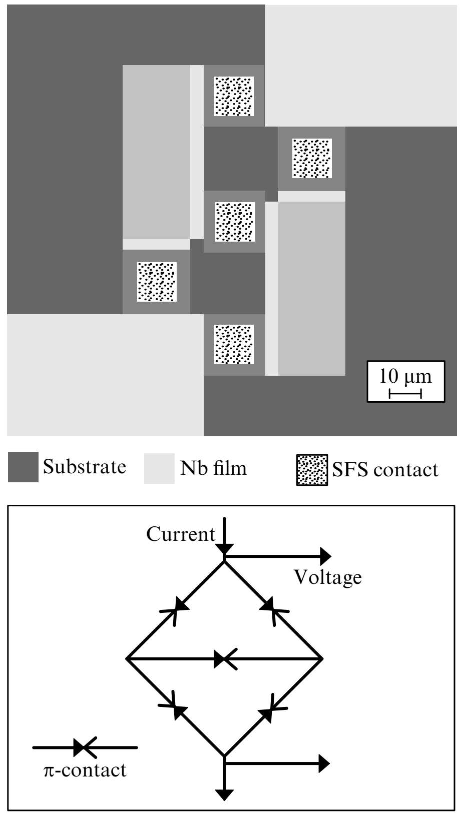

Figure 3. Real (top) and schematic (bottom) picture of the network with five SFS junctions Nb – $Cu_{0.46}Ni_{0.54}$ – Nb ($d_F = 19$ nm), which was used in the phase-sensitive experiments.

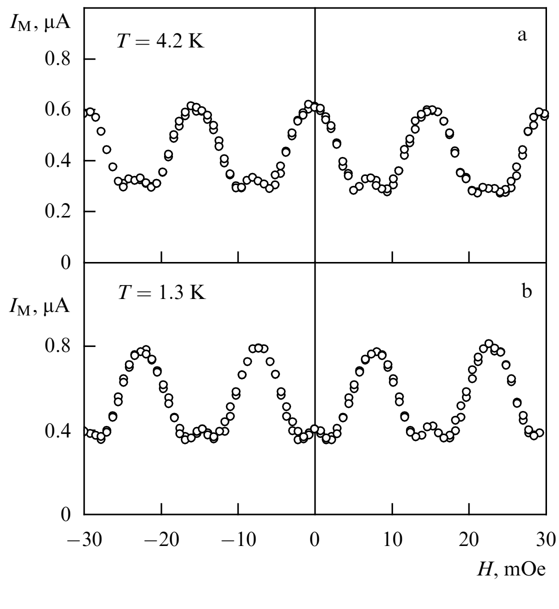

Figure 4. Dependence of the critical transport current of the structure depicted in Fig. 3 on the applied magnetic field at temperature above (a) and below (b) T_{cr}.

structure, therefore I_M reached maxima only at half-integer frustration parameters. The $I_M(T)$ dependence at $H = 0$, shown in Fig. 5, is similar to the $I_c(T)$ dependence for single junctions and demonstrates the sharp cusp at the temperature T_{cr} of the SFS junctions crossover to the π-state. However, one should take into account that while the high-temperature branch corresponds to the temperature dependence of the doubled critical current for the single junction, the low-temperature branch corresponds to the small peak amplitude dependence on temperature. Fig. 5b shows the maximal peaks positions before and after the crossover and demonstrates how abrupt it is.

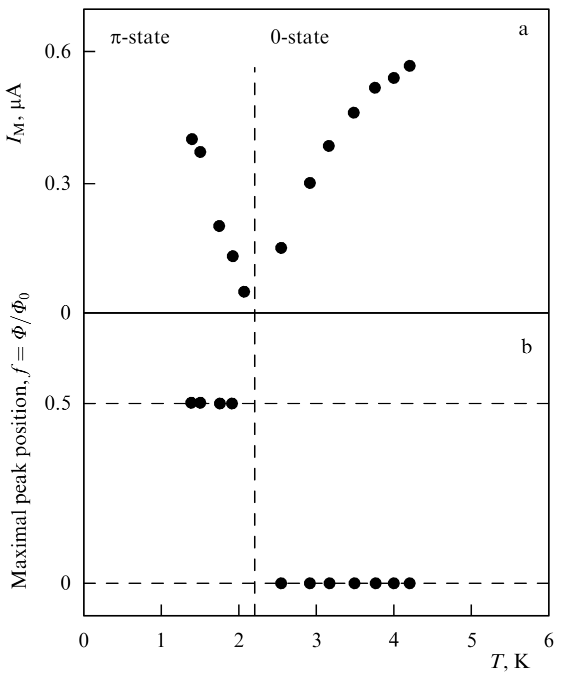

Figure 5. (a) Temperature dependence of the critical transport current for the structure depicted in Fig. 3 in the absence of magnetic field; (b) temperature dependence (jump) of the position of the maximal peak on the curves $I_M(H)$, corresponding to the two limiting temperatures noted in Fig. 4.

6. Conclusions

In this work, the peculiarities of the Josephson behavior of SFS sandwiches and superconducting 2D arrays based on these SFS junctions were experimentally and theoretically studied. It was found that the thin-film $Nb-Cu_{1-z}Ni_z-Nb$ sandwiches at definite values of Ni concentrations close to 50% and at specific values of the ferromagnet layer thickness, d_F, revealed the crossover to the Josephson π-state with decreasing temperature. These SFS sandwiches showed the reentrant oscillating behavior of temperature dependence of the critical current with vanishing amplitude at $T = T_{cr}$. The crossover to the π-state also manifested itself in the half-period shift of the external magnetic field dependence of the transport critical current in the triangular SFS arrays. This shift is associated with the appearance of spontaneous supercurrents in the array even in case of zero external field, with the ground state degenerated with respect to the two possible current flow directions. In conventional Josephson arrays such a state can be observed only by imposing the frustrating external field equal to a half-integer number of magnetic quanta per cell. Self-frustrated superconducting networks with π-junctions may be used for the realization of the superconducting quantum bit (qubit) as the basic element of hypothetical 'quantum computer', which produces calculations using quantum algorithms. Originally, the suggested superconducting 'phase' qubits [27] were based on magnetic-field frustrated superconducting networks. In this case it is necessary to imply half a quantum of the magnetic flux to create a degenerated two-level coherent quantum system, requested for the realization of the qubit. This system is not isolated from interference with environment and is estimated to have shorter coherence time with respect to the qubit using a π-junction [10]. Another possible application of a π-junction is related to the development of superconducting digital 'complementary' electronics where a π-junction is used as a superconducting phase inverter [11].

The authors would like to thank M V Feĭgel'man for the helpful discussion and N S Stepakov for the technical assistance. The work was supported by the State Program "Mesoscopic Electron Systems and Actual Fields in Condensed Matter Physics", RFBR (Grant 01-02-17581), and partially by Swiss National Research Foundation under project NFSF N7SUPJ062253.00.

References

1. Bulaevskiĭ L N, Kuziĭ V V, Sobyanin A A *Pis'ma Zh. Eksp. Teor. Fiz.* **25** 314 (1977) [*JETP Lett.* **25** 290 (1977)]
2. Andreev A V, Buzdin A I, Osgood R M *Phys. Rev. B* **43** 10124 (1991)
3. Tsuei C C et al. *Phys. Rev. Lett.* **73** 593 (1994)
4. Wollman D A et al. *Phys. Rev. Lett.* **74** 797 (1995); Van Harlingen D J *Rev. Mod. Phys.* **67** 515 (1995)
5. Baselmans J A et al. *Nature* **397** 43 (1999)
6. Avenel O, Mukharsky Yu, Varoquaux E *Nature* **397** 484 (1999)
7. Ryazanov V V *Usp. Fiz. Nauk* **169** 920 (1999) [*Phys. Usp.* **42** 825 (1999)]
8. Wollman D A et al. *Phys. Rev. Lett.* **71** 2134 (1993)
9. Buzdin A I, Bulaevskiĭ L N, Panyukov S V *Pis'ma Zh. Eksp. Teor. Fiz.* **35** 147 (1982) [*JETP Lett.* **35** 178 (1982)]
10. Ioffe L B et al. *Nature* **398** 679 (1999); Blatter G, Geshkenbein V B, Ioffe L B *Phys. Rev. B* **63** 174511 (2001)
11. Terzioglu E, Beasley M R *IEEE Trans. Appl. Supercond.* **8** 48 (1998)
12. Demler E A, Arnold G B, Beasley M R *Phys. Rev. B* **55** 15174 (1997)
13. Larkin A I, Ovchinnikov Yu N *Zh. Eksp. Teor. Fiz.* **47** 1136 (1964) [*Sov. Phys. JETP* **20** 745 (1965)]
14. Fulde P, Ferrel R A *Phys. Rev.* **135** A550 (1964)
15. Radovic Z et al. *Phys. Rev. B* **44** 759 (1991)
16. Buzdin A I, Vujicic B, Kupriyanov M Yu *Zh. Eksp. Teor. Fiz.* **101** 231 (1992) [*Sov. Phys. JETP* **74** 124 (1992)]
17. Jiang J S et al. *Phys. Rev. Lett.* **74** 314 (1995)
18 Mühge Th et al. *Phys. Rev. Lett.* **77** 1857 (1996)
19. Aarts J et al. *Phys. Rev. B* **56** 2779 (1997)
20. Lazar L et al. *Phys. Rev. B* **61** 3711 (2000)
21. Veretennikov A V et al. *Physica B* **284–288** 495 (2000)
22. Ryazanov V V et al. *Phys. Rev. Lett.* **86** 2427 (2001)
23. Usadel K D *Phys. Rev. Lett.* **25** 507 (1970)
24. Kupriyanov M Yu, Lukichev V F *Zh. Eksp. Teor. Fiz.* **94** 139 (1988) [*Sov. Phys. JETP* **67** 1163 (1988)]
25. Tsang Von-Tien, van Duzer T *J. Appl. Phys.* **46** 4573 (1975)
26. Likharev K K *Vvedenie v Dinamiku Dzhozefsonovskikh Kontaktov* (Introduction to Dynamics of Josephson's Junctions) Ch. 8 (Moscow: Nauka, 1985); *Dynamics of Josephson Junctions and Circuits* (New York: Gordon and Breach Sci. Publ., 1986)
27. Mooij J E et al. *Science* **285** 1036 (1999)