4. Conclusions

In summary, we have probed the proximity effect in two different systems. First, we have investigated the LDOS of a semi-infinite normal metal and found space-dependent energy spectra as a function of the distance to the NS interface. This behavior is in good agreement with the pseudo-gap model predicted by the theory of non-equilibrium superconductivity. But STM techniques allow also to address much smaller scales. Indeed, we have simultaneously investigated the local density of states in a confined geometry. In this case we have found a spectral structure which does not vary in space and which can be related to the Thouless energy. Similar experiments are under way at very low temperature in a dilution refrigerator. This should give more information on the temperature dependence of the scattering rates in the infinite case and on the value of the minigap in the finite one.

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Quantum entangled states and reduction of the wave packet

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Abstract. The paper deals with entangled electron states arising in a normal conductor located near a superconductor. Such paired entangled states can be treated as decomposed Cooper pairs. Some general problems of the theory of measurements are also considered.

1. Introduction

In recent years quantum entangled states have attracted considerable interest of experimenters and theoreticians, for which there are several reasons. On the one hand, the states can be used in quantum cryptography and in quantum computers. On the other hand, they are related to fundamental problems of the theory of measurements and to the possibility to verify the existence of hidden variables and nonlocal character of quantum mechanics, etc.

The entangled states of two (or more) particles can be defined as follows: the states are referred to as entangled if two-particle probabilities describing the system do not reduce to the product of the corresponding one-particle probabilities.

Let us consider, for example, the system of two spins, whose absolute value is equal to 1/2. When $P_{+-} \neq P_+ P_-$, the states are entangled. In particular, this means that the measurement of one spin affects *apriori* the probability of another spin.

The effect is especially surprising when two spins are spaced, and the expected time of measurements is much less than the time during which a light signal goes from one spin to another. It is just this phenomenon that has led to the conclusion about nonlocal character of quantum mechanics.

A gedanken experiment of this type was first considered by Einstein, Podolsky, and Rosen (EPR). As for real experiments with photons, they have been carried out only recently.

Some theoretical schemes for obtaining entangled electron states have been suggested this year [1, 2].

In Reference [2], an experiment with electron in the NS system was proposed. The main concept is simple and based on the use of Cooper pairs emitted by a superconductor into a normal metal in the form of EPR pairs. The superconductor is connected with two normal conducting wires. One or two electrons can be emitted into each conducting wire. To split the electron pair along 'arms' and to avoid entering of both the electrons into the same load, the contacts contain filters. The electrons in a pair are correlated by two variables, i.e., by the kinetic energy and the spin so that the energy of one electron is slightly higher and that of the other one is slightly less than the Fermi energy, while the electron spins are opposite (at s-pairing). Therefore, the filters can treat either the difference in kinetic energy or the difference in spins. In the first case, the electrons can be splitted with interferometers based on quantum dots, while in the second case — with interferometers based on ferromagnetic contacts. (In Ref. [3] it was suggested to use quantum dots, whose sizes are rather small to prevent tunneling of both the electrons due to high value of the Coulomb energy, as filters.) In order to detect the entanglement, it is proposed to study the correlators of the number of electrons entering each arm. The correlator of the number of electrons recorded during a long period t is expressed in terms of current correlators at zero frequency as $\langle\langle N_1 N_2 \rangle\rangle = t \langle\langle I_1 I_2 \rangle\rangle$.

Thus, the experimental task is to measure current correlator at low frequency. In the case of ideal filters the correlators are positive and their absolute values are equal to autocorrelators in each contact: $\langle\langle I_1 I_2 \rangle\rangle = \langle\langle I_2 I_2 \rangle\rangle = \langle\langle I_1 I_1 \rangle\rangle$.

Note that the entangled states can be different. In particular, the singlet state is entangled at any orientation of the axis of measurements, while for the triplet state there is a direction at which the measurements yield single-valued data. In our scheme with ferromagnetic filters, the effect could be checked by varying the polarization of ferromagnetics.

2. Measurements and reduction of the wave packet

Most of physicists believe that the EPR experiment implies that quantum mechanics is not local. The experiments on photons are considered to prove this standpoint, although some recent publications point to several logic loop-holes in these experiments, if we treat them as evidence in favor of the absence of hidden variables. The key qualitative effect being verified in this case is the so-called Bell inequality.

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Here I will briefly consider the process of measurements, which implies that the local nature is caused by the local origin of laws of quantum mechanics, in particular, the Schrödinger equation rather than by the presence of hidden variables.

The experiments carried out so far seem to be consistent with this standpoint. Pathos of the theory of hidden variables, which was proposed by Bohm, has been to reconcile the probabilistic interpretation of quantum mechanics, revealed in practice and an intuitive desire to know the cause of a particular experimental result. The key question can sound as: who decides where the electron will move — to the right or to the left? The possible answer is "the reservoir" (more exactly, "the reservoir, as a rule," see below). Therefore, I suppose that during the measurement the wave function of the particle, which is entangled by the degrees of freedom of the detector, evolves so that a certain wave function corresponds to a certain time interval of the measurements and the result of the measurement is single-valued if the initial wave function of the detector is given. This version is partly supported by the known results on the damping of non-diagonal coefficients of the density matrix, for example, for a particle coupled with a reservoir. Let us consider the question in detail and suppose the initial density matrix to be a product of the density matrix of the particle in a pure state $\rho_0(x, x') = \phi(x)\phi^*(x)$ and a density matrix of the reservoir, which is considered, for simplicity, to be diagonal in a $\{\beta\}$ set:

$$\rho_0(\alpha, \alpha') = \sum a_{\beta,\beta}\psi_\beta(\alpha)\psi^*_\beta(\alpha') .$$

For definiteness let us assume the particle to be in a two-well potential, and besides let us be interested in the presence of the particle in a well rather than in the exact coordinate of the particle. After a long time (and by no means instantaneous) the nondiagonal elements of the density matrix approach zero:

$$\rho_t(1, 2) = \sum_{\alpha,\beta} a_{\beta,\beta}\Psi_\beta(1, \alpha)\Psi^*_\beta(2, \alpha) = 0 . \tag{1}$$

Formally, such a damping can occur in various ways:

(i) The wave function localizes as a function of the particle coordinate x.

(ii) The states of Schrödinger's cat arise, i.e. superposition of localized states.

(iii) The wave function does not localize, but the phase difference of the wave function at $x = 1, 2$ depends strongly on the set of variables α, β, the latter results only in an infinitesimal contribution after summing-up.

We hold the first viewpoint. At least, the everyday observations of classical objects evidence in favor of the first way. We could validate this point of view if we prove the equality

$$\sum_{\alpha,\alpha',\beta} |\Psi_\beta(1, \alpha)|^2\, |\Psi^*_\beta(2, \alpha')|^2 = 0 . \tag{2}$$

The summation can certainly be carried out with a certain weight, for example, specified by a density matrix of the reservoir. In this case the states like Schrödinger's cat (SC) are excluded. Or more precisely, we could hold that the measure of the subspace of initial states resulting in the SC is equal to zero due to averaging over the initial states. In this sense, the situation is similar to classical tossing of a coin: the probability to chose the initial conditions such that the coin falls on the edge is infinitesimal. The localization of the wave function of a quasiclassical object interacting with a reservoir of soft modes is the important phenomenon relating quantum physics to the classical one. Nevertheless, at present there is no strong evidence in favor of this view. The above standpoint can more adequately be confirmed by direct calculations describing the evolution of the wave function of a particle and (detector) reservoir rather than the density matrix or averages like (2). However, this is not a simple problem. In fact, we have to determine the results of measurements as a function of a great number of variables describing the detector (reservoir).

Let us now discuss another intricate question of the theory of measurements, i.e., the rate of reduction of the wave packet. As is indicated above, the problem is most clear in the EPR experiment. Nevertheless, the EPR experiment does not differ in principle from the usual measurement of the coordinate of a particle. If the time required to measure is considered to be finite and independent of the type of wave packets, the wave packet reduces at the rate exceeding the speed of light in both the cases. Broadly speaking, this fact does not contradict to the relativistic invariance and does not allow us to transfer information faster than the speed of light. Nevertheless, in this case, too, it is necessary to demonstrate how it happens in fact. By analogy with Eqn (2), the time-difference correlator

$$\sum_{\alpha,\alpha',\beta} |\Psi_\beta(t_1, 1, \alpha)|^2\, |\Psi^*_\beta(t_2, 2, \alpha')|^2$$

should be studied.

The reduction of the wave packet can be considered as a tunneling process. Such an analogy is particularly appealing for the case of splitting of the wave packet impacting through a one-dimensional conductor on the junction with two other conductors. Assume that the particle that has passed can be recorded in a conductor with a detector located far from the junction. If the particle can be recorded in one detector it can be recorded in the other one only due to a malfunction. In the course of entangling of the degrees of freedom of the first detector with those of the particle, the wave packet near the second detector should reduce with respect to its absolute value and deform. Nevertheless, the deformation cannot result in the collapse of the wave packet near the first detector; it contradicts to dynamics of propagation and the probability of the measurements obtained in a conventional way. It remains to believe that the tunneling is a process when the wave packet does not arise in any intermediate positions, but simply disappears in one of the channels.

Now let us return to the problem of description of measurements as a unitary evolution of the complex including the particle and the detector. Above I have remarked that the result of a concrete measurement is determined by the reservoir, as a rule. The addition of 'as a rule' is caused by the reluctance to sign to the manifest a man is a quantum computer (robot), a part of a large quantum computer like 'our Metagalaxy'. Indeed, if we believe that everything including the process of measurements with inherent probability of the outcome is determined unambiguously by initial conditions and an ensuring unitary evolution, we should make just this conclusion. A man seems to be a computer in part, however, I hope he does more than that. A real construction of high-performance devices (quantum computers) evolving unitary may allow us to reveal the measure of a known physical system to be unitary. If the Universe is not only a computer, there must be a phenomenon, which could be experimentally detected as a weak inherent phase malfunction. The effect is most probably so weak that it could hardly be distinguished among usual causes of the malfunction, but the experiment is to answer this question.

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Proximity Action theory of superconductive nanostructures

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Abstract. We review a novel approach to the superconductive proximity effect in disordered normal–superconducting (N-S) structures. The method is based on the multicharge Keldysh action and is suitable for the treatment of interaction and fluctuation effects. As an application of the formalism, we study the subgap conductance and noise in two-dimensional N-S systems in the presence of the electron–electron interaction in the Cooper channel. It is shown that singular nature of the interaction correction at large scales leads to a nonmonotonuos temperature, voltage and magnetic field dependence of the Andreev conductance.

1. Introduction

A superconductor in contact with a normal metal induces Cooper correlations between electrons in the normal region, the phenomenon known as the proximity effect. Its microscopic origin lies in Andreev reflection [1] of an electron into a hole at the normal metal–superconducting interface. The probability of Andreev reflection and thus the strength of the proximity effect is determined by the transparency of the N-S interface and the nature of electron propagation in the N part of the structure. Disorder in the normal conductor near the N-S contact was shown theoretically [2–5] to increase considerably the effective probability of Andreev reflection (see Ref. [6] for a recent review from the experimental viewpoint).

The standard semiclassical theory of N-S conductivity [2–5], based either on the traditional nonequilibrium superconductivity approach [7] or on the scattering formalism [5, 8], usually neglects interaction effects in the N part of the structure. However, in low-dimensional structures, Coulomb interaction in the normal diffusive region gets enhanced [9], which may affect strongly the Andreev conductance and noise.

In this paper we address the effect of interaction between electrons in the normal part of an N-S structure on the charge transport through the system. To study a system with interaction a novel theoretical method should be developed since neither of the above-mentioned approaches can handle interaction corrections. Indeed, the scattering matrix formalism relying on the linear relation between the outgoing and incoming states is *a priori* a one-particle description. On the other hand, Larkin–Ovchinnikov kinetic equation [7] can be generalized to allow for (at least some part of) interaction corrections, but its practical solution seems hardly possible beyond the first order of perturbation theory in interaction strength [10, 11].

An appropriate formalism has been developed in Ref. [12] in the framework of the Keldysh action for disordered superonductors [13]. We start from the fully microscopic Lagrangian describing interacting electrons in the diffusive conductor. Then, successively integrating over electronic degrees of freedom in the normal conductor we end up with the *Proximity Action*, $S_{\text{prox}}[Q_S, Q_N]$, which is a functional of two matrices, Q_S and Q_N, describing the states of the superconductive and external normal terminals of the N-S structure (cf. Fig. 1). Once the form of the Proximity Action is known, one can easily calculate the conductivity of the system, current noise, and, in principle, higher correlators of current and even the full statistics of transmitted charge [14, 15]. The Proximity Action approach bears an obvious analogy with the scattering matrix approach [3, 5] as both describe transport properties in terms of the characteristics of the terminals (stationary-state Green functions of the terminals $Q_{S,N}$ in the former versus asymptotic scattering states in the latter approach). In this respect, the Proximity Action method also shares the logic of Nazarov's circuit theory of Andreev conductance [4]. On the other hand, the Keldysh action approach is a natural generalization of the kinetic equation for dirty superconductors in the case of fluctuating fields. The Larkin–Ovchinnikov kinetic equation then emerges as a saddle point equation for the Keldysh action [13].

As an application of the formalism, we will study charge transport in two-dimensional (2D) N-I-S structures shown in Fig. 1 at low (compared to the S gap Δ) temperature and voltages, and arbitrary ratio $t = R_D/R_T$, where R_D and R_T are the resistances of the diffusive normal conductor, and of the tunnel barrier in the normal state, correspondingly. We will calculate the Andreev conductance and noise of such systems in the presence of Cooper interaction in the normal conductor modified by the Coulomb interaction [16–18], as a function of the 'decoherence time' of an electron and the Andreev-reflected hole, $\hbar/\Omega_*$, where

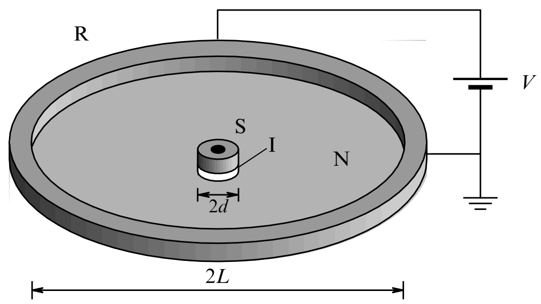

Figure 1. A small superconductive island (S) of size $2d$ connected to a reservoir (R) through a tunnel barrier (I) and a dirty normal film (N) of size $2L \gg 2d$.

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