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Accurate frequency and time dissemination in the optical domain

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Contents

Abstract. The development of the optical frequency comb technique has enabled a wide use of atomic optical clocks by allowing frequency conversion from the optical to the radio frequency range. Today, the fractional instability of such clocks has reached the record eighteen-digit level, two orders of magnitude better than for cesium fountains representing the primary frequency standard. This is paralleled by the development of techniques for transferring accurate time and optical frequency signals, including fiber links. With this technology, the fractional instability of transferred frequency can be lowered to below 10^{-18} with an averaging time of 1000 s for a 1000 km optical link. At a distance of 500 km, a time signal uncertainty of 250 ps has been achieved. Optical links allow comparing optical clocks and creating a synchronized time and frequency standard network at a new level of precision. Prospects for solving new problems arise, including the determination of the gravitational potential, the measurement of the continental Sagnac effect, and precise tests of fundamental theories.

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1. Introduction

Currently, one of the most important tasks for science is the formation of accurate time and frequency signals. Navigation technologies, geodesic measurements, global communication networks, or high-speed data transmission channels used worldwide are based on high-precision time and frequency signals. There are numerous applications where accurate measurements of time and frequency are needed: global space navigation, very-long-baseline interferometers, determination of fundamental constants, development of new metrology standards for physical quantities, and verification of fundamental physical theories.

The appearance of femtosecond frequency comb synthesizers stimulated the development of optical domain metrology ($10^{14} - 10^{15}$ Hz) [1]. Progress in laser cooling methods, laser stabilization, and optical frequency measurement led to the development of optical clocks with a fractional instability reaching 10^{-18} for a 1000 s averaging time [2, 3]. The stability and reproducibility of the optical clock frequency overcame the characteristics of the best microwave standards (cesium fountains) by two orders of magnitude, which pushed optical clocks to the leading positions.

Wide practical applications of high-precision optical frequency standards are limited due to the difficulties in transmission and comparison and propagation of frequency and time signals without losses. Currently, the main method that provides the highest accuracy of frequency comparison in the radio frequency (RF) range is the two-way satellite time and frequency transfer (TWSTFT) [4], which provides a fractional instability of the transmission at the level of 10^{-15} per one-day averaging time. This is sufficient for comparing most microwave clocks, but is unacceptable for optical frequency standards. For example, one of the new applications of optical clocks is related to gravimetry [5], which uses a gravity-induced frequency shift. To achieve a 1 cm accuracy for a height measurement, two clocks must be compared with a fractional inaccuracy of 10^{-18}, while the distance between them can reach several thousand kilometers.

Currently, there are plans to create a Global Geodetic Observing System (GGOS) [6], which would allow studying and predicting phenomena such as earthquakes and tsunamis on a global scale. Geodetic observatories, which are located all around the world, also need accurate reference in time and frequency. One essential condition for the GGOS operation is the existence of a common reference height level, which would not be affected by tidal, geological, or atmospheric processes. This means that reference atomic clocks are needed that would define the zero level of the gravitational potential. For example, such clocks can be located in Earth's orbit. A network of ground-based optical clocks — either mobile or connected with optical fibers — would then link and synchronize geodetic laboratories around the world with an uncertainty of the order of 10^{-18}. To realize these projects, a network of atomic clocks is currently being formed. This network includes optical clocks and uses various synchronization channels as well as fiber links [7–9]. The European project is called International Timescales with Optical Clocks (ITOC) [10].

Several laboratories around the world are attempting to solve the problems of high-precision time and frequency dissemination by transferring optical signals via optical fiber channels using an optical carrier. To increase the information transmission accuracy, the phase of the optical carrier is used instead of the RF modulation of the optical field. This method requires completely new approaches: a significant improvement in the radiation monochromaticity for telecommunication band lasers [11–13], the use of femtosecond optical frequency combs [1], and the use of specific feedback systems that can compensate channel noise [14–18].

Fiber links have high costs, and it is therefore not always possible to use dedicated connection lines. Internet fiber links are widespread and are common communication channels for optical frequencies, but it is difficult to directly transfer the technology due to the presence of optical routers and repeaters, which distort the carrier phase. One of the solutions could be hybrid fiber links, which would combine the standard Internet protocol and a dedicated channel in the same telecommunication C-band, allowing frequency dissemination [19]. Currently, there are already fiber links that connect laboratories in different cities or even different countries, and these links are used to transfer ultrastable frequency signals. Figure 1 shows the operational and planned European fiber links.

Significant progress has been achieved not only in the field of frequency dissemination but also in the field of time signal transfer using similar approaches [20–24]. The obtained results demonstrate significantly higher stability than in the case of two-way comparison, which can improve the accuracy of global navigation satellite systems (GNSSs). To realize such technologies, it is very important to increase the distance at which synchronization can be achieved (up to several thousand kilometers).

We also note the experiments performed in 2016, in which the time signals were disseminated via a free-space link at a distance of several kilometers with stability of the order of several femtoseconds [25]. There is also work in progress on developing a free-space link for frequency dissemination through satellites [7]. The method of time and frequency free-space transmission is based on the same mechanisms as in the case of signal dissemination via fiber links.

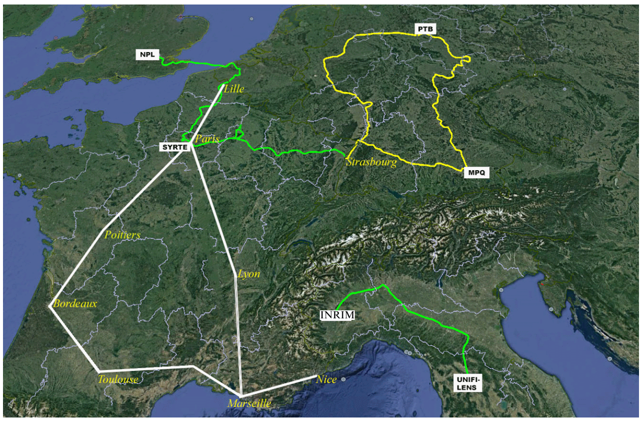

Figure 1. (Color online.) European fiber links used for the dissemination of precise frequency signals using the carrier phase. Yellow color corresponds to individual links, green to public Internet links, and white to links whose construction is being discussed. INRIM — Istituto Nazionale di Ricerca Metrologica (Turin, Italy), SYRTE — SYsteme de References Temps Espace (Paris, France), MPQ — Max-Planck Institute for Quantum Optics (Garching, Germany), NPL — National Physical Laboratory (Teddington, Great Britain), PTB — Physikalisch-Technische Bundesanstalt (Braunschweig, Germany), UNIFI–LENS — Universita di Firenze–Laboratorio Europeo di Spettroscopia non lineare (Florence, Italy).

Here, we review the time and frequency transfer methods using fiber links, discuss examples of possible applications of these methods, and describe some results we have obtained previously. In Section 2, we describe the main methods for precise frequency dissemination via optical fibers and discuss their advantages and disadvantages. Section 3 is devoted to the scheme of optical frequency dissemination via a dedicated channel. In Section 4, we discuss the possibility of using public Internet links for precise frequency dissemination and review recent achievements in this field. In Section 5, we describe methods for precise time transfer via fiber links. In Section 6, we analyse the possibility of using the described frequency dissemination methods in gravimetry and for Sagnac effect measurements. We also present examples of applications in spectroscopy and metrology.

2. Precise frequency dissemination via fiber links

In transferring an ultrastable frequency, it is important for the transmitting medium not to introduce phase and amplitude noises in the signal. Current technologies allow fabricating optical clocks with fractional instability on the level of $10^{-16}/\sqrt{\tau}$ [2, 3, 26], where τ is the averaging time. Simultaneously, impressive results are being obtained in the field of telecommunications in studying optical fibers. Many countries around the world have a dense optical fiber network used for Internet traffic. A solution that suggests itself is to use these networks for coherent dissemination of an optical carrier.

Optical fibers are the best medium for the transfer of optical signals over long distances due to their flexibility, outstanding noise characteristics, and low absorption of the optical signal (0.23 dB km^{-1} for radiation with a 1550 nm wavelength). This allows the transfer of signals without additional amplification over the distances up to 100 km. However, if the signal has to be disseminated over a distance much longer than 100 km, it is necessary to use intermediate amplifiers in order to compensate the absorption; straightforward enhancement of the input signal does not help because it would lead to inverse Brillouin scattering [27–29].

Intermediate amplification of the signal is realized using various methods. One of the most common is based on erbium-doped fiber amplifiers (EDFAs), in which the signal is amplified in the active region of the fiber, with the resultant enhancement up to 25 dB in the broad spectral band 1530–1580 nm. The amplification is limited by Rayleigh scattering and saturation of the amplifier active medium [30], and therefore a cascade of amplifiers spaced by no more than 100–120 km should be used for long distances between the transmitter and the receiver.

The radiation undergoes much less perturbation in the fiber than in free space [31]. However, the fiber introduces some phase perturbations. The most significant ones are the temperature fluctuations and acoustic vibrations, which lead to a change in the optical path in the fiber and to refractive index fluctuations. This results in phase shifts, which are often called Doppler noise.

Temperature variations influence the length of the optical path in the fiber with a refractive index n and a length L in accordance with

$$\frac{\mathrm{d}(nL)_T}{\mathrm{d}t}=\left(L\,\frac{\mathrm{d}n}{\mathrm{d}T}+n\,\frac{\mathrm{d}L}{\mathrm{d}T}\right)\frac{\mathrm{d}T}{\mathrm{d}t}\,. \tag{1}$$

For fused silica, the thermo-optic coefficient is $\mathrm{d}n/\mathrm{d}T\approx 1\times 10^{-5}$ K^{-1} and the thermal expansion coefficient is $\mathrm{d}L/\mathrm{d}T\approx 6\times 10^{-7}$ m K^{-1} [32]. The signal at a frequency ν_0 and with a wavelength λ_0 in the vacuum experiences the frequency shift

$$\delta y(t)=\frac{1}{c}\,\frac{\mathrm{d}(nL)_T}{\mathrm{d}t} \tag{2}$$

in relative units, where c is the speed of light in the vacuum. For example, if the temperature of a standard single-mode fiber ($n=1.468$) with a length of 1000 km changes by 1 K in 1 day, the signal at a wavelength of 1542 nm acquires a relative frequency shift of 4×10^{-13}, which corresponds to almost 80 Hz.

Acoustic waves acting on an optical fiber can induce tensions due to changes in the pressure P, which also leads to phase fluctuations in the transferred signal. As in the case of temperature fluctuations, pressure variations lead to fluctuations in the optical path length:

$$\frac{\mathrm{d}(nL)_P}{\mathrm{d}t}=\left(L\,\frac{\mathrm{d}n}{\mathrm{d}P}+n\,\frac{\mathrm{d}L}{\mathrm{d}P}\right)\frac{\mathrm{d}P}{\mathrm{d}t}\,. \tag{3}$$

The coefficients $\mathrm{d}n/\mathrm{d}P$ and $\mathrm{d}L/\mathrm{d}P$ are known to be respectively equal to 5×10^{-11} Pa^{-1} and 10^{-11} m Pa^{-1} [34, 35]. For a signal with a 1542 nm wavelength, a pressure variation of 0.01 Pa in a 1000 km long fiber would lead to a relative frequency shift of up to 1.5×10^{-14} or 3 Hz. We note that for both temperature and phase fluctuations, the frequency shift can be compensated using active elements.

Besides Doppler noises that lead to the appearance of phase white noise, the transferred signal can also contain noises of other nature. The signal can include flicker noise of the phase, which originates from the electronics in the transmitter and receiver, white noise of the frequency due to shot and thermal noises, random frequency walks, and flicker noise of the frequency. Various types of noises are characterized by different dependences of the phase fluctuation power spectral density on the frequency and are averaged with different rates. The noise type can be deduced from the exponent in the Allan deviation dependence on the averaging time τ [33]:

$$\sigma(\tau)\sim\tau^{\alpha}\,. \tag{4}$$

There are various methods for frequency dissemination using optical signals. Continuous laser radiation can be amplitude modulated, which allows signal transfer in the RF range. By disseminating femtosecond pulses from a laser with mode synchronization, simultaneous transfer of the RF and optical frequency can be achieved [39]. Femtosecond pulses can also be used for time signal dissemination [40]. Optical frequencies can be transferred directly using phase information from the continuous laser radiation carrier. We discuss these methods separately in what follows.

2.1 Amplitude modulation of an optical wave

The most common method of RF signal transfer via fiber links is based on the amplitude modulation of continuous laser radiation with a strictly periodic signal at a given frequency. A remote user needs only a fast photodiode for optical signal demodulation and RF signal reconstruction [36]. The scheme of this method is shown in Fig. 2. The transfer stability for the amplitude modulation method is much higher than that for methods involving satellite systems, and the required characteristics can already be achieved after averaging over several minutes (Fig. 3).

Achieving maximum stability requires stabilizing the fiber so as to compensate various shifts, including RF signal distortions caused by the group delay. For optical path length stabilization, either fiber piezoceramic elements, which change the fiber length, or thermally controlled fiber spools are typically used. This method is sensitive both to polarization mode dispersion (PMD) caused by the birefringence of the fiber and to chromatic dispersion, which leads to additional phase noises. Polarization mixing is used to reduce the PMD influence, and the chromatic dispersion effects can be significantly suppressed by the spectral narrowing of the transfer laser radiation and its frequency stabilization [36].

The most accurate RF signal dissemination using amplitude-modulated lasers was demonstrated in Paris between the Laboratoire de Physique des Lasers (LPL) and SYRTE via an optical fiber at a distance of 86 km. A 1 GHz signal was

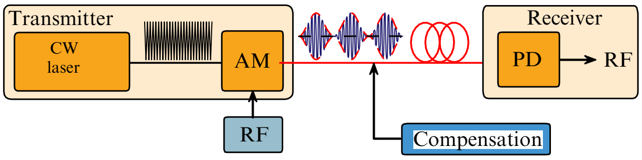

Figure 2. Scheme of the RF signal dissemination using the amplitude modulation of an optical wave. AM — amplitude modulation unit, PD — photodetector, RF — radio-frequency signal.

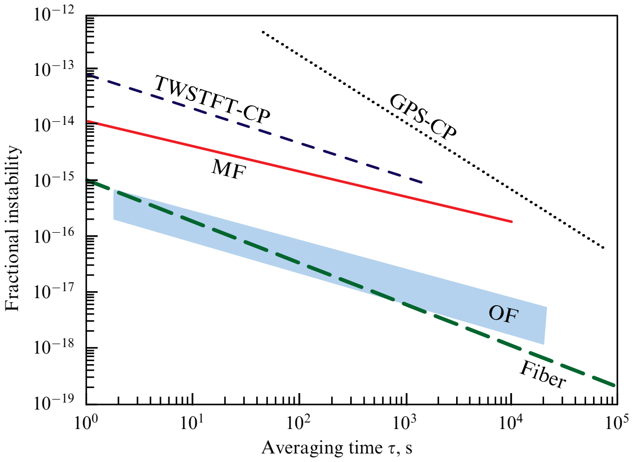

Figure 3. Fractional instability for frequency dissemination using the TWSTFT carrier-phase (TWSTFT-CP) method, the global positioning system carrier phase method (GPS-CP), and a compensated fiber link (Fiber) in comparison with the frequency fractional instability of the best microwave standards (MF) and optical clocks (OF).

transferred with a fractional instability of 5×10^{-15} for a 1 s averaging time and a fractional instability of the order of 10^{-18} for a 1 day averaging time. The method involved fast polarization mixing and advanced filtering [37]. In Russia, research on reference RF signal dissemination using amplitude modulation is performed at the Scientific Research Institute for Physicotechnical and Radiotechnical Measurements (VNIIFTRI) [38].

2.2 Simultaneous dissemination of RF and optical signals

In some cases, a remote user needs to simultaneously receive both RF and optical signals. For these purposes, pulsed femtosecond radiation with a stable repetition rate is used. In this case, the pulse repetition rate is a source of a stable RF signal, while the Fourier components of ultrashort pulses are the optical signal. Figure 4 shows a simplified scheme of the dissemination of such signals. The high stability of the signals is achieved by linking one of the Fourier components of the femtosecond laser to an atomic frequency standard, which provides stability of all other components, including those that correspond to the telecommunication band and are transmitted via a fiber link.

Dissemination of femtosecond pulses for a distance of 50 km with a 100 MHz repetition rate and fractional instability of the repetition rate equal to $4.6 \times 10^{-15}\,\tau^{-1}$ was demonstrated in [39]. The quality of the transferred signal in this case is influenced by chromatic dispersion, which leads to spectral broadening and overlapping of the pulses. This effect can be suppressed by incorporating a compensating fiber into the line, which typically allows obtaining a pulse shorter than 200 ps. Although this pulse duration is several orders of magnitude longer than the initial laser pulse length, this does not significantly influence the stability of the transferred signals. But this method does not allow disseminating signals over long distances due to the inevitable spreading of the pulses and loss of information.

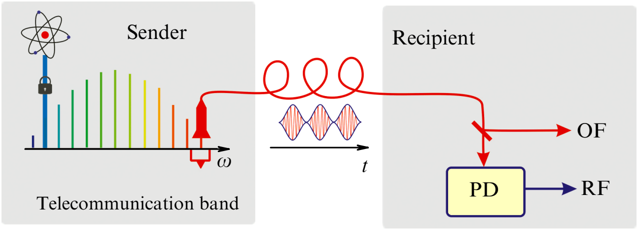

Figure 4. Schematic for simultaneous dissemination of the RF and optical signals using a femtosecond laser. PD — Photodiode, RF — radio-frequency signal, OF — optical frequency signal.

Another noise source in the case of pulsed radiation transfer is the receiver electronics. For low powers of the detected signal, a significant contribution to noise is made by photodetectors. At the same time, an increase in the signal power using intermediate amplifiers can lead to other undesirable effects, such as conversion of amplitude noise into phase noise by electronic devices, that is, the amplitude modulation (AM) to phase modulation (PM) conversion [41–43].

2.3 Optical frequency dissemination

The considered method uses the optical carrier phase as the disseminated information. The chromatic and polarization mode dispersions, as well as the AM–PM conversion, have minimal effect in this case. However, even for well-isolated fiber links located deep under the ground, temperature and pressure fluctuations can result in a fractional frequency shift of the order of 10^{-14}. Therefore, achieving better stability requires detecting and compensating the phase fluctuations that appear when the signal travels in an optical fiber. For this, the line on the sender side has an active phase element, such as an acousto-optical modulator (AOM), which corrects the phase according to the acquired phase noises. A phase autotuning loop on the sender side allows compensating phase distortions in the frequency band of the feedback loop, which is determined by the signal transmission time for the forward and backward directions [44].

All three methods described provide means for frequency signal dissemination via optical fibers. However, depending on the type of task, one of the methods can be more advantageous. For example, in the case of optical clock comparison, the method of direct transfer of the optical carrier has clear advantages because it does not need a broad spectral band as in the case of pulsed radiation transfer. This method is simpler than the AM method and does not have the disadvantages of the latter. When the 'dark' line is accessible and optical and RF signals have to be simultaneously disseminated at moderate distances, it is more appropriate to use a femtosecond pulse train. It can also be used to disseminate time signals [46].

Due to the development of optical clocks and the construction of a fiber link network on a continental scale, the problem of signal transfer over distances of several thousand kilometers becomes a primary one. In what follows, we describe in detail the method for frequency and time dissemination using information about the optical carrier phase, because this method appears to be the most promising for solving the distance problem.

3. Dissemination scheme for the optical carrier phase

As was mentioned in Section 2, the optical carrier phase dissemination method involves registering and compensating phase distortions that the signal acquires during the transfer

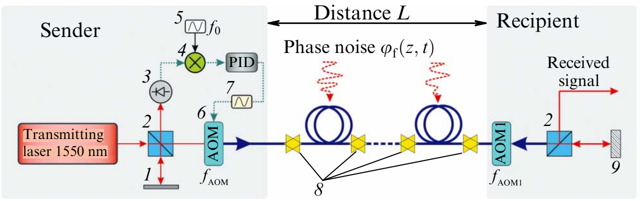

Figure 5. Simplified scheme of signal dissemination and compensation of the phase noise introduced by a fiber link. The signal that propagates through the fiber with length L acquires a time delay τ_d. AOM — acousto-optic modulator, PID — proportional-integrating-differentiating controller, *1* — mirror, *2* — 50:50 splitter, *3* — photodetector, *4* — mixer, *5* — stable RF source, *6* — AOM frequency control signal, *7* — voltage controlled oscillator, *8* — erbium bidirectional amplifiers, *9* — Faraday mirror.

through the fiber. Phase noise detection is usually performed with a Michelson interferometer one of whose arms is located on the sender side, as shown in Fig. 5. Radiation from a transmitting laser is split into two parts. One stays on the sender side and is directed to the short arm of the interferometer as a reference signal. The other part is directed into the long arm of the interferometer, which contains the fiber link. An acousto-optical modulator is placed on the sender side in front of the fiber and actuates the radiation frequency in such a way that the signal phase distortions are compensated. A Faraday mirror is placed at the fiber output on the remote user side in order to reflect part of the transferred light and rotate its polarization through 90°. In this case, the polarization fluctuations that occur in the fiber are significantly compensated, such that the amplitude of the beat note generated by the reference and feedback radiation remains constant [45]. A heterodyne RF beat note that contains phase shifts is registered with a photodetector on the sender side. This beat note contains information about the phase noise $\varphi_f(z,t)$ introduced by the fiber link, and this information is used to calculate the error signal and control the AOM frequency, which allows phase distortion compensation.

In some cases, another AOM is placed in front of the Faraday mirror (indicated as AOM1 in Fig. 5) and operates at a fixed frequency. It is used to uniquely distinguish the signal reflected by the Faraday mirror from other reflections that occur in long fiber links (for example, at the amplifier inputs or in the fiber welding regions). In this case, we can write

$$f_0 = 2\big(f_{\mathrm{AOM}}(t) + \Delta f_{\mathrm{fiber}}(t) + f_{\mathrm{AOM1}}\big)\,, \tag{5}$$

where f_0 is the registered beat note, f_{AOM} is the AOM compensation frequency, $\Delta f_{\mathrm{fiber}}$ is the frequency fluctuations in the fiber, and f_{AOM1} is the AOM1 frequency.

A fundamental limitation on the possible suppression of phase noises comes from the finiteness of the speed of light in the fiber, because the noises can be detected only after the light passes the fiber twice. This means that higher-frequency phase noise is not suppressed. Moreover, the light is received by the remote user before the corrections are made, and it is therefore impossible to fully suppress the signal noise after a single transmission through the fiber. Any difference in noise for the forward and backward directions leads to an imperfect compensation on the receiver side. These limitations start to play a decisive role for signal transmission over distances longer than 100 km.

We note that the effective noise suppression using the heterodyne method can be performed only if the beat note is not affected by the transmitting laser fluctuations. The laser coherence length L_c is determined by its spectrum: $L_c = c/(\pi\Delta\nu) = c\tau_c$, where $\Delta\nu$ is the laser line spectral width, c is the speed of light in the fiber, and τ_c is the coherence time. If the laser coherence length is less than the transmission line length, the reference signal and the transmitted signal become uncorrelated. This means that the beat note registered by the photodetector contains not only the fiber noise but also fluctuations of the transmitting laser, and the compensation is ineffective. Therefore, the transmitting laser usually has a narrow spectral width (less than 1 Hz), which corresponds to the coherence length $\sim 10^5$ km. In this case, the information obtained with the heterodyne method can be reliably used for the compensation of the phase noise introduced by the fiber.

Inverse Brillouin scattering in optical fibers limits the power of the transferred signal: the maximal radiation power at the fiber input is limited by a value of about 5 mW [30]. Therefore, it is necessary to use intermediate bidirectional amplifiers that would not distort the phase of the signal. This problem is typically solved by using a cascade of low-noise erbium-doped amplifiers and especially developed Brillouin amplifiers [30].

To stabilize the fiber link, the self-heterodyne method is used. A beat note generated by the signal with a frequency ν_0 and its time-delayed copy are used for active suppression of the noise introduced by the fiber. The Doppler noise suppression schematic is shown in Fig. 5.

In propagating through a fiber link of length L, the signal acquires the phase noise $\phi_{\mathrm{fiber}}(z,t)$, which is defined as [15]

$$\phi_{\mathrm{fiber}}(t) = \int_0^L \delta\phi\left(z, t - \left(\tau - \frac{nz}{c}\right)\right) \mathrm{d}z\,, \tag{6}$$

where n is the medium refractive index and $\tau = nL/c$ is the signal propagation time through the fiber. To compensate the Doppler noise, part of the light is reflected back into the same fiber and therefore accumulates a double phase noise after the round trip:

$$\begin{aligned}\phi_{\mathrm{fiber_{RT}}}(t) = \int_0^L \bigg[\delta\phi\left(z, t - \left(\tau - \frac{nz}{c}\right)\right)\\ + \delta\phi\left(z, t - \left(2\tau - \frac{nz}{c}\right)\right)\bigg] \mathrm{d}z\,.\end{aligned} \tag{7}$$

In the Fourier representation, the time delay is converted into phase shifts:

$$\tilde{\phi}_{\mathrm{fiber}}(\omega) = \int_0^L \exp\left[\mathrm{i}\omega\left(2\tau - \frac{nz}{c}\right)\right] \delta\tilde{\phi}(z,\omega)\,\mathrm{d}z\,, \tag{8}$$

$$\tilde{\phi}_{\mathrm{fiber_{RT}}}(\omega) = \int_0^L \left\{\exp\left(\mathrm{i}\omega\tau\right) \cos\left[\omega\left(t - \frac{nz}{c}\right)\right] \tilde{\phi}(z,\omega)\right\} \mathrm{d}z\,. \tag{9}$$

We can now obtain spectral densities of the phase noise power for the signal transmitted in one direction, $S_{\mathrm{fiber}}(\omega)$, and for the round-trip signal, $S_{\mathrm{fiber_{RT}}}(\omega)$:

$$S_{\mathrm{fiber}}(\omega) = \int_0^L \left\langle \left|\delta\phi(z,\omega)\right|^2 \right\rangle \mathrm{d}z\,, \tag{10}$$

$$S_{\mathrm{fiber_{RT}}}(\omega) = 2S_{\mathrm{fiber}}(\omega)\left(1 + \operatorname{sinc}\frac{2Ln\omega}{c}\right). \tag{11}$$

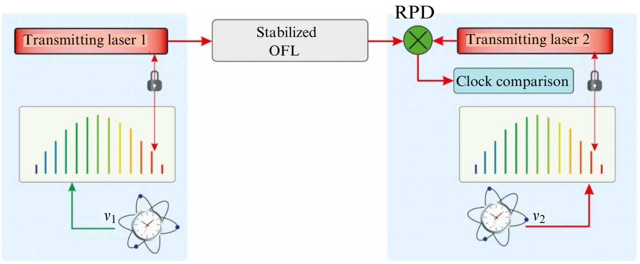

Figure 6. Scheme of the comparison of two atomic frequency standards using a stabilized OFL (see Fig. 5) and FOFC. RPD — relative phase detector for the laser radiation; ν_1 is the frequency of the first atomic clock and ν_2 is the frequency of the second atomic clock.

For low frequencies, $\operatorname{sinc} 0 \approx 1$, and the phase noise acquired after the round trip turns out to be four times larger than the noise of the signal received by the remote user. The round-trip signal is mixed with its copy without the time delay and with negligible phase noise. A phase detector converts the phase fluctuations into voltage, which is then used to form the error signal in the feedback loop for the fiber noise compensation.

A general fiber-link scheme for the comparison of two optical frequencies ν_1 and ν_2 includes the fiber link itself, ultrastable continuous lasers for information transfer, and two femtosecond optical frequency combs (FOFCs) (Fig. 6). The introduction of FOFCs provides a means for the comparison of not only different frequencies in the transparency window of an optical fiber link (OFL), but also frequencies from different spectral bands (RF and optical).

Currently, the longest distance over which an ultrastable frequency signal has been disseminated is 1840 km [18]. A signal at a frequency of 194 THz was transferred via an optical link between the Max-Planck-Institute for Quantum Optics (MPQ) in Garching and the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, Germany. The fiber link was built along an underground gas pipeline. In the experiment, two 'dark lines' were used; these are two backup lines that are free from Internet traffic and can be used purely for the investigated frequency signal dissemination. The length of the link between MPQ and PTB is 920 km. In one experiment, the scientists performed the direct transfer of an ultrastable frequency signal (see Section 5.3), while in the other series of experiments the ends of two fibers were connected at the PTB side and the distance traveled by the signal was doubled. Despite the long distance, the fiber link was stabilized as a whole, which limited the maximal frequency of the effectively suppressed noise due to the large travel time of the signal. The maximal noise frequency compensated by the feedback loop was 27 Hz. Nevertheless, the achieved fractional instability of the frequency was 2×10^{-14} for a 1 s averaging time and less than 1×10^{-19} for a 1000 s averaging time, which makes it possible to solve all current problems regarding optical clock comparison.

4. Frequency dissemination via Internet links

The method described in Section 3 uses a 'dark' fiber line that is dedicated to signal dissemination. The cost of rent of such fiber links is very high. On the other hand, one Internet line is separated into many frequency channels, which have different carrier frequencies, which allows more efficient information transfer at lower costs. Due to the global use of Internet links, it is reasonable to use them for the dissemination of stable frequency and time signals. However, the realization of such technology encounters several difficulties.

During transmission in an OFL, the signal decays. This problem is solved in Internet links using the following method, which is different from the analog scheme with a cascaded EDFA (see Fig. 5). The signal is being demultiplexed, distributed over channels, decoded, and recorded again on a higher-power carrier. Naturally, the carrier phase is not preserved in this process, and the system introduces delays. The carrier phase information is used in the dissemination of ultrastable frequency signals, and it therefore becomes necessary to distinguish the signal among all other signals transmitted through the link before it reaches the repeater. After that, the signal has to be extracted from the line, transmitted through a bidirectional amplifier, which preserves the phase, and injected back into the line for further transfer. The same procedure must be performed with the signal traveling in the opposite direction (Fig. 7).

Average power losses in public Internet links reach 0.29 dB km^{-1}, which is almost one and a half times more than those in a 'dark' line. The losses are mainly increased by the large number of fiber connectors with a flat surface (physical contact (PC) standard), which partially reflect light and form parasitic etalons. Consequently, bidirectional erbium amplifiers cannot operate at high amplification levels, because self-excitation can occur above a certain threshold. This means that signal attenuation using erbium amplifiers can be fully compensated only if the spacing of the intermediate stations is less than 50 km. Using bidirectional Brillouin amplifiers [30] is also impossible because their operation is based on a strong optical pump, which can

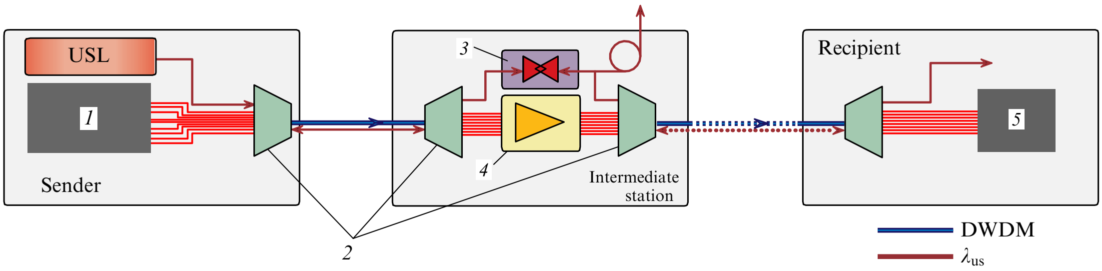

Figure 7. (Color online.) Frequency dissemination scheme using Internet links. The distance between the intermediate stations is less than 50 km. *1* — transmitter, *2* — optical add drop multiplexor (OADM), *3* — bidirectional erbium-doped fiber amplifier, *4* — unidirectional optical amplifier, *5* — detector. USL — ultrastable laser, DWDM — traffic of data converted using the dense wavelength division multiplexing method; λ_{us} is the ultrastable frequency signal.

hardly be implemented in public telecommunication network devices.

The described limitation can be overcome by using a cascaded transmission line: one separated into a number of segments that are connected through intermediate repeater laser stations [19, 47]. These remote intermediate stations are the most complicated element of the line because they should operate autonomously. In reality, every intermediate station is a separate laser system with a spectrally narrow radiation line, with its phase being locked to the phase of the incoming signal, which is transmitted from the preceding station using the method of frequency auto-tuning through the phase. The signal phase fluctuations are recorded in the phase of the radiation of the repeater laser located at an intermediate station N. Part of the repeater laser system radiation is directed back to station $N-1$, where it is compared with the incoming signal in order to compensate the phase noise that the signal acquired during the propagation from station $N-1$ to station N. The same operation is repeated at the next intermediate station $N+1$. This results in a consequential compensation of the transmitted signal phase noise. Finally, the repeater laser can operate as a stable frequency signal source if its radiation is directed into a router that sends a signal to the user via an additional short stabilized line.

The remote station, including the microcontrollers, is entirely controlled with a built-in computer, which is accessible via the Internet protocol. This allows remote optimization of the signal transfer process. Moreover, one needs to remotely control the feedback loop error signals in the phase noise compensation systems in order to optimize the amplification and avoid the undesirable self-excitation of the amplifiers. Finally, it is necessary to monitor possible cycle slips in frequency autotuning systems.

Currently, ultrastable frequency dissemination via public Internet links is realized using the following route: LPL (Villetaneuse, a northern suburb of Paris)–Nancy (eastern France)–LPL, over a distance of 1100 km. The line was separated into four segments and two parallel optical fibers were used, which were a part of a public Internet link. The fact that the sender and the recipient were located in the same laboratory allowed investigating the stability of the signal dissemination through the cascaded line and comparing it to the dissemination stability for the same signal using a short laboratory line. Fractional instability of the optical frequency signal dissemination using this line reached 4×10^{-16} for a 1 s averaging time and 5×10^{-20} for a 60,000 s averaging time [19].

5. Time dissemination

To maintain national time scales, perform very-long-baseline radio-interferometry (VLBI), and support GNSS systems, clocks separated by long distances have to be synchronized. The problem that has to be solved in this case is related to control over the time delay that the signal acquires after its transfer to a remote user. Time signal dissemination methods using satellites (GNSS, TWSTFT) allow transferring time signals over distances of up to 10,000 km and are currently the most popular ones. However, their precision is limited due to the delay time fluctuations introduced by the troposphere and the ionosphere, and this noise is difficult to compensate [48].

Methods for time dissemination via fiber links and the corresponding operating devices are in most cases similar to the ones used for time dissemination via satellite channels [49]. The system for time transfer via optical fiber (TTTOF) [20] is a version of the standard TWSTFT system in which the signal is transmitted via an optical fiber link between two ground-based stations. Time marker generation and marker synchronization with the clocks are performed using satellite time and ranging equipment (SATRE) modems [50]. The same modems measure the delay time. Among the additional advantages of fiber links is the broadband signal transmission, which allows using the maximum possible speed for binary modulation of the phase shift and thus increasing the accuracy compared with the standard TWSTFT systems. For time dissemination, fiber phase noise compensation is not necessary because the main task is to be able to measure and control the time delays of the signal that are acquired not only during the transmission through the fiber but also at every step on the sender and recipient sides.

Currently, the fiber-link time dissemination method is competing in accuracy and stability of the transferred signals with satellite methods, but the transmission distances of the first method are much shorter, not more than 1000 km.

A combination of technologies allows performing simultaneous dissemination of precise time and frequency signals via stabilized OFLs using information about the optical carrier phase. This can be done using public Internet links [21, 51]. A group of researchers in France demonstrated a simultaneous dissemination of time and frequency signals over a distance of 540 km. Fractional instability of the optical frequency in this experiment reached 2×10^{-18} for a 30,000 s averaging time. The absolute precision and long-term stability of time signals were respectively equal to 250 ps and 50 ps [21].

6. Application examples for precise frequency dissemination via fiber links

6.1 Gravitational potential measurement and the Sagnac effect

According to the general relativity theory, clocks that are located closer to massive objects run more slowly than those situated far from them. This effect is called the gravitational time dilation (GTD). Although the relativistic effects are extremely small on scales of distances and velocities in our everyday life, these effects should be taken into account when working with modern atomic clocks. For example, relativistic GTD corrections are introduced into on-board clocks of satellite navigation systems. The realization of optical clocks with stability on the eighteen-digit level allows observing the GTD and directly measuring the gravitational potential variations at different points on Earth [10]. Close to Earth's surface, the size of the effect in relative units is 10^{-16} per m.

In most modern experiments and applications, gravitational corrections are calculated based on measurements of the altitude difference, which are performed using classic geodetic methods (leveling and range-finding). The appearance of optical clocks has made it possible to 'invert' the problem and use precise frequency measurements to calculate the altitude difference. The first experiments of this type were performed by the Wineland [52] and Katori [53] groups.

Using fiber links to compare ultrastable optical clocks allows developing a new method for mapping Earth's gravitational potential. Having a network of base stations equipped with optical clocks that can be compared via fiber links and using a mobile frequency standard that can also be

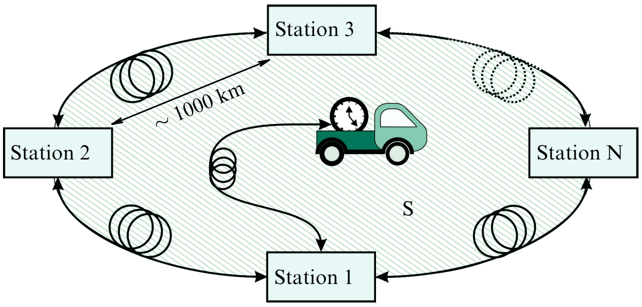

Figure 8. Scheme of the gravitational potential mapping using mobile clocks and a network of base stations using compensated fiber links.

connected to the fiber network, we can measure point-to-point gravitational potential variations (Fig. 8). Currently, the stability of a mobile frequency standard based on strontium atoms has already reached 8×10^{-17} [7], which is enough to measure the altitude variation with a precision of better than 1 m.

Another relativistic effect that can be observed using fiber links is the Sagnac effect. In a rotating reference frame, the times of electromagnetic radiation propagation along a closed path are different for the forward and backward directions. The difference between the two times is

$$\Delta t_{\Omega} = \frac{4\mathbf{\Omega S}}{c^2}\,, \tag{12}$$

where c is the speed of light in the vacuum, $\mathbf{\Omega}$ is the angular velocity of the reference frame rotation, and $\mathbf{S}$ is the vector of the area encompassed by the closed path.

Fiber links connect laboratories that are separated by more than 1000 km. Creating a closed circuit (see Fig. 1) will allow creating a ring interferometer with an area of several million square kilometers. By measuring the Sagnac-effect-induced phase shift of the counter-propagating optical radiation, it will be possible to precisely determine some parameters of Earth's rotation.

6.2 Spectroscopy of the 1S–2S hydrogen atom transition

Over the last 20 years, the authors of this paper have taken part in experiments on the spectroscopy of the 1S – 2S transition in the hydrogen atom, which were performed by the Hänsch group at Garching, in part with the aim to refine the value of the Rydberg constant R_{∞} [54]. In 1999, the measurement precision for the 1S – 2S transition in hydrogen exceeded the accuracy of any commercially available cesium clocks; therefore, in the period from 1999 to 2011, the measurements were performed using a cesium fountain mobile (FOM) [55]. A series of experiments [56–58] took place in 1999, 2003, and 2010. At the end of 2010, after a successful demonstration of stable frequency dissemination using an optical carrier via a fiber link between MPQ and PTB, 1S – 2S hydrogen transition spectroscopy was performed using a reference signal from the cesium fountain FO1 located at PTB [59]. The difference between the elevations above sea level of the MPQ and PTB laboratories, which is approximately 150 m, was taken into account in the calculations. The differential red shift between remote hydrogen atoms and the cesium frequency standard was determined from GPS measurements using the Earth Gravitational Model 2008 (EGM2008), and its accuracy was determined by tidal effects.

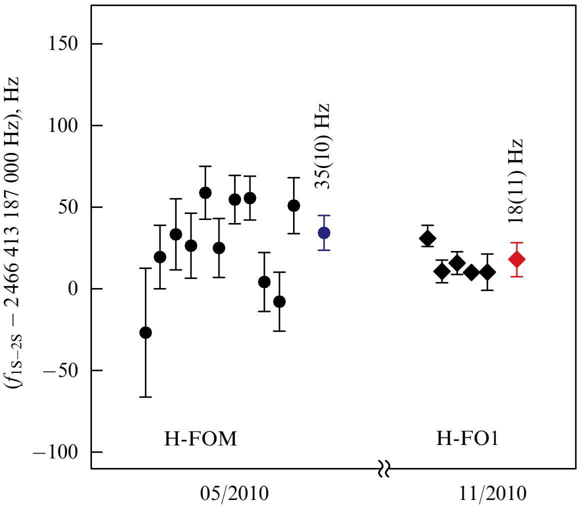

Figure 9. (Color online.) Comparison of 1S – 2S hydrogen transition centroid frequency measurements performed using the FOM mobile cesium fountain frequency standard (shown in blue) and the remote FO1 cesium fountain (shown in red). Each black symbol corresponds to the result of data averaging over one day. The red diamond corresponds to the average value, including the uncertainty. (Data taken from [59].)

Results of absolute measurements of the 1S – 2S transition frequency obtained in the papers cited above are presented in Fig. 9. Fiber noise was efficiently compensated and did not influence the stability and accuracy of the FO1 cesium fountain signal, which exceeded the stability and accuracy of the mobile standard FOM. This resulted in an improvement in the absolute 1S – 2S transition frequency uncertainty.

Joint analysis of the results obtained in 2010 and 2013 allowed constraining a linear combination of the Lorentz symmetry breaking parameters in the framework of the extended Standard Model [60]:

$$c_{(TX)} = (3.1 \pm 1.9) \times 10^{-11}\,, \tag{13}$$

$$0.92 c_{(TY)} + 0.40 c_{(TZ)} = (2.6 \pm 5.3) \times 10^{-11}\,. \tag{14}$$

Investigating bounds for possible Lorentz invariance violations using optical clocks is one important issue in modern physics [61, 62].

The experiments performed have shown the possibility of realizing precise measurements between laboratories separated by several thousand kilometers.

7. Conclusions

We have described the currently available methods for stable frequency and time signal dissemination via fiber links. Modern optical frequency standards, which demonstrate record high accuracy and stability at the level of 10^{-18}, imply new requirements for signal transfer stability and open new possibilities for their application. Fiber links, including public Internet lines, fully meet these requirements and allow the transmission of ultrastable frequency signals at distances of several thousand kilometers. Optical frequency standards can now be used to solve applied and fundamental problems: mapping Earth's gravitational potential, determining the parameters of Earth's rotation from Sagnac effect measurements, and realizing experiments on the precise spectroscopy of spectrally narrow transitions. Given the ubiquity of fiber links, we can expect that a method for precise frequency and time dissemination through OFLs will soon be used together with satellite methods, complementing and expanding their potential.

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