ON THE NON-LINEARITY OF ELECTROMAGNETIC PROCESSES IN SUPERCONDUCTORS

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It is customary to consider at present the electromagnetic processes in superconductors, as being described by the equation

$$\operatorname{rot} \Lambda \mathbf{j} = -\frac{1}{c}\mathbf{H}, \tag{1}$$

where $\mathbf{j}$ — the density of superconducting current and Λ — a constant [1,2]. Equation (1) together with the equation of the type $\partial \Lambda \mathbf{j}/\partial t = \mathbf{E}$, which has to be used in a variable field, has a hydrodynamical character. Namely, in the case of a charged liquid in a magnetic field the generalized condition of the absence of vortices of the velocity $\mathbf{v}$ is

$$\operatorname{rot} \mathbf{v} = -\frac{e}{mc}\mathbf{H}. \tag{2}$$

Since $\mathbf{j}=en\mathbf{v}$, where n for superconductors is the number of "superconducting electrons", equation (1) is immediately obtained from (2) if we put $\Lambda = m/e^2 n$. The number n, and hence Λ, depends upon the temperature and at the transition temperature $n=0$ and $\Lambda=\infty$ [2].

We note here that within the frame of the theory based on equation (2), the number n must depend upon v or, which is the same, upon H and, therefore, the equations for the current and the field in superconductors are non-linear, the fact already emphasized by the author [3,2].

This conclusion may be simply reached, starting from Landau's qualitative scheme [4] [see also [2]]. The superconducting charge decreases when the temperature is increased, owing to the increase of the number of excitations in the electronic liquid. In the presence of a magnetic field and, therefore, when $v \neq 0$ excitations are on the average at rest with respect to the lattice (if the normal current is absent). Thus they move with respect to the superconducting liquid with a velocity v. In this case the number of excitations and the number n depend upon v as it follows from the Gibbs' distribution for the gas, moving as a whole [see [4], § 5].

In the case of superconductors the dependence of n upon v may appear to be strong, since in the field of the order of the critical field H_c the velocity v is comparable with the excitation velocity. In the linear theory on the plane interface $v=(e/mc)\,\delta H$, where δ is the penetration depth. Therefore, in the field of the order of several hundreds gauss and for $\delta \sim 10^{-5}$ cm [3], we have $v \sim 10^4$—10^5 cm/sec.

Preserving equation (1) and using the equation $\operatorname{rot}\mathbf{H}=(4\pi/c)\,n\mathbf{v}$, we get for $\mathbf{v}$:

$$\Delta \mathbf{v} - \frac{1}{\delta^2}\mathbf{v} = 0, \quad \delta^2 = \frac{mc^2}{4\pi e^2 n(v)}. \tag{3}$$

If n=const this equation coincides, of course, with that obtained from (1). The function $n(v)$ can be found if the excitation spectum is given. We shall not consider this problem here. The very general conclusion consists in the weak dependence of n upon v in weak fields; this dependence, however, becomes a sharp one in strong fields.

The non-linearity of the electromagnetic phenomena in superconductors must manifest itself in the following experiment. A solenoid with a superconducting core is fed with the current of the frequency ω; then an emf must be induced in the coil surrounding the solenoid having a frequency spectrum ω, 3ω, 5ω, 7ω, and so on. Even overtones are absent, since the field in the superconductors is an odd function of the external field. The ratio of amplitude of the successive overtones in the field $H \sim H_c$ may be of the order of unity and in general of the order of $(H/H_c)^2$ If $n(v)=n_0(1-a^2v^2)$, where $a^2v^2 \ll 1$, the problem can be solved exactly. In this case the ratio of the amplitude of the harmonic 3ω to that of the principal one ω is

$$\frac{3}{16}a^2\left(\frac{e\delta_0}{mc}H_0\right)^2 \tag{4}$$

with $\delta_0^2 = mc^2/4\pi e^2 n_0$ and H_0 — the amplitude of the field of the solenoid.

It is to be noted that not only equation (1) but also the more general equations (2), (3) cannot be considered as being proved either by experiment or in a theoretical way. It is proved only that the magnetic field does not penetrate into a superconductor, which confirms equations (1)—(3) only qualitatively. The use of equation (2) requires a possibility to describe the superconducting current classically which cannot be grounded *a priori*. An attempt to construct a self-consistent theory based on equation (2) has been made by L. Landau and the author. Such an attempt meets difficulties since the surface energy of the boundaries between the normal and superconducting regions turns out to be negative.

A certain indication of the failure of the classical description of the superconducting current gives the fact that the zero energy of excitations in superconductors is of the order of $\hbar^2 n/m\delta \sim 1$ (for $\delta \sim 10^{-5}$ and $n \sim 10^{22}$) and, therefore, is larger than the magnetic energy $\delta H^2/8\pi \sim 0.1$ (at $H \sim 500$).

Thus, the penetration depth in superconductors is comparatively so small, that the classical description may appear to be inconsistent.

The conclusion, concerning the existence of the non-linearity in the superconductors, is obviously much wider than the conclusions obtained directly from equation (2).

I am very obliged to Prof. L. Landau for the discussion of the problem.

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